QUESTION PAPER CODE 65/1/MT

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 3 \\ -2 & -3 & 0 \end{pmatrix}$ or any other correct example ½ + ½ m

2. Order : 2, degree : 2, Product : 4 ½ + ½ m

3. $\frac{dy}{dx} = -\alpha A \sin \alpha x + \alpha B \cos \alpha x$ ½ m

$\frac{d^2y}{dx^2} = -\alpha^2 (A \cos \alpha x + B \sin \alpha x)$

$\frac{d^2y}{dx^2} + \alpha^2 y = 0$ ½ m

4. Projection of $\vec{a}$ on $\vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$ ½ m

Projection $= \frac{5}{\sqrt{2}}$ ½ m

5. Value $= 3$ 1 m

6. Writing dr's correctly ½ m

D.C'S $\frac{3}{13}, \frac{4}{13}, \frac{12}{13}$ ½ m

SECTION - B

7.

	M W C	Expenses		Family expenses
Family A Family B Family C	$\begin{pmatrix} 2 & 3 & 1 \\ 2 & 1 & 3 \\ 4 & 2 & 6 \end{pmatrix}$	$\begin{pmatrix} 200 \\ 150 \\ 200 \end{pmatrix}$	=	$\begin{pmatrix} 1050 \\ 1150 \\ 2300 \end{pmatrix}$

2 m

Expenses for family A = ₹ 1050

Expenses for family B = ₹ 1150

Expenses for family C = ₹ 2300 } 1 m

Any relevant impact 1 m

8. $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{2} - \tan^{-1}z$ 1 m

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \cot^{-1}z$ 1 m

$\tan^{-1}\left(\frac{x+y}{1-xy}\right) = \tan^{-1}\left(\frac{1}{z}\right)$ as $z > 0$ 1 m

$\frac{x+y}{1-xy} = \frac{1}{z}$ ½ m

$xy + yz + zx = 1$ ½ m

9. $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$

$R_1 \to R_1 + R_2 + R_3$

$(a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ 1 m

$C_1 \to C_1 - C_2,\ C_2 \to C_2 - C_3$

$$(a+b+c)\begin{vmatrix} 0 & 0 & 1 \\ b-c & c-a & a \\ c-a & a-b & b \end{vmatrix} = 0$$ 2 m

$(a + b + c)(ab + bc + ca - a^2 - b^2 - c^2) = 0$

given $a \neq b \neq c$, so $ab+bc+ca-a^2-b^2-c^2 \neq 0$ ½ m

$\Rightarrow (a+b+c) = 0$ ½ m

10. Let $x = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ 1 m

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$

$$\begin{pmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$$ 1½ m

$a + 4b = -7,\ c + 4d = 2,\ 2a + 5b = -8,\ 2c + 5d = 4$

Solving $a = 1,\ b = -2,\ c = 2,\ d = 0$ 1 m

$\therefore x = \begin{pmatrix} 1 & -2 \\ 2 & 0 \end{pmatrix}$ ½ m

OR

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$|A| = 1 \neq 0$, A^{-1} will exist ½ m

$$\text{adj } A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$ (Any four correct Cofactors : 1 mark) 2 m

$$A^{-1} = \frac{\text{adj } A}{|A|} = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix}$$ ½ m

$$A^{-1} A = \begin{pmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ 1 m

11. $f(x) = |x-3| + |x-4|$

$$= \begin{cases} 7-2x, & x<3 \\ 1, & 3 \le x < 4 \\ 2x-7, & x \ge 4 \end{cases}$$ 1 m

L. H. D at $x = 3$ $\lim_{x\to 3^-} \frac{f(x) - f(3)}{x-3}$

$$\lim_{x\to 3^-} \frac{6-2x}{x-3} = -2$$

R. H. D at $x = 3$ $\lim_{x\to 3^+} \frac{f(x) - f(3)}{x-3}$

$$= \frac{1-1}{x-3} = 0$$

L. H. D $\neq$ R. H. D $\therefore$ $f(x)$ is not diffrentiable at $x = 3$ 1½ m

L. H. D at x = 4 $\lim\limits_{x \to 4^-} \dfrac{f(x) - f(4)}{x - 4}$

$= \dfrac{1 - 1}{x - 4} = 0$

R. H. D at x = 4 $\lim\limits_{x \to 4^+} \dfrac{f(x) - f(4)}{x - 4}$

$\lim\limits_{x \to 4^+} \dfrac{2x - 7 - 1}{x - 4} = 2$

L. H. D at x = 4 $\neq$ R.H.D at x = 4

f (x) is not differentiable at x = 4 1½ m

12. $y = x^{e^{-x^2}}$

$\log y = e^{-x^2} \log x$ 1 m

Diff. w. r. t x

$\dfrac{1}{y} \dfrac{dy}{dx} = \dfrac{e^{-x^2}}{x} + \log x \, e^{-x^2} (-2x)$ 2 m

$\dfrac{dy}{dx} = y \left(\dfrac{e^{-x^2}}{x} - 2x \log x \, e^{-x^2} \right)$ ½ m

$= x^{e^{-x^2}} e^{-x^2} \left(\dfrac{1}{x} - 2x \log x \right)$ ½ m

OR

$\log \sqrt{x^2 + y^2} = \tan^{-1} \dfrac{x}{y}$

Diff. w. r. t. x

$$\frac{1}{2\left(x^2+y^2\right)}\left(2x+2y\frac{dy}{dx}\right) = \frac{1}{1+\frac{x^2}{y^2}}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)$$ 2 m

$$\frac{x+y\frac{dy}{dx}}{x^2+y^2} = \frac{y^2}{x^2+y^2}\left(\frac{y-x\frac{dy}{dx}}{y^2}\right)$$ 1 m

$$\frac{dy}{dx}\left(y+x\right) = y-x$$ ½ m

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$ ½ m

13. $$y = \sqrt{x+1} - \sqrt{x-1}$$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}} - \frac{1}{2\sqrt{x-1}}$$ 1 m

$$= \frac{\sqrt{x-1}-\sqrt{x+1}}{2\sqrt{x^2-1}}$$ ½ m

$$4\left(x^2-1\right)\left(\frac{dy}{dx}\right)^2 = y^2$$ ½ m

$$4\left(x^2-1\right)2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2} + 8x\left(\frac{dy}{dx}\right)^2 = 2y\frac{dy}{dx}$$ 1 m

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} = \frac{y}{4}$$ ½ m

$$\left(x^2-1\right)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - \frac{y}{4} = 0$$ ½ m

14. $\int \frac{1-\cos x}{\cos x\,(1+\cos x)}\,dx$

$= \int \frac{1+\cos x - 2\cos x}{\cos x\,(1+\cos x)}\,dx$ 1½ m

$\int \frac{dx}{\cos x} - 2\int \frac{dx}{1+\cos x}$ ½ m

$\int \sec x\,dx - \int \sec^2 \frac{x}{2}\,dx$ 1 m

$\log|\sec x + \tan x| - 2\tan\frac{x}{2} + c$ 1 m

15. $\int x \sin^{-1} x\,dx$

$\frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\,dx$ 1 m

$\frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int \frac{1-x^2-1}{\sqrt{1-x^2}}\,dx$ ½ m

$\Rightarrow \frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\int\sqrt{1-x^2}\,dx - \frac{1}{2}\int\frac{dx}{\sqrt{1-x^2}}\,dx$ 1 m

$\frac{x^2}{2}\sin^{-1}x + \frac{1}{2}\left(\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right) - \frac{1}{2}\sin^{-1}x + c$ 1½

or $\frac{x^2}{2}\sin^{-1}x + \frac{x}{4}\sqrt{1-x^2} - \frac{1}{4}\sin^{-1}x + c$

16. $\int_0^2 \left(x^2 + e^{2x+1}\right) dx$

$h = \frac{2}{n}$ ½ m

$\int_0^2 \left(x^2 + e^{2x+1}\right) dx = \lim_{h\to 0} h\ [f(0) + f(0+h) + f(0+2h) + ...$

$+ ....f(0+n-1)h\ ]$ 1 m

$= \lim_{h\to 0} h\ [h^2\left(1^2 + 2^2 + ... + (n-1)^2\right)$

$+ e\left(1 + e^{2h} + e^{4h} + ...e^{2(n-1)h}\right)]$ 1 m

$= \lim_{h\to 0} \frac{(nh)(nh-h)(2nh-h)}{6}$ ½ m

$+ \lim_{h\to 0} e.h.\left(\frac{e^{2nh} - 1}{e^{2h} - 1}\right)$ ½ m

$= \frac{8}{3} + \frac{(e^4 - 1)e}{2} = \frac{8}{3} + \frac{e^5 - e}{2}$ ½ m

OR

$\int_0^\pi \frac{x \tan x\ dx}{\sec x\ \text{cosec}\ x}$

$\int_0^\pi x \sin^2 x\ dx$ 1 m

Let $I = \int_0^\pi x \sin^2 x\ dx$

$$= \int_0^{\pi} (\pi - x) \sin^2 (\pi - x)\ dx$$ ½ m

$$= \int_0^{\pi} (\pi - x) \sin^2 x\ dx$$ ½ m

$$2I = \pi \int_0^{\pi} \sin^2 x\, dx = \pi \int_0^{\pi} \frac{1 - \cos 2x}{2}\ dx$$ ½ m

$$= \frac{\pi}{2} \left[x - \frac{\sin 2x}{2} \right]_0^{\pi}$$ 1 m

$$= \frac{\pi^2}{2}$$

$$I = \frac{\pi^2}{4}$$ ½ m

17. $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda$ ½ m

$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} = \mu$ ½ m

$x = 3\lambda + 1,\ y = -\lambda + 1,\ z = -1$ 1 m

$x = 2\mu + 4,\ y = 0,\ z = 3\mu - 1$

At the point of intersection

$\lambda = 1,\ \mu = 0$ 1 m

so $3\lambda + 1 = 4 = 2\mu + 4$ ½

Hence the lines are intersecting

Point of intersection is (4, 0, –1) ½ m

18. Coordinats of Q are $-3\mu+1,\ \mu-1,\ 5\mu+2$ ½ m

D.R's of $\vec{PQ}$ $-3\mu-2,\ \mu-3,\ 5\mu-4$ 1 m

as $\vec{PQ}$ is parallel to the plane $x-4y+3z=1$

$1(-3\mu-2)-4(\mu-3)+3(5\mu-4)=0$ 1½ m

$\mu=\frac{1}{4}$ 1 m

OR

The D.R's of the line are 2, –6, 4 1 m

mid point of the line 2, 1, –1 1 m

The plane passes through (2, 1, –1) and is perpendicular to the plane

eqn. : $2(x-2)-6(y-1)+4(z+1)=0$

$x-3y+2z+3=0$ 1 m

Vector from : $\vec{r}\cdot\left(\hat{i}-3\hat{j}+2\hat{k}\right)+3=0$ 1 m

19. No's divisible by 6 .... = 16 1m

No's divisible by 8 .... = 12 1m

No's not divisible by 24 .... = 20 1m

Required probabilty $=\frac{20}{100}=\frac{1}{5}$ 1 m

SECTION - C

20. For every $a \in A$, $(a, a) \in R$

$\because \ |a-a| = 0$ is divisible by 2

$\therefore$ R is reflexive — 1 m

For all $a, b \in A$

$(a, b) \in R \Rightarrow |a-b|$ is divisible by 2

$\Rightarrow |b-a|$ is divisible by 2

$\therefore \ (b, a) \in R \quad \therefore$ R is symmetric — 1 m

For all $a, b, c \in A$

$(a, b) \in R \Rightarrow |a-b|$ is divisible by 2

$(b, c) \in R \Rightarrow |b-c|$ is divisible by 2

So, $a-b = \pm 2k$ — 1 m

$\underline{b-c = \pm 2\ell}$

$a-c = \pm 2m$

$\Rightarrow |a-c|$ is divisible by 2

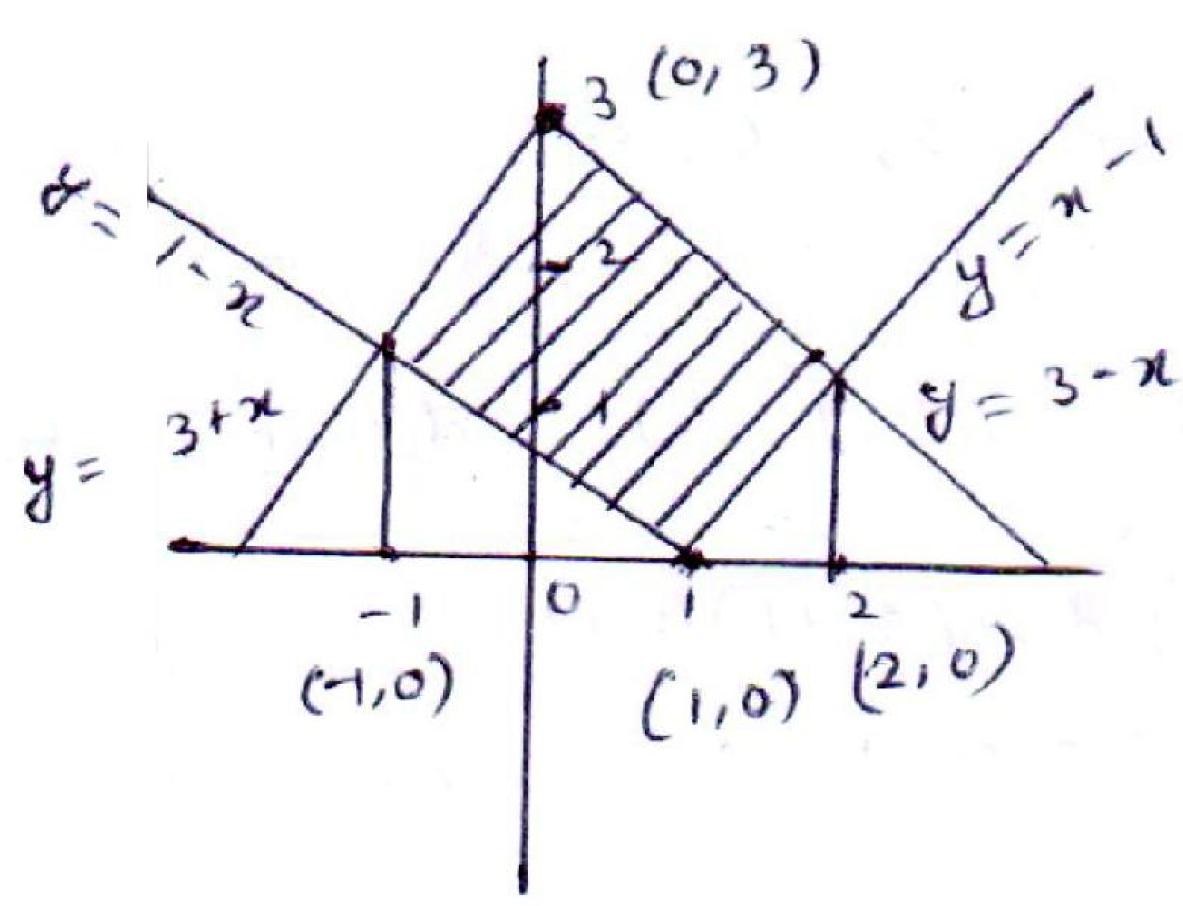
$\Rightarrow (a, c) \in R$

$\Rightarrow$ R is transitive — 1 m

Showing elements of $\{1, 3, 5\}$ and — 1 m

$\{2, 4\}$ are related to each other

and $\{1, 3, 5\}$ and $\{2, 4\}$ are not related to each other — 1 m

21. 

Graph — 2 + 2 m

Area of shaded reigon

$$= \int_{-1}^{0}(3+x+x-1)\,dx + \int_{0}^{2}(3-x)\,dx - 2\int_{1}^{2}(x-1)\,dx$$ 1 m

$$= 2\left.\frac{(x+1)^2}{2}\right]_{-1}^{0} - \left.\frac{(3-x)^2}{2}\right]_{0}^{2} - 2\left.\frac{(x-1)^2}{2}\right]_{1}^{2}$$

$$= 1 - \frac{1}{2}(1-9) - 1 = 4 \text{ sq. units}$$ 1 m

22. $y = \frac{x}{1+x^2}$

$$\frac{dy}{dx} = \frac{1-x^2}{(1+x^2)^2}$$ 2 m

Let $f(x) = \frac{1-x^2}{(1+x^2)^2}$

$$f'(x) = 0 \Rightarrow \frac{-2x(3-x^2)}{(1+x^2)^3} = 0$$

For max or min $x(3-x^2) = 0 \Rightarrow x = 0$ or $x = \pm\sqrt{3}$ 2 m

Calculating $\frac{d^2f(x)}{dx^2}$ at $x = 0 < 0$

at $x = \pm\sqrt{3} > 0$ } 1 m

$\Rightarrow$ $x = 0$ is the point of local maxima

$\Rightarrow$ the required pt is (0, 0) } 1 m

23. $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$

Let $y = vx, \frac{dy}{dx} = v + x\frac{dv}{dx}$ ½ m

$v + x\frac{dv}{dx} = \frac{v^2}{v-1}$ 1½ m

$x\frac{dv}{dx} = \frac{v}{v-1}$

$\frac{dx}{x} = \left(\frac{v-1}{v}\right) dv$ 1½ m

$\int \frac{dx}{x} = \int \left(1 - \frac{1}{v}\right) dv$

$\log x = v - \log v + c$ 1 m

$\log y = \frac{y}{x} + c$ or $x \log y - y = c x$ 1½ m

OR

$\sin 2x \frac{dy}{dx} - y = \tan x$

$\frac{dy}{dx} - \frac{y}{\sin 2x} = \frac{\tan x}{\sin 2x}$ 1 m

$\frac{dy}{dx} - y(\text{cosec } 2x) = \frac{\sec^2 x}{2}$

$P = -\text{cosec } 2x, \; Q = \frac{1}{2} \sec^2 x$

$\int P\, dx = -\int \text{cosec } 2x\, dx$

$= -\frac{1}{2} \log |\tan x|$

So $e^{\int P\,dx} = \dfrac{1}{\sqrt{\tan x}}$ 1½ m

Solution is

$$\frac{y}{\sqrt{\tan x}} = \frac{1}{2}\int \frac{\sec^2 x\,dx}{\sqrt{\tan x}} \quad \left(\begin{array}{c} \sqrt{\tan x} = t \\ \Rightarrow \dfrac{1}{2}\dfrac{\sec^2 x\,dx}{\sqrt{\tan x}} = dt \end{array}\right)$$ 1½ m

$$\frac{y}{\sqrt{\tan x}} = \sqrt{\tan x} + c$$ 1 m

Getting $c = 1$ ½ m

$\Rightarrow y = \tan x - \sqrt{\tan x}$ ½ m

24. Eqn. of plane

$(x+y+z-6)+\lambda(2x+3y+4z+5) = 0$ 2 m

it passes through $(1,1,1)$

$-3+14\lambda = 0 \Rightarrow \lambda = \dfrac{3}{14}$ 2 m

Eqn. of plane will be

$20x+23y+26z-69=0$ 1 m

vector from : $\vec{r}\cdot\left(20\hat{i}+23\hat{j}+26\hat{k}\right) = 69$ 1 m

25. Let E_1 be the event of following course of meditation and yoga and E_2 be the event of following course of drugs 1 m

$P(E_1) = \frac{1}{2}, \ P(E_2) = \frac{1}{2}$ 1 m

$P(A|E_1) = \frac{70 \times 40}{100 \times 100} \quad P(A|E_2) = \frac{75}{100} \times \frac{40}{100}$ 1 m

Formula 1 m

$$P(E_1|A) = \frac{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100}\right)}{\frac{40}{100}\left(\frac{1}{2} \times \frac{70}{100} + \frac{1}{2} \times \frac{75}{100}\right)}$$

$$= \frac{70}{145} = \frac{14}{29}$$ 2 m

26. Let the no. of items in the item $A = x$

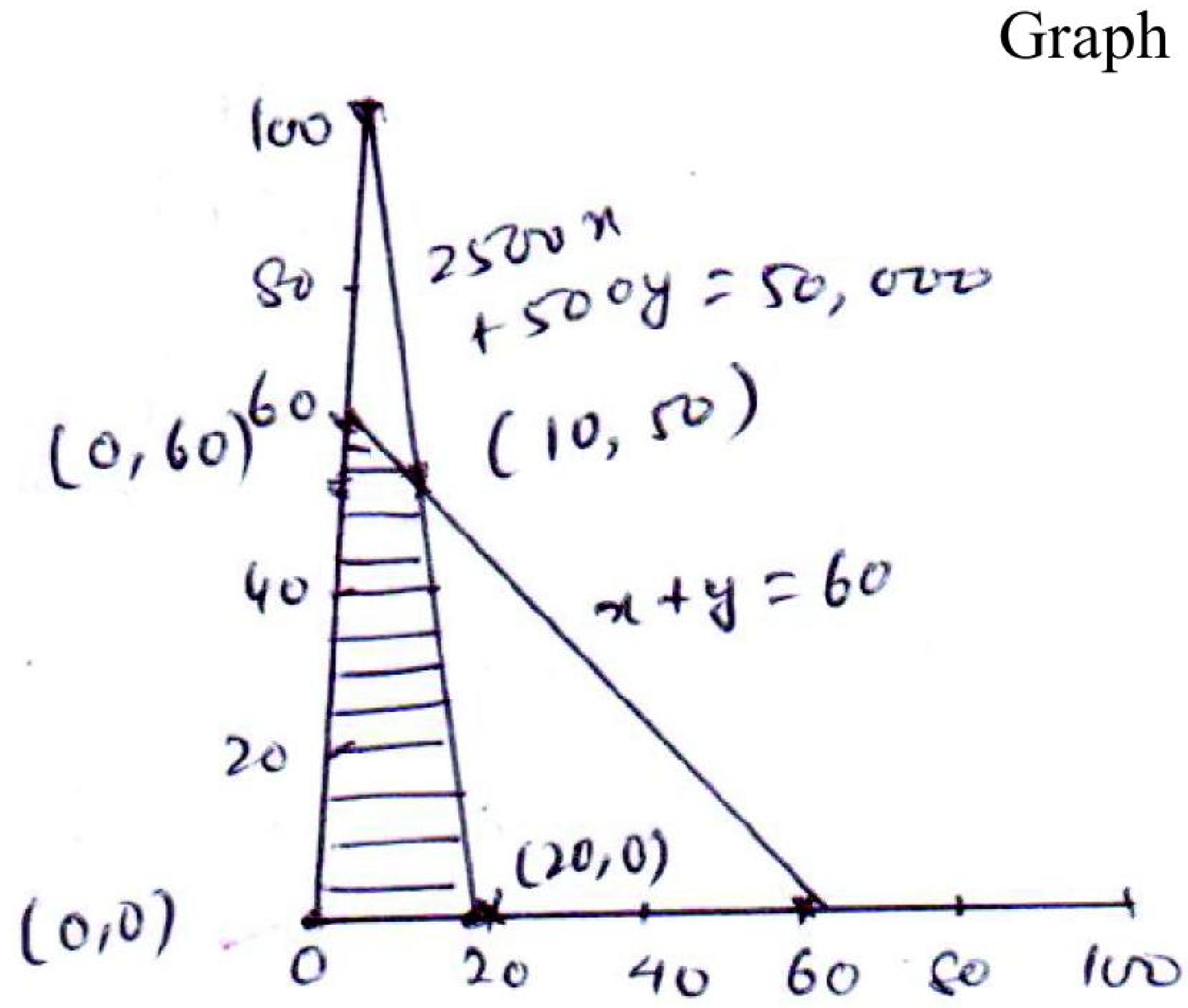
Let the no. of items in the item $B = y$

(Maximize) $z = 500x + 150y$ 1 m

$x + y \leq 60$

$2500x + 500y \leq 50,000$ Graph 2 m

$x, y \geq 0$



$z(0,0) = 0$

$z(10, 50) = 12,500$

$z(20, 0) = 10,000$ 2 m

$z(0, 60) = 9,000$

Max. Profit = Rs. 12,500 1 m

OR

Let the no. of packets of food X = x

Let the no. of packets of food Y = y

(minimize) $P = (6x + 3y)$ 1 m

subject to

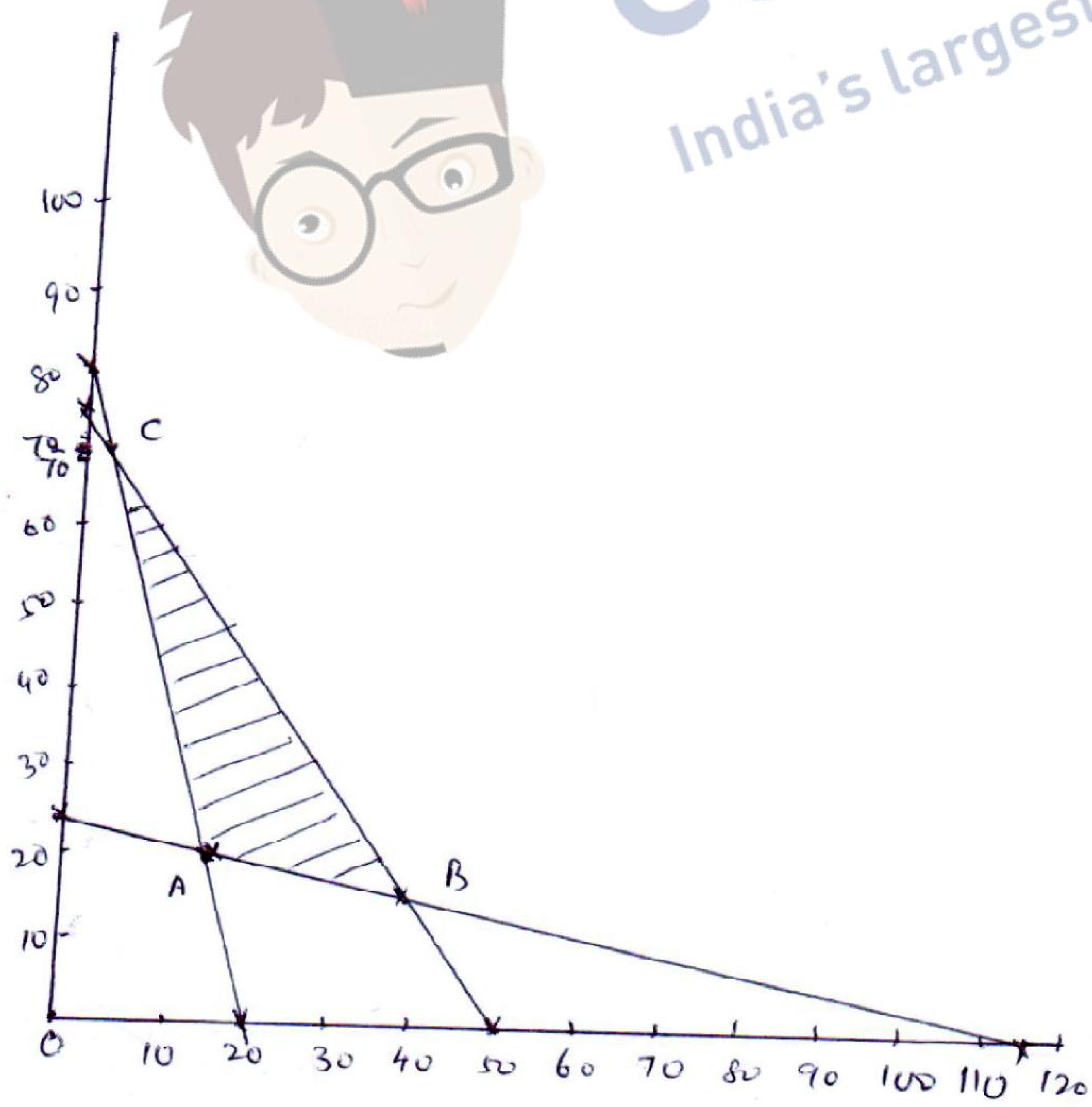
$12x + 3y \geq 240$

$4x + 20y \geq 460$

$6x + 4y \leq 300, \; x, y \geq 0$

or

$$\left.\begin{aligned} 4x + y &\geq 80 \\ x + 5y &\geq 115 \\ 3x + 2y &\leq 150 \\ x, y &\geq 0 \end{aligned}\right\} \quad \text{2 m}$$



Correct points of feasible region

A (15, 20), B (40, 15), C (2, 72)

So P (15, 20) = 150

P (40, 15) = 285

P (2, 72) = 228

Graph 2 m

minimum amount of vitamin A = 150 units when 15 packets of food X and 20 packets of food Y are used 1 m