Sample Paper 5

ANSWERKEY																			
1	(b)	2	(d)	3	(d)	4	(c)	5	(b)	6	(b)	7	(c)	8	(c)	9	(a)	10	(a)
11	(b)	12	(a)	13	(c)	14	(c)	15	(b)	16	(a)	17	(d)	18	(c)	19	(c)	20	(c)
21	(d)	22	(c)	23	(c)	24	(c)	25	(a)	26	(b)	27	(d)	28	(d)	29	(a)	30	(c)
31	(d)	32	(c)	33	(b)	34	(a)	35	(d)	36	(a)	37	(b)	38	(c)	39	(a)	40	(d)
41	(c)	42	(a)	43	(b)	44	(a)	45	(b)	46	(b)	47	(a)	48	(c)	49	(a)	50	(d)

SOLUTIONS

1. **(b)** $x = 3 + 3^{2/3} + 3^{1/3}$

$$(x-3)^3 = \left(3^{\frac{2}{3}} + 3^{\frac{1}{3}}\right)^3$$

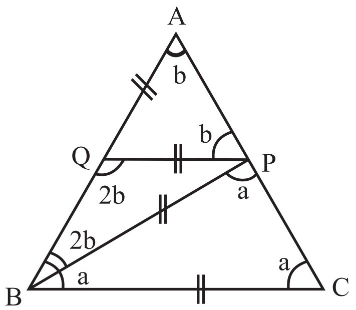
$x^3 - 27 - 9x^2 + 27x = 3^2 + 3 + 3 \times 3^{2/3} \times 3^{1/3} (3^{2/3} + 3^{1/3})$

$x^3 - 27 - 9x^2 + 27x - 9 - 3 = 9(x - 3)$

$x^3 - 39 - 9x^2 + 27x - 9x + 27 = 0$

$x^3 - 9x^2 + 18x - 12 = 0$

2. **(d)**



In ΔABC

$AB = AC$

$\Rightarrow \angle C = \angle B \Rightarrow \angle B = \angle C = a$

By angle sum properly in ΔABC,

$b + a + a = 180$

$\Rightarrow b + 2a = 180°$ ...(i)

In ΔQPB

$\Rightarrow \angle QPB = 180 - 4b$

Since 'APC' is a straight line

$\Rightarrow 180 - 4b + a + b = 180$

$\Rightarrow a = 3b$ ...(ii)

From equations (i) & (ii)

$b + 2(3b) = 180 \Rightarrow b = \dfrac{180}{7}$

$\angle AQP = 180° - 2\left(\dfrac{180}{7}\right) = \dfrac{5}{7}\pi$

3. **(d)** $2\pi r = 4\pi \Rightarrow r = 2$

Area $= \pi(2)^2 = 4\pi$

When, $2\pi r = 8\pi$

$\Rightarrow r = 4$

Area $= 16\pi$

4. **(c)** $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$= \left\{1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right\} \times \left\{1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right\}$$

$$= \frac{\{(\cos\theta + \sin\theta) + 1\} \times \{(\cos\theta + \sin\theta) - 1\}}{\cos\theta \times \sin\theta}$$

$$= \frac{(\cos\theta + \sin\theta)^2 - (1)^2}{\cos\theta \times \sin\theta} \quad \{\because (a+b)(a-b) = a^2 - b^2\}$$

$$= \frac{1 + 2\cos\theta\sin\theta - 1}{\cos\theta \times \sin\theta} = 2.$$

5. **(b)**

6. **(b)** Let the number of boys and girls in classroom is x and y.

According to question

$$\frac{x - x/5}{y} = \frac{2}{3} \Rightarrow \frac{4x}{5y} = \frac{2}{3} \Rightarrow \frac{x}{y} = \frac{5}{6} \quad \text{...(i)}$$

Also, $\dfrac{x - x/5}{y - 44} = \dfrac{5}{2} \Rightarrow \dfrac{4x}{5\,(y - 44)} = \dfrac{5}{2}$

$\Rightarrow 8x = 25y - 1100$ ...(ii)

From Eqs. (i) and (ii), we get, $x = 50$, $y = 60$

Let n number of boy leaves the class so number of boys and number of girls become equal.

$\therefore\ 50 - 10 - n = 60 - 44$

$n = 40 - 16 = 24$

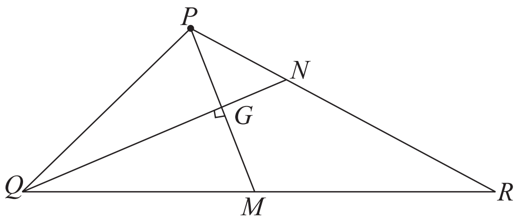
7. **(c)** Let ΔPQR

Given, $QR^2 + PR^2 = 5PQ^2$

Median PM and QN intersect at G.

$\Rightarrow\ PN = NR = \dfrac{1}{2}PR$ &

$QM = MR = \dfrac{1}{2}QR$



$QG = \dfrac{2}{3}QN, GM = \dfrac{1}{3}PM$

$\Rightarrow QG^2 + GM^2 = \left(\dfrac{2}{3}QN\right)^2 + \left(\dfrac{1}{3}PM\right)^2$

$= \dfrac{4}{9}QN^2 + \dfrac{1}{9}PM^2$

$= \dfrac{4}{9}\left(\dfrac{2PQ^2 + 2QR^2 - PR^2}{4}\right)$

$+ \dfrac{1}{9}\left(\dfrac{2PQ^2 + 2PR^2 - QR^2}{4}\right)$

$= \dfrac{1}{9}\left[\dfrac{8PQ^2 + 8QR^2 - 4PR^2 + 2PQ^2 + 2PR^2 - QR^2}{4}\right]$

$= \dfrac{1}{9}\left[\dfrac{10PQ^2 + 7QR^2 - 2PR^2}{4}\right]$

$= \dfrac{1}{9}\left[\dfrac{2QR^2 + 7QR^2}{4}\right] = \dfrac{1}{4}QR^2 = QM^2$

$QG^2 + GM^2 = QM^2 \qquad \therefore \angle QGM = 90^\circ$

8. **(c)** Perimeter $= \dfrac{1}{4} \times 2\pi r + 2r$

$= \left(\dfrac{1}{2} \times \dfrac{22}{7} \times 7 + 2 \times 7\right)$ cm = 25 cm

9. **(a)** $\Delta ABC \sim \Delta ANM$

$\therefore\ \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ANM} = \dfrac{AC^2}{AM^2}$ ...(i)

$\Delta ABC \sim \Delta MPC$

$\therefore\ \dfrac{\text{Area of } \Delta ABC}{\text{Area of } \Delta MPC} = \dfrac{AC^2}{MC^2}$ ...(ii)

From Eqs. (i) and (ii,) we get

$\dfrac{\text{Area of } \Delta ANM}{\text{Area of } \Delta MPC} = \dfrac{AM^2}{MC^2}$

$\dfrac{\text{Area of } \Delta ANM + \text{Area of } \Delta MPC}{\text{Area of } \Delta MPC} = \dfrac{AM^2 + MC^2}{MC^2}$

Now, Area of ΔANM + Area of ΔMPC

= Area of ΔABC – Area of $BNMP$

Using Area of BNMP $= \dfrac{5}{18}$ of area of ΔABC

$\therefore\ \dfrac{13}{18}\dfrac{(\text{Area of } \Delta ABC)}{(\text{Area of } \Delta MPC)} = \dfrac{AM^2 + MC^2}{MC^2}$ ...(iii)

From Eq. (iii), $\dfrac{13}{18}\left(\dfrac{AC^2}{MC^2}\right) = \dfrac{AM^2 + MC^2}{MC^2}$

$\Rightarrow\ 13\,(AM + MC)^2 = 18\,(AM^2 + MC^2)$

$\Rightarrow\ \dfrac{AM}{MC} = 5, \dfrac{1}{5}$. Hence, option (a) is correct.

10. **(a)** Let $a_1, a_2, a_3, ..., a_{100}$ be non-zero real number and

$a_1 + a_2 + a_3 + ... + a_{100} = 0$

$a_i \cdot 2^{a_i} > a_i$ and $a_i \cdot 2^{-a_i} < a_i$

$\therefore\ \displaystyle\sum_{i=1}^{100} a_1 \cdot 2^{a_i} > \sum_{i=1}^{100} a_i$ and $\displaystyle\sum_{i=1}^{100} a_1 \cdot 2^{-a_i} < \sum_{i=1}^{100} a_i$

$\Rightarrow\ \displaystyle\sum_{i=1}^{100} a_1 \cdot 2^{a_i} > 0$ and $\displaystyle\sum_{i=1}^{100} a_1 \cdot 2^{-a_i} < 0$

Hence, option (a) is correct.

11. **(b)**

12. **(a)** Condition for infinite many solutions.

$$\frac{p}{12}=\frac{3}{p}=\frac{p-3}{p}\left\{\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}\right\}$$

$p^2 = 36$; $p =$ {From I and II}

$p^2 - 3p = 3p$ {From II and III}

$p = 6$

$\therefore$ $p = 6$

13. **(c)** $x^2 - 4 = (x - 2)(x + 2)$ are the factors

$\therefore$ $x = 2, -2$ are roots of polynomial

$\therefore$ at $x = 2$; $P(2) = 2(2)^3 + k_1(2)^2 + k_2(2) + 12 = 0$

$\Rightarrow 16 + 4k_1 + 2k_2 + 12 = 0 \Rightarrow 2k_1 + k^2 = -14$ ...(i)

at $x = 2$; $P(-2) = 2(-2)^3 + k_1(-2)^2 + k_2(-2) + 12 = 0$

$\Rightarrow -16 + 4k_1 - 2k_2 + 12 = 0$

$\Rightarrow 2k_1 - k_2 = 2$ ...(ii)

From (i) & (ii), $k_1 = -3$ $\therefore$ $k_1 + k_2 = -11$

14. **(c)** Radius of outer concentric circle = (35 + 7) m = 42 m.

Area of path $= \pi\,(42^2 - 35^2)\text{ m}^2 = \frac{22}{7}(42^2 - 35^2)\text{ m}^2$

15. **(b)** $9\sec^2 A - 9\tan^2 A = 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1 = 9$.

16. **(a)** Total three digit number are : $3 \times 3 \times 2 = 18$

Now, numbers divisible by 5 are :

$2 \times 3 \times 1 + 2 \times 2 \times 1 = 10$

So, probability that the slip bears a number divisible by $5 = \frac{10}{18} = \frac{5}{9}$

17. **(d)** $x - y = 2$ .... (i)

$kx + y = 3$ .... (ii)

Adding (i) and (ii), we have

$kx + x = 5 \Rightarrow x(k + 1) = 5 \Rightarrow x = \frac{5}{k+1}$

Putting the value of x in equation (i), we have

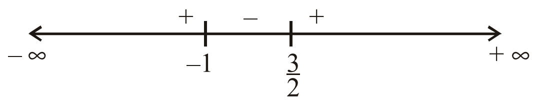
$$\frac{5}{k+1} - y = 2$$

$$\Rightarrow \frac{5}{k+1} - 2 = y \Rightarrow \frac{5-2k-2}{k+1} = y \Rightarrow y = \frac{3-2k}{k+1}$$

y should be positive as they intersect in 1st quadrant

Therefore, $y > 0$

$$\frac{3-2k}{k+1} > 0 \Rightarrow \frac{2k-3}{k+1} < 0$$



$\therefore$ k should lie between – 1 and 3/2

18. **(c)** Let $x = 0.\overline{235}$ ...(i)

$1000x = 235.\overline{235}$ ...(ii)

Subtract (i) from (ii), $999x = 235 \Rightarrow x = \frac{235}{999}$

19. **(c)** Joining B to O and C to O

Let the radius of the outer cirlce be r

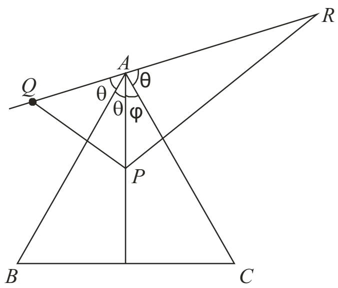
$\therefore$ perimeter $= 2\pi r$

But $OQ = BC = r$ [diagonals of the square BQCO]

$\therefore$ Perimeter of $ABCD = 4r$.

Hence, ratio $= \frac{2\pi r}{4r} = \frac{\pi}{2}$

20. **(c)** Here, ABC is a triangle & P be interior point of a ΔABC, Q and R be the reflections of P in AB and AC, respectively.



As QAR are collinear

$\therefore$ $\angle QAR = 180°$

Q is reflection of P on AB

$\therefore$ $\angle QAB = \angle BAP$

R is reflection of P on AC

$\therefore$ $\angle RAC = \angle CAP$

$\angle QAR = 180°$

$\therefore$ $2\angle BAP + 2\angle CAP = 180°$

$\angle BAP + \angle CAP = 90° \Rightarrow \angle BAC = 90°$

21. **(d)** Let us consider that $n^2 + 3$ is divisible by 17

$\therefore$ $n^2 + 3 = 17K$ $[K \in N]$

$\Rightarrow n^2 = 17K - 3 \Rightarrow n^2 = 3(17m - 1)$ $[\because K = 3m]$

$3(17m - 1)$ is a perfect square, which is not possible.

$\therefore$ $n^2 + 3$ is never divisible by 17.

In, $n^2 + 4$, put $n = 9$

So, $(9)^2 + 4 = 81 + 4 = 85$ which is divisible by 17.

$\therefore$ I is true and II is false.

22. **(c)**

23. **(c)** $\dfrac{2\tan 30°}{1-\tan^2 30°} = \dfrac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2}$

$$= \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \sqrt{3} = \tan 60°.$$

24. **(c)** Let the number of people in first and second village be x and y respectively.

According to given condition,

average income of x people = P and

average income of y people = Q, where $P \neq Q$

$\therefore$ Total income of people in two villages are Px and Qy respectively.

When, one person moves from first village to second village.

Then, number of people in first village = $x - 1$ and in second village = $y + 1$.

New average income = P' and Q'

(Total income = no. of persons × average income)

$\therefore$ Total income = $P'(x - 1)$ and $Q'(y + 1)$

Total income in both cases are same

$\therefore$ $Px + Qy = P'(x - 1) + Q'(y + 1)$

$\Rightarrow$ $Px - P'(x - 1) = Q'(y + 1) - Qy$

$\Rightarrow$ $x(P - P') + P' = y(Q' - Q) + Q'$

$\therefore$ $P' \neq P$ and $Q' \neq Q$

Hence, option (c) is not possible.

25. **(a)** $x = -1$ is the root of the quadratic polynomial $p(x)$

So, quadratic polynomial $p(x) = k(x + 1)^2$

$p(-2) = k(-2 + 1)^2 = 2 \Rightarrow k = 2 \;\therefore\; p(x) = 2(x + 1)^2$

Also, $p(2) = 2(2 + 1)^2 = 2 \times 3 \times 3 = 18$

26. **(b)**

27. **(d)** Given $2x + y = 10$

on adding y both sides, we get, $2x + y + y = 10 + y$

$\Rightarrow$ $2(x + y) = 10 + y \Rightarrow x + y = 5 + \dfrac{y}{2}$

Now, $(x + y)_{max}$ when y is maximum & maximum value of y will be 10. ($\because$ $y = 10 - 2x$)

So $(x + y)_{max} = 5 + 5 = 10$ & $(x + y)_{min}$ when $y = 0$

$\therefore$ minimum value of $x + y = 5$

So, sum of $(x + y)_{max}$ & $(x + y)$min = 15

28. **(d)**

29. **(a)** Here, when A = 0°

LHS = sin 2 A = sin 0° = 0

and RHS = 2 sin A= 2 sin 0° = 2 × 0 = 0

In the other options, we will find that

LHS ≠ RHS

30. **(c)** $\dfrac{1}{7} = 0.\overline{142857}$

The second positive integer whose reciprocal have six different repeating decimals is

$$\frac{1}{13} = 0.\overline{076923}$$

And the third positive integer whose reciprocal have six different repeating decimals is

$$\frac{1}{21} = 0.\overline{047619}$$

Therefore, the values of x are 7, 13, 21

Hence, the required sum is = 7 + 13 + 21 = 41

31. **(d)**

32. **(c)** Let distance = d,

Time taken upstream = $\dfrac{d}{15-5} = \dfrac{d}{10}$

Time taken downstream = $\dfrac{d}{15+5} = \dfrac{d}{20}$

Hence, average speed

$$= \frac{2d}{\frac{d}{10}+\frac{d}{20}} = \frac{2d \times 20}{3d} = \frac{40}{3}\text{km/hr}$$

Ratio = $\dfrac{40}{3} : 15 = 40 : 45 = 8 : 9$

33. **(b)** $P(x) = ax^3 + 4x^2 + 3x - 4$

$P(3) = 27a + 36 + 9 - 4 = 27a + 41$

$P(x) = x^3 - 4x + a$; $P(3) = 27 - 12 + a = 15 + a$

$\therefore 27a + 41 = 15 + a \Rightarrow a = -1$

34. **(a)** $P(E) + P(\overline{E}) = 1$

35. **(d)** $\dfrac{1-\tan^2 45°}{1+\tan^2 45°} = \dfrac{1-(1)^2}{1+(1)^2} = 0.$

36. **(a)** Let the required number be 33a and 33b.
Then 33a + 33b = 528 $\Rightarrow$ a + b =16.
Now, co-primes with sum 16 are (1, 15), (3, 13), (5, 11) and (7, 9).
$\therefore$ Required numbers are (33 × 1, 33 × 15), (33 × 3, 33 × 13), (33 × 5, 33 × 11), (33 × 7, 33 × 9).
The number of such pairs is 4.

37. **(b)** Upstream speed = 4 km/hr and time = x hrs.
Downstream speed = 8 km/hr and
time taken = $x/2$ hrs.
Hence average speed = $\frac{4x+8\times x/2}{x+x/2}=\frac{16}{3}$ km/hr.

38. **(c)**

39. **(a)** $\frac{2\tan 30°}{1+\tan^2 30°}=\frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$

$=\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}}=\frac{2}{\sqrt{3}}\times\frac{3}{4}=\frac{\sqrt{3}}{2}=\sin 60°.$

40. **(d)** For given numbers,
$(55)^{725}$, unit digit = 5; $(73)^{5810}$, unit digit = 9
$(22)^{853}$, unit digit = 2
Unit digit in the expression
$55^{725}+735^{810}+22^{853}$ is 6

41. **(c)** $\frac{AB}{A'B'}=\frac{AC}{A'C'}\Rightarrow\frac{5}{15}=\frac{3}{A'C'}$
$\Rightarrow$ A′C′ = 9 cm

42. **(a)** $\frac{AB}{A'B'}=\frac{BC}{B'C'}\Rightarrow\frac{5}{15}=\frac{BC}{12}$
$\Rightarrow$ BC = 4 cm

43. **(b)** $\because \angle A = \angle A' = 80°$

44. **(a)** $\because \angle B = \angle B' = 60°$

45. **(b)** $\because \angle A + \angle B + \angle C = 180°$
$80° + 60° + \angle C = 180°$
$\angle C = 40°$

46. **(b)** **47.** **(a)** **48.** **(c)**

49. **(a)** **50.** **(d)**