DU MA MSc Mathematics

Topic:- MATHS MA

1) Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : If $S = \{\frac{1}{n^2} : n \in \mathbb{N}\}$ then $\inf S = 0$.

Reason R : If $x \in \mathbb{R}$, then there exists $n_x \in \mathbb{N}$ such that $x < n_x$.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9578]

1. Both **A** and **R** are true and **R** is the correct explanation of **A**

   [Option ID = 38309]
2. Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

   [Option ID = 38310]
3. **A** is true but **R** is false

   [Option ID = 38311]
4. **A** is false but **R** is true

   [Option ID = 38312]

2) Which of the following series converge?

A. $\sum_{n=1}^{\infty} \sin(\frac{1}{n})$.

B. $\sum_{n=1}^{\infty} \cos(\frac{1}{n})$.

C. $\sum_{n=1}^{\infty} (-1)^n e^{\frac{1}{n}}$.

D. $\sum_{n=4}^{\infty} \frac{1}{\ln(n)^{\ln(n)}}$.

E. $\sum_{n=1}^{\infty} \sin(e^{-n})$.

Choose the ***correct*** answer from the options given below:

[Question ID = 9579]

1. B, C, and D only

   [Option ID = 38313]
2. A, C and D only

   [Option ID = 38314]
3. D and E only

   [Option ID = 38315]
4. C, D and E only

   [Option ID = 38316]

3) Given below are two statements.

Statement I: Suppose $a_n > 0$ for every n and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} < 1$ then $\lim a_n = 0$.

Statement II: Suppose $a_n > 0$ for every n and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} > 1$ then $\lim a_n = \infty$.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9580]

1. Both Statement I and Statement II are true

   [Option ID = 38317]
2. Both Statement I and Statement II are false

   [Option ID = 38318]
3. Statement I is true but Statement II is false

   [Option ID = 38319]
4. Statement I is false but Statement II is true

   [Option ID = 38320]

4) Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : If for a real sequence (a_n), there exists a $k \in [0, 1)$ such that $|a_{n+2} - a_{n+1}| \leq k|a_{n+1} - a_n|$ for all n then (a_n) is convergent in $\mathbb{R}$.

Reason R : Every Cauchy sequence is convergent.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9581]

1. Both **A** and **R** are true and **R** is the correct explanation of **A**

   [Option ID = 38321]
2. Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

   [Option ID = 38322]
3. **A** is true but **R** is false

[Option ID = 38323]
4. A is false but R is true

[Option ID = 38324]

5) Given below are two statements.

Statement I: Every compact subset $S \subseteq \mathbb{R}$ contains a maximum and a minimum element.

Statement II: If $S \subseteq \mathbb{R}$ contains a maximum and a minimum element, then S is compact.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9582]
1. Both Statement I and Statement II are true

[Option ID = 38325]
2. Both Statement I and Statement II are false

[Option ID = 38326]
3. Statement I is true but Statement II is false

[Option ID = 38327]
4. Statement I is false but Statement II is true

[Option ID = 38328]

6)

Given below are two statements.

Statement I: The function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=\begin{cases} x, & x \text{ is irrational} \\ -x, & x \text{ is rational}\end{cases}$ has a discontinuity of second kind at every rational number.

Statement II: The function $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=1-[x]$, where $[\cdot]$ is the greatest integer function, has a countable number of discontinuities of the second kind.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9583]
1. Both Statement I and Statement II are true

[Option ID = 38329]
2. Both Statement I and Statement II are false

[Option ID = 38330]
3. Statement I is true but Statement II is false

[Option ID = 38331]
4. Statement I is false but Statement II is true

[Option ID = 38332]

7) Which of the following statements are true?

A. Every monotonic function defined on a finite interval is bounded.

B. If $f:[a,b]\to[c,d]$ is continuous and monotonic, then it is surjective.

C. Every monotonic function defined on a finite interval attains its bounds.

D. If f is monotonic on finite interval (a,b), then set of discontinuities of f in (a,b) is at most countable.

Choose the ***correct*** answer from the options given below.

[Question ID = 9584]
1. A and C only

[Option ID = 38333]
2. A and B only

[Option ID = 38334]
3. A, C and D only

[Option ID = 38335]
4. B and D only

[Option ID = 38336]

8) Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : The series $\sum_{n=1}^{\infty} x^n(1-x)$ converges uniformly on $[0,1)$.

Reason R : There exists a convergent series $\sum_{n=1}^{\infty} M_n$ of real numbers such that for all $x\in[0,1]$, $|x^n(1-x)|\le M_n$, for all n.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9585]
1. Both A and R are true and R is the correct explanation of A

[Option ID = 38337]
2. Both A and R are true but R is NOT the correct explanation of A

[Option ID = 38338]
3. A is true but R is false

[Option ID = 38339]
4. A is false but R is true

[Option ID = 38340]

9) The sequence $\left(\frac{\sin nx}{\sqrt{n}}\right)$ is

[Question ID = 9586]

1. uniformly convergent on $[0, \pi]$

[Option ID = 38341]

2. uniformly convergent on $(0, \pi]$, but not on $[0, \pi]$

[Option ID = 38342]

3. uniformly convergent on $(0, \pi)$, but not on $[0, \pi]$

[Option ID = 38343]

4. uniformly convergent on $[0, \pi)$, but not on $[0, \pi]$

[Option ID = 38344]

10) Given below are two statements.

Statement I: If $f : [a, \infty) \to \mathbb{R}$ is uniformly continuous and $\lim_{x\to\infty} f(x)$ is finite then f is bounded on $[a, \infty)$.

Statement II: If f is uniformly continuous on every finite subinterval of $[a, \infty)$, then f is uniformly continuous on $[a, \infty)$.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9587]

1. Both Statement I and Statement II are true

[Option ID = 38345]

2. Both Statement I and Statement II are false

[Option ID = 38346]

3. Statement I is true but Statement II is false

[Option ID = 38347]

4. Statement I is false but Statement II is true

[Option ID = 38348]

11) Let $f : [0, 1] \to \mathbb{R}$ be a non-constant polynomial function such that $f(0) = f(1) = 2$. Which of the following is always true?

[Question ID = 9588]

1. There exists a unique $c \in (0, 1)$ such that $f'(c) = 0$

[Option ID = 38349]

2. There exist infinitely many points $c \in (0, 1)$ where $f^{(2)}(c) = 0$

[Option ID = 38350]

3. f has atleast one root in $(0, 1)$

[Option ID = 38351]

4. The set $\{c \in (0, 1) : f'(c) = 0\}$ is a non-empty finite set

[Option ID = 38352]

12) For a function $f : [-1, 1] \to \mathbb{R}$, consider the following statements.

Statement I: If $\lim_{n\to\infty} f(\frac{1}{n}) = f(0) = \lim_{n\to\infty} f(-\frac{1}{n})$, then f is continuous at 0.

Statement II: If f is continuous at 0, then $\lim_{n\to\infty} f(\frac{1}{n}) = \lim_{n\to\infty} f(-\frac{1}{n}) = \lim_{n\to\infty} f(e^{\frac{1}{n}} - 1) = f(0)$.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9589]

1. Both Statement I and Statement II are true

[Option ID = 38353]

2. Both Statement I and Statement II are false

[Option ID = 38354]

3. Statement I is true but Statement II is false

[Option ID = 38355]

4. Statement I is false but Statement II is true

[Option ID = 38356]

13) Let a_n denote the coefficient of x^n in the Maclaurin Series expansion of a function f.
If $a_n = \frac{a_{n-1}}{n}$ and $a_0 = 2$, then $f(x)$ is equal to

[Question ID = 9590]

1. e^x

[Option ID = 38357]

2. e^{2x}

[Option ID = 38358]

3. $2e^x$

[Option ID = 38359]

4. $2e^{2x}$

[Option ID = 38360]

14) Let $f(x, y) = x^2 + y^2 - 6x - 2y + 13$. Then f has

[Question ID = 9591]

1. an absolute minimum at $(3, 1)$ and the minimum value is 13

[Option ID = 38361]

2. an absolute maximum at $(3, 1)$ and the maximum value is 13

[Option ID = 38362]

3. no critical point [Option ID = 38363]
4. an absolute minimum at $(3,1)$ and the minimum value is 3

[Option ID = 38364]

15) The $\lim_{(x,y)\to(0,0)} \frac{x^2ye^y}{x^4+4y^2}$

[Question ID = 9592]
1. exists and is equal to 0

[Option ID = 38365]
2. exists and is equal to $\frac{1}{4}$

[Option ID = 38366]
3. does not exist

[Option ID = 38367]
4. exists and is equal to $\frac{1}{5}$

[Option ID = 38368]

16) If $f(x,y,z) = \sqrt{\sin^2 x + \sin^2 y + \sin^2 z}$, then the value of $f_z(0,0,\frac{\pi}{4})$ is

[Question ID = 9593]
1. $\frac{1}{\sqrt{2}}$

[Option ID = 38369]
2. $\frac{1}{3}$

[Option ID = 38370]
3. 0

[Option ID = 38371]
4. $\frac{1}{2}$

[Option ID = 38372]

17) The Riemann sum for $f(x) = x^2$ on the interval $[0,3]$ for the partition $\{0, \frac{3}{2}, 2, 3\}$, which uses the left end points as sample points, is

[Question ID = 9594]
1. 5

[Option ID = 38373]
2. $\frac{45}{4}$

[Option ID = 38374]
3. $\frac{25}{4}$

[Option ID = 38375]
4. $\frac{41}{8}$

[Option ID = 38376]

18)

On the interval $[0,1]$, the function $f(x) = \begin{cases} x-1, & \text{if } x \text{ is rational} \\ x, & \text{if } x \text{ is irrational} \end{cases}$ is

[Question ID = 9595]
1. Riemann integrable and $\int_0^1 f dx = \frac{1}{2}$

[Option ID = 38377]
2. Riemann integrable and $\int_0^1 f dx = -\frac{1}{2}$

[Option ID = 38378]
3. not Riemann integrable

[Option ID = 38379]
4. Riemann integrable and $\int_0^1 f dx = 0$

[Option ID = 38380]

19) Which of the following is not a metric on $\mathbb{R}$?

[Question ID = 9596]
1. $d(x,y) = |x-y|$

.

[Option ID = 38381]
2. $d(x,y) = |x^2 - y^2|$

.

[Option ID = 38382]
3. $d(x,y) = |x^3 - y^3|$

.

[Option ID = 38383]
4. $d(x,y) = \frac{|x-y|}{1+|x-y|}$

.

[Option ID = 38384]

20) Given below are two statements:

Statement I: Let X be an uncountable set with the discrete metric. Then X is separable.

Statement II: Let X be the set of all bounded sequences of real numbers with the metric $d(x, y) = \sup\{|\xi_n - \eta_n| \mid n = 1, 2, \ldots\}$, where $x = (\xi_n), y = (\eta_n) \in X, \xi_n, \eta_n \in \mathbb{R}$ for $n = 1, 2, \ldots$ Then X is separable.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9597]

1. Both Statement I and Statement II are correct

   [Option ID = 38385]
2. Both Statement I and Statement II are incorrect

   [Option ID = 38386]
3. Statement I is correct but Statement II is incorrect

   [Option ID = 38387]
4. Statement I is incorrect but Statement II is correct

   [Option ID = 38388]

21) Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R .**

Assertion A : Let A be a square matrix of order 2 with eigenvalues ± 1 and let $p(x) = x^4 + x^3 - x^2 + x + 1$ be a polynomial in x. Then $p(A) = 2A + I$, where I is the identity matrix of order 2.

Reason R : Let A be a square matrix with characteristic polynomial $\Delta(x) = x^n + a_1 x^{n-1} + \cdots + a_n$. Then A is invertible if and only if $a_n \neq 0$.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9598]

1. Both A and R are true and R is the correct explanation of A

   [Option ID = 38389]
2. Both A and R are true but R is NOT the correct explanation of A

   [Option ID = 38390]
3. A is true but R is false

   [Option ID = 38391]
4. A is false but R is true

   [Option ID = 38392]

22) Let $t_n = \binom{n+1}{2}, \quad \geq 1$. For what values of n does t_n divides the sum $t_1 + t_2 + \cdots + t_n$

[Question ID = 9599]

1. $2k$ for each positive integer k

   [Option ID = 38393]
2. $2k + 1$ for each non-negative integer k

   [Option ID = 38394]
3. $3k$ for each positive integer k

   [Option ID = 38395]
4. $3k + 1$ for each non-negative integer k

   [Option ID = 38396]

23)

Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Let $N = a_0 + a_1 10 + \cdots + a_{m-1} 10^{m-1} + a_m 10^m$ be the decimal representation of the positive integer N, $0 \leq a_k \leq 9$ and let $S = a_0 + a_1 + \cdots + a_{m-1} + a_m$. The 9 divides N if and only if 9 divides S.

Reason R : Let $P(X) = \sum_{k=0}^{m} c_k X^k$ be a polynomial function of X with integer coefficients c_k. If $a \equiv b \pmod{n}$, then $P(a) \equiv P(b) \pmod{n}$.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9600]

1. Both A and R are true and R is the correct explanation of A

   [Option ID = 38397]
2. Both A and R are true but R is NOT the correct explanation of A

   [Option ID = 38398]
3. A is true but R is false

   [Option ID = 38399]
4. A is false but R is true

   [Option ID = 38400]

24) The positive integer that leaves remainder 2,3,2 when divided by 3,5,7 respectively is

[Question ID = 9601]

1. 58

   [Option ID = 38401]
2. 44

   [Option ID = 38402]
3. 38

[Option ID = 38403]

4. $\mathbf{23}$

[Option ID = 38404]

25) An element a of a group G is called idempotent if $a^2 = a$. If G is the group of all non-singular matrices over the reals of order 2, then the number of idempotent elements in G are

[Question ID = 9602]

1. $\mathbf{1}$

[Option ID = 38405]

2. $\mathbf{2}$

[Option ID = 38406]

3. $\mathbf{3}$

[Option ID = 38407]

4. $\mathbf{4}$

[Option ID = 38408]

26) Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : A group G can not be expressed as a union of two proper subgroups.

Reason R : It is not necessary that a group G has two proper subgroups.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9603]

1. Both A and R are true and R is the correct explanation of A

[Option ID = 38409]

2. Both A and R are true but R is NOT the correct explanation of A

[Option ID = 38410]

3. A is true but R is false

[Option ID = 38411]

4. A is false but R is true

[Option ID = 38412]

27) Given below are two statements.

Statement I: The order of a finite cyclic group G, in which for every pair of distinct subgroups H and K of G either $H \subset K$ or $K \subset H$, is a prime power.

Statement II: The order of a finite cyclic group with one generator is a prime power.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9604]

1. Both Statement I and Statement II are true

[Option ID = 38413]

2. Both Statement I and Statement II are false

[Option ID = 38414]

3. Statement I is true but Statement II is false

[Option ID = 38415]

4. Statement I is false but Statement II is true

[Option ID = 38416]

28) Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Let $G = \{z \in \mathbb{C} : z^{p^n} = 1\}$ be a group of $p-$power roots of unity in $\mathbb{C}$, where p is a prime. Then G is isomorphic to a proper quotient of G itself.

Reason R : G has a normal series in which all factor groups are abelian.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9605]

1. Both A and R are true and R is the correct explanation of A

[Option ID = 38417]

2. Both A and R are true but R is NOT the correct explanation of A

[Option ID = 38418]

3. A is true but R is false

[Option ID = 38419]

4. A is false but R is true

[Option ID = 38420]

29) Let R be a ring and S be the set of all distinct homomorphic images (up to isomorphism) of R. If $|S| = n$, then the number of ideals of R is

[Question ID = 9606]

1. greater than ***n***

[Option ID = 38421]

2. less than ***n***

[Option ID = 38422]

3. equal to ***n***

[Option ID = 38423]
4. infinite

[Option ID = 38424]

30) Consider the following statements.

A. The additive order of all nonzero elements in a ring is the same.

B. The additive order of all nonzero elements in an integral domain is the same.

C. The additive order of all nonzero elements in a field is the same.

Choose the *correct* answer from the options given below.

[Question ID = 9607]
1. A and B only

[Option ID = 38425]
2. B and C only

[Option ID = 38426]
3. C and A only

[Option ID = 38427]
4. A, B and C

[Option ID = 38428]

31) Let R be a ring such that $x^3 = x$ for all $x \in R$. Then for all nonzero $x \in R$

[Question ID = 9608]
1. $4x = 0$

[Option ID = 38429]
2. $5x = 0$

[Option ID = 38430]
3. $6x = 0$

[Option ID = 38431]
4. $nx \neq 0$ for all $n \in \mathbb{N}$

[Option ID = 38432]

32) Let F be a field. Then the ring $\frac{F[x]}{\langle x^3 \rangle}$ has exactly

[Question ID = 9609]
1. four maximal ideals

[Option ID = 38433]
2. three maximal ideals

[Option ID = 38434]
3. two maximal ideals

[Option ID = 38435]
4. one maximal ideal

[Option ID = 38436]

33) Let $\{a, b, c\}$ be a basis of a vector space V over $\mathbb{R}$. Which of the following sets are bases of V?

$A = \{2a + 3b, 2a - c, a + b\}$

$B = \{2a + 3b, 3a - c, a - 3b - c\}$

$C = \{a + 2b - 2c, a + b + c, 3a + 4b\}$

$D = \{6a - 3b + c, 3a + 4b + c, a + c\}$

Choose the ***correct*** answer from the options given below.

[Question ID = 9610]
1. A, B and D only

[Option ID = 38437]
2. A and D only

[Option ID = 38438]
3. A, C and D only

[Option ID = 38439]
4. B and C only

[Option ID = 38440]

34) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that $T(1, 2) = (3, 4, 5)$ and $T(6, 7) = (8, 9, 10)$. Then $T(12, 10)$ is

[Question ID = 9611]
1. (8,10,12) [Option ID = 38441]
2. (9,7,5) [Option ID = 38442]
3. (8,6,2) [Option ID = 38443]
4. (8,6,4) [Option ID = 38444]

35) Given below are two statements.

Statement I: Consider the product defined on $\mathbb{R}^3$ as

$\langle x, y \rangle = x_1y_1 - x_1y_2 - x_2y_1 + kx_2y_2 + 3x_3y_3$,

where $x = (x_1, x_2, x_3)$, $y = (y_1, y_2, y_3) \in \mathbb{R}^3$. The space $\mathbb{R}^3$ is an inner product space for each $k > 0$.

Statement II: Consider the product defined on $\mathbb{R}^3$ as

$\langle u, v \rangle = u_1v_1 - u_1v_2 - u_2v_1 + 2u_2v_2 + mu_3v_3$, where $u = (u_1, u_2, u_3)$, $v = (v_1, v_2, v_3) \in \mathbb{R}^3$. The space $\mathbb{R}^3$ is an inner product space for each $m > 0$.

In light of the above statements, choose the *correct* answer from the options given below.

[Question ID = 9612]
1. Both Statement I and Statement II are true

   [Option ID = 38445]
2. Both Statement I and Statement II are false

   [Option ID = 38446]
3. Statement I is true but Statement II is false

   [Option ID = 38447]
4. Statement I is false but Statement II is true

   [Option ID = 38448]

36) Steady state solution to the equation

$$u_t - a^2 u_{xx} = 0,\ 0 < x < L,\ t > 0,$$

$u(0,t) = T_1,\ u(L,t) = T_2,\ u(x,0) = f(x),$ is given by

is given by[Question ID = 9613]
1. $T_2 + \frac{T_1 - T_2}{L} x$

   [Option ID = 38449]
2. $T_1 + \frac{T_1 - T_2}{L} x$

   [Option ID = 38450]
3. $T_1 + \frac{T_2 - T_1}{L} x$

   [Option ID = 38451]
4. $T_2 + \frac{T_2 - T_1}{L} x$

   [Option ID = 38452]

37)

Given below are two statements, one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : If u and z_1 are complementary function and particular integral respectively of the linear partial differential equation

$F(D, D')z = f(x, y)$, where $D = \frac{\partial}{\partial x}$, $D' = \frac{\partial}{\partial y}$, then $u + z_1$ is a general solution of the equation.

Reason R : The function $u + z_1$ satisfies $F(D, D')z = f(x, y)$ and contains correct number of arbitrary elements to qualify as a general solution.

In light of the above statements, choose the *correct* answer from the options given below.

[Question ID = 9614]
1. Both **A** and **R** are true and **R** is the correct explanation of **A**

   [Option ID = 38453]
2. Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

   [Option ID = 38454]
3. **A** is true but **R** is false

   [Option ID = 38455]
4. **A** is false but **R** is true

   [Option ID = 38456]

38) Given below are two statements, one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Method of separation of variables reduces a linear boundary value problem into an eigen value problem.

Reason R : Linear boundary value problem admits the superposition principle.

In light of the above statements, choose the *correct* answer from the options given below.

[Question ID = 9615]
1. Both **A** and **R** are true and **R** is the correct explanation of **A**

   [Option ID = 38457]
2. Both **A** and **R** are true but **R** is NOT the correct explanation of **A**

   [Option ID = 38458]
3. **A** is true but **R** is false

   [Option ID = 38459]
4. **A** is false but **R** is true

   [Option ID = 38460]

39) For the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y},$$

A. $z = \varphi_1(x+y) + \varphi_2(x-y) + \varphi_3(2x+y)$ is a complementary function.

B. $z = -\frac{1}{2} x e^{(x+y)}$ is a particular solution.

C. $z = \varphi_1(x+y) + \varphi_2(x-y) + \varphi_3(2x-y)$ is a complementary function.

D. $z = \frac{1}{2} x e^{x+y}$ is a particular solution.

Choose the ***correct*** answer from the options given below:

[Question ID = 9616]

1. C only

   [Option ID = 38461]
2. D only

   [Option ID = 38462]
3. A and D only

   [Option ID = 38463]
4. A and B only

   [Option ID = 38464]

40) The eigen values and eigen functions in the solution of

$u_{tt} = c^2 u_{xx}, \quad 0 < x < L, \quad t > 0,$

$u(0,t) = 0, \quad u(L,t) = 0, \quad u(x,0) = f(x), \;\; u_t(x,0) = g(x),$

$0 \le x \le L, \quad t \ge 0$

are, respectively,

are, respectively,[Question ID = 9617]

1. $\left(\frac{n\pi}{L}\right)^2$, $\sin\frac{n\pi x}{L}, n = 1, 2, 3, \ldots$

   [Option ID = 38465]
2. $\left(\frac{n\pi}{L}\right)^2$, $\cos\frac{n\pi x}{L}, n = 1, 2, 3, \ldots$

   [Option ID = 38466]
3. $\frac{n\pi}{L}$, $\sin\frac{n\pi x}{L}, n = 1, 2, 3, \ldots$

   [Option ID = 38467]
4. $\frac{n\pi}{L}$, $\cos\frac{n\pi x}{L}, n = 1, 2, 3, \ldots$

   [Option ID = 38468]

41) The complete integral of

$z^2p^2y + 6zpxy + 2zqx^2 + 4x^2y = 0$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$, is given by

[Question ID = 9618]

1. $z^2 = ax^2 - \left(2 + 3a + \frac{a^2}{2}\right)y^2 + b$, where a and b are arbitrary constants

   [Option ID = 38469]
2. $z^2 = ax - \left(2 + 3a + \frac{a^2}{2}\right)y^3 + b$, where a and b are arbitrary constants

   [Option ID = 38470]
3. $z^2 = ax^2 + \left(2 - 3a - \frac{a^2}{2}\right)y^2 + b$, where a and b are arbitrary constants

   [Option ID = 38471]
4. $z^2 = -ax^2 + \left(2 + 3a + \frac{a^2}{2}\right)y^3 + b$, where a and b are arbitrary constants

   [Option ID = 38472]

42) The partial differential equation

$x^2\frac{\partial^2 z}{\partial x^2} - x(y^2 - 1)\frac{\partial^2 z}{\partial x \partial y} + y(y-1)^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 0$

is hyperbolic in a region in the xy-plane if

[Question ID = 9619]

1. $x \neq 0$ and $y = 1$.

   [Option ID = 38473]
2. $x = 0$ and $y \neq 1$.

   [Option ID = 38474]
3. $x \neq 0$ and $y \neq 1$.

   [Option ID = 38475]
4. $x = 0$ and $y = 1$.

   [Option ID = 38476]

43) The solution of the differential equation

$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = 0$

using the method of variation of parameters, is given by

[Question ID = 9620]

1. $y(x) = ae^{-x} + be^{2x} - \frac{1}{2}e^{3x}$, where a and b are arbitrary constants.

   [Option ID = 38477]
2. $y(x) = ae^{x} + be^{3x} + \frac{1}{6}e^{3x}$, where a and b are arbitrary constants.

   [Option ID = 38478]
3. $y(x) = ae^{x} + be^{2x} + \frac{1}{2}e^{3x}$, where a and b are arbitrary constants.

   [Option ID = 38479]
4. $y(x) = ae^{x} + be^{-2x} + \frac{1}{2}e^{3x}$, where a and b are arbitrary constants.

   [Option ID = 38480]

44) The solution of the partial differential equation

$$x(y^2+z)p - y(x^2+z) = z(x^2-y^2),$$

where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$, is given by

[Question ID = 9621]

1. $\varphi(x^2+y^2+2z, x+y+z) = 0.$

   [Option ID = 38481]
2. $\varphi(x^2-y^2-2z, xyz) = 0.$

   [Option ID = 38482]
3. $\varphi(x^2+y^2-2z, xyz) = 0.$

   [Option ID = 38483]
4. $\varphi(x^2+y^2-2z, x+y+z) = 0.$

   [Option ID = 38484]

45) The order of convergence of fixed point iteration method is[Question ID = 9622]

1. 1 [Option ID = 38485]
2. 2 [Option ID = 38486]
3. 3 [Option ID = 38487]
4. 4 [Option ID = 38488]

46) The Newton-Raphson formula for finding the cube root of N is

[Question ID = 9623]

1. $\dfrac{2x_n^3+N}{3x_n^2}$

   [Option ID = 38489]
2. $\dfrac{2x_n^3-N}{3x_n^2}$

   [Option ID = 38490]
3. $\dfrac{2x_n^3+N^2}{3x_n^2}$

   [Option ID = 38491]
4. $\dfrac{2x_n^3-N^2}{3x_n^2}$

   [Option ID = 38492]

47) For the given initial value problem

$$\frac{dy}{dx} = y - x, \quad y(0) = 2,$$

with $h = 0.1$, the value of $y(0.1)$ by using the Runge-Kutta 2nd order formula correct upto 4 decimal places is

[Question ID = 9624]

1. 2.8909

   [Option ID = 38493]
2. 2.6492

   [Option ID = 38494]
3. 2.2050

   [Option ID = 38495]
4. 2.4210

   [Option ID = 38496]

48) Given below are two statements.

Statement I: The condition for convergence of the Gauss-Seidel method for solving $AX = B$, where A is a square matrix of order n, is $\sum_{j=1}^{n} a_{ij} < |a_{ii}|, \ \forall i$.

Statement II: The Gauss elimination method reduces the system of equations to an upper triangular system, which can be solved by forward substitution.

In light of the above statements, choose the ***correct*** answer from the options given below.

[Question ID = 9625]

1. Both Statement I and Statement II are true

   [Option ID = 38497]
2. Both Statement I and Statement II are false

   [Option ID = 38498]
3. Statement I is true but Statement II is false

   [Option ID = 38499]
4. Statement I is false but Statement II is true

[Option ID = 38500]

49) The singular solution of the ordinary differential equation

$$\left(\frac{dy}{dx}\right)^2 + 1 = \frac{1}{y^2}$$

is given by

A. $y = 1.$

B. $y = -1.$

C. $(x + c)^2 + y^2 = 1,$ where c is an arbitrary constant.

D. $y = 0.$

Choose the ***correct*** answer from the options given below:

[Question ID = 9626]

1. A and B only

   [Option ID = 38501]
2. B and C only

   [Option ID = 38502]
3. C and D only

   [Option ID = 38503]
4. A and D only

   [Option ID = 38504]

50) The unique solution of the initial value problem

$$\frac{dy}{dx} = y^2, \ y(1) = -1,$$

using the existence and uniqueness theorem, exists on the interval

[Question ID = 9627]

1. $\left[\frac{3}{4}, \frac{5}{4}\right]$

   [Option ID = 38505]
2. $\left[\frac{5}{4}, \frac{7}{4}\right]$

   [Option ID = 38506]
3. $[1,2]$

   [Option ID = 38507]
4. $[-1,0]$

   [Option ID = 38508]