ME - 2006

Test Booklet Code **B** **Test Booklet No.** 348210

This booklet contains 12 pages.
DO NOT open this Test Booklet until you are asked to do so.

Important Instructions :-

1. The MATHEMATICS test is consist of **40** questions. Each question carries **1** mark. For each correct response the candidate will get **1** mark. For each incorrect response, ¼ mark will be deducted. The maximum marks are **40**.
2. The Test is of **1 hour** duration.
3. Use **Black Ball Point Pen only** for writing particulars on OMR Answer Sheet marking ● responses.
4. Rough work is to be done on the space provided for this purpose in the Test Booklet only.
5. **On completion of the test, the candidate must handover the Answer Sheet to the Invigilator in the Room / Hall. The candidates are allowed to take away this Test Booklet with them.**
6. The CODE for this Booklet is **B.** Make sure that the CODE printed on the Answer Sheet is the same as that on this booklet . In case of discrepancy, the candidate should immediately report the matter to the Invigiltor for replacement of both the Test Booklet and the Answer Sheet.
7. The candidate should ensure that the Answer Sheet is not folded. Do not make any stray marks on the Answer Sheet.
8. Do not write your Seat No. anywhere else, except in the specified space in the Test Booklet / Answer Sheet.
9. Use of white fluid for correction is not permissible on the Answer Sheet.
10. Each candidate must show, on demand his / her Admission Card to the Invigilator.
11. No candidate, without special permission of the Superintendent or Invigilator, should leave his / her seat.
12. Use of Manual Calculator is permissible.
13. The candidate should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and must sign the Attendance Sheet (Patrak-01). Cases where a candidate has **not** signed the Attendance Sheet (Patrak-01) be deemed not to have handed over the Answer Sheet and dealt with as a unfair means case.
14. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules ans Regulations of the Board.
15. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.
16. The candidates will write the Correct Test Booklet Code as given in the Test Booklet / Answer Sheet in the Attendance Sheet. (Patrak-01)

SEAL

Candidate's Name:.....

Exam.Seat No. (in figures).....(in words).....

Name of Exam. Centre:.....Exam. Centre No:.....

Test Booklet Code : ..... Test Booklet No.: .....

Candidate's Sign..... *Block Supt. Sign* .....

1. What is the equation of the auxillary circle of $\frac{x^2}{16} - \frac{y^2}{9} = 1$?

A) $x^2 + y^2 = 9$
B) $x^2 + y^2 = 25$
C) $x^2 + y^2 = 7$
D) $x^2 + y^2 = 16$

2. Find the equation of tangent to the hyperbola $4x^2 - y^2 = 64$, which is parallel to the line $8x - 6y + 11 = 0$.

A) $2x + y = 1$
B) Tangent does not exists
C) $3x + y = 1$
D) $x + 3y = 1$

3. For non-zero vectors $\bar{x}, \bar{y} \in R^3$ if $\bar{x} \cdot \bar{y} = |\bar{x}| \cdot |\bar{y}|$, then what is $\bar{x} \times \bar{y} = ?$

A) $\bar{0}$
B) Unit vector
C) 0
D) $|\bar{x}|\,|\bar{y}|$

4. Obtain $\lim_{x \to 2} \frac{(Cos\,\alpha)^x + (Sin\,\alpha)^x - 1}{x - 2}$, where $\left(0 < \alpha < \frac{\pi}{2}\right)$

A) $Sin^2\alpha\, Log_e\, Sin\alpha - Cos^2\alpha\, Log_e\, Cos\alpha$
B) $Cos^2\alpha\, Log_e\, Sin\alpha - Sin^2\alpha\, Log_e\, Cos\alpha$
C) $Cos^2\alpha\, Log_e\, Cos\alpha - Sin^2\alpha\, Log_e\, Sin\alpha$
D) $Cos^2\alpha\, Log_e\, Cos\alpha + Sin^2\alpha\, Log_e\, Sin\alpha$

5. For the unit vectors $\bar{a}$ and $\bar{b}$, if $\bar{a} + \bar{b} = \bar{0}$, then what is $\bar{a} \cdot \bar{b}$?

A) -1
B) 2
C) 0
D) 1

(Space for Rough Work)

6. What is the area of ΔABC if $\overrightarrow{AB} = \bar{i} + 2\bar{j} + 3\bar{k}$ and $\overrightarrow{AC} = -3\bar{i} + 2\bar{j} + \bar{k}$ in ΔABC?

A) $3\sqrt{5}$ B) $\frac{3}{2}\sqrt{5}$

C) 45 D) $5\sqrt{3}$

7. What is the magnitude of the resultant force of the forces $(1, 2, -1)$ and $(-3, 0, 2)$?

A) 5 B) $\sqrt{5}$

C) 9 D) 3

8. If $f(x) = 2^x + 2^{x+1} + 2^{x+2} + \ldots\ldots + 2^{x+9}$, then find $f'(2)$

A) $2^x \, Log\, 2$ B) None of these

C) $1023\, Log_e 16$ D) $1023\, Log_e 2$

9. What is the direction of the line passing through the points $(2, 1, 3)$ and $(3, 2, -1)$?

A) $(-1, -1, 4)$ B) $(1, 1, -2)$

C) $(1, 1, 4)$ D) $(-1, -1, -4)$

10. What is the equation of line passing through $(1, 2, 3)$ and perpendicular to $3x + 4y - 5z = 6$?

A) $\frac{x-1}{3} = \frac{y-2}{4} = \frac{3-z}{5}$ B) $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z+5}{3}$

C) $\frac{1-x}{3} = \frac{y-2}{4} = \frac{z-3}{-5}$ D) $\frac{x-1}{3} = \frac{2-y}{4} = \frac{3-z}{5}$

(Space for Rough Work)

11. What is the perpendicular distance between $2x + 2y - z + 1 = 0$ and $x + y - \frac{z}{2} + 2 = 0$?

A) $\sqrt{5}$ B) $\sqrt{2}$

C) 2 D) 1

12. What is the radius of the sphere whose extrimities of the diameter are $(1, -1, 1)$ and $(-1, 1, 1)$?

A) $2\sqrt{2}$ B) 1

C) $\sqrt{2}$ D) 2

13. Find $\int e^{3Log\,x}\left(x^4 + 1\right)^{-1} dx$

A) $-Log\left(x^4 + 1\right) + c$ B) None of these

C) $Log\left(x^4 + 1\right) + c$ D) $\frac{1}{4} Log\left(x^4 + 1\right) + c$

14. $\underset{x \to 0+}{Lim} \frac{Sin\sqrt{x}}{\sqrt[4]{x}} = ?$

A) -1 B) Does not exists

C) 0 D) 1

15. $\underset{x \to 0}{Lim} \frac{f(Cos\,x)}{x^2} = ?$ Where $f(x) = \frac{1-x}{1+x}$

A) $\frac{1}{4}$ B) $\frac{1}{5}$

C) $\frac{1}{2}$ D) $\frac{1}{3}$

(Space for Rough Work)

16. If $\left\{x \middle/ \frac{1}{|3x+2|} \leq \frac{1}{5} ; x \in R-\left(-\frac{2}{3}\right)\right\}$, then what is the complement set of this set ?

A) $\left(1, \frac{7}{3}\right)$ B) $\left(-\frac{7}{3}, 1\right)$

C) $R-\left(1, \frac{7}{3}\right)$ D) $R-\left(-\frac{7}{3}, 1\right)$

17. $\frac{d}{dx}\left[Sec^{-1} e^{2x}\right] = ?$

A) $\frac{2}{\sqrt{e^{4x}-1}}$ B) $\frac{2}{e^{2x}\sqrt{e^{4x}-1}}$

C) $\frac{1}{\sqrt{1-e^{4x}}}$ D) $-\frac{1}{\sqrt{1-e^{4x}}}$

18. $\frac{d}{dx}\left[Tan^{-1}\left(\frac{x+a}{1-ax}\right)\right] = ?$ (Where $x \in R^+$, $a \in R^+$, $ax < 1$)

A) $\frac{1}{1+a^2x^2}$ B) $-\frac{1}{1+a^2x^2}$

C) $\frac{1}{1+x^2}$ D) $\frac{-1}{1+x^2}$

19. $\frac{d}{dx}\left[e^{Sin^{-1}x + Cos^{-1}x}\right] = ?$ (Where $|x| \leq 1$)

A) $\frac{\pi}{2}$ B) $-\frac{\pi}{2}$

C) $e^{\pi/2}$ D) 0

20. If there remains an error of 4% in measuring the area of a circle, then what is the error in percentage in the radius of a circle ?

A) 6% B) 8%

C) 2% D) 4%

(Space for Rough Work)

21. If $(2, 3)$ lies on $y^2 = ax^3 + b$, slope of tangent at $(2, 3)$ is 4, then what is the value of a?

A) -7
B) 7
C) 2
D) -2

22. What is the rate of change of the volume of sphere w.r.t. its surface area when its radius is 2 units ?

A) 2 units
B) None of these
C) 1 unit
D) 3 units

23. $\int e^{-2Log\,x}\,dx = ?$ (where $x \neq 0$)

A) $-\frac{1}{x} + c$
B) $-\frac{2}{x} + c$
C) $\frac{1}{x} + c$
D) $\frac{x^3}{3} + c$

24. $\int \left(Sin^{-1}x + Cos^{-1}x\right) dx = ?$

A) $x + c$
B) $-\frac{\pi x}{2} + c$
C) Not possible
D) $\frac{\pi x}{2} + c$

25. $\int \frac{3Tan\frac{x}{3} - Tan^3\frac{x}{3}}{1 - 3\,Tan^2\frac{x}{3}}\,dx = ?$

A) $-Log\left|Cos\,x\right| + c$
B) $Sec^2 x + c$
C) $Log\left|Tan\,x\right| + c$
D) $-Log\left|Sec\,x\right| + c$

(Space for Rough Work)

26. $\int_0^{\pi/2} \frac{(Sin\,x)^{2006}}{(Sin\,x)^{2006} + (Cos\,x)^{2006}}\,dx = ?$

A) $\frac{\pi}{2}$

B) None of these

C) $2006 \cdot (Sin\,x)^{2007}$

D) $\frac{\pi}{4}$

27. If $\int_{\sqrt{2}}^{k} \frac{1}{x\sqrt{x^2-1}}\,dx = \frac{\pi}{12}$, then what is the value of k?

A) 0

B) 1

C) -2

D) 2

28. If $\int_0^a f(x)\,dx = m$ and fx is an even and continuous function defined on $[-a, a]$, then evaluate $\int_{-a}^{a} f(x)\,dx \quad (a \in R^+)$

A) $2m$

B) 0

C) $m/2$

D) m

29. Find the degree of $(y_2)^2 - \sqrt{y_1} = y^3$

A) 3

B) 4

C) $1/2$

D) 2

30. A particle is moving with a uniform acceleration. If it covers a distance of 7.25 and 9.25 meters in 5^{th} and 9^{th} seconds respectively, then what is its initial velocity ?

A) 5.5 m/s

B) 7 m/s

C) 10 m/s

D) 5 m/s

(Space for Rough Work)

31. For right angle triangle ABC, $AB = AC$. What is the circumcentre of ΔABC having the vertices are $A(1,1), B(5,1), C(1,4)$?

A) $(3, 5/2)$
B) $(3, 1)$
C) $(1, 5/2)$
D) $(6, 5)$

32. $A(2,-3)$ and $B(-5,1)$ are two vertices of the ΔABC. Its centroid is on x-axis and vertices C is on y-axis, then find the co-ordinates of C.

A) $(2, 0)$
B) $(0, 2)$
C) $(0, -2)$
D) $(-2, 0)$

33. If $A(11, 7), B(-1, k)$ and $C(5, -1)$ are the vertices of ΔABC and $m\angle ACB = \frac{\pi}{2}$, then what is k ?

A) $4/3$
B) None of these
C) -5
D) 5

34. Obtain the angle between the lines $\{(x,0) / x \in R\}$ and $\{(0, y) / y \in R\}$

A) $-\frac{\pi}{2}$
B) 0
C) π
D) $\frac{\pi}{2}$

35. If $kx + 2y - 1 = 0$ and $6x - 4y + 2 = 0$ are identical lines, then determine k.

A) 6
B) $1/6$
C) 3
D) -3

(Space for Rough Work)

36. Line $12x + 5y + 60 = 0$ intersects the axes in A and B respectively, then what is the equation of the circle whose diameter as $\overline{AB}$?

A) $x^2 + y^2 - 5x + 12y = 0$

B) $x^2 + y^2 + 5x + 12y = 0$

C) $x^2 + y^2 - 5x - 12y = 0$

D) $x^2 + y^2 + 5x - 12y = 0$

37. If $(1, 2)$ is the centre of a circle passing through the point $(4, 6)$, then what is the length of a circumference of this circle ?

A) 25π

B) None of these

C) 5π

D) 10π

38. What is the value of t such that $P(t)$ and $Q(3)$ are the end points of a focal chord of a parabola $y^2 = 4ax$?

A) $t = 3$

B) $t = -3$

C) $t = -1/3$

D) $t = 1/3$

39. What is the area of a triangle whose vertices are vertex and end points of latus-rectum of $x^2 = 12y$?

A) 36

B) None of these

C) 9

D) 18

40. Find the length of major axis and its eccentricity of $\frac{x^2}{16} + \frac{y^2}{25} = 1$.

A) $2/5$ and $3/5$

B) None of these

C) $5/2$ and $\sqrt{3/5}$

D) 10 and $3/5$

(Space for Rough Work)