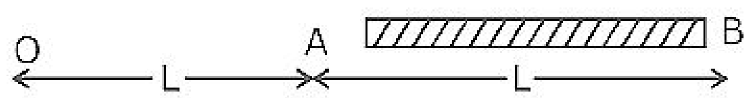
PART - I [PHYSICS]

Q.1 A charge Q is uniformly distributed over a long rod AB of length l as shown in the figure. The electric potential at the point O lying at a distance L from the end A is:

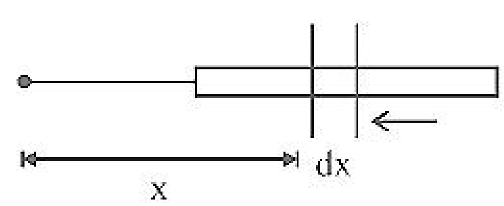


(1) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$ (2) $\frac{Q\ln 2}{4\pi\varepsilon_0 L}$

(3) $\frac{Q}{8\pi\varepsilon_0 L}$ (4) $\frac{3Q}{4\pi\varepsilon_0 L}$

Sol. **(2)**

Formula $v = \frac{KQ}{L}\ell n2$



$$dv = \frac{K(dQ)}{x} = \frac{K}{x}\frac{Q}{L}\times dx$$

$$v = \frac{KQ}{L}\int_{L}^{2L}\frac{dx}{x} = \frac{KQ}{L}\ell n2$$

Q.2 A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively?

(1) 200.5 Hz
(2) 770 Hz
(3) 188.5 Hz
(4) 178.2 Hz

Sol. **(4)**

$$\text{Strain} = \frac{1}{100}$$

$$\text{Stress} = (\text{strain})y = \frac{1}{100} \times 2.2 \times 10^{11} = 2.2 \times 10^9$$

$$\frac{T}{A} = \text{Stress} = 2.2 \times 10^9 \qquad T = 2.2 \times 10^9\ A$$

$$v = \sqrt{\frac{T}{M}} = \sqrt{\frac{T}{P.A}} = \sqrt{\frac{2.2\times10^9.A}{P.A}}$$

$$v = \sqrt{\frac{2.2\times10^9}{7.7\times10^3}} = \sqrt{\frac{2}{7}\times10^6}$$

$$f_1 = \frac{v}{2L} = \sqrt{\frac{2}{7}\times10^6}\times\frac{1}{2\times1.5} = 178.2\text{ Hz}$$

Q.3 A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is:

(1) $4y = 2x - 5x^2$
(2) $4y = 2x - 25x^2$
(3) $y = x - 5x^2$
(4) $y = 2x - 5x^2$

Sol. **(4)**

$\vec{v} = i + 2\hat{j} \Rightarrow v_x = 1$

$v_y = 2$

$\tan\theta = 2$

$\cos\theta = \frac{1}{\sqrt{5}}$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

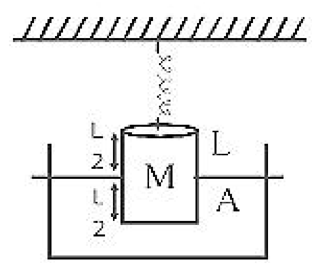
$$y = x(2) - \frac{gx^2}{2(5)\left(\frac{1}{5}\right)} = 2x - 5x^2$$

Q.4 A uniform cylinder of length L and mass M having cross - sectional area A is suspended, with its length vertical, from a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is:

(1) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (2) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

(3) $\frac{Mg}{k}$ (4) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$

Sol. **(1)**

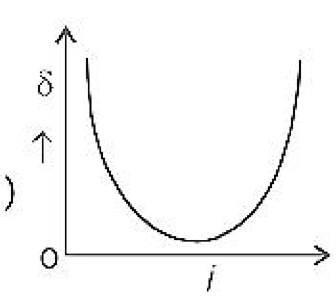


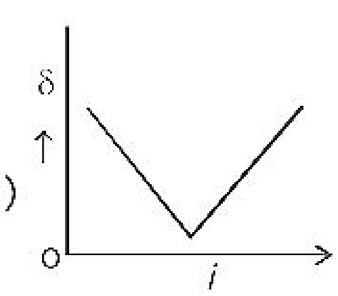
$Mg = Kx + B$

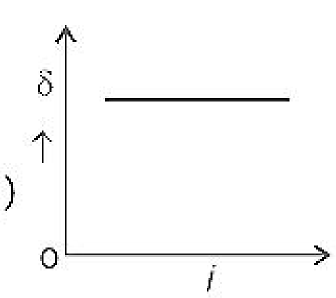
$Mg = Kx + \sigma \frac{AL}{2} g \quad x = \frac{Mg}{K} - \frac{ALg}{2K}\sigma$

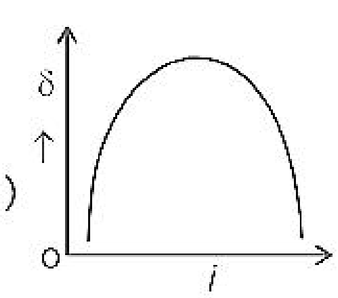
$= \frac{Mg}{K}\left(1 - \frac{\sigma AL}{2M}\right)$

Q.5 The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is respresented by:

(1) 

(2) 

(3) 

(4) 

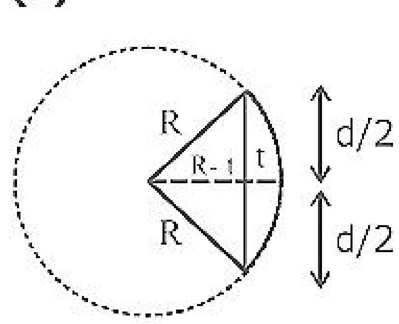
Sol. (1)

Q.6 Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is :

(1) 30 cm (2) 10 cm

(3) 15 cm (4) 20 cm

Sol. **(1)**



$R^2 = (R - t)^2 + \frac{d^2}{4}$

$R^2 = R^2 + t^2 - 2\,tR + \frac{d^2}{4}$ (t^2 is neglect)

$2tR = \frac{d^2}{4} \quad \Rightarrow \quad R = \frac{d^2}{8t}$

$R = \frac{36\,cm^2}{8 \times 0.3\,cm} = \frac{120}{8} = 15$ cm

$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R}\right]$

$\frac{1}{f} = \frac{(\mu - 1)}{15} = \frac{1}{30}$

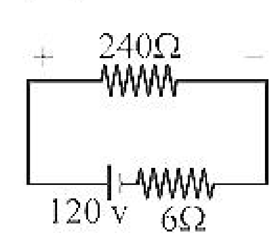
$f = 30$

Q.7 The supply voltage to a room is 120 V. The resistance of the lead wires is 6Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb?

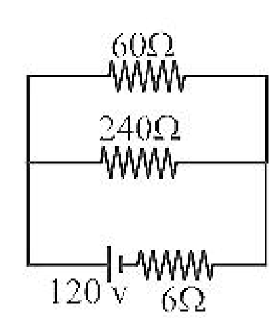
(1) 13.3 Volt (2) 10.4 Volt

(3) zero Volt (4) 2.9 Volt

Sol. **(2)**



$v_1 = \frac{120}{246} \times 240 = 117.07$



$v_2 = \frac{120}{54} \times 48 = 106.67$

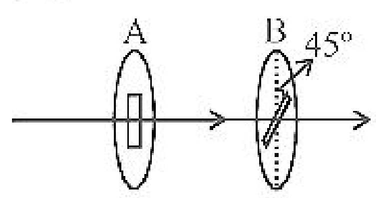
$\Delta v = v_1 - v_2 = 117.073 - 106.67 = 10.4$

Q.8 A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is:

(1) $I_0/4$ (2) $I_0/8$

(3) I_0 (4) $I_0/2$

Sol. **(1)**



Through A only component parallel to slit will pass so intensity after passing through A will be $\frac{I_0}{2}$.

After passing through B

$$I = \frac{I_0}{2}\cos^2\phi = \frac{I_0}{4}$$

Q.9 The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decreases to α times its original magnitude, where α equals :

(1) 0.729 (2) 0.6

(3) 0.7 (4) 0.81

Sol. **(1)**

From equation of damped oscillation

$A' = Ae^{-\frac{bt}{2}}$

$0.9\,A = A\,e^{-2.5\,b}$

solving b = 0.0421

In another 10 seconds i.e. at

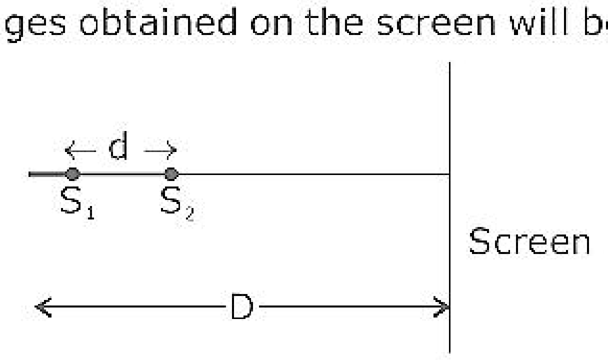
t = 15 seconds

$A'' = Ae^{-\frac{15b}{2}}$

$\alpha A = Ae^{-\frac{15b}{2}}$

$\alpha = 0.729$

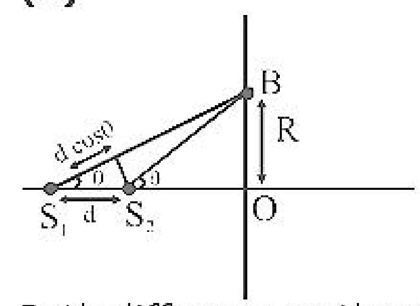
Q.10 Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be :



(1) semi-circles (2) concentric circles

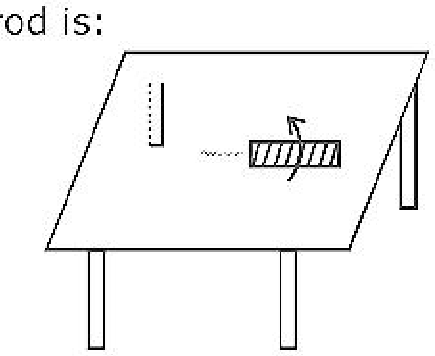
(3) points (4) straight lines

Sol. **(2)**



Path differnce on the circle of radius R around O on the wall will be same hence concentric circle.

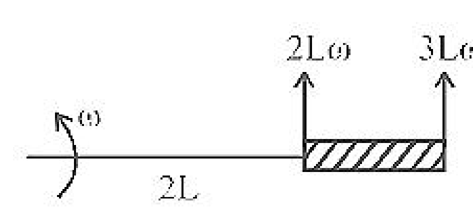
Q.11 A metallic rod of length 'l' is tied to a string of length 2 *l* and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is:



(1) $\frac{4B\omega l^2}{2}$ (2) $\frac{5B\omega l^2}{2}$

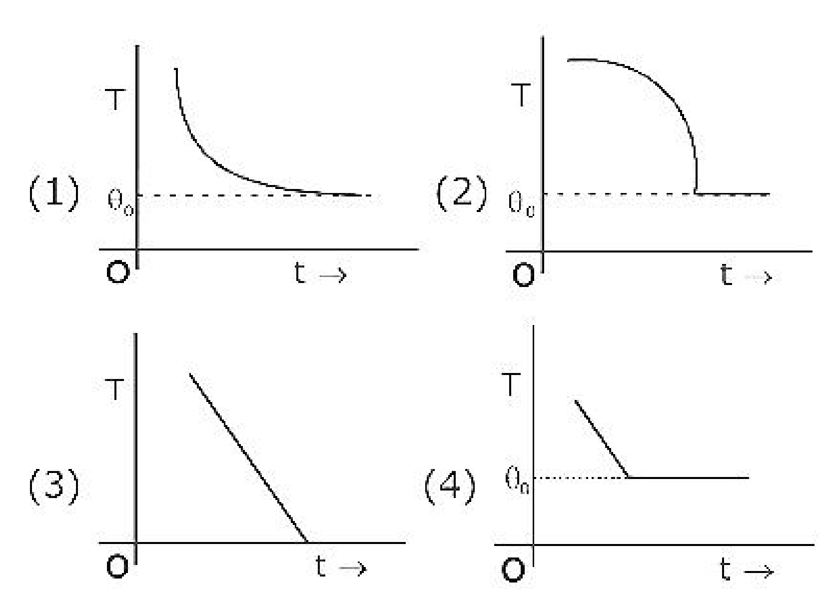
(3) $\frac{2B\omega l^2}{2}$ (4) $\frac{3B\omega l^2}{2}$

Sol. **(2)**



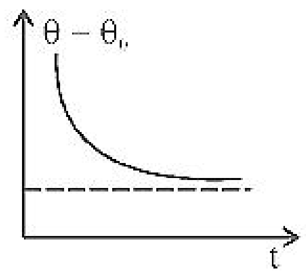
$$\text{EMF} = \left(\frac{2L\omega + 3L\omega}{2}\right)LB = \frac{5}{2}BL^2\omega$$

Q.12 If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closed to:



Sol. **(2)**

Newton's law of cooling



$$\frac{d\theta}{dt} = -K(\theta - \theta_0)$$

$$\int \frac{d\theta}{\theta - \theta_0} = -K\int dt$$

Q.13 This question has statement I and statement II. Of the four choices given after the statement, choose the one that best decribes the two statements.

Statement - I : Higher the range, greater is the resistance of ammeter.

Statement - II : To increase the range of ammeter, additional shunt needs to be used across it.

(1) Statement-I is true, Statement-II is false

(2) Statement-I is false, Statement-II is ture

(3) Statement-I is true, Statement-II is true, Statement-II is the correct explanation of statement-I.

(4) Statement-I is true, Statement-II is true, Statement-II is not the correct explanation of Statement-I.

Sol. **(2)**

Statement-I → (False)

$$I = I_g\left(1 + \frac{G}{R}\right)$$

If R → More

I → Less

Statement-II → (True)

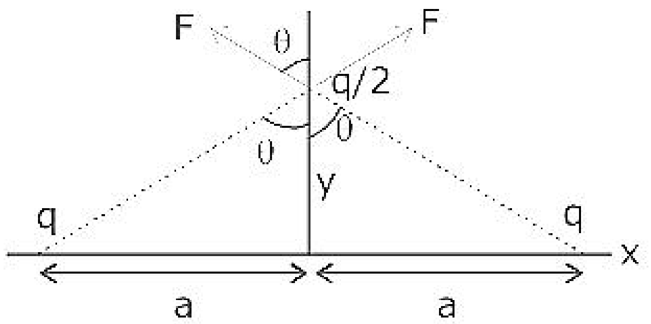
Addition shunt in parallel decreases the resistance.

Q.14 Two charges, each equal to q, are kept at x = – a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is given a small displacement (y << a) along the y-axis, the net force acting on the particle is proportional to :

(1) $\frac{1}{y}$ (2) $-\frac{1}{y}$

(3) y (4) –y

Sol. **(3)**



$$F_{net} = 2F\cos\theta$$

$$= 2\frac{kq(q/2)}{(a^2+y^2)}\cdot\frac{y}{(a^2+y^2)^{1/2}}$$

$$\simeq \frac{kq^2.y}{(a^2+y^2)^{3/2}} \simeq \frac{kq^2.y}{a^3}$$

$$F_{net} \propto y$$

Q.15 This queation has Statement I and StatementII. Of the four choices given after the Statements, choose the one that best describes the two statements.

Statement-I : A point particle of mass m moving with speed υ collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}m\upsilon^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement-II : Maximum energy loss occurs when the particles get stuck together as a result of the collision.

(1) Statement-I is true, Statement-II is false

(2) Statement-I is false, Statement-II is true.

(3) Statement-I is true, Statement-II is a correct explanation of Statement-I

(4) Statement-I is true, Statement-II is ture, Statement-II is not a correct explanation of Statement-I.

Sol. **(2)**

Statement–I → (False)

For max energy loss, e = 0

$$V = \frac{mv}{m+M}$$

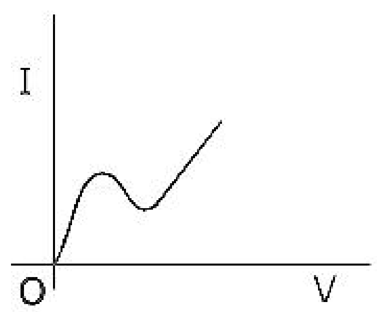
$$\text{loss} = \frac{1}{2}mv^2 - \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2$$

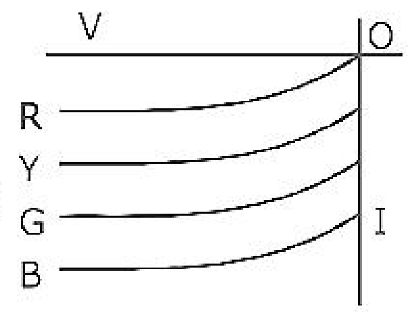
$$= \frac{1}{2}mv^2 - \frac{1}{2}\frac{m^2v^2}{m+M}$$

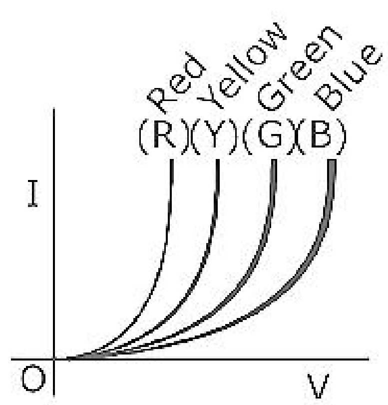
$$= \frac{1}{2}\frac{mv^2(m+M) - m^2v^2}{(m+M)} = \frac{1}{2}\frac{mMv^2}{m+M}$$

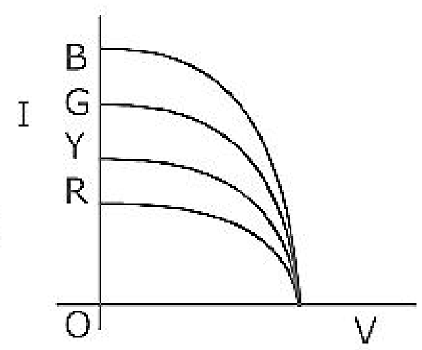
Statement–II → (True)

Q.16 The I – V characteristic of an LED is:

(1) 

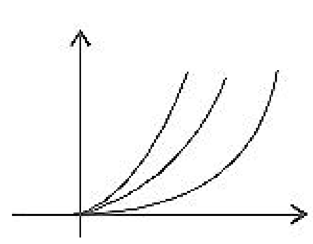
(2) 

(3) 

(4) 

Sol. **(3)**

LED work in forrward biased



Q.17 Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for that to be Possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization.

(1) $T/\rho L$ (2) $2T/\rho L$

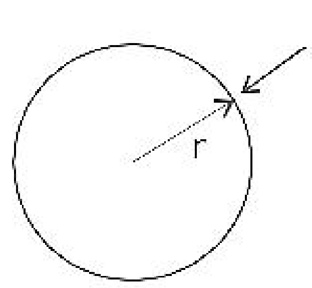
(3) $\rho L/T$ (4) $\sqrt{T/\rho L}$

Sol. **(2)**

$$s\left[4\pi(r+dr)^2 - 4\pi r^2\right] = (dm)L$$

$$\because dm = 4\pi r^2 dr\rho$$

$$r = \frac{2T}{\rho L}$$



Q.18 Two capacitors C_1 and C_2 are charged to 120 V and 200 V respectively. It is found that by connecting them together the potential on each one can be made zero. Then:

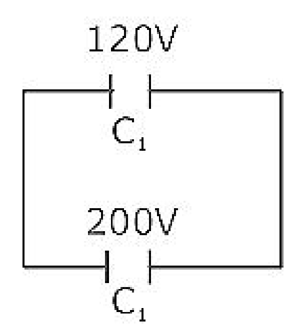
(1) $3C_1 + 5C_2 = 0$

(2) $9C_1 = 4C_2$

(3) $5C_1 = 3C_2$

(4) $3C_1 = 5C_2$

Sol. **(4)**



$$c_1 v_1 = c_2 v_2$$

$$c_1(120) = c_2(200)$$

$$3c_1 = 5c_2$$

Q.19 What is the minimum energy required to launch a statellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R?

(1) $\frac{GmM}{2R}$

(2) $\frac{GmM}{3R}$

(3) $\frac{5GmM}{6R}$

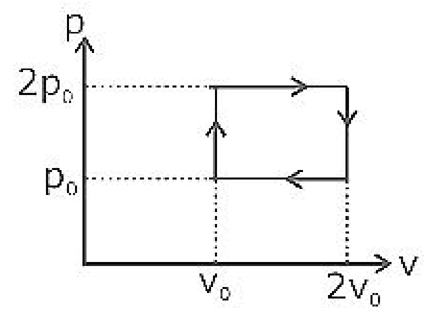
(4) $\frac{2GmM}{3R}$

Sol. **(3)**

Energy = Δu + KE

$$= \left[\frac{GMm}{R} - \frac{GMm}{3R}\right] + \frac{1}{2}M\left(\frac{Gm}{3R}\right) = \frac{5}{6}\ \frac{GMm}{R}$$

Q.20

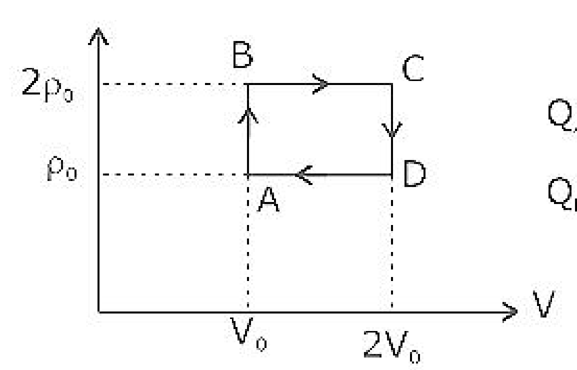


The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is:

(1) $\left(\frac{11}{2}\right)p_0v_0$ (2) $4p_0v_0$

(3) p_0v_0 (4) $\left(\frac{13}{2}\right)p_0v_0$

Sol. **(4)**



$Q_{AB} \rightarrow +ve$

$Q_{BC} \rightarrow +ve$

$$Q_{AB} = nc_v\Delta T = \frac{3}{2}nR\Delta T = \frac{3}{2}[P_2V_2 - P_1v_1]$$

$$= \frac{3P_0V_0}{2}$$

$$Q_{BC} = nC_p\Delta T = \frac{5}{2}nR\Delta T$$

$$= \frac{5}{2}(P_2V_2 - P_1V_1) \quad = 5P_0V_0$$

Total = (1- 5 + 5)P_0V_0 = 6.5 P_0V_0

21. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is :

(1) 3.3×10^{-11} weber
(2) 6.6×10^{-9} weber
(3) 9.1×10^{-11} weber
(4) 6×10^{-11} weber

Sol. **(3)**

$$\phi = \frac{\mu_0 IR^2}{2\left(R^2 + x^2\right)^{3/2}} \times \pi r^2$$

$$= \frac{\mu_0 IR^2 r^2 \pi}{2\left(R^2 + x^2\right)^{3/2}}$$

$$= \frac{4\pi \times 10^{-7} \times 2 \times (0.2)^2 (0.03)^2 \times \pi}{2\left[(0.2)^2 + (0.15)^2\right]^{3/2}}$$

$= 9.09 \times 10^{-11}$

22. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pico farad in parallel with a load resistance 100 kilo ohm. Find the maximum modulated frequency which could be detected by it.

(1) 5.31 MHz (2) 5.31 kHz
(3) 10.62 MHz (4) 10.62 kHz

Sol. **(4)**

$$V < \frac{1}{RC} < \frac{1}{(2.5 \times 10^{-12})(10^5)F} < 40 \text{ KHz}$$

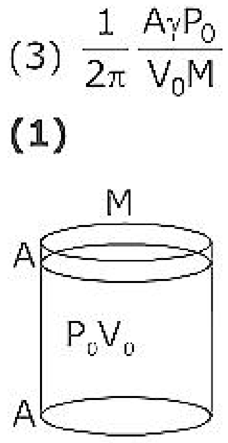
From given options the highest frequency , 40 KHz is 10.62 KHz.

23. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency :

(1) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

(3) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0 M}$ (4) $\frac{1}{2\pi}\frac{V_0 MP_0}{A^2\gamma}$

Sol. **(1)**



Adiabatic

$$Pv^r = \text{constant}$$

$$prv^{r-1}dv + v^r.dp = 0$$

$Pr\,v^{r-1}(A.dx) = -v^r(dp)$

$dp = \dfrac{-rpv^{r-1}Adx}{v^r}$ $dP = \dfrac{-rPA}{v}.dx$

$dF = \dfrac{-rPA^2}{V}.dx = -k.dx$

$T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{mv}{rpA^2}}$

$f = \dfrac{1}{2\pi}\sqrt{\dfrac{\gamma P_0 A^2}{mV_0}}$

24. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it cases to slip ?

(1) $\dfrac{r\omega_0}{2}$ (2) $r\omega_0$

(3) $\dfrac{r\omega_0}{4}$ (4) $\dfrac{r\omega_0}{3}$

Sol. **(1)**

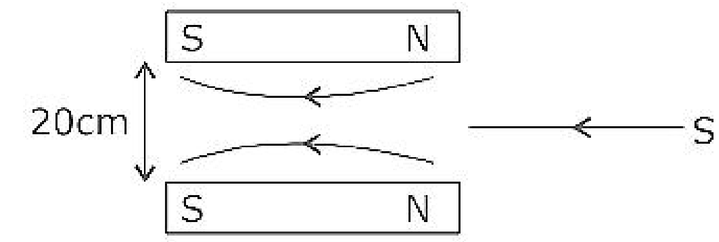
C.O.A.M. (abont bottom)

$mr^2\omega_0 = 2mr^2\dfrac{V_{cm}}{r}$

$V_{cm} = \dfrac{\omega_0 r}{2}$

25. Two short bar magnetic of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2 respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid - point O of the line joining their centre is close to :
(Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m^2)
(1) 3.50×10^{-4} Wb/m^2
(2) 5.80×10^{-4} Wb/m^2
(3) 3.6×10^{-5} Wb/m^2
(4) 2.56×10^{-4} Wb/m^2

Sol. **(4)**



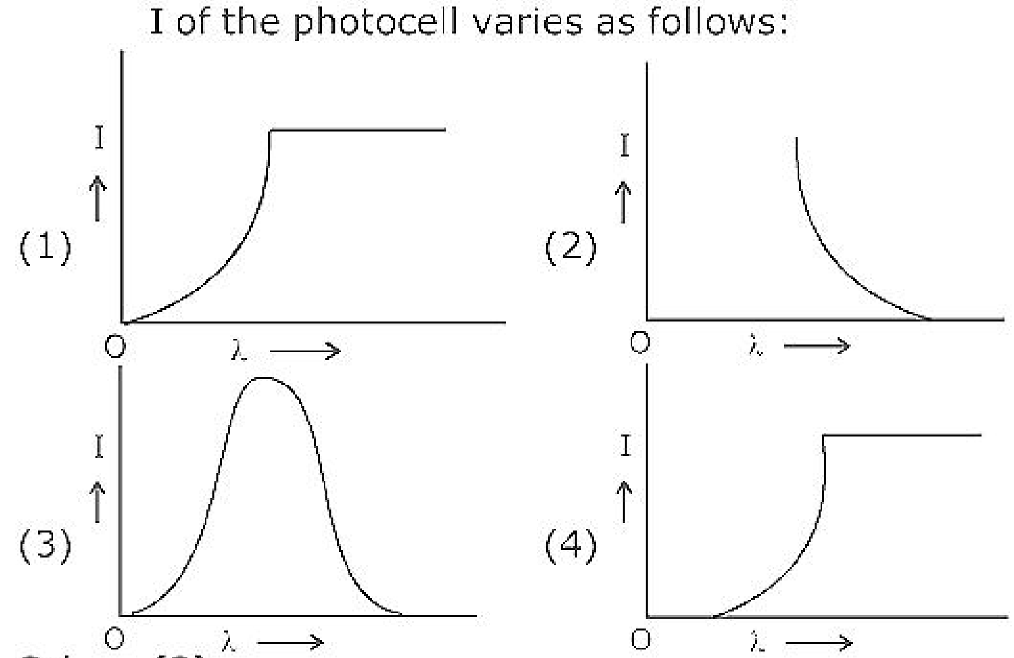
$B = B_1 + B_2 + B_2 = \dfrac{M_0}{4\pi}\left[\dfrac{M}{(r)^3} + \dfrac{M}{(r)^2}\right] + B$

$= 10^{-7}\left[\dfrac{1.2+1}{(0.1)^3}\right] + 3.6\times10^{-5}$

$= 2.2\times10^{-4} + 3.6\times10^{-5}$

$= 2.56 \times 10^{-4}$

26. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows:



Sol. **(2)**

Energy $\propto \dfrac{1}{\lambda}$

27. Let $[\varepsilon_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then :
(1) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A^{-2}]$
(2) $[\varepsilon_0] = [M^{-1}L^2T^{-1}A^2]$
(3) $[\varepsilon_0] = [M^{-1}L^{-3}T^2A]$
(4) $[\varepsilon_0] = [M^{-1}L^{-3}T^4A^2]$

Sol. $\dfrac{Energy}{Volume} = \dfrac{1}{2}\epsilon_0 E^2 = \dfrac{1}{2}\epsilon_0\left[\dfrac{F}{q}\right]^2$

$\dfrac{ML^2T^{-2}}{L^3} = \epsilon_0\left[\dfrac{MLT^{-2}}{AT}\right]^2$

$\epsilon_0 \rightarrow M^{-1}L^{-3}T^4A^2$

28. In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number (n – 1). If n >> 1, the frequency of radiation emitted it proportional to

(1) $\frac{1}{n^{3/2}}$ (2) $\frac{1}{n^3}$

(3) $\frac{1}{n}$ (4) $\frac{1}{n^2}$

Sol. **(2)**

Energy $\propto \frac{1}{n^2}$

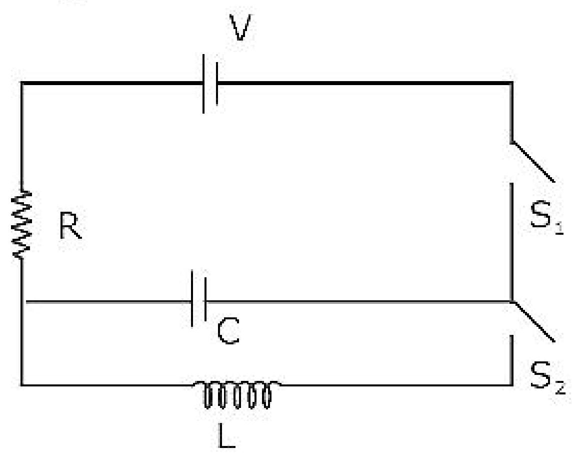
$h\nu \propto \frac{1}{(n-1)^2} - \frac{1}{n^2}$

$h\nu \propto \frac{n^2-(n-1)^2}{(n-1)^2(n)^2} = \frac{2n-1}{n^2(n-1)^2}$

$\cong \frac{2n}{n^2(n)^2}$

$h\nu \propto \frac{1}{n^3}$

29. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and τ = RC is Capacitive time constant). Which of the following statement is correct ?



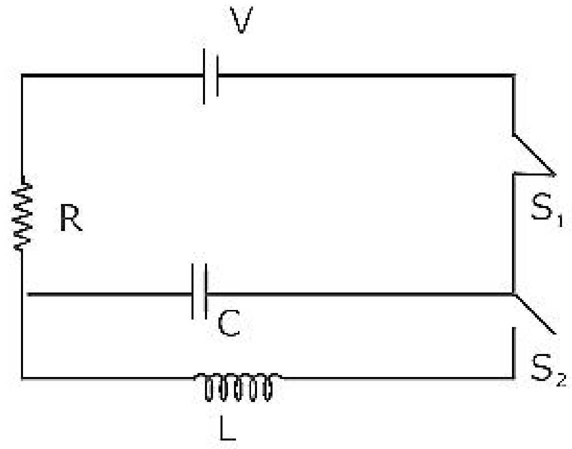
(1) At $t = 2\tau$, $q = CV(1 - e^{-2})$

(2) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$

(3) Work done by the battery is half of the energy dissipated in the resistor

(4) At $t = \tau$, $q = CV/2$

Sol. **(1)**



$q = CV\left(1 - e^{-\frac{t}{RC}}\right)$

At $t = 2RC$

$q = CV[1 - e^{-2}]$

30. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is :

(1) 9 V/m (2) 12 V/m

(2) 3 V/m (4) 6 V/m

Sol. **(4)**

$\frac{E}{B} = c$

$E = B.c = 20 \times 10^{-9} \times 3 \times 10^{8}$

= 6 v/m

PART - II [MATHEMATICS]

31. The real number k for which the equation, $2x^3 + 3x + k = 0$ has two distinct real roots in [0, 1]
(1) lies between –1 and 0.
(2) does not exist
(3) lies between 1 and 2
(4) lies between 2 and 3

Sol. **2**

$f'(x) = 6x^2 + 3 \quad \Rightarrow$ so f(x) is increasing $\forall$ x
equation can not have 2 distinct roots

32. The number of values of k, for which the system of equations :
$(k + 1)\,x + 8y = 4k$
$kx + (k + 3)y = 3k - 1$
has no solution, is :
(1) 2 (2) 3 (3) infinite (4) 1

Sol. **4**

$(k + 1)x + 8y = 4k$
$kx + (k + 3)y = 3k - 1$

$$-\frac{k+1}{8} = -\frac{k}{k+3}$$

$k^2 + 4k + 3 = 8k$
$k^2 - 4k + 3 = 0$
$k = 3, k = 1$
for k = 1 same lines
$\therefore$ k = 3 is only solution

33. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3 × 3 matrix A and |A| = 4, then α is equal to :
(1) 5 (2) 0 (3) 4 (4) 11

Sol. **4**

$$\left|\begin{matrix} 1 & \alpha & 3 & 1 & \alpha \\ 1 & 3 & 3 & 1 & 3 \\ 2 & 4 & 4 & 2 & 4 \end{matrix}\right| = 4^2$$

$12 + 6\alpha + 12 - 18 - 12 - 4\alpha = 16$
$\alpha = 11$

34. Let T_n be the number of all possible triangles formed by joining vertices of an n-sided regular polygon. If $T_{n+1} - T_n = 10$, then the value of n is:
(1) 10 (2) 8 (3) 7 (4) 5

Sol. **4**

$T_n = {}^nC_3$
${}^{n+1}C_3 - {}^nC_3 = 10$
${}^nC_2 = 10$ $\qquad {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$
$\therefore$ n = 5 $\qquad {}^nC_{r-1} = {}^{n+1}C_r - {}^nC_r$

35. At present, a firm is manufacturing 2000 items. It is estimated that rate of change of production P w.r.t additional number of workers x is given by $\frac{dP}{dx} = 100 - 12\sqrt{x}$. If the firm employs 25 more workers, then the new level of production of items is :
(1) 3500 (2) 4500 (3) 2500 (4) 3000

Sol. **1**

$$\frac{dp}{dx} = 100 - 12\sqrt{x}$$

$$\Rightarrow P = 100x - \frac{12x^{3/2}}{3/2} + c$$

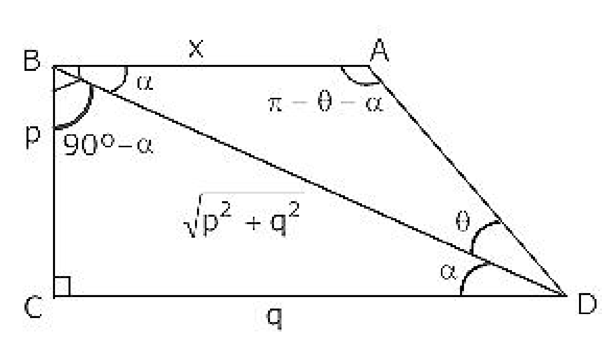
If x= 0 then P = 2000
$\Rightarrow P = 100\,x - 8x^{3/2} + 2000$
If x = 25
P = 2500 – 1000 + 2000 = 3500

36. ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If $\angle ADB = \theta$, BC = p and CD = q, then AB is equal to :

(1) $\frac{p^2 + q^2}{p^2\cos\theta + q^2\sin\theta}$ (2) $\frac{(p^2 + q^2)\sin\theta}{(p\cos\theta + q\sin\theta)^2}$

(3) $\frac{(p^2 + q^2)\sin\theta}{p\cos\theta + q\sin\theta}$ (4) $\frac{p^2 + q^2\cos\theta}{p\cos\theta + q\sin\theta}$

Sol. **3**



$$\frac{AB}{\sin\theta} = \frac{BD}{\sin(\pi - \theta - \alpha)}$$

$$AB = \frac{BD\sin\theta}{\sin(\theta + \alpha))} = \frac{BD\sin\theta}{\sin\theta\cos\alpha + \cos\theta\sin\alpha}$$

$$= \frac{\sqrt{p^2+q^2}\,\sin\theta}{\sin\theta\frac{q}{\sqrt{p^2+q^2}} + \cos\theta\frac{p}{\sqrt{p^2+q^2}}}$$

$$= \frac{(p^2+q^2)\sin\theta}{q\sin\theta + p\cos\theta}$$

37. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ?

(1) mode (2) Variance
(3) mean (4) median

Sol. **2**

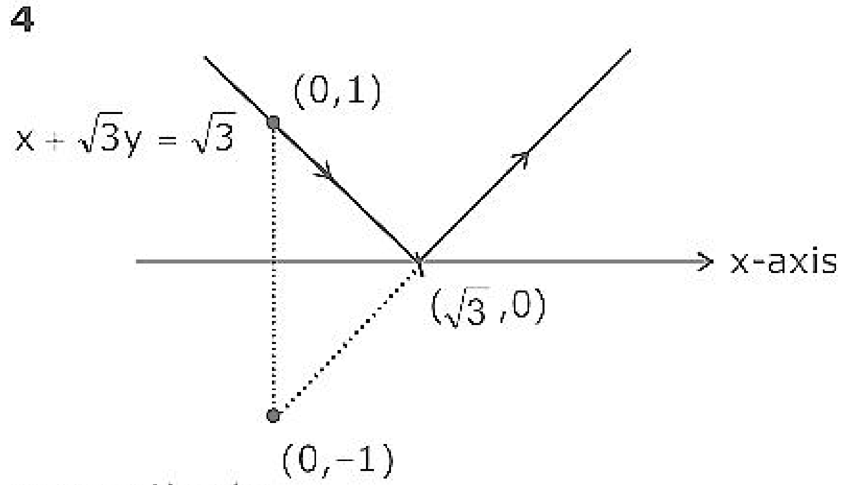
variance does not change

38. A ray of light along $x + \sqrt{3}y = \sqrt{3}$ gets reflected upon reaching x-axis, the equation of the reflected ray is :

(1) $y = \sqrt{3}x - \sqrt{3}$ (2) $\sqrt{3}y = x - 1$

(3) $y = x + \sqrt{3}$ (4) $\sqrt{3}y = x - \sqrt{3}$

Sol. **4**



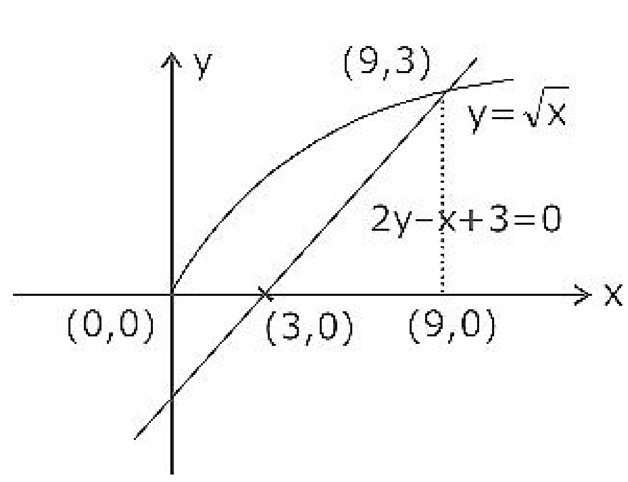
so equation is

$$\frac{x}{\sqrt{3}} + \frac{y}{-1} = 1$$

39. The area (in square units) bounded by the curves $y = \sqrt{x}$, $2y - x + 3 = 0$, x -axis, and lying in the first quadrant is:

(1) 18 (2) $\frac{27}{4}$ (3) 9 (4) 36

Sol. **3**



$y^2 = 2y + 3$
$y^2 - 2y - 3 = 0$
$y = 3 \quad y = -1$

$$\text{Area} = \int_0^9 \sqrt{x}\,dx - \frac{1}{2}.6.3$$

$$= \left(\frac{2x^{3/2}}{3}\right)_0^9 - 9 = \frac{2}{3}.27 - 9$$

$= 9$ sq. units.

40. If z is a complex number of unit modulus and argument θ, then $\arg\left(\frac{1+z}{1+\bar{z}}\right)$ equals.

(1) θ (2) $\pi - \theta$ (3) $-\theta$ (4) $\frac{\pi}{2} - \theta$

Sol. **1**

$|z| = 1$

$$\arg\left(\frac{1+z}{1+\bar{z}}\right) = \arg(1+z)^2$$

$$= 2\arg(1+z) = \theta$$

41. If $\int f(x)dx = \Psi(x)$, then $\int x^5 f(x^3)dx$ is equal to :

(1) $\frac{1}{3}x^3\Psi(x^3) - \int x^2\Psi(x^3)dx + C$

(2) $\frac{1}{3}\left[x^3\Psi(x^3) - \int x^3\Psi(x^3)dx\right] + C$

(3) $\frac{1}{3}\left[x^3\Psi(x^3) - \int x^2\Psi(x^3)dx\right] + C$

(4) $\frac{1}{3}x^3\Psi(x^3) - 3\int x^3\Psi(x^3)dx + C$

Sol. **1**

$$\int x^5 f(x^3)\,dx$$

$x^3 = t \Rightarrow 3x^2\,dx = dt$

$$\int t\ f(t)\frac{dt}{3} = \frac{1}{3}\left[t\int f(t)\,dt - \int \psi(t)\,dt\right]$$

$$= \frac{1}{3}\left[x^3\ \psi\ (x^3) - \int \psi(x^3)\ 3x^2dx\right] + c$$

42. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of A x B having 3 or more elements is :

(1) 219 (2) 211 (3) 256 (4) 220

Sol. **1**

$n(A \times B) = 8$

number of subsets $= {}^8C_3 + {}^8C_4 + \dots + {}^8C_8$

$= 2^8 - {}^8C_0 - {}^8C_1 - {}^8C_2$

$= 256 - 1 - 8 - 28 = 219$

43. If the lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplaner, then k can have :

(1) exactly two values (2) exactly three values

(3) any value (4) exactly one value

Sol. **1**

S.D = 0

$\vec{a}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{a}_2 = \hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{b}_1 = \hat{i} + \hat{j} - (k)\hat{k}$, $\vec{b}_2 = (k)\hat{i} + 2\hat{j} + \hat{k}$

$\Rightarrow (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$

$$\begin{vmatrix} 1 & -1 & -1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = 0$$

$1(1 + 2k) + 1(1 + k^2) - 1(2 - k) = 0$

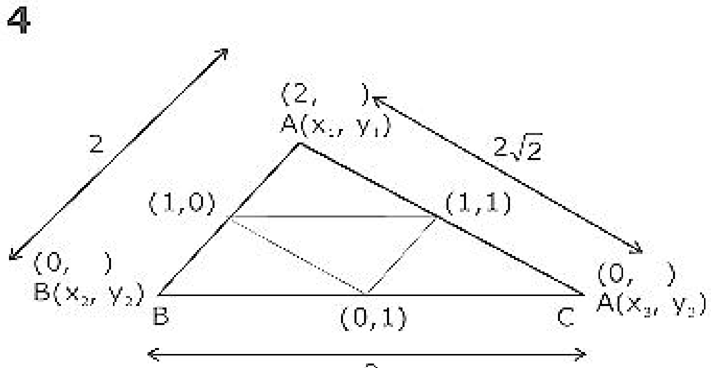
$k^2 + 3k = 0 \Rightarrow k = 0, -3$

44. The x-coordinate of the incentre of the triangle that has the coordinates of mid points of its sides as (0, 1) (1, 1) and (1, 0) is :

(1) $1 + \sqrt{2}$ (2) $1 - \sqrt{2}$

(3) $2 + \sqrt{2}$ (4) $2 - \sqrt{2}$

Sol. **4**



$x_2 + 1 = 1 + 0$

$x_1 + 0 = 2$

$x_3 + 1 = 1$

x-coordinate of incentre

$= \frac{4}{4 + 2\sqrt{2}} = \frac{2}{2 + \sqrt{2}} \times \frac{2 - \sqrt{2}}{2 - \sqrt{2}}$

45. Consider :

Statement - I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement - II : $(p \to q) \leftrightarrow (\sim q \to \sim p)$ is a tautology.

(1) Statement - I is true; Statement - II is false.

(2) Statement - I is false; Statement - II is true.

(3) Statement -I is true; Stetement - II is true; Statement - II is a correct explanation for Statement - I

(4) Statement - I is true; Stetement-II is true; Statement - II is not a correct explanation for Statement -I

Sol. **4**

p	q	$p \to q$	$\sim p$	$\sim q$	$\sim q \to \sim p$	S – II	$p \wedge \sim q$	$\sim p \wedge q$	S – I
T	T	T	F	F	T	T	F	F	F
T	F	F	F	T	F	T	T	F	F
F	T	T	T	F	T	T	F	T	F
F	F	T	F	T	T	T	F	F	F

Both S-I and S-II are true but S-II does not explan S-I

46. If the equations $x^2 + 2x + 3 = 0$ and $ax^2 + bx + c = 0$, a, b, c, $\in$ R, have a common root, then a : b : c is :

(1) 1 : 3 : 2 (2) 3 : 1 : 2

(3) 1 : 2 : 3 (4) 3 : 2 : 1

Sol. **3**

First equation has imaginary roots

$\Rightarrow$ Both roots are common

$\Rightarrow$ a : b : c = 1 : 2 : 3

47. The sum of first 20 terms of the sequence 0.7, 0.77, 0.777, ... is :

(1) $\frac{7}{81}(179 + 10^{-20})$ (2) $\frac{7}{9}(99 + 10^{-20})$

(3) $\frac{7}{81}(179 - 10^{-20})$ (4) $\frac{7}{9}(99 - 10^{-20})$

Sol. **1**

$S = 7\{0.1 + 0.11 + 0.111 + \dots\}$

$= \frac{7}{9}\{0.9 + 0.99 + 0.999 + \dots\}$

$= \frac{7}{9}\{(1 - 0.1) + (1 - 0.01) + (1 - 0.001) + \dots\}$

$$= \frac{7}{9}\left\{20 - \frac{0.1\left(\left(\frac{1}{10}\right)^{20} - 1\right)}{\frac{1}{10} - 1}\right\}$$

$$= \frac{7}{9}\left\{20 - \frac{(-10^{-20} + 1)}{9}\right\} = \frac{7}{81}\{179 + 10^{-20}\}$$

48. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is :

(1) 210 (2) 310 (3) 4 (4) 120

Sol. **1**

$$\left(\frac{(x+1)(x^{1/3}+1)}{(x^{2/3}-x^{1/3}+1)(x^{1/3}+1)}-\frac{(x^{1/2}-1)(x^{1/2}+1)}{x^{1/2}(x^{1/2}-1)}\right)^{10}$$

$$=\left[x^{1/3}+1-\frac{(x^{1/2}+1)}{x^{1/2}}\right]^{10}$$

$$=\left[x^{1/3}+1-(1+x^{-1/2})\right]^{10}=\left[x^{1/3}-x^{-1/2}\right]^{10}$$

$$T_{r+1} = {}^{10}C_r\,(x^{1/3})^r\,(-x^{-1/2})^{10-r}$$

$$= {}^{10}C_r\,(-1)^{10-r}\,x^{r/3}\,.\,x^{\frac{r-10}{2}}$$

$$= {}^{10}C_r\,(-1)^{10-r}\,.\,x^{\frac{r}{3}+\frac{r-10}{2}}$$

for independent term $\Rightarrow \frac{r}{3}+\frac{r-10}{2}=0$

$2r + 3r - 30 = 0$

$r = 6$

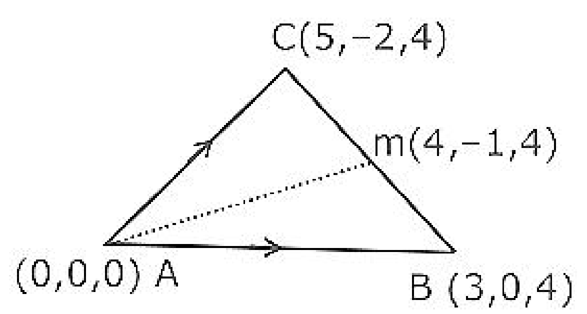
Hence $T_7 = {}^{10}C_6\,(-1)^4$

$$= {}^{10}C_6 = \frac{10.9.8.7}{4.3.2.1} = 210$$

49. If the vectors $\overrightarrow{AB} = 3\hat{i} + 4\hat{k}$ and $\overrightarrow{AC} = 5\hat{i}-2\hat{j}+4\hat{k}$ are the sides of a triangle ABC, then the length of the median through A is :

(1) $\sqrt{33}$ (2) $\sqrt{45}$ (3) $\sqrt{18}$ (4) $\sqrt{72}$

Sol. **1**



$\overrightarrow{AB} = 3\hat{i}+4\hat{k}$

$\overrightarrow{CA} = 5\hat{i}-2\hat{j}+4\hat{k}$

$|\overrightarrow{AM}| = \sqrt{16+1+16} = \sqrt{33}$

50. If x, y, z are in A.P and $\tan^{-1}x$, $\tan^{-1}y$ and $\tan^{-1}z$ are also in A.P., then :

(1) 6x = 3y = 2z (2) 6x = 4y = 3z

(3) x = y = z (4) 2x = 3y = 6z

Sol. **3**

$2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z$

$$\frac{2y}{1-y^2} = \frac{x+z}{1-xz}$$

$\Rightarrow y^2 = xz$ $(\because 2y = x + z)$

$\Rightarrow x = y = z$

51. The intercepts on x-axis made by tangents to the curve, $y=\int_0^x |t|\,dt, x \in R$, which are parallel to the line y = 2x, are equal to :

(1) ±3 (2) ±4 (3) ±1 (4) ±2

Sol. **3**

$\frac{dy}{dx} = x = 2.$ $\int_0^2 t\,dt = \frac{t^2}{2} = 2$

$y - 2 = 2(x - 2)$

$y = 2x - 2$

x – intercept = ± 1

52. Distance between two parallel planes 2x + y + 2z = 8 and 4x + 2y + 5 = 0 is :

(1) $\frac{7}{2}$ (2) $\frac{9}{2}$ (3) $\frac{3}{2}$ (4) $\frac{5}{2}$

Sol. **1**

Distance between parallel planes

$$D = \left|\frac{d_1-d_2}{\sqrt{a^2+b^2+c^2}}\right|$$

$$D = \left|\frac{(-8)-\frac{5}{2}}{\sqrt{4+1+4}}\right|$$

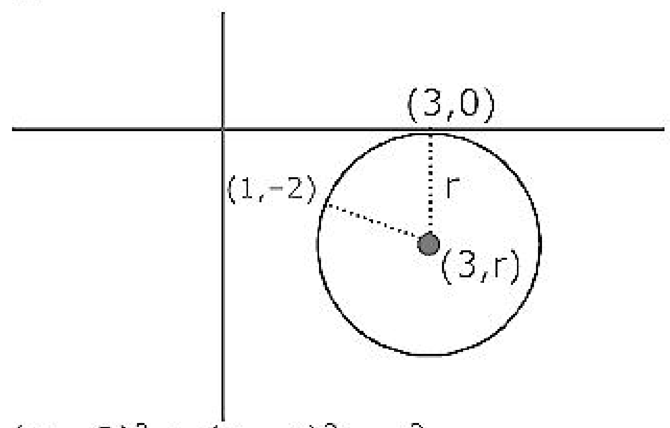
$$D = \left|\frac{-21}{6}\right|$$

$$D = \frac{7}{2}$$

53. The circle passing through (1, –2) and touching the axis of x at (3, 0) also passes through the point :

(1) (5, –2) (2) (–2, 5)
(3) (–5, 2) (4) (2, –5)

Sol. **1**



$(x - 3)^2 + (y - r)^2 = r^2$

passes through (1, –2)

$(1 - 3)^2 + (-2 - r)^2 = r^2$

$4 + 4 + r^2 + 4r = r^2$

$r = -2$

centre : (3, –2)

r = 2

Hence circle : $[(x - 3)^2 + (y + 2)^2 = 4]$

54. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, and having centre at (0, 3) is :

(1) $x^2 + y^2 - 6y - 5 = 0$
(2) $x^2 + y^2 - 6y + 5 = 0$
(3) $x^2 + y^2 - 6y - 7 = 0$
(4) $x^2 + y^2 - 6y + 7 = 0$

Sol. **3**

$9 = 16(1 - e^2) \Rightarrow 1 - e^2 = \frac{9}{16} \Rightarrow e^2 = \frac{7}{16}$

$\Rightarrow e = \frac{\sqrt{7}}{4}$

so foci are $(\pm \sqrt{7}, 0)$

clearly (B) satisfy the point

55. If $y = \sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at x = 1 is equal to:

(1) 1 (2) $\sqrt{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) $\frac{1}{2}$

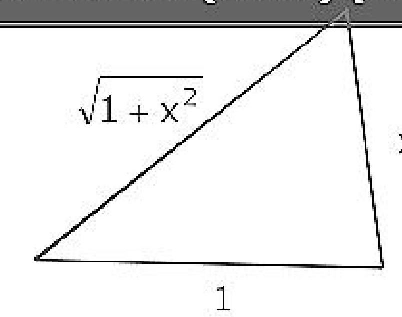
Sol. **3**

$y = \sec(\tan^{-1} x)$

$y = \sqrt{1 + x^2}$

$\frac{dy}{dx} = \frac{x}{\sqrt{1 + x^2}}$

$\left.\frac{dy}{dx}\right|_{at\ x=1} = \frac{1}{\sqrt{2}}$



56. The expression $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ can be written as :

(1) tanA + cotA (2) secA + cosecA
(3) sin A cos A + 1 (4) secA cosecA + 1

Sol. **4**

$\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$

$\frac{\sin^2 A}{\cos A(\sin A - \cos A)} + \frac{\cos^2 A}{\sin A(\cos A - \sin A)}$

$= \frac{\sin^2 A + \cos^2 A + \sin A \cos A}{\sin A \cos A}$

= 1 + secA cosec A

57. Given: A circle, $2x^2 + 2y^2 = 5$ and a parabola, $y^2 = 4\sqrt{5}x$.

Statement - I: An equation of a common tangent to these curves is $y = x + \sqrt{5}$.

Statement - II: If the line, $y = mx + \frac{\sqrt{5}}{m}$ $(m \neq 0)$ is their common tangent, then m satisfies $m^4 - 3m^2 + 2 = 0$,

(1) If Statement-I is true but Statement - II is false.
(2) If Statement-I is false but Statement-II is true.
(3) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.
(4) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I.

Sol. **1**

$y = mx + \frac{\sqrt{5}}{m}$ tangent to parabola $y^2 = 4\sqrt{5}x$

$x^2 + y^2 = \frac{5}{2} \rightarrow$ circle

$\perp^r$ from (0, 0) $= \frac{\sqrt{5}}{\sqrt{2}}$

$$\left|\frac{\sqrt{5}/m}{\sqrt{m^2+1}}\right| = \frac{\sqrt{5}}{\sqrt{2}}$$

$$\frac{5}{m^2(m^2+1)} = \frac{5}{2} \Rightarrow m^4 + m^2 - 2 = 0$$

$t^2 + t - 2 = 0$

$(t + 2)(t - 1) = 0$

$t = -2$ or $t = 1$

$\therefore\ m^2 = 1$

$m = \pm 1$

equation of tangent

$y = x + \sqrt{5}$

$y = -x - \sqrt{5}$

58. A multiple choice examination has 5 questions. Each question has three alternative answers of which exactly one is correct. The probability that a student will get 4 or more correct answers just by guessing is :

(1) $\frac{11}{3^5}$ (2) $\frac{10}{3^5}$ (3) $\frac{17}{3^5}$ (4) $\frac{13}{3^5}$

Sol. **1**

RRRRW or RRRRR

$$\frac{5!}{4!} \times \left(\frac{1}{3}\right)^4 \cdot \frac{2}{3} + \frac{5!}{5!} \cdot \left(\frac{1}{3}\right)^5$$

$$= \frac{10+1}{3^5} = \frac{11}{3^5}$$

59. **Statement - I :** The value of the integral $\int_{\pi/6}^{\pi/3} \frac{dx}{1+\sqrt{\tan x}}$ is equal to $\frac{\pi}{6}$.

Statement - II : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$.

(1) If Statement-I is true but Statement - II is false.
(2) If Statement-I is false but Statement-II is true.
(3) If both Statement - I and Statement - II are true, and Statement - II is the correct explanation of Statement- I.
(4) If both Statement-I and Statement - II are true but Statement - II is not the correct explanation of Statement-I.

Sol. **2**

$$I = \int_{\pi/6}^{\pi/3} \frac{\sqrt{\cos x}\,dx}{\sqrt{\sin x}+\sqrt{\cos x}}$$

$$\Rightarrow 2I = \int_{\pi/6}^{\pi/3} dx = \frac{\pi}{6} \Rightarrow I = \frac{\pi}{12}$$

60. $\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x\tan 4x}$ is equal to :

(1) 1 (2) 2 (3) $-\frac{1}{4}$ (4) $\frac{1}{2}$

Sol. **2**

$$\lim_{x\to 0} \frac{(1-\cos 2x)(3+\cos x)}{x \tan 4x}$$

$$= \lim_{x\to 0} \frac{4x^2\left[\frac{1-\cos 2x}{(2x)^2}\right](3+\cos x)}{4x\left(\frac{\tan 4x}{4x}\right)} = \frac{1}{2} \cdot 4 = 2$$

PART - III [CHEMISTRY]

61. Which of the following represents the correct order of increasing first ionization enthalpy for Ca, Ba, S, Se and Ar ?

(A) Ba < Ca < Se < S < Ar
(B) Ca < Ba < S < Sr < Ar
(C) Ca < S < Ba < Se < Ar
(D) S < Se < Ca < Ba < Ar

Sol. A

Ar highest ionization energy because noble gas. Ba Lowest ionization energy because 6 period & more metallic.

62. A gaseous hydrocarbon given upon combustion 0.72 g of water and 3.08 g of CO_2. The empirical formula of the hydrocarbon is

(A) C_6H_5 (B) C_7H_8 (C) C_2H_4 (D) C_3H_4

Sol. B

$$C_xH_y + O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

$CO_2 = 3.08$ gm

$$CO_2 \text{ (mole)} = \frac{3.08}{44} = 0.07 \text{ mole}$$

H_2O (moles) = 0.72 gm

$$H_2O \text{ (moles)} = \frac{0.72}{18} = 0.04 \text{ moles}$$

moles of C : H
0.07 : 0.08 (C_7H_8)
7 : 8

63. The rate of a reaction doubles when its temperature changes from 300 K to 310 K. Activation energy of such a reaction will be:
($R = 8.314$ JK^{-1} mol^{-1} and log 2 = 0.301)

(A) 58.5 kJ mol^{-1}
(B) 60.5 kJ mol^{-1}
(C) 53.6 kJ mol^{-1}
(D) 48.6 kJ mol^{-1}

Sol. C

$$\log \frac{k_2}{k_1} = \frac{E_a(T_2 - T_1)}{2.303R(T_1T_2)}$$

$$\log 2 = \frac{E_a \times 10}{2.303(8.314)(300)(310)}$$

$E_a = 53.42$ kJ /mole
$E_a = 53.6$ kJ /mole

64. The gas leaked from a storage tank of the Union Carbide plant in Bhopal gas tragedy was:

(A) Ammonia
(B) Phosgene
(C) Methylisocynate
(D) Methylamine

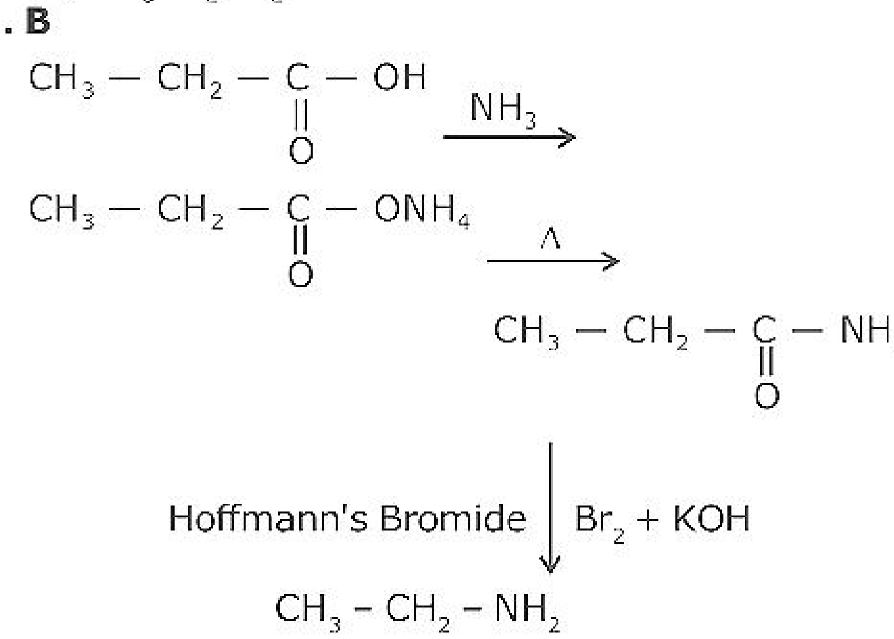
Sol. C

MIC is poisonous gas.

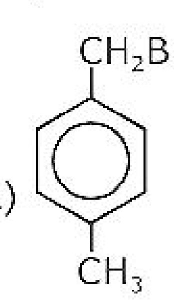
65. An organic compound A upon reacting and NH_3 gives B. On heating B gives C. in presence of KOH reacts is with Br_2 to given $CH_3CH_2NH_2$ A is

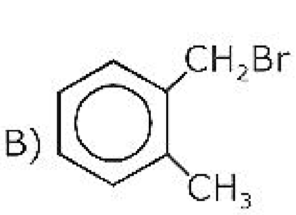
(A) $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-COOH$
(B) CH_3CH_2COOH
(C) CH_3COOH
(D) $CH_3CH_2CH_2COOH$

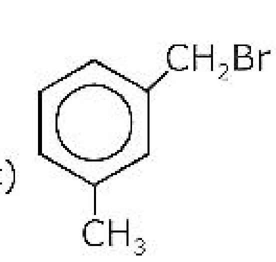
Sol. B

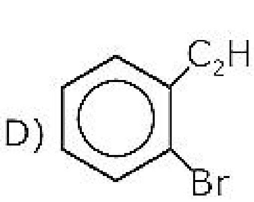


66. Compound (A), C_8H_9Br, gives a white precipitate when warmed with alcoholic $AgNO_3$. Oxidation of (A) gives an acid (B), $C_8H_6O_4$. (B) easily forms anhydride on heating. identify the compound (A).

(A) 

(B) 

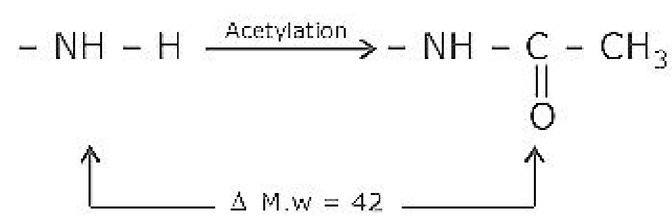
(c) 

(D) 

Sol. B

67. A compound with molecular mass 180 its acrylated with CH_3COCl to get a compound with molecular mass 390. the number of amino groups present per molecular of the former compound is
(A) 4 (B) 6 (C) 2 (D) 5

Sol. D



No. of $-NH_2$ group = $\dfrac{\Delta M.w}{42}$

$\dfrac{390-180}{42} = 5$

68. When one of the following molecules is expected to exhibit diamagnetic behaviour ?
(A) O_2 (B) S_2 (C) C_2 (D) N_2

Sol. C,D
S_2 Paramagnetic two unpaired electrons
C_2 Diamagnetic zero unpaired electrons
N_2 Diamagnetic zero unpaired electrons
O_2 Paramagnetic two unpaired electrons

69. In which of the following pairs of molecules/ions, both the species are not likely to exist?
(A) H_2^{2+}, He_2
(B) H_2^-, He_2^{2-}
(C) H_2^+, He_2^{2-}
(D) H_2^-, He_2^{2-}

Sol. A
Bond order zero mean molecule does not exist

	H_2	$H_2^{\oplus}$	$H_2^{\ominus}$	H_2^{2-}	$H_2^{2\oplus}$	He_2
bond order	1	.5	.5	0	0	0

70. Which of the following complex species is not expected to exhibit optical isomerism?
(A) $[Co(NH_3)_3Cl_3]$
(B) $[Co(en)(NH_3)_2Cl_2]^+$
(C) $[Co(en)_3]^{3+}$
(D) $[Co(en)_2Cl_2]^+$

Sol. A
It is a type of Ma_3b_3 octahedral complex so do not show optical isomerism therefore answer is A.

71. The coagulating power of electrolytes having ions Na^+, Al^{3+} and Ba^{2+} for arsenic sulphide sol increases in the order:
(A) Ba^{2+}, $Na^+ < Al^{3+}$ (B) $Al^{3+} < Na^+ < Ba^{2+}$
(C) $Al^{3+} < Ba^{2+} < Na^+$ (D) $Na^+ < Ba^{2+} < Al^{3+}$

Sol. D
As_2S_3
is negative colloide
Coagulation power α charge (+)
$Na^+ < Ba^{+2} < Al^{+3}$

72. Consider the following reaction
$x\,MnO_4^- + yC_2O_4^{2-} + zH^+ \rightarrow xMn^{2+} + 2yCO_2 + z/2H_2O$
The value of x,y and z in the reaction are respectively.
(A) 2, 5 and 16
(B) 5, 2 and 8
(C) 5, 2 and 16
(D) 2, 5 and 8

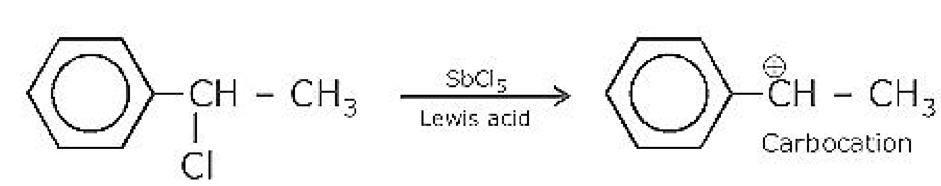
Sol. A
$2MnO_4^- + 5C_2O_4^{-2} + 16H^+ \rightarrow 2Mn^{+2} + 10CO_2 + 8H_2O$

73. Which of the following exists as covalent crystals in the solid state ?
(A) Sulphur (B) Phosphorus
(C) Iodine (D) Silicon

Sol. D

74. A solution of (–) –1–chloro–1–phenylethane in toluene racemizes slowly in the presence of a small amount of $SbCl_5$. due to formation of :
(A) carbocation (B) free radical
(C) carbanion (D) carbene

Sol. A



75. An unknown alcohol is treated with the "Lucas reagent" to determine whether the alcohol is primary, secondary or tertiary. Which alcohol reacts fastest and by what mechanism
(A) secondary alcohol by S_{N2}
(B) tertiary alcohol by S_{N2}
(C) Secondary alcohol by S_{N1}
(D) tertiary alcohol by S_{N1}

Sol. D
Reactivity of alcohol towards Luca's reagent
= 3º > 2º > 1º > CH_3OH [SN^1 Rx^n]
Intermediate is carbocation

76. How many litres of water must be added to litre of an aqueous solution of HCl with a pH of 1 to create an aqueous solution with pH of 2?
(A) 2.0 L (B) 9.0 L (C) 0.1 L (D) 0.9 L

Sol. B
$M_1V_1 = M_2V_2$
$10^{-1} \times 1 = 10^{-2} V_2$
$V_2 = 10$ L
water added = 10L – 1 L = 9 L

77. The molarity of a solution obtained by mixing 750 mL of 0.5(M) HCl with 250 mL of 2(M) HCl will be
(A) 1.75 M (B) 0.975 M
(C) 0.875 M (D) 1 M

Sol. C
$M_1V_1 + M_2V_2 = M_3V_3$
$$\frac{2 \times 250 + 0.5 \times 750}{1000} = M_3$$
$M_3 = 0.875$

78. A piston filled with 0.04 mol of an ideal gas expands reversibly from 50.0 mL to 375 mL at a constant temperature of 37.0°C. As it does so, it absorbs 208 J of heat. The values of q and w for the process will be: (R = 3.314 J/mol K) (Ln 7.5 = 2.01)
(A) q = – 208 J, w = + 208 J
(B) q = + 208 J, w = + 208 J
(C) q = + 208 J, w = – 208 J
(D) q = – 208 J, w = – 208 J

Sol. C
$$W_{rev}, \text{exp.} = - \mid 2.303 \text{ mRT} \log \frac{V_2}{V_1}$$
$$= -2.303\ (0.04)\ (8.3)\ (310) \log \frac{375}{50}$$
= – 208 J
($\Delta E = 0$)
$\Delta E = Q + w.$, $Q = -w = +208$ J

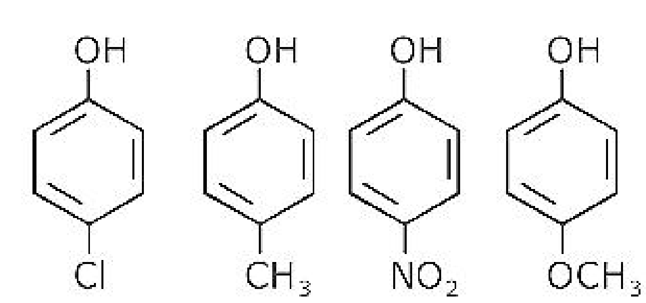
79. Experimentally it was found that a metal oxide has formula $M_{0.98}O$. Metal M, is present as M^{2+} and M^{3+} in its oxide. Fraction of the metal which exists as M^{3+} would be:
(A) 6.05 % (B) 5.08 %
(C) 7.01 % (D) 4.08 %

Sol. D
$M_{0.98}O$
$0.98(x) - 2 = 0$
$$x = \frac{200}{98} = \text{Average O.N.}$$
Let M^{+2} M^{+3}
(100 – y) % y %
$$\frac{2(100 - y) + 3y}{100} = \frac{200}{98} \Rightarrow y = 4.08\ \%$$

80. For gaseous state if most probable speed is denoted C*, average speed by $\bar{C}$ and mean square speed by C, then for a large number of molecules the ratios of these speeds are :
(A) $C^* : \bar{C} : C = 1 : 128 : 1.225$
(B) $C^* : \bar{C} = 1 : 1.225 : 1.128$
(C) $C^* : \bar{C} : C = 1.225 : 1.128 : 1$
(D) $C^* : \bar{C} : C = 1.128 : 1.225 : 1$

Sol. A
$C^* : \bar{C} : C = 1 : 1.128 : 1.225$
$$C_{mps} = \sqrt{\frac{2RT}{M}}$$
$$C_{AS} = \sqrt{\frac{8RT}{\pi M}}$$
$$C_{rms} = \sqrt{\frac{3RT}{M}}$$
$$C^* : \bar{C} : C = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}$$
= 1 : 1.128 : 1.225

81. Arrange the following compound in order of decreasing acidity :



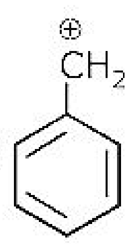
(I) (II) (III) (IV)

(A) III > I > II > IV
(B) IV > III > I > II
(C) II > IV > I > III
(D) I > II > III > IV

Sol. A
$$\text{Order of Acidity } \alpha \frac{-M/-H/-I}{+M/+H/+I}$$
$-Cl \Rightarrow -I > +M$ $-CH_3 \Rightarrow +I < +H$
$-NO_2 \Rightarrow -I < -M$ $-OCH_3 \Rightarrow -I < +M$
III > I > II > IV

82. The order of stability of the following carbocations:

(I) $CH_2=CH-\overset{\oplus}{C}H_2$ (II) $CH_3-CH_2-\overset{\oplus}{C}H_2$

(III) 

is :

(A) I > II > III (B) III > I > II

(C) III > II > I (D) II > III > I

Sol. B

Benzylic > allylic > 1° alkyl

83. The first ionisation potential of Na is 5.1 eV. The value of electron gain enthalpy of Na^+ will be:

(A) – 10.2 eV (B) + 2.55 eV

(C) – 2.55 eV (D) – 5.1 eV

Sol. D

First I.P.

$Na + I\,IP \rightarrow Na^+ + e^-$ $\Delta H = +5.1$ eV

$Na^+ + 1e^- \rightarrow Na$ $\Delta H = -5.1$ eV

(e^- gain enthalpy)

84. Four successive member of the first now transition elements are listed below with atoms number. which one of them is expected to have the highest $E^0_{M^{3+}/M^{2+}}$ value ?

(A) Fe(Z = 26) (B) Co(Z = 27)

(C) Cr(Z = 24) (D) Mn(Z = 25)

Sol. B

Co (Z = 27)	+ 1.80
Cr	–0.407
Mn	+ 1.54
Fe	+ 0.771

85. Stability of the species Li_2, Li_2^- and Li_2^+ increases in the order of

(A) $Li_2 < Li_2^- < Li_2^+$

(B) $Li_2^- < Li_2 < Li_2^+$

(C) $Li_2 < Li_2^+ < Li_2^-$

(D) $Li_2^- < Li_2^+ < Li_2$

Sol. D

$Li < Li^{\oplus} < Li^{\ominus}$

bond order 1 .5 .5

more bond order more stability, $Li^{\ominus}$ is less stable than $Li^{\oplus}$ because it contain more antibonding electron.

86. Energy of an electron is given by

$E = -2.178 \times 10^{-18}\left(\frac{Z^2}{n^2}\right)$. Wavelength of ligh required to excite an electron in an hydrogen atom from level n = 1 to n = 2 will be

($h = 6.62 \times 10^{-34}$ Js and $c = 3.0 \times 10^8\ ms^{-1}$)

(A) 6.500×10^{-7} m (B) 8.500×10^{-7} m

(C) 1.214×10^{-7} m (D) 2.816×10^{-7} m

Sol. C

$$E_2 - E_1 = \frac{hc}{\lambda}$$

$$+2.178 \times 10^{-8}\left(1-\frac{1}{4}\right) = \frac{6.6\times10^{-34}\times3\times10^{8}}{\lambda}$$

$$= \frac{6.6\times4}{2.178} \times 10^{-8}$$

$= 12.12 \times 10^{-8}$

$= 1.212 \times 10^{-7}$ m

87. Synthesis of each molecule of glucose in photosynthesis involves:

(A) 8 molecules of ATP

(B) 6 molecules of ATP

(C) 18 molecules of ATP

(D) 10 molecules of ATP

Sol. C

Per CO_2 require 3 molecule of ATP.

1 molecule of Glucose

gives 6 CO_2 so Ans is 18 ATP.

88. Which of the following is the wrong statement ?

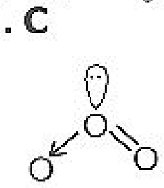
(A) Ozone is violet black in solid state

(B) Ozone is diamagnetic gas

(C) ONCl and ONO^- are not isoelectron

(D) O_3 molecule is bent

Sol. C



sp^2 bent molecule

diamagnetic

* ONCl & $ONO^{\ominus}$ are isoelectronic because total valence electron is same

6 + 5 + 7 = 18 6 + 5 + 6 + 1 = 18

89. Which of the following arrangements does not represent the correct order of the property stated against it?

(A) $Co^{3+} < Fe^{3+} < Cr^{3+} < Sc^{3+}$ stability in aqueous solution.

(B) Sc < Ti < Cr < Mn : number of oxidation states.

(C) $V^{2+} < Cr^{2+} < Mn^{2+} < Fe^{2+}$: Paramagnetic behaviour.

(D) $Ni^{2+} < Co^{2+} < Fe^{2+} < Mn^{2+}$: ionic size

Sol. C

More the unpaired electron more paramagnetic

magnetic moment = $\sqrt{n(n+2)}$

n = no of unpaired electron

	unpaired electron
V^{+2}	3
Cr^{+2}	4
Mn^{+2}	5
Fe^{+2}	4

90. Given

$E^0_{Cr^{3+}/Cr} = -0.74V$; $E^0_{MnO_4^-/Mn^{2+}} = 1.51V$

$E^0_{Cr_2O_7^{2-}/Cr^{3+}} = 1.33V$; $E^0_{Cl/Cl} = 1.36V$

Based on the data given above, strongest oxidisting agent will be

(A) Mn^{2+} (B) MnO_4^-

(C) Cl^- (D) Cr^{3+}

Sol. B

Oxidising power $\times$ E^o_R self reduction

$MnO_4^-/Mn^{+2} = 1.51$

MnO_4^-