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JKPSC Exam

Mains Previous Paper

Simplifying
Government Exams

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STATISTICS-I
[23]

Time Allowed : 3 Hours

Maximum Marks : 300

INSTRUCTIONS

- i) Answers must be written in English.
- ii) The number of marks carried by each question is indicated at the end of the question.
- iii) The answer to each question or part there of should begin on a fresh page.
- iv) Your answer should be precise and coherent.
- v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.
- vi) Candidates should attempt **any Five** questions choosing atmost **Two** from each section
- vii) If you encounter any typographical error, please read it as it appears in the text book.
- viii) Candidates are in their own interest advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.
- ix) No Continuation sheets shall be provided to any candidate under any circumstances.
- x) Candidates shall put a cross (X) on blank pages of Answer Script.
- xi) No blank page be left in between answer to various questions.
- xii) No programmable Calculator is allowed.
- xiii) No stencil (with different markings) is allowed.
- xiv) In no circumstances help of scribe will be allowed.

SECTION-I

1. a) Let $f(x,y) = k(x+y); 0 < x < 1, 0 < y < 1, 0 < x+y < 1$. (35,25)
 - (i) Determine the value of k such that f(x,y) is a probability distribution.
 - (ii) Find $f_x(x)$ and
 - (iii) Find the joint and marginal distribution of X + Y and X - Y.
- b) An urn contains n red and n blue balls. Two balls are removed from the urn together, at random.
 - i) What is the Sample Space?
 - ii) What is the Probability of drawing two balls of different colour?
 - iii) Find the Probability P_n that the balls are the same colour, and hence evaluate $\lim_{n \rightarrow \infty} P_n$.

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(1)

[Turn Over

2. a) Let X and Y have joint density $f(x,y) = \begin{cases} \frac{1}{4a^2} [1 + xy(x^2 - y^2)], & |x| \leq a, |y| \leq a, a > 0 \\ 0, & \text{otherwise} \end{cases}$

show that the characteristic functions $\phi_{X+Y}(t) = \phi_X(t)\phi_Y(t)$, although X and Y are dependent variates. (40,20)

- b) An urn contains N balls numbered 1 through N, where the first D balls are defective and the remaining N-D are non-defective. A sample of size n balls is drawn from the urn. Let A_k be the event that the sample of n balls contains exactly k defectives. Find the probability $P(A_k)$ when the sample is drawn without replacement.
3. a) Examine whether the WLLN holds for the sequence of independent random variables $\{X_k\}$, having $P(X_k = \pm 2^k) = 1/2$ (25,15,20)
- b) Stating clearly the reason, arrange the following in increasing order of magnitude: $P(A), P(A \cup B), P(A \cap B), P(A) + P(B)$
- c) If m students born in 1991 are attending a lecture, what is the probability that at least two of them share a birth day? What difference would it make if they were born in 1992?
4. a) Consider the geometric distribution in the form $P(x) = 2^{-x}, x = 1, 2, 3, \dots$. Show that Chebyshev's inequality gives $P(|X - 2| \leq 2) > \frac{1}{2}$, while the exact probability is $\frac{15}{16}$.
- b) Let X be a random variable denoting the number of tosses required to get two consecutive heads (E_2) when a fair coin is tossed. Show that the p.g.f. of X is $\frac{S^2}{4} \left\{ 1 - \frac{S}{2} - \frac{S^2}{4} \right\}^{-1}$. Show further that, if a fair coin is tossed indefinitely, the event E_2 is certain to occur and that the expected number of trials required is 6. (20,40)

SECTION-II

5. a) Show that the most general form of a distribution for which the sample harmonic mean is the ML estimator of θ , has p.d.f. (or p.m.f.) $f_\theta(x) = \exp \left[\frac{1}{x} \{ \theta A'(\theta) - A(\theta) \} - A'(\theta) + C(x) \right]$. (30,30)
- b) If $x_i (i = 1, 2, \dots, n)$ is an observed random sample from the distribution having p.d.f. $f_\lambda(x) = \frac{\lambda^{k+1} x^k \exp(-\lambda x)}{\Gamma(k+1)}, x > 0$. Where $\lambda > 0$ and k is a known constant.

Obtain the ML estimate for λ . Show that the corresponding estimator is biased but consistent and hence obtain its asymptotic distribution.

6. a) Discuss complete sufficient statistics and suggest the method to find MVUE for $e^{-\lambda}$ in Poisson distribution with parameter λ . (30,30)
 b) Let X_1, X_2, \dots, X_n be a sample from Gamma distribution with parameters (r, β) . Find the shortest length confidence interval for β at level $(1-\alpha)$, based on a sufficient statistic for β .

7. a) Find the most powerful test for testing $H_0 : f(x) = \begin{cases} 2x, & \text{if } 0 < x < \frac{1}{2} \\ 2(1-x)x, & \text{if } \frac{1}{2} \leq x < 1 \end{cases}$ against

the alternative $H_1 : f(x) = 1, \text{if } 0 < x < 1$, based on a sample of size $n = 1$. Also find the power of the test. (25,20,15)

- b) Let X_1, X_2, \dots, X_n be a random sample from p.d.f.

$$f(x, \theta) = \begin{cases} \theta x^{\theta-1}, & 0 \leq x \leq 1, \theta > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Obtain a level α UMP test for testing

$H_0 : \theta = 2$, against $H_1 : \theta > 2$. Further, obtain power function of the test and sketch the power curve.

- c) Describe the sign test for testing the location of a probability distribution. What is the parametric counterpart of this test?
 8. a) Consider observations from Poisson distribution with parameter λ . Develop a sequential probability ratio test (SPRT) to test $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = k\lambda_0, k > 1$. Obtain OC and ASN functions. (30+30)
 b) Let X_1, X_2, \dots, X_n be a random sample from p.d.f.

$$f(x, \theta) = \begin{cases} e^{-(x-\theta)}, & x \geq \theta, -\infty < \theta < \infty \\ 0, & \text{elsewhere} \end{cases}$$

. Find an estimator of θ by the method of

moments.

SECTION-III

9. a) If \underline{y} is a random vector with mean $\underline{\mu}$ and covariance matrix Σ and if A is a symmetric matrix of constants, then

(i) Show that $E(\underline{y}' A \underline{y}) = \text{tr}(A \Sigma) + \underline{\mu}' A \underline{\mu}$. Hence

(ii) Considering $\underline{y} = (y_1, y_2, \dots, y_n)'$ and a suitable application of the

result, show that $E(s^2) = \sigma^2$, where $s^2 = \frac{1}{n'-1} \sum_{i=1}^n (y_i - \bar{y})^2$. (30+30)

b) Let Y_1, Y_2, Y_3 be uncorrelated observations having common variance σ^2 and

$$E(y_1) = \beta_1 + \beta_2$$

$$E(y_2) = \beta_1 + \beta_3$$

$$E(y_3) = \beta_1 + \beta_2.$$

(i) Show that $\lambda_1\beta_1 + \lambda_2\beta_2 + \lambda_3\beta_3$ is estimable if and only if $\lambda_1 = \lambda_2 + \lambda_3$.

(ii) Obtain an unbiased estimate of σ^2 .

10. a) When is a two-way ANOVA model said to be mixed? How will you formulate the different hypotheses of this model? Describe how the hypotheses are tested. (25,35)

b) Let the random vector $\underline{y} \sim N_4(\underline{\mu}, \Sigma)$, where

$$\underline{\mu} = \begin{bmatrix} 2 \\ 5 \\ -2 \\ 1 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 9 & 0 & 3 & 3 \\ 0 & 1 & -1 & 2 \\ 3 & -1 & 6 & -3 \\ 3 & 2 & -3 & 7 \end{bmatrix}$$

If \underline{y} is partitioned as $\underline{y} = (y_1, y_2, x_1, x_2)'$, then $\underline{\mu}_y = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $\underline{\mu}_x = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$,

$\Sigma_{yy} = \begin{bmatrix} 9 & 0 \\ 0 & 1 \end{bmatrix}$, $\Sigma_{yx} = \begin{bmatrix} 3 & 3 \\ -1 & 2 \end{bmatrix}$ and $\Sigma_{xx} = \begin{bmatrix} 6 & -3 \\ -3 & 7 \end{bmatrix}$. Then obtain $E(\underline{y} / \underline{x})$ and $\text{Cov}(\underline{y} / \underline{x})$.

11. a) If $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$ (MVND), then show that its moment generating function is given by $M_{\underline{X}}(t) = e^{t'\underline{\mu} + \frac{1}{2}t'\Sigma t}$. (30,30)

b) Define Hotelling's T^2 . Show that it is invariant under non-singular linear transformations. Mention some uses of the T^2 statistic.

12. a) Explain the concept of partial correlation. Show in the usual notations that

$$r_{12.3} = \frac{r_{12} - r_{13}r_{23}}{\sqrt{(1-r_{13}^2)(1-r_{23}^2)}} \text{ Further, if } r_{12} \text{ and } r_{13} \text{ are given, then show that } r_{23}$$

must lie in the range: (30,30)

$$r_{12}r_{13} \pm (1-r_{12}^2 - r_{13}^2 + r_{12}^2r_{13}^2)^{1/2}.$$

b) Let $\underline{X} \sim N_p(\underline{\mu}, \Sigma)$ and let $\underline{X} = \begin{bmatrix} X^{(1)} \\ X^{(2)} \end{bmatrix}$, $\underline{\mu} = \begin{bmatrix} \underline{\mu}^{(1)} \\ \underline{\mu}^{(2)} \end{bmatrix}$, $\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix}$

Where $X^{(1)}_{q \times 1}$. Show that $X^{(1)} \sim N_q(\underline{\mu}^{(1)}, \Sigma_{11})$, even if $X^{(1)}$ and $X^{(2)}$ are not independent.



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