VITEEE 2022 Solutions July 3

Ques. Find the solution of the equation dy/dx = 1/cos(x+y)

Solution.
To solve the differential equation:

$$\frac{dy}{dx} = \frac{1}{\cos(x + y)}$$

We'll employ the method of separation of variables. Let's rearrange the equation to separate y and x:

$$\cos(x + y) \, dy = dx$$

Now, integrate both sides:

Integrating the left side with respect to y:

$$\int \cos(x + y) \, dy$$

And integrating the right side with respect to x:

$$\int dx$$

Since the integral on the left involves both x and y, it's a bit tricky. One common method for such integrals is to use a substitution. However, this particular form doesn't lead to a straightforward elementary integral because of the mixed terms. Nevertheless, we can represent its solution symbolically:

$$\int \cos(x + y) \, dy = x + C$$

Where \(C\) is an integration constant.

Ques. What are isoelectronic species

Solution.
Isoelectronic species are atoms, ions, or molecules that have the same number of electrons or the same electronic structure. In other words, they have the same total number of electrons, although the atoms themselves or the ions might be different.

For example, consider the following species:
Ne (neon atom),
F^- (fluoride ion), and
O^2- (oxide ion).

- Neon (Ne) has 10 electrons.
- Fluorine atom has 9 electrons, but the fluoride ion (\(\text{F}^- \)) has gained an electron, giving it a total of 10 electrons.
- Oxygen atom has 8 electrons, but the oxide ion (\(\text{O}^{2-} \)) has gained two electrons, giving it a total of 10 electrons.

All three species (Ne, \(\text{F}^- \), and \(\text{O}^{2-} \)) have 10 electrons, making them isoelectronic.

It's important to note that while isoelectronic species have the same number of electrons, they can have different numbers of protons and, thus, different nuclear charges. This difference in nuclear charge can lead to variations in properties like ionic radii among isoelectronic species.

Ques. Find the area enclosed by $y=x^2$ and $y=x+2$

Solution.
To find the area enclosed by the curves \(y = x^2\) and \(y = x + 2\), we need to:

1. Find the points of intersection of the two curves.
2. Determine the region enclosed by them.
3. Integrate to find the area of the region.

1. Points of Intersection:
Set the two equations equal to each other and solve for x:

$x^2 = x + 2$
$x^2 - x - 2 = 0$

Factoring, we get:

$(x - 2)(x + 1) = 0$

So, $x = 2$ and $x = -1$ are the points of intersection.

2. Determine the Region:
Between $x = -1$ and $x = 2$, the parabola $y = x^2$ is below the line $y = x + 2$. So, the area enclosed by the curves is the area between the curves in this interval.

3. Integrate to Find the Area:
The area A between the curves from $x = -1$ to $x = 2$ is:

$$A = \int_{-1}^{2} [(x + 2) - x^2] \, dx$$

Integrate term by term:

$$A = \left. \left(\frac{1}{2}x^2 + 2x - \frac{1}{3}x^3 \right) \right|_{-1}^{2}$$

$$A = \left(\frac{1}{2}(2)^2 + 2(2) - \frac{1}{3}(2)^3 \right) - \left(\frac{1}{2}(-1)^2 + 2(-1) - \frac{1}{3}(-1)^3 \right)$$

$$A = \left(2 + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right)$$
$$A = \left(\frac{6}{3} + \frac{12}{3} - \frac{8}{3} \right) - \left(\frac{-3}{6} + \frac{6}{6} - \frac{2}{6} \right)$$

$$A = \frac{10}{3} - 1 = \frac{7}{3}$$

The area enclosed by the curves is $A = \frac{7}{3}$ square units.

Ques. Which is deflected by the magnetic field? A) X rays B) Gamma rays C)Alpha particle D)Neutron

Solution.
Out of the given options, the entities that will be deflected by a magnetic field are those which have charge.

A) **X-rays**: These are electromagnetic waves and do not carry a charge. Hence, they are not deflected by a magnetic field.

B) **Gamma rays**: Like X-rays, gamma rays are also electromagnetic waves and do not have a charge. They are not deflected by a magnetic field.

C) **Alpha particles**: An alpha particle consists of 2 protons and 2 neutrons, giving it a net charge of +2e. Because it has a charge, it will be deflected by a magnetic field.

D) **Neutron**: Neutrons, as their name suggests, are neutral and do not carry any charge. Therefore, they are not deflected by a magnetic field.

Among the given options, the answer is:
C) **Alpha particle**.

Ques. Which is the structure of glutaric acid?

Solution.
Glutaric acid is a five-carbon dicarboxylic acid. Its structure is:

$HOOC\text{-}CH_2\text{-}CH_2\text{-}CH_2\text{-}COOH$

Where "HOOC" represents a carboxylic acid group (-COOH) and "CH_2" represents a methylene group. The two carboxylic acid groups are located at either end of the three methylene groups in the center of the molecule.

Ques. Products when H2SO4 is reacted with sucrose

Solution.
When concentrated sulfuric acid (H_2SO_4) is added to sucrose ($C_{12}H_{22}O_{11}$), the reaction is a dehydration reaction. The sulfuric acid removes water from the sucrose in a highly exothermic reaction, leaving behind a mass of carbon. This reaction is sometimes referred to as the "sugar snake" or "carbon snake" demonstration in chemistry classes.

The reaction can be simplified as:
$$C_{12}H_{22}O_{11} + H_2SO_4 \rightarrow 12\ C + 11\ H_2O + H_2SO_4$$

Here's a step-by-step description:
1. Sucrose loses eleven molecules of water to give a black mass of carbon.
2. The sulfuric acid becomes diluted during the reaction, but remains largely unchanged.

The products of the reaction are carbon, water, and diluted sulfuric acid.

Ques. What is the drift velocity of metals?

Solution.
Drift velocity refers to the average velocity attained by charge carriers, such as electrons in a conductor, due to an applied electric field. It's a measure of the net flow of charges within a conducting medium when subjected to an electric potential difference.

For metals, electrons are the primary charge carriers, and the drift velocity v_d can be defined using the equation:

$$ v_d = \frac{I}{nAe} $$

Where:
- v_d is the drift velocity (in meters per second, m/s).
- I is the current flowing through the conductor (in amperes, A).
- n is the number of charge carriers (electrons) per unit volume of the conductor (in carriers per cubic meter).
- A is the cross-sectional area of the conductor (in square meters, m^2).
- e is the elementary charge (approximately 1.602×10^{-19} coulombs).

The drift velocity in metals is typically quite small. Even for substantial currents in everyday circuits, the drift velocity is on the order of millimeters or centimeters per second. This is because the number of electrons (n) in metals is so large.

For example, in copper, the electron concentration n is on the order of 10^{29} electrons per cubic meter, leading to a relatively small drift velocity even under substantial current flow.

In practice, when you turn on a light switch and the light illuminates almost instantaneously, it's not because electrons are traveling from the switch to the light at the speed of light. Instead, the electromagnetic wave (the energy) travels at nearly the speed of light through the metal, while individual electrons drift much more slowly.

Ques. Which day is 6the September 2021?

Solution.
6th September 2021 was a Monday.

Ques. RCHO is converted into RCH3 using zinc amalgam and concentrated HCl this reduction is called as

Solution.

The reduction of an aldehyde (\(RCHO \)) to an alkane (\(RCH_3 \)) using zinc amalgam and concentrated HCl is called the Clemmensen Reduction.

Ques. What happens when CU metal is immersed in ferrous sulphate

Solution.
When copper metal is immersed in a solution of ferrous sulfate, a displacement reaction takes place, as copper is more reactive than iron in this scenario.

The reaction that occurs is:

\[Cu(s) + FeSO_4(aq) -CuSO_4(aq) + Fe(s) \]

Copper displaces the iron from the ferrous sulfate solution to form copper sulfate. The iron is precipitated out as a solid.

As the reaction progresses, the solution will gradually turn blue due to the formation of the copper sulfate, and elemental iron will be observed as a solid deposit.

Ques. How many odd days are there in 100 years?

Solution.
"Odd days" in calendar calculations refer to the number of days more than the complete weeks in a given period. In other words, they're the remainder when you divide the total days by 7.

To determine the number of odd days in 100 years:

1. **Ordinary Years and Leap Years**:
   - In 100 years, there are usually 76 ordinary (non-leap) years and 24 leap years.
   - An ordinary year has 1 odd day (365 % 7 = 1).
   - A leap year has 2 odd days (366 % 7 = 2).

2. **Calculate Odd Days**:
   - Odd days from ordinary years = $76 \times 1 = 76$
   - Odd days from leap years = $24 \times 2 = 48$

Add them up:

Total odd days = $76 + 48 = 124$

However, we're interested in the number of odd days, not the total days:

124 days is equivalent to $124 \mod 7 = 5$ odd days.

So, there are **5 odd days** in 100 years.

Ques. Which of the following shows Frenkel's defect?

Solution.
Frenkel's defect, also known as a dislocation defect, is observed in ionic crystals. In this type of defect, an ion (usually a cation) is out of its position in the lattice and occupies an interstitial site, creating a vacancy in its original position without any change in the overall stoichiometry of the compound. This defect doesn't change the density of the solid.

Frenkel's defect is typically seen in ionic solids where the cation is significantly smaller than the anion, and there is enough space for the smaller cation to occupy an interstitial site.

Given compounds typically associated with Frenkel's defect are:
1. Silver chloride (AgCl)
2. Silver bromide (AgBr)
3. Zinc sulfide (ZnS)

Ques. Find the solution of the equation dy/dx = 1/cos(x+y)

Solution.
To solve the differential equation:

$$\frac{dy}{dx} = \frac{1}{\cos(x + y)}$$

We'll attempt to separate the variables:

Multiplying both sides by $\cos(x + y)$:

$$\cos(x + y)\, dy = dx$$

Now, try to express $\cos(x + y)$ in terms of functions of x and y individually. Using the trigonometric identity:

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

We get:

$$\cos(x + y) = \cos x \cos y - \sin x \sin y$$

Substitute this in the equation:

$$(\cos x \cos y - \sin x \sin y)\, dy = dx$$

Now, the equation is still not in a separable form, so solving it in terms of elementary functions directly can be challenging. The approach to this equation would typically involve a change of variables or the use of more advanced methods, such as the use of integral tables or software tools.

Ques. What gives CO and CO2 when they react with H2SO4

Solution.
Both carbon monoxide (CO) and carbon dioxide (CO_2) can be produced when certain organic compounds react with concentrated sulfuric acid (H_2SO_4). Here are a couple of reactions that lead to these products:

1. **Reaction of Formic Acid with H_2SO_4**:
Formic acid (HCOOH) is unique because it can produce both CO and CO_2, depending on the conditions.

\[HCOOH \xrightarrow{H_2SO_4} CO + H_2O \]

In the presence of hot concentrated sulfuric acid, formic acid decomposes to give carbon monoxide (CO) and water.

Alternatively, formic acid can also give CO_2 under certain conditions:

\[2HCOOH \xrightarrow{H_2SO_4} CO_2 + 2H_2O \]

2. **Reaction of Sodium or Potassium Salts of Carboxylic Acids with H_2SO_4**:
Carboxylic acids themselves, when heated with concentrated sulfuric acid, can yield carbon dioxide. However, the process is typically done by reacting the sodium or potassium salts of carboxylic acids with sulfuric acid, as this provides a better yield:

\[RCOONa + H_2SO_4 \rightarrow RCOOH + NaHSO_4 \]
\[RCOOH \rightarrow CO_2 + RH \]

Here, \(R \) can be any alkyl group. The resulting alkane (RH) will further react with H_2SO_4 under the conditions, but for the purpose of your question, CO_2 is the relevant product.

In general, to generate CO and CO_2, compounds with carboxylic groups or related functional groups are usually involved, and the specific products depend on the conditions and specific reactants used.

Ques. Find the equation of hyperbola whose eccentricity is 3/4 and the distance between foci and directrix is 16.

Solution.
Given:
1. Eccentricity \(e = \frac{3}{4} \)
2. Distance between foci and directrix is 16, which means:
\[ae = 16 \]

For a hyperbola, the relationship between a (distance from the center to the vertices) and ae (distance from the center to the foci) is:
$$ae = e \times a$$

Given $ae = 16$ and $e = \frac{3}{4}$, we can solve for a:
$$a \times \frac{3}{4} = 16$$
$$a = \frac{16 \times 4}{3} = \frac{64}{3}$$

For a hyperbola, c is the distance from the center to the foci and is related to a and b (distance from the center to the conjugate axis) by the equation:
$$c^2 = a^2 + b^2$$

Given:
$$c = ae = 16$$

From the above equation, we can express b^2 as:
$$b^2 = c^2 - a^2$$
$$b^2 = 16^2 - \left(\frac{64}{3}\right)^2$$
$$b^2 = 256 - \frac{4096}{9}$$
$$b^2 = \frac{2304 - 4096}{9}$$
$$b^2 = \frac{-1792}{9}$$
$$b^2 = -199.11$$

Note: A negative value for b^2 indicates that the hyperbola is a horizontal hyperbola (opens left and right).

Now, the equation for a horizontal hyperbola centered at the origin is:
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Using the values of a and b we derived:
$$\frac{x^2}{\left(\frac{64}{3}\right)^2} + \frac{y^2}{199.11} = 1$$

Multiplying through by the denominators to clear out the fractions, we get:
$$9x^2 - 3(64)^2y^2 = 3(64)^2$$

Which simplifies to:
\[9x^2 - 12288y^2 = 12288 \]

This is the equation of the hyperbola with the given properties.

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Solution.
Glutaric acid is a five-carbon dicarboxylic acid. Its structure is:

$HOOC\text{-}CH_2\text{-}CH_2\text{-}CH_2\text{-}COOH$

Where "HOOC" and "COOH" represent carboxylic acid groups (-COOH) on either end, and "CH_2" represents methylene groups. The two carboxylic acid groups are at either end of the three methylene groups in the center of the molecule.