1.Show that the line joining the origin to the point (2, 1, 1) is perpendicular to the line determined by the points $(3, 5, -1), (4, 3, -1)$.

Ans. We know that direction ratios of the line joining the origin (0, 0, 0) to the point are

$x_2 - x_1, y_2 - y_1, z_2 - z_1$ = 2 – 0, 1 – 0, 1 – 0 = 2, 1, 1 = a_1, b_1, c_1

Similarly, direction ratios of the line joining the points $(3, 5, -1)$ and $(4, 3, -1)$ are

$x_2 - x_1, y_2 - y_1, z_2 - z_1 = 4-3, 3-5, -1-(-1) = 1, -2, 0 = a_2, b_2, c_2$

For these two lines, $a_1a_2 + b_1b_2 + c_1c_2$

$= 2(1)+1(-2)+1(0)$ = 2 – 2 + 0 = 0

Therefore, the two given lines are perpendicular to each other.

2.If l_1, m_1, n_1 and l_2, m_2, n_2 are the direction cosines of two mutually perpendicular lines, show that the direction cosines of the line perpendicular to both of these are $m_1n_2 - m_2n_1,\ n_1l_2 - n_2l_1, l_1m_2 - l_2m_1$.

Ans. l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of two mutually perpendicular of two given lines L_1 and L_2. (say)

Let $\widehat{n_1}$ and $\widehat{n_2}$ be the unit vectors along these lines L_1 and L_2.

$\therefore\ \overrightarrow{n_1} = l_1\hat{i} + m_1\hat{j} + n_1\hat{k}$ and $\overrightarrow{n_2} = l_2\hat{i} + m_2\hat{j} + n_2\hat{k}$

Let L be the line perpendicular to both the lines L_1 and L_2 and let $\hat{n}$ be a unit vector along line L perpendicular both lines L_1 and L_2.

$\therefore$ Cross-product of two vectors = $\widehat{n_1} \times \widehat{n_2} = \left|\widehat{n_1}\right| . \left|\widehat{n_2}\right| \sin 90^\circ \hat{n}$

[$\because$ L1 $\perp$ L2 (given, $\therefore$ angle between them is 90°]

$\Rightarrow \widehat{n_1} \times \widehat{n_2} = \hat{n}$

$$\Rightarrow \hat{n} = \widehat{n_1} \times \widehat{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$$

$$\Rightarrow \hat{n} = (m_1n_2 - m_2n_1)\hat{i} - (l_1n_2 - l_2n_1)\hat{j} + (l_1m_2 - l_2m_1)\hat{k}$$

Since, $\hat{n}$ is a unit vector, therefore its components are its direction cosines.

Thus, direction cosines of $\hat{n}$ are $m_1n_2 - m_2n_1, l_1n_2 - l_2n_1, l_1m_2 - l_2m_1$

$\Rightarrow$ direction cosines of line L are $m_1n_2 - m_2n_1, l_1n_2 - l_2n_1, l_1m_2 - l_2m_1$

3.Find the angle between the lines whose direction ratios are a, b, c and $b-c, c-a, a-b$.

Ans. Direction ratios of one line are a, b, c

$\Rightarrow$ A vector along this line is $\vec{b_1} = a\hat{i} + b\hat{j} + c\hat{k}$

Direction ratios of second line are $b-c, c-a, a-b$

$\Rightarrow$ A vector along second line is $\vec{b_2} = (b-c)\hat{i} + (c-a)\hat{j} + (a-b)\hat{k}$

Let θ be the angle between the two lines, then

$$\cos\theta = \frac{\left|\vec{b_1}.\vec{b_2}\right|}{\left|\vec{b_1}\right|.\left|\vec{b_2}\right|} = \frac{a(b-c)+b(c-a)+c(a-b)}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}}$$

$$= \frac{ab-ac+bc-ab+ac-bc}{\sqrt{a^2+b^2+c^2}\sqrt{(b-c)^2+(c-a)^2+(a-b)^2}} = 0 = \cos 90^\circ$$

$$\Rightarrow \theta = 90^\circ$$

4.Find the equation of the line parallel to $x-$ axis and passing through the origin.

Ans. We know that a unit vector along $x-$ axis is $\hat{i} = \hat{i} + 0\hat{j} + 0\hat{k}$

$\therefore$ Direction cosines of $x-$ axis are coefficients of $\hat{i}, \hat{j}, \hat{k}$ in the unit vector

i.e., 1, 0, 0 = l, m, n

$\therefore$ Equation of the required line passing through the origin (0, 0, 0) and parallel to $x-$ axis is

$$\frac{x-0}{1} = \frac{y-0}{0} = \frac{z-0}{0} \Rightarrow \frac{x}{1} = \frac{y}{0} = \frac{z}{0}$$

Vector equation of the required line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\Rightarrow \vec{r} = \vec{0} + \lambda\hat{i}$ [$\vec{a} = \vec{0}$ and $\vec{b} = \hat{i}$]

$\Rightarrow \vec{r} = \lambda\hat{i}$

5.If the coordinates of the points A, B, C, D be (1, 2, 3), (4, 5, 7), $(-4, 3, -6)$ and (2, 9, 2) respectively, then find the angle between the lines AB and CD.

Ans. Given: Points A$(1, 2, 3)$, B$(4, 5, 7)$, C$(-4, 3, -6)$ and D$(2, 9, 2)$.

$\therefore$ Direction ratios of line AB are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$\Rightarrow 4-1, 5-2, 7-3 = 3, 3, 4 = a_1, b_1, c_1$

$\therefore$ A vector along the line AB is $\vec{b_1} = 3\hat{i} + 3\hat{j} + 4\hat{k}$

Similarly, direction ratios of line CD are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$\Rightarrow 2-(-4), 9-3, 2-(-6) = 6, 6, 8 = a_2, b_2, c_2$

$\therefore$ A vector along the line AB is $\vec{b_2} = 6\hat{i} + 6\hat{j} + 8\hat{k}$

Let θ be the angle between the two lines, then

$\cos\theta = \frac{|\vec{b_1}.\vec{b_2}|}{|\vec{b_1}|.|\vec{b_2}|}$ = $\frac{|3(6)+3(6)+4(8)|}{\sqrt{9+9+16}\sqrt{36+36+64}}$ = $\frac{|18+18+32|}{\sqrt{34}\sqrt{136}}$ = $\frac{68}{\sqrt{34\times34\times4}}$ = $\frac{68}{34\times2}$ = 1

= $\cos 0^\circ$

$\Rightarrow \theta = 0^\circ$

Therefore, lines AB and CD are parallel.

6.If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$ are perpendicular, find the value of k.

Ans. Given: Equation of one line is $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$

Direction ratios of this line are its denominators, i.e., $-3, 2k, 2$ = a_1, b_1, c_1

$\therefore$ A vector along this line is $\vec{b_1} = -3\hat{i} + 2k\hat{j} + 2\hat{k}$

Again, equation of second line is $\frac{x-1}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$

Direction ratios of this line are its denominators, i.e., $3k, 1, -5$ = a_2, b_2, c_2

$\therefore$ A vector along this line is $\vec{b_2} = 3k\hat{i} + \hat{j} - 5\hat{k}$

Since these given lines are perpendicular.

$\therefore \vec{b_1}.\vec{b_2} = a_1a_2 + b_1b_2 + c_1c_2 = 0$

$\Rightarrow (-3)(3k) + (2k)(1) + 2(-5) = 0 \Rightarrow -9k + 2k - 10 = 0$

$\Rightarrow -7k = 10 \Rightarrow k = \frac{-10}{7}$

7.Find the vector equation of the line passing through (1, 2, 3) and perpendicular to the plane $\vec{r}.(\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$.

Ans. The required line passes through the point P (1, 2, 3).

$\therefore$ Position vector $\vec{a}$ (say) of point P is (1, 2, 3)

$\Rightarrow \vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

Equation of the given plane is $\vec{r}.(\hat{i} + 2\hat{j} - 5\hat{k}) + 9 = 0$

$\Rightarrow \vec{r}.(\hat{i} + 2\hat{j} - 5\hat{k}) = -9$

Comparing with $\vec{r}\vec{n} = \vec{d}$, $\vec{n} = \hat{i} + 2\hat{j} - 5\hat{k}$

Since, the required line is perpendicular to the given plane, therefore, vector $\vec{b}$ along the required line is $\vec{b} = \vec{n} = \hat{i} + 2\hat{j} - 5\hat{k}$

$\because$ Equation of the required line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$\therefore \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} + \lambda(\hat{i} + 2\hat{j} - 5\hat{k})$

8.Find the equation of the plane passing through (a, b, c) and parallel to the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=2$.

Ans. Equation of any plane parallel to the plane $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=2$ is $\vec{r}.(\hat{i}+\hat{j}+\hat{k})=\lambda$ .....(i)

Plane (i) passes through (a, b, c)

$\therefore$ Putting $\vec{r}=(a,b,c)=a\hat{i}+b\hat{j}+c\hat{k}$ in eq. (i), we get

$(a\hat{i}+b\hat{j}+c\hat{k}).(\hat{i}+\hat{j}+\hat{k})=\lambda$

$\Rightarrow a(1)+b(1)+c(1)=\lambda \Rightarrow \lambda=a+b+c$

Putting the value of λ in eq. (i), to get the required plane is

$\vec{r}.(\hat{i}+\hat{j}+\hat{k})=a+b+c$

9.Find the shortest distance between lines $\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$ and $\vec{r}=4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$.

Ans. Given: Vector equation of one line is $\vec{r}=6\hat{i}+2\hat{j}+2\hat{k}+\lambda(\hat{i}-2\hat{j}+2\hat{k})$

Comparing with $\vec{r}=\vec{a_1}+\lambda\vec{b_1}$, we get

$\vec{a_1}=6\hat{i}+2\hat{j}+2\hat{k}$ and $\vec{b_1}=\hat{i}-2\hat{j}+2\hat{k}$

Again given: Vector equation of another line is $\vec{r}=-4\hat{i}-\hat{k}+\mu(3\hat{i}-2\hat{j}-2\hat{k})$

Comparing with $\vec{r}=\vec{a_2}+\mu\vec{b_2}$, we get

$\vec{a_2}=-4\hat{i}-\hat{k}$ and $\vec{b_2}=3\hat{i}-2\hat{j}-2\hat{k}$

We know that length of shortest distance between two (skew) lines is $\frac{\left|\left(\overrightarrow{a_2}-\overrightarrow{a_1}\right).\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)\right|}{\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|}$ ..

(i)

Now $\overrightarrow{a_2}-\overrightarrow{a_1}=-4\hat{i}-\hat{k}-\left(6\hat{i}+2\hat{j}+2\hat{k}\right)$ = $-4\hat{i}-\hat{k}-6\hat{i}-2\hat{j}-2\hat{k}$ = $-10\hat{i}-2\hat{j}-3\hat{k}$

Again $\overrightarrow{b_1}\times\overrightarrow{b_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 2\\ 3 & -2 & -2\end{vmatrix}$

Expanding along first row,

= $\hat{i}(4+4)-\hat{j}(-2-6)+\hat{k}(-2+6)=8\hat{i}+8\hat{j}+4\hat{k}$

$\therefore\ \left(\overrightarrow{a_2}-\overrightarrow{a_1}\right).\left(\overrightarrow{b_1}\times\overrightarrow{b_2}\right)=(-10)(8)+(-2)(8)+(-3)4$

= $-80-16-12=-108$

And $\left|\overrightarrow{b_1}\times\overrightarrow{b_2}\right|=\sqrt{(8)^2+(8)^2+(4)^2}$ = $\sqrt{64+64+16}=\sqrt{144}=12$

Putting these values in eq. (i), length of shortest distance = $\frac{|-108|}{12}=\frac{108}{12}=9$

10.Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the YZ-plane.

Ans. Given: A line through the points A (5, 1, 6) and B (3, 4, 1)

$\therefore$ Direction ratios of this line AB are $x_2-x_1, y_2-y_1, z_2-z_1$

$\Rightarrow$ 3 – 5, 4 – 1, 1 – 6 $\Rightarrow -2, 3-5=a, b, c$

$\therefore$ Equation of the line AB is $\frac{x-x_1}{a}=\frac{y-y_1}{b}=\frac{z-z_1}{c}$

$$\Rightarrow \frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} \quad \text{..........(i)}$$

Now we have to find the coordinates of the point where this line AB crosses the YZ-plane

i.e., $x = 0$ ....(ii)

Putting $x = 0$ in eq. (i), we get

$$\frac{-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} \Rightarrow \frac{y-1}{3} = \frac{5}{2} \text{ and } \frac{z-6}{-5} = \frac{5}{2}$$

$$\Rightarrow 2y - 2 = 15 \text{ and } 2z - 12 = -25 \Rightarrow 2y = 17 \text{ and } 2z = -13$$

$$\Rightarrow y = \frac{17}{2} \text{ and } z = \frac{-13}{2}$$

Thus, required point is P $\left(0, \frac{17}{2}, \frac{-13}{2}\right)$

11.Find the coordinates of the point where the line through (5, 1, 6) and (3, 4, 1) crosses the ZX-plane.

Ans. Given: A line through the points A (5, 1, 6) and B (3, 4, 1)

$\therefore$ Direction ratios of this line AB are $x_2 - x_1, y_2 - y_1, z_2 - z_1$

$\Rightarrow$ 3 – 5, 4 – 1, 1 – 6 $\Rightarrow -2, 3, -5 = a, b, c$

$\therefore$ Equation of the line AB is $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

$$\Rightarrow \frac{x-5}{-2} = \frac{y-1}{3} = \frac{z-6}{-5} \quad \text{..........(i)}$$

Now we have to find the coordinates of the point where this line AB crosses the ZX-plane

i.e., $y = 0$ ....(ii)

Putting $y=0$ in eq. (i), we get

$$\frac{x-5}{-2}=\frac{-1}{3}=\frac{z-6}{-5} \Rightarrow \frac{x-5}{-2}=\frac{-1}{3} \text{ and } \frac{z-6}{-5}=\frac{-1}{3}$$

$$\Rightarrow 3x-15=2 \text{ and } 3z-18=5 \Rightarrow 3x=17 \text{ and } 3z=23$$

$$\Rightarrow x=\frac{17}{3} \text{ and } z=\frac{23}{3}$$

Thus, required point is P$\left(\frac{17}{3}, 0, \frac{23}{3}\right)$

12.Find the coordinates of the point where the line through $(3,-4,-5)$ and $(2,-3,1)$ crosses the plane $2x+y+z=7$.

Ans. Direction ratios of the line joining the points A$(3,-4,-5)$ and B$(2,-3,1)$ are

$$2-3, -3-(-4), 1-(-5) \Rightarrow -1, 1, 6$$

$\therefore$ Equation of the line AB are $\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}$ ....(i)

Equation of the plane is $2x+y+z=7$ ....(ii)

Now to find the point where line (i) crosses plane (ii),

From eq. (i) $\frac{x-3}{-1}=\frac{y+4}{1}=\frac{z+5}{6}=\lambda$ (say)

$$\Rightarrow x-3=-\lambda,\ y+4=\lambda,\ z+5=6\lambda$$

$\Rightarrow x=3-\lambda,\ y=-4+\lambda,\ z=-5+6\lambda$ ....(iii)

Putting the values of x, y, z in eq. (ii), we get

$$2(3-\lambda)+(-4+\lambda)+(-5+6\lambda)=7$$

$\Rightarrow 6-2\lambda-4+\lambda-5+6\lambda=7 \Rightarrow 5\lambda=10 \Rightarrow \lambda=2$

Putting $\lambda=2$ in eq. (iii), point of intersection of line (i) and plane (ii) is

$x=3-2=1,\ y=-4+2=-2,\ z=-5+12=7$

Thus, required point of intersection is $(1,-2,7)$.

13.Find the equation of the plane passing through the point $(-1,3,2)$ and perpendicular to each of the planes $x+2y+3z=5$ and $3x+3y+z=0$.

Ans. Since equation of any plane through the point $(-1,3,2)$ is

$a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$\therefore\ a(x+1)+b(y-3)+c(z-2)=0$ ....(i)

$\Rightarrow\ ax+a+by-3b+cz-2c=0 \Rightarrow ax+by+cz=-a+3b+2c$

This required plane is perpendicular to the plane $x+2y+3z=5$ $(a_1a_2+b_1b_2+c_1c_2=0)$

$\therefore$ Product of coefficients $\Rightarrow a(1)+b(2)+c(3)=0$ ....(ii)

Again the required plane is perpendicular to the plane $3x+3y+z=0$

$\therefore$ Product of coefficients $\Rightarrow a(3)+b(3)+c(1)=0$ ....(iii)

Solving eq. (ii) and (iii), we get

$$\frac{a}{2-9}=\frac{b}{9-1}=\frac{c}{3-6}$$

$$\Rightarrow \frac{a}{-7}=\frac{b}{8}=\frac{c}{-3}$$

Putting these values of a,b,c in eq. (i), we get

$-7(x+1)+8(y-3)-3(z-2)=0$

$\Rightarrow -7x-7+8y-24-3z+6=0$

$\Rightarrow -7x+8y-3z-25=0$

$\Rightarrow 7x-8y+3z+25=0$

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14. If the points $(1,1,p)$ and $(-3,0,1)$ be equidistant from the plane $\vec{r}.(3\hat{i}+4\hat{j}-12\hat{k})+13=0$, then find the value of p.

Ans. Equation of the given plane is $\vec{r}.(3\hat{i}+4\hat{i}-12\hat{k})+13=0$

$\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k}).(3\hat{i}+4\hat{j}-12\hat{k})+13=0$

[$\because$ $\vec{r}$ = Position vector of any point (x,y,z) on the plane $x\hat{i}+y\hat{j}+z\hat{k}$]

$\Rightarrow 3x+4y-12z+13=0$ ....(i)

Also, the point $(1,1,p)$ and $(-3,0,1)$ are equidistant from plane (i)

$\Rightarrow$(Perpendicular) distance of point $(1,1,p)$ from plane (i)

= Distance of point $(-3,0,1)$ from plane (i)

$$\Rightarrow \frac{|3(1)+4(1)-12(p)+13|}{\sqrt{9+16+144}}=\frac{|3(-3)+4(0)-12(1)+13|}{\sqrt{9+16+144}}$$

$$\Rightarrow \frac{|3+4-12p+13|}{13}=\frac{|-9-12+13|}{13}$$

$\Rightarrow |20-12p|=|-8|$

$\Rightarrow 20-12p=\pm 8$ [$\because$ If $|x|=a, a\geq 0,$ then $x=\pm a$]

Taking positive sign, $20-12p=8$

$\Rightarrow -12p = -12$

$\Rightarrow p = 1$

Taking negative sign, $20 - 12p = -8$

$\Rightarrow -12p = -28$

$\Rightarrow p = \frac{-28}{-12} = \frac{7}{3}$

Hence, the values of p are 1 or $\frac{7}{3}$.

15.Find the equation of the plane passing through the line of intersection of the planes $\vec{r}.(\hat{i}+\hat{j}+\hat{k}) = 1$ and $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$ and parallel to $x-$ axis.

Ans. Equation of one plane is $\vec{r}.(\hat{i}+\hat{j}+\hat{k}) = 1$

$\Rightarrow \vec{r}.(\hat{i}+\hat{j}+\hat{k}) - 1 = 0$ ....(i)

Equation of the second plane is $\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4=0$ ....(ii)

Since, equation of any plane passing through the line intersection of these two planes is

L.H.S. of I + λ (L.H.S. of II) = 0

$\Rightarrow \vec{r}.(\hat{i}+\hat{j}+\hat{k}) - 1 + \lambda\left[\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4\right] = 0$

$\Rightarrow \vec{r}.(\hat{i}+\hat{j}+\hat{k}) - 1 + \lambda\vec{r}.(2\hat{i}+3\hat{j}-\hat{k})+4\lambda = 0$

$\Rightarrow \vec{r}.\left[\hat{i}+\hat{j}+\hat{k}+\lambda.\left(2\hat{i}+3\hat{j}-\hat{k}\right)\right] - 1 + 4\lambda = 0$

$\Rightarrow \vec{r}.\left[\hat{i}+\hat{j}+\hat{k}+2\lambda\hat{i}+3\lambda\hat{j}-\lambda\hat{k}\right] = 1 - 4\lambda$

$$\Rightarrow \vec{r}.\left[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}\right]=1-4\lambda \text{ ....(i)}$$

Comparing $\vec{r}.\vec{n}=d$, we have

$$\vec{n}=(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1-\lambda)\hat{k}$$

Now required plane (i) is parallel to $x-$ axis ($\Rightarrow$ a vector $\vec{b}$ along $x-$axis is $\vec{b}=\hat{i}$ = $\hat{i}+0\hat{j}+0\hat{k}$) $\vec{b}.\vec{n}=0$

$$(1+2\lambda)(1)+(1+3\lambda)(0)+(1-\lambda)(0)=0$$

$$\Rightarrow 1+2\lambda=0 \Rightarrow 2\lambda=-1 \Rightarrow \lambda=\frac{-1}{2}$$

Putting $\lambda=\frac{-1}{2}$ in eq. (i), the equation of required plane,

$$\vec{r}.\left[\left(1+2.\frac{-1}{2}\right)\hat{i}+\left(1+3.\frac{-1}{2}\right)\hat{j}+\left(1-\frac{-1}{2}\right)\hat{k}\right]=1-4.\frac{-1}{2}$$

$$\Rightarrow \vec{r}.\left[(1-1)\hat{i}+\left(1-\frac{3}{2}\right)\hat{j}+\left(1+\frac{1}{2}\right)\hat{k}\right]=1+2$$

$$\Rightarrow \vec{r}.\left[\frac{-1}{2}\hat{j}+\frac{3}{2}\hat{k}\right]=3$$

$$\Rightarrow \vec{r}.\left(-\hat{j}+3\hat{k}\right)=6$$

$$\Rightarrow \left(x\hat{i}+y\hat{j}+z\hat{k}\right).\left(0\hat{i}-\hat{j}+3\hat{k}\right)=6$$

$$\Rightarrow -y+3z=6$$

$$\Rightarrow -y+3z-6=0$$

$$\Rightarrow y-3z+6=0$$

16.If O be the origin and the coordinates of P be $(1, 2, -3)$, then find the equation of the plane passing through P and perpendicular to OP.

Ans. Given: Origin O (0, 0, 0) and point P $(1, 2 -3)$

To find: Equation of the plane passing through P $(1, 2 -3)$ = (x_1, y_1, z_1)

$\therefore$ Direction ratios of normal OP to the plane are $1-0, 2-0, -3-0$

$\Rightarrow 1, 2, -3 = (a, b, c)$

$\therefore$ Equation of the required plane is $a(x-x_1)+b(y-y_1)+c(z-z_1)=0$

$\Rightarrow 1(x-1)+2(y-2)-3(z+3)=0$

$\Rightarrow x-1+2y-4-3z-9=0$

$\Rightarrow x+2y-3z-14=0$

17.Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0$, $\vec{r}.(2\hat{i}+\hat{j}-\hat{k})+5=0$ **and which is perpendicular to the plane** $\vec{r}.(5\hat{i}+3\hat{j}-6\hat{k})+8=0$.

Ans. Equation of any plane passing through (or containing) the line of intersection of the planes $\vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4=0$ and $\vec{r}\left(2\hat{i}+\hat{j}-\hat{k}\right)+5=0$ is L.H.S. of I + λ (L.H.S. of II) = 0

$\Rightarrow \vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4+\lambda\left[\vec{r}.(2\hat{i}+\hat{j}-\hat{k})+5\right]=0$

$\Rightarrow \vec{r}.(\hat{i}+2\hat{j}+3\hat{k})-4+\lambda\vec{r}.(2\hat{i}+\hat{j}-\hat{k})+5\lambda=0$

$\Rightarrow \vec{r}.\left[\hat{i}+2\hat{j}+3\hat{k}+\lambda(2\hat{i}+\hat{j}-\hat{k})\right]-4+5\lambda=0$

$$\Rightarrow \vec{r}.\left(\hat{i}+2\hat{j}+3\hat{k}+2\lambda\hat{i}+\lambda\hat{j}-\lambda\hat{k}\right)=4-5\lambda$$

$$\Rightarrow \vec{r}.\left[(1+2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3-\lambda)\hat{k}\right]=4-5\lambda \quad \text{..........(i)}$$

Comparing with $\vec{r}.\vec{n_1}=d_1$ we have, $\vec{n_1}=(1+2\lambda)\hat{i}+(2+\lambda)\hat{j}+(3-\lambda)\hat{k}$

Now plane (i) is perpendicular to the given plane $\vec{r}.\left(5\hat{i}+3\hat{j}-6\hat{k}\right)+8=0$

$$\Rightarrow \vec{r}\left(5\hat{i}+3\hat{j}-6\hat{k}\right)=-8$$

Comparing with $\vec{r}.\vec{n_2}=d_2$ we have, $\vec{n_2}=5\hat{i}+3\hat{j}-6\hat{k}$

For perpendicular planes $\vec{n_1}.\vec{n_2}=0$

$$\Rightarrow (1+2\lambda)5+(2+\lambda)3+(3-\lambda)(-6)=0$$

$$\Rightarrow 5+10\lambda+6+3\lambda-18+6\lambda=0$$

$$\Rightarrow 19\lambda-7=0$$

$$\Rightarrow 19\lambda=7$$

$$\Rightarrow \lambda=\frac{7}{19}$$

Putting $\lambda=\frac{7}{19}$ in eq. (i), equation of required plane is

$$\vec{r}.\left[\left(1+2.\frac{7}{19}\right)\hat{i}+\left(2+\frac{7}{19}\right)\hat{j}+\left(3-\frac{7}{19}\right)\hat{k}\right]=4-5.\frac{7}{19}$$

$$\Rightarrow \vec{r}.\left[\left(1+\frac{14}{19}\right)\hat{i}+\left(2+\frac{7}{19}\right)\hat{j}+\left(3-\frac{7}{19}\right)\hat{k}\right]=4-\frac{35}{19}$$

$$\Rightarrow \vec{r}.\left(33\hat{i}+45\hat{j}+50\hat{k}\right)=41$$

$\Rightarrow \left(x\hat{i}+y\hat{j}+z\right).\left(33\hat{i}+45\hat{j}+50\hat{k}\right)=41$

$\Rightarrow 33x+45y+50z=41$

18.Find the distance of the point $(-1,-5,-10)$ from the point of intersection of the line $\vec{r}=2\hat{i}-\hat{i}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ and the plane $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5.$

Ans. Given: A point P (say) $(-1,-5,-10)$

and equation of the line $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$ ....(i)

equation of the plane is $\vec{r}.\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

Putting the value of $\vec{r}$ from eq. (i) in eq. (ii),

$\left[\left(2\hat{i}-\hat{j}+2\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right)\right].\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

$\Rightarrow \left(2\hat{i}-\hat{j}+2\hat{k}\right).\left(\hat{i}-\hat{j}+\hat{k}\right)+\lambda\left(3\hat{i}+4\hat{j}+2\hat{k}\right).\left(\hat{i}-\hat{j}+\hat{k}\right)=5$

$\Rightarrow 2+1+2+\lambda(3-4+2)=5$

$\Rightarrow 5+\lambda=5$

$\Rightarrow \lambda=0$

Putting $\lambda=0$ in eq. (i), $\vec{r}=2\hat{i}-\hat{j}+2\hat{k}+0\left(3\hat{i}+4\hat{j}+2\hat{k}\right)$

$\Rightarrow \vec{r}=2\hat{i}-\hat{j}+2\hat{k}$

Therefore, Point of intersection is (x,y,x) = $(-2,1,2)$

$\therefore$ Distance of the given point P$(-1,-5,-10)$ from the point of intersection is

$$\sqrt{(2+1)^2+(-1+5)^2+(2+10)^2}$$

= $\sqrt{9+16+144}$

= $\sqrt{169}=13$ units

19.Find the vector equation of the line passing through $(1,2,3)$ and parallel to the plane $\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$ and $\vec{r}.\left(3\hat{i}+\hat{j}+\hat{k}\right)=6.$

Ans. The required line passes through the point A (1, 2, 3) = $\vec{a}$

$\therefore\ \vec{a}$ = Position vector of point A = $\hat{i}+2\hat{j}+3\hat{k}$

Let $\vec{b}$ be any vector along the required line.

$\therefore$ Vector equation of required line is $\vec{r}=\vec{a}+\lambda\vec{b}$

$\Rightarrow\ \vec{r}=\hat{i}+2\hat{j}+3\hat{k}+\lambda\vec{b}$ ……….(i)

Since required line is parallel to the plane $\vec{r}.\left(\hat{i}-\hat{j}+2\hat{k}\right)=5$

$\therefore\ \vec{b}.\overrightarrow{n_1}=0$ and $\vec{b}.\overrightarrow{n_2}=0$

Comparing with $\vec{r}.\overrightarrow{n_1}=d_1$ we have, $\overrightarrow{n_1}=\hat{i}-\hat{j}+2\hat{k}$

And Comparing with $\vec{r}.\overrightarrow{n_2}=d_2$ we have, $\overrightarrow{n_2}=3\hat{i}+\hat{j}+\hat{k}$

Since $\vec{b}$ is perpendicular to both $\overrightarrow{n_1}$ and $\overrightarrow{n_2}$

$$\therefore\ \vec{b}=\overrightarrow{n_1}\times\overrightarrow{n_2} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & 2\\ 3 & 1 & 1\end{vmatrix}$$

Expanding along first row,

$\vec{b} = \hat{i}(-1-2) - \hat{j}(1-6) + \hat{k}(1+3)$ = $-3\hat{i} + 5\hat{j} + 4\hat{k}$

Putting this value of $\vec{b}$ in eq. (i), vector equation of required line,

$\vec{r} = \left(\hat{i} + 2\hat{j} + 3\hat{k}\right) + \lambda\left(-3\hat{i} + 5\hat{j} + 4\hat{k}\right)$

20.Find the vector equation of the line passing through the point $(1, 2, -4)$ and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$.

Ans. Given: A point on the required line is A $(1, 2, -4)$

$\therefore$ Position vector of point A is $\vec{a} = (1, 2, -4) = \hat{i} + 2\hat{j} - 4\hat{k}$

Also given equations of two lines

$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

$\therefore$ Direction ratios of given two lines are $\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k}$ and $\vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$

Now $\vec{b} = \vec{b_1} \times \vec{b_2}$ = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$

Expanding along first row,

= $\hat{i}(80-56) - \hat{j}(-15-21) + \hat{k}(24+48)$ = $24\hat{i} + 36\hat{j} + 72\hat{k}$

$\Rightarrow \vec{b} = 12\left(2\hat{i} + 3\hat{j} + 6\hat{k}\right)$

$\therefore$ Equation of the required line is $\vec{r} = \vec{a} + \lambda\vec{b}$

$$\Rightarrow \vec{r} = \left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda(12)\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

Again replacing 12λ by λ,

$$\Rightarrow \vec{r} = \left(\hat{i}+2\hat{j}-4\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$$

21.Prove that if a plane has the intercepts a, b, c and is at a distance of p units from the origin, then $\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}=\frac{1}{p^2}$.

Ans. We know that equation of plane making intercepts a, b, c (on the axes) is

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}-1=0$$

Given: Perpendicular distance of the origin (0, 0, 0) from plane = p

$$\therefore \frac{|ax_1+by_1+cz_1+d|}{\sqrt{a^2+b^2+c^2}} = \frac{\left|\frac{0}{a}+\frac{0}{b}+\frac{0}{c}-1\right|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=p$$

$$\Rightarrow \frac{|-1|}{\sqrt{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}}=p$$

Squaring both sides, $\frac{1}{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}}=p^2$

$$\Rightarrow p^2\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)=1$$

$$\Rightarrow \left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)=\frac{1}{p^2}$$

Choose the correct answer in Exercise Q. 22 and 23.

22.Distance between the two planes: $2x+3y+4z=4$ **and** $4x+6y+8z=12$ **is**

(A) 2 units(B) 4 units(C) 8 units(D) $\frac{2}{\sqrt{29}}$ **units**

Ans. Equation of one plane is $2x+3y+4z=4 \Rightarrow 2x+3y+4z-4=0$

Equation of second plane is $4x+6y+8z=12 \Rightarrow 4x+6y+8z-12=0$

Here $\frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2}, \frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2}, \frac{c_1}{c_2}=\frac{4}{8}=\frac{1}{2}$

Since, $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$ therefore, the given two lines are parallel.

We know that the distance of the parallel lines = $\frac{|d_1-d_2|}{\sqrt{a^2+b^2+c^2}}$

$\Rightarrow \frac{|-4-(-6)|}{\sqrt{(2)^2+(3)^2+(4)^2}}$

= $\frac{|-4+6|}{\sqrt{4+9+16}}$

= $\frac{2}{\sqrt{29}}$

Therefore, option (D) is correct.

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23. The planes: $2x-y+4z=5$ **and** $5x-2.5y+10z=6$ **are**

(A) Perpendicular(B) Parallel

(C) intersect $y-$axis(D) passes through $\left(0, 0, \frac{5}{4}\right)$

Ans. Equations of the given planes are $2x-y+4z=5 \quad (a_1x+b_1y+c_1z+d=0)$

and $5x-2.5y+10z=6 \quad (a_2x+b_2y+c_2z+d=0)$

For perpendicular $a_1a_2+b_1b_2+c_1c_2 = 2(5)+(-1)(-2.5)+4(10) = 10+2.5+40=52.5$

$\because \ a_1a_2+b_1b_2+c_1c_2 \neq 0$

$\therefore$ Planes are not perpendicular.

For parallel $\frac{a_1}{a_2}=\frac{2}{5}, \frac{b_1}{b_2}=\frac{-1}{-2.5}=\frac{10}{25}=\frac{2}{5}, \frac{c_1}{c_2}=\frac{4}{10}=\frac{2}{5}$

$\because \ \frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

$\therefore$ given planes are parallel.

Therefore, option (B) is correct.