Sample Paper 8

ANSWERKEY																			
1	(c)	2	(d)	3	(d)	4	(d)	5	(b)	6	(d)	7	(c)	8	(a)	9	(c)	10	(c)
11	(d)	12	(d)	13	(c)	14	(a)	15	(a)	16	(b)	17	(b)	18	(b)	19	(c)	20	(d)
21	(b)	22	(a)	23	(b)	24	(d)	25	(a)	26	(b)	27	(d)	28	(b)	29	(b)	30	(a)
31	(a)	32	(d)	33	(b)	34	(d)	35	(a)	36	(a)	37	(b)	38	(c)	39	(d)	40	(b)
41	(d)	42	(a)	43	(d)	44	(c)	45	(b)	46	(a)	47	(b)	48	(a)	49	(c)	50	(a)

SOLUTIONS

1. **(c)** Let the speed of the boat in still water be x km/hr and the speed of the stream be y km/hr then speed of boat in downstream is (x + y) km/hr and the speed of boat upstream is (x – y) km/hr.

Ist case : Distance covered upstream = 12 km

$\therefore$ time $= \frac{12}{x-y}$ hr

Distance covered downstream = 40 km

$\therefore$ time $= \frac{40}{x+y}$ hr

Total time is 8 hr $\therefore \frac{12}{x-y}+\frac{40}{x+y}=8$ ...(i)

IInd case :

Distance covered upstream = 16 km

$\therefore$ time $= \frac{16}{x-y}$ hr

Distance covered downstream

= 32 km $\therefore$ time $= \frac{32}{x+y}$ hr

Total time taken = 8 hr

$\therefore \frac{16}{x-y}+\frac{32}{x+y}=8$ ...(ii)

Solving (i) and (ii), we get,

x = speed of boat in still water = 6 km/hr,

y = speed of stream = 2 km/hr

2. **(d)** $A\left(\sqrt{3}+1,\sqrt{2}-1\right), B\left(\sqrt{3}-1,\sqrt{2}+1\right)$

$$AB=\sqrt{\left(\sqrt{3}-1-\sqrt{3}-1\right)^2+\left(\sqrt{2}+1-\sqrt{2}+1\right)^2}$$

$$=\sqrt{(-2)^2+(2)^2}$$

$$=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

3. **(d)** isosceles and similar

4. **(d)** Let us first find the H.C.F. of 210 and 55.

Applying Euclid's division lemma on 210 and 55, we get

210 = 55 × 3 + 45 ..... (i)

Since, the remainder 45 $\neq$ 0. So, we now apply division lemma on the divisor 55 and the remainder 45 to get

55 = 45 × 1 + 10 ..... (ii)

We consider the divisor 45 and the remainder 10 and apply division lemma to get

45 = 4 × 10 + 5 ..... (iii)

We consider the divisor 10 and the remainder 5 and apply division lemma to get

10 = 5 × 2 + 0 ..... (iv)

We observe that the remainder at this stage is zero. So, the last divisor i.e., 5 is the H.C.F of 210 and 55.

$\therefore$ 5 = 210 × 5 + 55y $\Rightarrow$ y $=\frac{-1045}{55}=-19$

5. **(b)** There are a total of six digits (1, 2, 2, 3, 4, 6) out of which four are even (2, 2, 4, 6)

So, required probility $=\frac{4}{6}=\frac{2}{3}$

6. **(d)**

(a) It is quadratic polynomial

[$\because$ the graph meets the x-axis in two points]

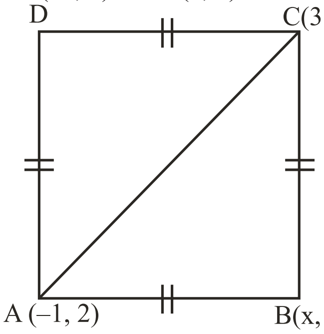
(b) It is a quadratic polynomial

[$\because$ the graph meets the x-axis in two points]

(c) It is a quadratic polynomial
[∵ the graph meets the *x*-axis in two points]

(d) It is a not quadratic polynomial
[∵ the graph meets the *x*-axis in one point]

7. **(c)** Let ABCD be a square and two opposite vertices of it are A(–1, 2) and C(3, 2). ABCD is square.



$\Rightarrow$ AB = BC
$\Rightarrow$ $AB^2 = BC^2$
$\Rightarrow$ $(x + 1)^2 + (y - 2)^2 = (x - 3)^2 + (y - 2)^2$
$\Rightarrow$ $x^2 + 2x + 1 = x^2 - 6x + 9$
$\Rightarrow$ $2x + 6x = 9 - 1 = 8$
$\Rightarrow$ $8x = 8 \Rightarrow x = 1$
ABC is right Δ at B, then
$AC^2 = AB^2 + BC^2$ (Pythagoras theorem)
$\Rightarrow$ $(3 + 1)^2 + (2 - 2)^2$
$= (x + 1)^2 + (y - 2)^2 + (x - 3)^2 + (y - 2)^2$
$\Rightarrow$ $16 = 2(y - 2)^2 + (1 + 1)^2 + (1 - 3)^2$
$\Rightarrow$ $16 = 2(y - 2)^2 + 4 + 4$
$\Rightarrow$ $(y - 2)^2 = 4 \Rightarrow y - 2 = \pm 2$
$\Rightarrow$ y = 4 and 0
i.e., when x = 1 then y = 4 and 0
Coordinates of the opposite vertices are :
B(1, 0) or D(1, 4)

8. **(a)**

9. **(c)** In right angled triangle POR
$PR^2 = PO^2 + OR^2 = (6)^2 + (8)^2 = 36 + 64 = 100$
∴ PR = 10 cm
Again in right angled triangle
PQR, $QR^2 = (26)^2 = 676$
$PQ^2 + PR^2 = (24)^2 + (10)^2 = 576 + 100 = 676$
∴ $QR^2 = PQ^2 + PR^2$
∴ ΔPQR is a right angled triangle with right angle at P.
i.e., ∠QPR = 90°

10. **(c)** $\frac{\pi r_1^2}{\pi r_2^2} = \frac{25}{16} \Rightarrow \frac{r_1}{r_2} = \frac{5}{4} \Rightarrow \frac{2\pi r_1}{2\pi r_2} = \frac{5}{4} = \frac{5\times125}{4\times125} = \frac{625}{500}$

11. **(d)** For any rational number $\frac{p}{q}$, where prime factorization of q is of the form $2^n.5^m$, where n and m are non-negative integers, the decimal representation is terminating.

12. **(d)** Let the y-axis divides the line segment joining (4, 5) and (– 10, 2) in the ratio k : 1.
x coordinate will be zero on y-axis.
We know that the coordinates of the point dividing the line segment joining (x_1, y_1) and (x_2, y_2) in the ratio m : n are given by $\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

Here, x coordinate of the point dividing the line segment joining (4, 5) and (– 10, 2) is equal to zero.
So, $\frac{k\times(-10)+1\times4}{k+1} = 0 \Rightarrow -10k = -4 \Rightarrow k = \frac{2}{5}$
Therefore, the line segment joining (4, 5) and (– 10, 2) is cut by the y-axis in the ratio 2 : 5.

13. **(c)** There are 4 cards of king and 4 cards of Jack n(S) = 52, n(E) = 4 + 4 = 8
$P(E) = \frac{n(E)}{n(S)} = \frac{8}{52} = \frac{2}{13}$

14. **(a)** The graph of $y = ax^2 + bx + c$ is a parabola open upward if a > 0. So, for $y = x^2 - 6x + 9$, a = 1 > 0, the graph is a parabola open upward.

15. **(a)** If 1, 1 and 2 are sides of a right triangle then sum of squares of any two sides is equal to square of third side.
Case 1 $(1)^2 + (1)^2 = 2 \neq (2)^2$
Case 2 $(1)^2 + (2)^2 = 1 + 4 = 5 \neq (1)^2$
Case 3 $(2)^2 + (1)^2 = 5 \neq (1)^2$

16. **(b)** n(S) = 6 × 6 = 36
E = {(1, 2), (2, 1), (2, 3), (3, 2), (3, 4), (4, 3), (4, 5), (5, 4), (5, 6), (6, 5)}
n(E) = 10
$P(E) = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$

17. **(a)** $\sqrt{2}$ is not a rational number. It can't be expressed in the fractional form.

18. **(b)** Area $= \frac{\theta}{360°}\times\pi r^2 = \frac{30°}{360°}\times\pi(7)^2 = \frac{49\pi}{12}$

19. **(c)** Inconsistent system

20. **(d)** S = {1, 2, 3, 4......, 25}
n(S) = 25
E = {2, 3, 5, 7, 11, 13, 17, 19, 23}
n(E) = 9
∴ $P(E) = \frac{9}{25}$

21. **(b)** $\frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{\alpha^3+\beta^3}{(\alpha\beta)^3} = \frac{\frac{3abc-b^3}{a^3}}{\left(\frac{c}{a}\right)^3}$

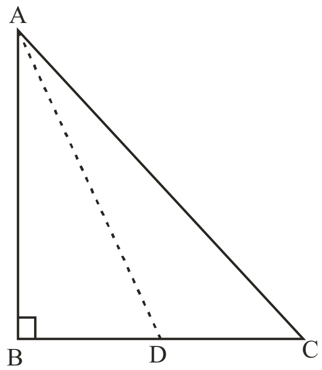
$\Rightarrow \frac{1}{\alpha^3} + \frac{1}{\beta^3} = \frac{3abc-b^3}{c^3}$

22. **(a)** S = {S, M, T, W, Th, F, Sa}
n(S) = 7
A non-leap year contains 365 days,
i.e., 52 weeks + 1 day.
E = {Sa}
n(E) = 1
$\therefore \quad P(E) = \frac{n(E)}{n(S)} = \frac{1}{7}$

23. **(b)** Let the given points be A(4, 3) and B(x, 5)
Since A and B lies on the circumference of a circle with centre O(2, 3), we have
OA = OB
$\Rightarrow \quad OA^2 = OB^2$
$\Rightarrow \quad (4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$
$\Rightarrow \quad 4 + 0 = (x-2)^2 + 4$
$\Rightarrow \quad (x-2)^2 = 0 \Rightarrow x = 2$

24. **(d)** Since
$$\frac{13}{125} = \frac{13}{53} = \frac{13\ 2^3}{(2)^3(5)^3} = \frac{104}{1000} = 0.104$$

25. **(a)**



Given : A ΔABC in which ∠B = 90° and D is the midpoint of BC.
Join AD.
In ΔABC, ∠B = 90°.
$\therefore \quad AC^2 = AB^2 + BC^2$ ....(i)
[by Pythagoras' theorem]
In ΔABD, ∠B = 90°
$\therefore \quad AD^2 = AB^2 + BD^2$ ....(ii)
[by Pythagoras' theorem]
$\Rightarrow \quad AB^2 = (AD^2 - BD^2)$.
$\therefore \quad AC^2 = (AD^2 - BD^2) + BC^2$ [using (i)]
$\Rightarrow \quad AC^2 = AD^2 - CD^2 + (2CD)^2$
[∵ BD = CD and BC = 2CD]
$\Rightarrow \quad AC^2 = AD^2 + 3CD^2$
Hence, $AC^2 = AD^2 + 3CD^2$

26. **(b)** $\frac{3}{x} + \frac{4}{y} = 1$ ...(i) $\frac{4}{x} + \frac{2}{y} = \frac{11}{12}$ ...(ii)
Multiplying (ii) by 2
$\Rightarrow \quad \frac{8}{x} + \frac{4}{y} = \frac{22}{12}$ ...(iii)
Subtracting (i) from (iii) $\Rightarrow \frac{5}{x} = \frac{10}{12}$
$\therefore \quad x = \frac{5 \times 12}{10} = 6$
Substituting x = 6 in (i)
$\Rightarrow \frac{3}{6} + \frac{4}{y} = 1 \Rightarrow \frac{4}{y} = 1 - \frac{1}{2} = \frac{1}{2} \quad \therefore y = 8.$
Hence, x = 6 and y = 8

27. **(d)**

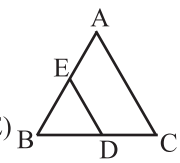
28. **(b)** n(S) = 6× 6 = 36, E = {(1, 5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5), (1, 6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)}
n(E) = 12
$P(E) = \frac{n(E)}{n(S)} = \frac{12}{36} = \frac{1}{3}$

29. **(b)** Put x + 1 = 0 or x = – 1 and x + 2 = 0 or x = – 2 in p (x)
Then, p (–1) = 0 and p (–2) = 0
$\Rightarrow \quad p(-1) =$
$\Rightarrow \quad -1 + 3 + 2\alpha + \beta = 0 \Rightarrow \beta = -2\alpha - 2$ .... (i)
$p(-2) = (-2)^3 + 3(-2)^2 - 2\alpha(-2) + \beta = 0$
$\Rightarrow \quad -8 + 12 + 4\alpha + \beta = 0 \Rightarrow \beta = -4\alpha - 4$ .... (ii)
By equalising both of the above equations, we get
$-2\alpha - 2 = -4\alpha - 4$
$\Rightarrow \quad 2\alpha = -2 \quad \Rightarrow \alpha = -1$
put α in eq. (i)
$\Rightarrow \quad \beta = -2(-1) - 2 = 2 - 2 = 0$
Hence, $\alpha = -1, \ \beta = 0$

30. **(a)** Let D be the window at a height of 9m on one side of the street and E be the another window at a height of 12 m on the other side.
In ΔADC
$AC^2 = 152 - 92 = 225 - 81$
AC = 12 m
In ΔECB
$CB^2 = 15^2 - 12^2 = 225 - 144$
CB = 9 m
Width of the street = (12 + 9)m = 21 m

31. **(a)**

32. **(d)** All equilateral triangles are similar
∴ Δ ABC~ Δ EBD
$\Rightarrow \quad \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta BDE} = \frac{BC^2}{BD^2}$
$= \frac{2BD^2}{BD^2} = \frac{4}{1}$



$\Rightarrow$ Area (DABC) : Area (DBDE) = 4 : 1

33. **(b)** As A lies on x-axis and B lies on y-axis, so their coordinates are (x, 0) and (0, y), respectively. Then,
$\frac{x+0}{2} = 4$ and $\frac{0+y}{2} = -3 \Rightarrow x = 8$ and $y = -6$
Hence, the points A and B are (8, 0) and (0, –6).

34. **(d)** If 6^x ends with 5, then 6^x would contain the prime 5. But $6^x = (2 \times 3)^x = 2^x \times 3^x$.
$\Rightarrow$ The only prime numbers in the factorization of 6x are 2 and 3.
$\therefore$ By uniqueness of fundamental theorem, there are no primes other than 2 & 3 in 6^x. So, 6^x will never end with 5.

35. **(a)** (a) Area $= \frac{\theta}{360°} \times \pi r^2 = \frac{60°}{360°} \times \frac{22}{7} \times (6)^2 = \frac{132}{7}$ cm^2

(b) Area of minor sector $= \frac{\theta}{360°} \times \pi r^2$

$= \frac{60°}{360°} \times \frac{22}{7} \times 14 \times 14 = 102.57$ cm^2

Area of major sector

= Area of circle – Area of minor sector

$= \frac{22}{7}(14)^2 - 102.57$

$= 615.44 - 102.57 = 512.87$ cm^2

(c) $\frac{C}{A} = \frac{2\pi(5)}{\pi(5)^2} = \frac{2}{5}$

(d) Given, $\left(\frac{\theta}{360°}\right) 2\pi r = 22$

$\therefore$ Area of sector $= \left(\frac{\theta}{360°}\right)\pi r^2 = \left(\frac{\theta}{360°}\right)\frac{\pi r}{2}(2r)$

$= \left(\frac{\theta}{360°}\right) 2\pi r \left(\frac{r}{2}\right) = \frac{22 \times 6}{2} = 66$ cm^2

36. **(a)** Let AD = 5x cm and DB = 4x cm.
Then,
AB = (AD + DB) = (5x + 4x) cm = 9x cm.
In ΔADE and ΔABC, we have
$\angle ADE = \angle ABC$ (corres. $\angle$s)
$\angle AED = \angle ACB$ (corres, $\angle$s)
$\therefore$ $\Delta ADE \sim \Delta ABC$ [by AA-similarity]

$\Rightarrow \frac{DE}{BC} = \frac{AD}{AB} = \frac{5x}{9x} = \frac{5}{9}$

$\Rightarrow \frac{DE}{BC} = \frac{5}{9}$ ...(i)

In ΔDFE and ΔCFB, we have
$\angle EDF = \angle BCF$ (alt. int. $\angle$s)
$\angle DEF = \angle CBF$ (alt. int. $\angle$s)
$\therefore$ $\Delta DFE \sim \Delta CBF$

$\Rightarrow \frac{ar(\Delta DFE)}{ar(\Delta CFB)} = \frac{DE^2}{CB^2} = \frac{DE^2}{BC^2} = \left(\frac{DE}{BC}\right)^2 = \left(\frac{5}{9}\right)^2 = \frac{25}{81}$

$\Rightarrow$ ar (ΔDFE) : ar (ΔCFB) = 25 : 81

37. **(b)** Let $f(x) = 6x^3 - 11x^2 + kx - 20$

$f\left(\frac{4}{3}\right) = 6\left(\frac{4}{3}\right)^3 - 11\left(\frac{4}{3}\right)^2 + k\left(\frac{4}{3}\right) - 20 = 0$

$\Rightarrow 6.\frac{64}{27} - 11.\frac{16}{9} + \frac{4k}{3} - 20 = 0$

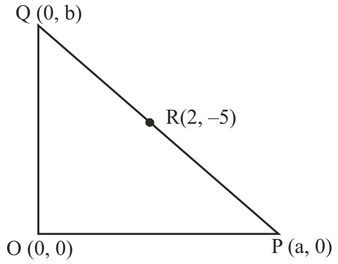
$\Rightarrow 128 - 176 + 12k - 180 = 0$

$\Rightarrow 12k + 128 - 356 = 0 \quad 12k = 228 \Rightarrow k = 19$

38. **(c)** For coincident lines, $\frac{3}{6} = \frac{-1}{-k} = \frac{8}{16}$

$\frac{1}{2} = \frac{1}{k} \Rightarrow k = 2$

39. **(d)** Let the line $\frac{x}{a} + \frac{y}{b} = 1$ meet x-axis at P(a, 0) and y-axis at Q(0, b). Since R is mid point at PQ.



$\therefore \frac{a+0}{2} = 2, \quad \frac{0+b}{2} = -5$

$\therefore$ a = 4, b = –10 $\therefore$ P is (4, 0), Q is (0, –10)

40. **(b)** $\frac{21}{45} = \frac{21}{9 \times 5} = \frac{21}{3^2 \times 5}$

Clearly, 45 is not of the form 2m × 5n. So the decimal expansion of $\frac{21}{45}$ is non-terminating and repeating.

41. **(d)** Sample space = {HH, HT, TH, TT}
Total number of elementary events = 4
Favourable event E = HH
n (E) = 1
P(E) = $\frac{1}{4}$

42. **(a)** Favourable event E = {TH, HT}
n(E) = 2
P(E) = $\frac{2}{4} = \frac{1}{2}$

43. **(d)** Favourable event E = {TT}
n(E) = 1
P(E) = $\frac{1}{4}$

44. **(c)** At most one head = {HT, TH, TT}
P = $\frac{3}{4}$

45. **(b)** At least one head
{HH, HT, TH}
P = $\frac{3}{4}$

46. (a) **47.** (b) **48.** (a)
49. (c) **50.** (a)