Sample Paper 1

ANSWERKEY																			
1	(a)	2	(a)	3	(d)	4	(b)	5	(a)	6	(c)	7	(b)	8	(c)	9	(a)	10	(d)
11	(a)	12	(d)	13	(d)	14	(b)	15	(b)	16	(c)	17	(a)	18	(b)	19	(a)	20	(a)
21	(c)	22	(a)	23	(d)	24	(c)	25	(d)	26	(d)	27	(a)	28	(b)	29	(b)	30	(c)
31	(a)	32	(c)	33	(a)	34	(b)	35	(a)	36	(d)	37	(c)	38	(d)	39	(b)	40	(d)
41	(c)	42	(d)	43	(c)	44	(d)	45	(c)	46	(a)	47	(c)	48	(a)	49	(b)	50	(b)

SOLUTIONS

1. (a) Let the required numbers be 15x and 11x.
Then, their H.C.F. is x. So, x = 13.
$\therefore$ The numbers are 5 × 13 and 11 × 13 i.e., 195 and 143.

2. (a) Terminating

3. (d) (a) is false $\because$ π is the ratio of the circumference of a circle to the length of the diameter.
It is nearly equal to $\frac{22}{7}$ but not exactly.
(b) False [$\because$ real numbers can be irrationals also]
(c) False [$\because$ non-terminating decimal can be recurring or non-recurring]
(d) Since 0.2 < 0.21 < 0.3

4. (b) Degree of quotient = degree of dividend – degree of divisor
Degree of quotient = 7 – 4 = 3.

5. (a) 1 is zero of p(x)
$\Rightarrow$ p (1) = 0
$\Rightarrow$ $a(1)^2 - 3(a - 1)(1) - 1, = 0$
$\Rightarrow$ $-2a + 2 = 0$
$\Rightarrow$ a = 1

6. (c) Here, the two triangles are similar.
Ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding altitudes.

So, $\frac{h_1^2}{h_2^2} = \frac{25}{36}$

$\therefore \quad \frac{h_1}{h_2} = \frac{5}{6}$

7. (b) n(S) = 6 × 6 = 36
E= {(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5), (5, 4), (5, 6), (6, 5)}
n(E) = 10

$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

8. (c) For reflection of a point with respect to x-axis change sign of y-coordinate and with respect to y-axis change sign of x-coordinate.

9. (a) It is given that AD is the bisector of $\angle$A.

$\frac{AB}{AC} = \frac{BD}{DC} \Rightarrow AC = \frac{6 \times 3}{4} = 4.5$ cm

10. (d) Given, $\tan\theta = \frac{a}{b}$

$\therefore \quad \frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a\tan\theta - b}{a\tan\theta + b} = \frac{a^2 - b^2}{a^2 + b^2}$

11. (a) Let the age of father be 'x' years and the age of son be 'y' years

According to question, $x + y = 65$ ...(i)

and $2(x - y) = 50 \Rightarrow x - y = 25$ ...(ii)

Adding eqs. (i) and (ii), we get, $2x = 90 \Rightarrow x = 45$

Hence, the age of father = 45 years

12. (d) $P(6, 2) = \left(\frac{4\times3+1\times6}{3+1}, \frac{3\times y+1\times5}{3+1}\right)$

$\because \quad 6 \neq \frac{18}{4}$ (Question is wrong)

$2 = \frac{3y+5}{4} \Rightarrow 3y + 5 = 8$

$3y = 3 \Rightarrow y = 1$

13. **(d)** We know that $\sec^2\theta - \tan^2\theta = 1$ and $\sec\theta = \frac{x}{p}$, $\tan\theta = \frac{y}{q}$

$\therefore$ $x^2q^2 - p^2y^2 = p^2q^2$

14. **(b)** Substitute $x = 1$ in $f(x)$ and $x = -2$ in $g(x)$, and add

$f(1) = 2(1) - 6(1) + 4(1) - 5 = -5 \Rightarrow g(-2) = 3(4) - 9 = 3$

$f(1) + g(-2) = -2$

15. **(b)** S = {1, 2, 3, ..., 100}

n(S) = 100

E = {11, 22, 33, 44, 55, 66, 77, 88, 99}

n(E) = 9

$\therefore \quad P(E) = \frac{9}{100}$

16. **(c)**

17. **(a)** Let the speeds of the cars starting from A and B be x km/hr and y km/hr respectively

According to problem,

$9x - 90 = 9y$ .... (i)

$\frac{9}{7}x + \frac{9}{7}y = 90$ .... (ii)

Solving we get $x = 40$ km/hr, $y = 30$ km/hr,

speed of car $A = 40$ km/hr

& speed of car $B = 30$ km/hr

18. **(b)** Given $(x)^2 + (x + 2)^2 = 290$

$\Rightarrow x^2 + x^2 + 4x + 4 = 290$

$\Rightarrow 2x^2 + 4x - 286 = 0$

$\Rightarrow x^2 + 2x - 143 = 0$

$\Rightarrow x^2 + 13x - 11x - 143 = 0$

$\Rightarrow (x + 13)(x - 11) = 0$

$\Rightarrow x = -13, \ x = 11$

x cannot be negative, discard $x = -13$, so $x = 11$

Hence the two consecutive positive integers are 11, 13

19. **(a)** Given equations are :

$7x - y = 5$ and $21x - 3y = k$

Here $a_1 = 7, b_1 = -1, c_1 = 5$

$a_2 = 21, b_2 = -3, c_2 = k$

We know that the equations are consistent with unique solution

if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

Also, the equations are consistent with many solutions

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

$\therefore \frac{7}{21} = \frac{-1}{-3} = \frac{5}{k} \Rightarrow \frac{1}{3} = \frac{5}{k} \Rightarrow k = 15$

Hence, for $k = 15$, the system becomes consistent.

20. **(a)** Let the numbers be 37a and 37b. Then

$37a \times 37b = 4107 \Rightarrow ab = 3$

Now, co-primes with product 3 are (1, 3)

So, the required numbers are

(37 × 1, 37 × 3) i.e., (37, 111).

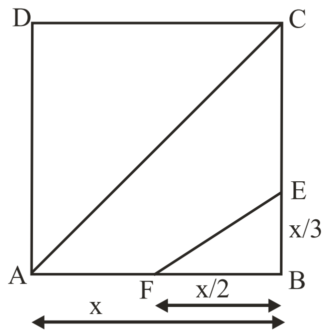
$\therefore$ Greater number = 111

21. **(c)** We have ABCD is square,

$AF = BF, BE = \frac{1}{3} BC$,

Area $\Delta FBE = 108$ sq cm.

Let $AB = x \Rightarrow BF = \frac{x}{2}$ and $BE = \frac{x}{3}$



Area of $\Delta FBE = \frac{1}{2} BF \times BE$

$= \frac{1}{2} \times \frac{x}{2} \times \frac{x}{3} = \frac{x^2}{12}$

= 108 sq. cm. (Given)

$\Rightarrow x^2 = 12 \times 108$

$\Rightarrow x^2 = 12 \times 12 \times 3 \times 3$

$\Rightarrow x = 12 \times 3 = 36$ cm

In rt. ΔABC, $AC = \sqrt{AB^2 + BC^2}$

(By Pythagoras Theorem)

$= \sqrt{36^2 + 36^2} = 36\sqrt{2}$ cm

22. **(a)** Let D be the window at a height of 9m on one side of the street and E be the another window at a height of 12 m on the other side.

In ΔADC

$AC^2 = 15^2 - 9^2$

$= 225 - 81$

AC = 12 m

In ΔECB

$CB^2 = 15^2 - 12^2$

$= 225 - 144$

CB = 9 m

Width of the street = (12 + 9)m = 21 m

23. **(d)** $x - y = 2$ ... (i)

$kx + y = 3$ ... (ii)

by adding (i) and (ii)

$kx + x = 5$

$x(k + 1) = 5$

$x = \frac{5}{k+1}$

putting value of x in equation (i)

$\frac{5}{k+1} - y = 2$

$\frac{5}{k+1} - 2 = y$

$\frac{5-2k-2}{k+1} = y$

$y = \frac{3-2k}{k+1}$

y should be positive as they intersect in 1st quadrant therfore

$y > 0$

$\frac{3-2k}{k+1} > 0 \Rightarrow \frac{2k-3}{k+1} < 0$

$$\begin{array}{ccccccc} & + & & - & & + & \\ -\infty & & -1 & & 3/2 & & \infty \end{array}$$

$\therefore$ k should lie between – 1 and 3/2

24. **(c)** We have, sin x + cosec x

$\Rightarrow \sin x + \frac{1}{\sin x}$

$\therefore$ We know that sum of the number and its reciprocal is greater than or equal to 2.

25. **(d)** We have, $\sin 5\theta = \cos 4\theta$

$\Rightarrow 5\theta + 4\theta = 90°$

[$\because$ $\sin\alpha = \cos\beta$, than $\alpha + \beta = 90°$]

$\Rightarrow 9\theta = 90° \Rightarrow \theta = 10°$

Now, $2\sin 3\theta - \sqrt{3}\tan 3\theta$

$= 2\sin 30° - \sqrt{3}\tan 30°$

$= 2 \times \frac{1}{2} - \sqrt{3} \times \frac{1}{\sqrt{3}} = 1 - 1 = 0$

26. **(d)** If two similar triangles have equal area then triangles are necessarily congruent.

27. **(a)** Here, the two lines are $2x + 3y = 7$ and $2ax + (a + b) y = 28$. The above lines are coincident.

Two lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are coincident if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, $\frac{2}{2a} = \frac{3}{a+b} = \frac{-7}{28}$

$\Rightarrow$ a = 4, b = 8

$\therefore$ b = 2a

28. **(b)** $2ax - 2by + a + 4b = 0$ .... (i)

and $2bx + 2ay + b - 4a = 0$ .... (ii)

Multiplying eq. (i) with b and eq. (ii) with a, we get

$2abx - 2b^2y + ab + 4b^2 = 0$ .... (iii)

and $2abx + 2a^2y + ab - 4a^2 = 0$ .... (iv)

Subtracting (iv) from (iii), we get

$-(2b^2 + 2a^2)\,y + 4b^2 + 4a^2 = 0$

$\Rightarrow -(2b^2 + 2a^2)\,y = -4b^2 - 4a^2 \Rightarrow y = 2$

Substituting y = 2 in eq. (i), we get

$2ax - 2b \times 2 + a + 4b = 0$

$\Rightarrow x = -1/2$ $\quad\therefore x = -1/2,\ y = 2$

29. **(b)** Put $x + 1 = 0$ or $x = -1$ and $x + 2 = 0$ or $x = -2$ in $p(x)$

Then, $p(-1) = 0$ and $p(-2) = 0$

$\Rightarrow p(-1) = (-1)^3 + 3(-1)^2 - 2\alpha(-1) + \beta = 0$

$\Rightarrow -1 + 3 + 2\alpha + \beta = 0 \Rightarrow \beta = -2\alpha - 2$ .... (i)

$p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta = 0$

$\Rightarrow -8 + 12 + 4\alpha + \beta = 0 \Rightarrow \beta = -4\alpha - 4$ .... (ii)

By equalising both of the above equations, we get

$-2\alpha - 2 = -4\alpha - 4$

$\Rightarrow 2\alpha = -2 \Rightarrow \alpha = -1$

put $\alpha = -1$ in eq. (i)

$\Rightarrow \beta = -2(-1) - 2 = 2 - 2 = 0$

Hence, $\alpha = -1$, $\beta = 0$

30. **(c)** Let α, β be two zeroes of $2x^2 - 8x - m$, where $\alpha = \frac{5}{2}$.

$\therefore \alpha + \beta = \frac{(-\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

$\Rightarrow \frac{5}{2} + \beta = \frac{8}{2} \Rightarrow \beta = \frac{8}{2} - \frac{5}{2} = \frac{3}{2}$.

31. **(a)** Let $f(x) = 2x^3 - 5x^2 + ax + b$

$f(2) = 2(2)^3 - 5(2)^2 + a(2) + b = 0$

$\Rightarrow 16 - 20 + 2a + b = 0 \Rightarrow 2a + b = 4$

$f(0) = 2(0)^3 - 5(0)^2 + a(0) + b = 0 \Rightarrow b = 0$

$\Rightarrow 2a = 4 \Rightarrow a = 2, b = 0$

32. **(c)** $\cos A = \frac{3}{5} \Rightarrow \sin A = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$

consider

$$9\cot^2 A - 1 = \frac{9\cos^2 A}{\sin^2 A} - 1 = \frac{9\cos^2 A - \sin^2 A}{\sin^2 A}$$

$$= \frac{9\left(\frac{9}{25}\right) - \left(\frac{16}{25}\right)}{\frac{16}{25}} = \frac{(81-16)}{25} \times \frac{25}{16} = \frac{65}{16}$$

33. **(a)** All isosceles triangles are not similar.

34. **(b)** Let α and 6α be roots of equation.

Sum of roots : $\alpha + 6\alpha = \frac{14}{p}$

$\Rightarrow 7\alpha = \frac{14}{p} \Rightarrow p = \frac{2}{\alpha}$

Product of roots : $(\alpha)(6\alpha) = \frac{8}{p} \Rightarrow p = \frac{4}{3\alpha^2}$

$\Rightarrow \quad \frac{2}{\alpha} = \frac{4}{3\alpha^2}$

$\Rightarrow \quad \alpha = \frac{2}{3}$

Therefore, $p = \frac{2}{\alpha} = 3$

35. (a) The equations $3x - (a+1)y = 2b - 1$
$5x + (1 - 2a)y = 3b$
The system will have infinite number of solutions

if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Here, $a_1 = 3,\ b_1 = -(a+1),\ c_1 = 2b - 1$
$a_2 = 5,\ b_2 = 1 - 2a,\ c_2 = 3b$

$\therefore \quad \frac{3}{5} = \frac{-(a+1)}{1-2a} = \frac{2b-1}{3b}$

Taking I and II: $\frac{3}{5} = \frac{-(a+1)}{1-2a}$

$\Rightarrow -5a - 5 = 3 - 6a$

$\Rightarrow -5a + 6a = 3 + 5$

$a = 8$

Taking I and III: $\frac{3}{5} = \frac{2b-1}{3b}$

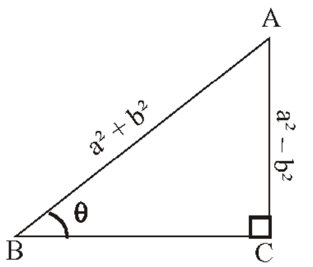
$\Rightarrow 10b - 5 = 9b$

$\Rightarrow 10b - 9b = 5$

$b = 5$

$\therefore \quad a = 8, b = 5$

36. (d) $\sin\theta = \frac{a^2 - b^2}{a^2 + b^2}$



Since, $\sin\theta = \frac{\text{perpendicular}}{\text{base}}$

$\therefore \frac{AC}{AB} = \frac{a^2 - b^2}{a^2 + b^2}$

Now in ΔABC,
$\angle B = \theta$ and $\angle C = 90°$
$(a^2 + b^2)^2 = BC^2 + (a^2 - b^2)^2$
$\therefore BC = 2ab$

$\text{cosec}\,\theta = \frac{a^2 + b^2}{a^2 - b^2}$,

$\cot\theta = \frac{BC}{AC} = \frac{2ab}{a^2 - b^2}$

$\text{cosec}\,\theta + \cot\theta = \frac{a^2 + b^2}{a^2 - b^2} + \frac{2ab}{a^2 - b^2} = \frac{a+b}{a-b}$

37. (c) 3, because it is the exponent of the highest degree term in the polynomial $y^3 - 2y^2 - \sqrt{3}y + \frac{1}{2}$.

38. (d) $ax + by = c,\ bx - ay = c$
Using the cross-multiplication method,

$\frac{x}{-ac - bc} = \frac{y}{ac - bc} = \frac{1}{-a^2 - b^2}$

$\Rightarrow \quad x = \frac{-ac - bc}{-a^2 - b^2} = \frac{-c(a+b)}{-(a^2 + b^2)} = \frac{c(a+b)}{a^2 + b^2}$

and

$y = \frac{ac - bc}{-a^2 - b^2} = \frac{c(a-b)}{-(a^2 + b^2)} = -\frac{c(a-b)}{a^2 + b^2}$

Therefore, $x = \frac{c(a+b)}{a^2 + b^2},\ y = -\frac{c(a-b)}{a^2 + b^2}$

39. (b) $\frac{21}{45} = \frac{21}{9 \times 5} = \frac{21}{3^2 \times 5}$.

Clearly, 45 is not of the form $2^m \times 5^n$. So the decimal expansion of $\frac{21}{45}$ is non-terminating and repeating.

40. (d) $(\sin A + \text{cosec} A)^2 + (\cos A + \sec A)^2$

$\Rightarrow \quad \sin^2 A + \text{cosec}^2 A + 2\sin A\ \text{cosec}\ A + \cos^2 A + \sec^2 A + 2\sec A \cos A$

$\Rightarrow \quad (\sin^2 A + \cos^2 A) + \text{cosec}^2 A + 2 + \sec^2 A + 2$

$\Rightarrow \quad 1 + 4 + 1 + \cot^2 A + 1 + \tan^2 A$

$\Rightarrow \quad 7 + \cot^2 A + \tan^2 A$

$\therefore \quad (\sin A + \text{cosec} A)^2 + (\cos A + \sec A)^2$

$= 7 + \cot^2 A + \tan^2 A$

Hence, a = 7

41. (c) **42. (d)** **43. (c)** **44. (d)** **45. (c)**
46. (a) **47. (c)** **48. (a)** **49. (b)** **50. (b)**