CBSE Class–12 Mathematics

NCERT solution

Chapter - 11

Three Dimensional Geometry - Exercise 11.1

1.If a line makes angles $90^\circ, 135^\circ, 45^\circ$ with the x, y and $z-$axes respectively, find its direction cosines.

Ans. Here $\alpha = 90^\circ, \beta = 135^\circ$ and $\gamma = 45^\circ$

Since direction cosines of a line making angles α, β, γ with the x, y and $z-$axes respectively are $\cos\alpha, \cos\beta, \cos\gamma$.

Therefore, the direction cosines of the required line are:

$\cos 90^\circ = 0; \cos 135^\circ = \frac{-1}{\sqrt{2}}; \cos 45^\circ = \frac{1}{\sqrt{2}}$

$$\left[\because \cos 135^\circ = \cos\left(180^\circ - 45^\circ\right) = -\cos 45^\circ = \frac{-1}{\sqrt{2}}\right]$$

2.Find the direction cosines of a line which makes equal angles with the co-ordinate axes.

Ans. Let a line make equal angles α, α, α with the co-ordinate axes.

$\therefore$ Direction cosines of the line are $\cos\alpha, \cos\alpha, \cos\alpha$ ....(i)

$\therefore \cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1 \left[\because \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1\right]$

$\Rightarrow 3\cos^2\alpha = 1$

$\Rightarrow \cos^2\alpha = \frac{1}{3}$

$\Rightarrow \cos\alpha = \pm\frac{1}{\sqrt{3}}$

Putting $\cos\alpha = \pm\frac{1}{\sqrt{3}}$ in eq. (i), direction cosines of the required line making equal angles with the co-ordinate axes are $\pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}, \pm\frac{1}{\sqrt{3}}$.

Direction cosines of a line making equal angles with the co-ordinate axes in the positive i.e., first octant are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$.

3.If a line has direction ratios $-18, 12, -4$, then what are its direction cosines?

Ans. We know that if a, b, c are direction ratios of a line, then direction cosines of the line are:

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}} \quad \text{..........(i)}$$

Here direction ratios of the line are $-18, 12, -4$

Putting the values in eq. (i),

$$\frac{-18}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{12}{\sqrt{(-18)^2+(12)^2+(-4)^2}}, \frac{-4}{\sqrt{(-18)^2+(12)^2+(-4)^2}}$$

$$\Rightarrow \frac{-18}{\sqrt{324+144+16}}, \frac{12}{\sqrt{324+144+16}}, \frac{-4}{\sqrt{324+144+16}}$$

$$\Rightarrow \frac{-18}{\sqrt{484}}, \frac{12}{\sqrt{484}}, \frac{-4}{\sqrt{484}}$$

$$\Rightarrow \frac{-18}{22}, \frac{12}{22}, \frac{-4}{22}$$

$$\Rightarrow \frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$$

Hence, direction cosines of required line are $\frac{-9}{11}, \frac{6}{11}, \frac{-2}{11}$.

4.Show that the points (2, 3, 4), $(-1, -2, 1)$, (5, 8, 7) are collinear.

Ans. The given points are A (2, 3, 4), B $(-1, -2, 1)$ and C (5, 8, 7)

$\therefore$ Direction ratios of the line joining A and B are

$-1-2, -2-3, 1-4 \left[\because x_2 - x_1, y_2 - y_1, z_2 - z_1\right]$ ....(i)

$\Rightarrow -3, -5, -3 = a_1, b_1, c_1$ (say)

Again Direction ratios of the line joining B and C are

$5-(-1), 8-(-2), 7-1 = 6, 10, 6 = a_2, b_2, c_2$ (say)..........(ii)

From eq. (i) and (ii),

$$\frac{-3}{6} = \frac{-1}{2}, \frac{-5}{10} = \frac{-1}{2}, \frac{-3}{6} = \frac{-1}{2}$$

$$\Rightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, AB is parallel to BC. But point B is common to both AB and BC. Hence points A, B, C are collinear.

5.Find the direction cosines of the sides of the triangle whose vertices are $(3, 5, -4), (-1, 1, 2)$ and $(-5, -5, -2)$.

Ans. Direction ratios of the line joining A and B are $-1-3, 1-5, 2-(-4)$

$\Rightarrow -4, -4, 6 \left[\because x_2 - x_1, y_2 - y_1, z_2 - z_1\right]$

$\therefore$ Direction cosines of line AB are

$$\frac{a}{\sqrt{a^2+b^2+c^2}} : \frac{b}{\sqrt{a^2+b^2+c^2}} : \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}} : \frac{-4}{\sqrt{(-4)^2+(-4)^2+(6)^2}} : \frac{6}{\sqrt{(-4)^2+(-4)^2+(6)^2}}$$

$$\Rightarrow \frac{-4}{\sqrt{16+16+36}} : \frac{-4}{\sqrt{16+16+36}} : \frac{6}{\sqrt{16+16+36}}$$

$$\Rightarrow \frac{-4}{\sqrt{68}} : \frac{-4}{\sqrt{68}} : \frac{6}{\sqrt{68}}$$

$$\Rightarrow \frac{-4}{2\sqrt{17}} : \frac{-4}{2\sqrt{17}} : \frac{6}{2\sqrt{17}}$$

$$\Rightarrow \frac{-2}{\sqrt{17}} : \frac{-2}{\sqrt{17}} : \frac{3}{\sqrt{17}}$$

Now Direction ratios of the line joining B and C are $-5-(-1), -5-1, -2-2 = -4, -6, -4$

$\therefore$ Direction cosines of line BC are

$$\frac{a}{\sqrt{a^2+b^2+c^2}} : \frac{b}{\sqrt{a^2+b^2+c^2}} : \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}} : \frac{-6}{\sqrt{(-4)^2+(-6)^2+(-4)^2}} : \frac{-4}{\sqrt{(-4)^2+(-6)^2+(-4)^2}}$$

$$\Rightarrow \frac{-4}{\sqrt{16+36+16}}, \frac{-6}{\sqrt{16+36+16}}, \frac{-4}{\sqrt{16+36+16}}$$

$$\Rightarrow \frac{-4}{\sqrt{68}} : \frac{-6}{\sqrt{68}} : \frac{-4}{\sqrt{68}}$$

$$\Rightarrow \frac{-4}{2\sqrt{17}} : \frac{-6}{2\sqrt{17}} : \frac{-4}{2\sqrt{17}}$$

$$\Rightarrow \frac{-2}{\sqrt{17}}, \frac{-3}{\sqrt{17}}, \frac{-2}{\sqrt{17}}$$

Direction ratios of the line joining C and A are $3-(-5), 5-(-5), -4-(-2) = 8, 10, -2$

∴ Direction cosines of line CA are

$$\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}$$

$$\Rightarrow \frac{8}{\sqrt{(8)^2+(10)^2+(-2)^2}}, \frac{10}{\sqrt{(8)^2+(10)^2+(-2)^2}}, \frac{-2}{\sqrt{(8)^2+(10)^2+(-2)^2}}$$

$$\Rightarrow \frac{8}{\sqrt{64+100+4}}, \frac{10}{\sqrt{64+100+4}}, \frac{-2}{\sqrt{64+100+4}}$$

$$\Rightarrow \frac{8}{\sqrt{168}}, \frac{10}{\sqrt{168}}, \frac{-2}{\sqrt{168}}$$

$$\Rightarrow \frac{8}{2\sqrt{42}}, \frac{10}{2\sqrt{42}}, \frac{-2}{2\sqrt{42}}$$

$$\Rightarrow \frac{4}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{-1}{\sqrt{42}}$$