6/10/17

Date :
Page No :

$2+3+5 = 10m$

DIFFERENTIAL EQUATION

Definition: DE : An equation involving derivatives of the dependent variable wrt independent variable is called a DE.

Eg: $3\frac{d^3y}{dx^3} - 2y\frac{dy}{dx} + 7y = 0$

$2y\frac{dy}{dx} - y = 0$

There are 2 types of DE's

(a) Partial differential equation (PDE) :

DE's involving derivatives of the dependent variable wrt more than one indepent variables, are called PDEs

Eg: $\frac{\partial p}{\partial x} + \frac{\partial p}{\partial y} + \frac{\partial p}{\partial z} = 7$; ∂ = doe

(b) Ordinary differential equation (ODE) :

DE's involving derivatives of the dependent variable wrt to only one indepent variable, are called ODE's

Order and degree of DE :

(a) Order : Order of a DE is defined as the order of highest order derivative involved in the given DE.

(b) Degree : Degree of DE is the highest power of the highest order derivative in the given DE.

NOTE: Order and degree should be found only in the absence of radical powers (powers should be only in whole no's).

** If the derivative is present, as variables $\cos\left(\frac{dy}{dx}\right)$ or $e^{\frac{dy}{dx}}$ or $\log\left(\frac{dy}{dx}\right)$, then degree is not defined. ($\because$ such derivatives do not form a polynomial in y')

EXERCISE 9.1

Determine order and degree:

1] $\frac{d^4y}{dx^4} + \sin(y''') = 0$

Order (O) - 4

Degree (D) - not defined.

2] $(y')' + 5y = 0$ O - 1, D - 1

3] $\left(\frac{ds}{dt}\right)^4 + 3s\frac{d^2s}{dt^2} = 0$ O - 2, D - 1

4] $\left(\frac{d^2y}{dx^2}\right)^2 + \cos\left(\frac{dy}{dx}\right) = 0$ O - 2, D - not defined

5] $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$ O - 2, D - 1

6] $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$ O - 3, D - 2

7] $y''' + 2y'' + y' = 0$, O = 3, D - 1

8] $y' + y = e^x$ O - 1, D - 1

9] $y'' + (y')^2 + 2y = 0$ O - 2, D - 1

10] $y'' + 2y' + \sin y = 0$ O - 2, D - 1

11] $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ O - 2, D - not defined

12] $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ O - 2, D - 1

Eg1] (a) $\frac{dy}{dx} - \cos x = 0$ O - 1 , D - 1

(b) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$ O - 2 , D - 1

(c) $y''' + y^2 e^{y'} = 0$ O - 3 , D - not defined

GS and PS of a DE :

GS - General solution ; PS - Particular solution

* GS is the solution consisting of the constant c / c and c_1
* Using initial conditions, substituting for c and c_1 in GS, we get PS

EXERCISE 9.2

In the following problems, verify that the given functions is a solution of the corresponding DE

1] $y = e^x + 1$: $y'' - y' = 0$

$y' = e^x$

$y'' = e^x$

RHS $= y'' - y' = e^x - e^x = 0 =$ RHS

Hence verified.

2] $y = x^2 + 2x + C$: $y' - 2x - 2 = 0$

$y' = 2x + 2$

$y' - 2x - 2 = 0$

Hence verified

3] $y = \cos x + C$: $y' + \sin x = 0$

$y' = -\sin x$

$y' + \sin x = 0$ Hence verified

7] $xy = \log y + C$; $y' = \dfrac{y^2}{1-xy}$

$xy' + y = \dfrac{1}{y} \cdot y'$

$y'\left(\dfrac{1}{y} - x\right) = y$

$y'\left(\dfrac{1-xy}{y}\right) = y$

$y' = \dfrac{y^2}{1-xy}$ Hence verified

9] $x + y = \tan^{-1} y$: $y^2y' + y^2 + 1 = 0$

$1 + y' = \dfrac{1}{1+y^2} \cdot y'$

$1 = \left[\dfrac{1}{1+y^2} - 1\right] y'$

$1 = \left(\dfrac{1 - 1 - y^2}{1+y^2}\right) y'$

$1 + y^2 = -y^2 y'$

$y^2 y' + y^2 + 1 = 0$

** 11] The no. of arbitrary constants in the GS of a DE of 4^{th} order are :

(a) 0 (c) 3

(b) 2 (d) 4

12] The no. of arbitrary constants in PS of a DE of 3^{rd} order are :

(a) 3 (c) 1

(b) 2 (d) 0

4] $y = \sqrt{1+x^2}$: $y' = \frac{xy}{1+x^2}$

$y' = \frac{\cancel{2}x}{\cancel{2}\sqrt{1+x^2}}$

$y' = \frac{x}{\sqrt{1+x^2}} \cdot \frac{\sqrt{1+x^2}}{\sqrt{1+x^2}} = \frac{xy}{1+x^2}$

5] $y = ax$: $xy' = y$

$y' = a$

$y' = \frac{y}{x}$ $\quad y = ax \Rightarrow a = \frac{y}{x}$

$xy' = y$

6] $y = x \cdot \sin x$: $xy' = y + x\sqrt{x^2 - y^2}$

$y' = \sin x + x\cos x$

$y' = \sin x + x\sqrt{1-\sin^2 x}$

$y' = \sin x + \sqrt{x}\sqrt{x}\sqrt{1-\sin^2 x}$

$y' = \sin x + \sqrt{x^2 - x^2\sin^2 x}$

Multiply by x throughout

$xy' = x\sin x + x\sqrt{x^2 - x^2\sin^2 x}$

$xy' = y + x\sqrt{x^2 - y^2}$

8] $y - \cos y = x \quad : (y\sin y + \cos y + x)y' = y$

$y' + \sin y\, y' = 1$

$y'[\sin y + 1] = 1$

Multiply by y throughout , $[y = x + \cos y]$

$yy'(\sin y + 1) = y$

$y'(x + \cos y)(\sin y + 1) = y$

$y'[x\sin y + x + \cos y \sin y + \cos y] = y$

$y'[(y - \cos y)\sin y + x + \cos y \sin y + \cos y] = y$

$y'[y\sin y - \cancel{\cos y \sin y} + x + \cancel{\cos y \sin y} + \cos y] = y$

$y'[y\sin y + \cos y + x] = y$

10] $y = \sqrt{a^2 - x^2} \quad : x + y\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{-\cancel{2}x}{\cancel{2}\sqrt{a^2 - x^2}}$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{a^2 - x^2}}$$

Multiply by y on both sides,

$$y\frac{dy}{dx} = \frac{-x}{\cancel{\sqrt{a^2 - x^2}}}\cancel{\sqrt{a^2 - x^2}}$$

$$x + y\frac{dy}{dx} = 0$$

(3m) Formation of DE :

To eliminate one constant , differentiate once.

To eliminate two constants , differentiate twice.

EXERCISE 9.3

1] $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{1}{a} + \frac{1}{b}y' = 0$

$y' = \frac{-b}{a}$

$y'' = 0$

2] $y^2 = a(b^2 - x^2)$

$\cancel{2}yy' = a(-\cancel{2}x)$

$\frac{yy'}{x} = -a$

$\frac{x\left[yy'' + (y')^2\right] - yy'}{x^2} = 0$

$xyy'' + x(y')^2 - yy' = 0$

** 6] Form DE of the family of circles ~~such~~ touching y-axis at origin.

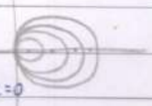
NOTE: General circle : $x^2+y^2+2gx+2fy+c=0$; center $\equiv (-g,-f)$

• circle passes through origin $\Rightarrow c = 0$

• centre on x-axis $\Rightarrow f = 0$

• centre on y-axis $\Rightarrow g = 0$

Sol] Coaxial system :



general eq: $x^2+y^2+2gx+2fy+c=0$

center on x-axis $\Rightarrow f=0$, passing through $(0,0) \Rightarrow c=0$

$\therefore$ Req circle : $x^2+y^2+2gx+0=0 \longrightarrow (1)$

$2x+2y\frac{dy}{dx}+2g=0 \longrightarrow (2)$

Using (2) in (1)

$x^2+y^2+\left(-2x-2y\frac{dy}{dx}\right)x=0$

$y^2-x^2-2xy\frac{dy}{dx}=0$

** Eg 7] Form DE of family of circles touching x-axis at origin.

Sol] Passes through $(0,0) \Rightarrow c=0$

Centre on y-axis $\Rightarrow g=0$

$\therefore$ Family of 0^{les} : $x^2+y^2+2fy=0 \longrightarrow (1)$

$2x+2y\frac{dy}{dx}+2f\frac{dy}{dx} \longrightarrow (2)$

Using ① in ② :

$$2x + 2y\frac{dy}{dx} + \frac{dy}{dx}\left(\frac{-x^2-y^2}{y}\right) = 0$$

multiply by 'y'

$$2xy + 2y^2\frac{dy}{dx} - x^2\frac{dy}{dx} - y^2\frac{dy}{dx} = 0$$

$$2xy + (y^2 - x^2)\frac{dy}{dx} = 0$$

** Eg 6] Form DE of family of ellipses having foci on x-axis and centre at origin.

Sol] Gn: Family of ellipes :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (a>b)$$

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\cancel{2}x}{a^2}\cdot\frac{b^2}{\cancel{2}y}$$

$$\frac{yy'}{x} = -\frac{b^2}{a^2}$$

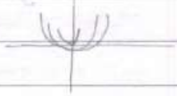
$$\frac{x[yy'' + (y')^2] - yy'}{x^2} = 0$$

$$xyy'' + x(y')^2 - yy' = 0$$

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

7] Form DE of family of parbolas having vertex at origin and axis along +ve y-axis.

Sol] Gn : Family of parabola's



$$x^2 = 4ay \rightarrow (1)$$

$$2x = 4a\frac{dy}{dx}$$

$$2x = \frac{x^2}{y}\frac{dy}{dx}$$

$$2xy - x^2\frac{dy}{dx} = 0$$

8] Form DE of family of ellipes having foci on y-axis and centre at origin.

Sol] Gn: family of ellipes :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a < b)$$

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{\cancel{2}x}{a^2}\cdot\frac{b^2}{\cancel{2}y}$$

$$\frac{yy_1}{x} = \frac{-b^2}{a^2}$$

$$\frac{x(yy_2 + y_1^2) - yy_1}{x^2} = 0$$

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

9] Form DE of family of hyperbolas having foci on x-axis and center at origin

Sol] Gn : Family of hyperbola :

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{2x}{a^2} - \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{\cancel{2}x}{a^2}\cdot\frac{b^2}{\cancel{2}y}$$

$$\frac{yy_1}{x} = \frac{b^2}{a^2}$$

$$\frac{x(y_0 y_2 + y_1^2) - y y_1}{x^2} = 0$$

$$xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$$

10] Form DE of family of circles having center on y-axis and radius 3 units.

Sol] Centre on y-axis $= (0, k)$, $r = 3$

Eq: $(x-h)^2 + (y-k)^2 = r^2$

$$x^2 + (y-k)^2 = 9 \Rightarrow y - k = \sqrt{9 - x^2}$$

$$x^2 + y^2 + k^2 - 2yk - 9 = 0$$

$$2x + 2y\frac{dy}{dx} - 2k\frac{dy}{dx} = 0$$

$$x + y\frac{dy}{dx} - k\frac{dy}{dx} = 0$$

$$x + \frac{dy}{dx}(y - k) = 0$$

$$x + \frac{dy}{dx}\left(\sqrt{9 - x^2}\right) = 0$$

$$\left(\frac{dy}{dx}\right)^2 (9 - x^2) = x^2$$

(or)

$$y - \sqrt{9 - x^2} = k$$

$$\frac{dy}{dx} + \frac{2x}{2\sqrt{9 - x^2}} = 0$$

DE's can be solved in many methods :

→ Variable separable ~~form~~ method :

- Here we seperate the variables in given DE, integrate wrt corresponding variables and find G.S. Substituting initial values, get value of C and write P.S.

→ Homogeneous DE :

(•) If every term is of the same degree in given DE, then it is homogeneous DE.

(a) If $\left(\frac{y}{x}\right)$ is dominative, then put $y = vx$

$$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$

Now, DE reduces to variable seperable form.

(b) If $\left(\frac{x}{y}\right)$ is dominative, the put $x = vy$

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

(5m) → Linear DE (LDE):

(a) Form: $\frac{dy}{dx} + Py = Q$

Integrating factor (IF) $= e^{\int P dx}$

GS : $y(IF) = \int Q(IF)dx$

(b) Form : $\frac{dx}{dy} + Px = Q$

IF : $e^{\int P dy}$

GS : $x(IF) = \int Q(IF)dy$

EXERCISE 9.4

1] $\frac{dy}{dx} = \frac{1-\cos x}{1+\cos x}$

$$\int dy = \int \frac{1-\cos x}{1+\cos x}dx$$

$$y = \int \frac{\cancel{2}\sin^2\frac{x}{2}}{\cancel{2}\cos^2\frac{x}{2}}dx = \int \tan^2(x/2)dx = \int \sec^2\frac{x}{2} - 1\, dx$$

GS : $y = 2\tan\frac{x}{2} - x + c$

2] $\frac{dy}{dx} = \sqrt{4-y^2}$

$$\int \frac{dy}{\sqrt{4-y^2}} = \int dx$$

GS : $\sin^{-1}\left(\frac{y}{2}\right) = x + c$

3] $\frac{dy}{dx} + y = 1$

$\frac{dy}{dx} = 1 - y$

$\int \frac{dy}{1-y} = \int dx$

$\frac{\log|1-y|}{-1} = x + c$

GS : $\log\left|\frac{1}{1-y}\right| = x + c$

** 4] $\sec^2 x \tan y \, dx + \sec^2 y \cdot \tan x \, dy = 0$

$\sec^2 x \tan y \, dx = -\sec^2 y \cdot \tan x \, dy$

$\int \frac{\overbrace{\sec^2 x}^{f'(x)}}{\underbrace{\tan x}_{f(x)}} dx = \int -\frac{\sec^2 y}{\tan y} dy$

$\log|\tan x| + \log|\tan y| = \log c$

$\log|\tan x \tan y| = \log c$

$\tan x \tan y = c$

5] $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$

$(e^x + e^{-x})dy = (e^x - e^{-x})dx$

$\int dy = \int \left(\frac{\overbrace{e^x - e^{-x}}^{f'(x)}}{\underbrace{e^x + e^{-x}}_{f(x)}}\right) dx$

GS : $y = \log|e^x + e^{-x}| + c$

6] $\frac{dy}{dx} = (1 - x^2)(1 + y^2)$

$\int \frac{dy}{1+y^2} = \int (1 - x^2) dx$

GS: $\tan^{-1} y = x - \frac{x^3}{3} + C$

7] $y \log y \, dx - x \, dy = 0$

$y \log y \, dx = x \, dy$

$\int \frac{1}{x} dx = \int \frac{1}{y \log y} dy$

$\log|x| = \log|\log y| + \log C \Rightarrow \log|x| = \log|C \log y|$

$|x| = C \log y$

8] $x^5 \frac{dy}{dx} = -y^5$

$\int \frac{dy}{y^5} = -\int \frac{dx}{x^5}$

$\frac{y^{-4}}{-4} + \frac{x^{-4}}{-4} = C$

GS: $\frac{1}{y^4} + \frac{1}{x^4} = -4C = C$

9] $\frac{dy}{dx} = \sin^{-1} x$

$\int dy = \int \sin^{-1} x \cdot 1 \, dx$

$y = \sin^{-1} x \cdot x - \int x \frac{1}{\sqrt{1-x^2}} dx$

$y = \sin^{-1} x \cdot x - \frac{1}{2} \int \frac{2x}{\sqrt{1-x^2}} dx$ $\quad 1 - x^2 = t$, $-2x \, dx = dt$

$y = x \cdot \sin^{-1} x + \frac{1}{2} \int \frac{dt}{\sqrt{t}}$

GS: $y = x \sin^{-1} x + \sqrt{1-x^2} + C$

10] $e^x \tan y\, dx + (1-e^x)\sec^2 y\, dy = 0$

$e^x \tan y\, dx = -(1-e^x)\sec^2 y\, dy$

$$\int \frac{-e^x}{(1-e^x)}dx = \int \frac{+\sec^2 y}{\tan y}dy$$

$$\log|1-e^x| = \log|\tan y| + C$$

$$\log\left|\frac{1-e^x}{\tan y}\right| = \overset{\log}{C}$$

$$\frac{1-e^x}{\tan y} = C$$

For each of the following DE's, find PS satisfing the given conditio

11] $(x^3+x^2+x+1)\frac{dy}{dx} = 2x^2+x \quad ; \; y=1,\; x=0$

$$dy = \frac{2x^2+x}{x^2(x+1)+1(x+1)}dx$$

$$\int dy = \int \frac{2x^2+x}{(x^2+1)(x+1)}dx \longrightarrow (1)$$

$$\frac{2x^2+x}{(x^2+1)(x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$2x^2+x = A(x^2+1) + (Bx+C)(x+1)$$

$x=-1$: $\quad 1 = A(2) \Rightarrow A = \frac{1}{2}$

coeff x^2: $\quad 2 = A+B \Rightarrow B = \frac{3}{2}$

coeff x: $\quad 1 = B+C \Rightarrow C = -\frac{1}{2}$

(1) $\Rightarrow \displaystyle\int dy = \frac{1}{2}\int\frac{dx}{x+1} + \frac{1}{2}\int\frac{3x-1}{x^2+1}dx$

$$y = \frac{1}{2}\log|x+1| + \frac{3}{(2)2}\int\frac{2x}{x^2+1}dx - \frac{1}{2}\int\frac{1}{x^2+1}dx$$

GS: $y = \frac{1}{2}\log|x+1| + \frac{3}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + C$

$x=0,\; y=1$

$1 = 0+0-0+C \Rightarrow C=1$

PS: $y = \frac{1}{2}\log|x+1| + \frac{3}{4}\log|x^2+1| - \frac{1}{2}\tan^{-1}x + 1$

12] $x(x^2-1)\frac{dy}{dx} = 1 \quad ; y=0, x=2$

$$dy = \frac{dx}{x(x^2-1)} = \frac{dx}{x(x-1)(x+1)}$$

$$\int dy = \int \frac{-1}{x}dx + \frac{1}{2}\int\frac{dx}{x-1} + \frac{1}{2}\int\frac{dx}{x+1}$$

$$y = -\log|x| + \frac{1}{2}\log|x-1| + \frac{1}{2}\log|x+1| + C$$

$$y = \log\left|\frac{\sqrt{(x+1)(x-1)}}{x}\right| + C$$

GS: $y = \log\left|\frac{\sqrt{x^2-1}}{x}\right| + C$

$x=2, y=0$ $\quad 0 = \log\left|\frac{\sqrt{4-1}}{2}\right| + C$

$$C = \log\left(\frac{2}{\sqrt{3}}\right)$$

PS: $y = \log\left|\frac{\sqrt{x^2-1}}{x}\right| + \log\left(\frac{2}{\sqrt{3}}\right)$

13] $\cos\frac{dy}{dx} = A \quad ; y=2, x=0$

$$\frac{dy}{dx} = \cos^{-1}A$$

$$\int dy = \int \cos^{-1}A\, dx$$

GS: $y = x\cdot\cos^{-1}A + C$

$x=0, y=2 \quad , \quad 2 = 0 + C \quad \Rightarrow C = 2$

PS: $y = x\cos^{-1}A + 2$

14] $\frac{dy}{dx} = y\tan x \quad ; y=1, x=0$

$$\int\frac{dy}{y} = \int\tan x\, dx$$

~~$\log y = \sec^2 x + C$~~

$\log y = \log|\sec x| + \log C$

GS: $y = \sec x\cdot C$

$x=0, y=1$

$1 = \sec 0\, C$

$C = 1$

PS: $y = \sec x$

15] Find the eq. of a curve passing through origin whose DE is $y' = e^x \sin x$

Sol] Gn: $\frac{dy}{dx} = e^x \sin x$, pt = (0,0)

$$\int dy = \int e^x \sin x \, dx \longrightarrow ①$$

$$I = \int e^x \sin x \, dx$$
$$= -e^x \cos x + \int \cos x \, e^x \, dx$$
$$I = -e^x \cos x + e^x \sin x - \int \sin x \, e^x \, dx$$
$$2I = e^x(\sin x - \cos x)$$
$$I = \frac{e^x}{2}(\sin x - \cos x)$$

① ⇒ $y = \frac{e^x}{2}(\sin x - \cos x) + c$

$x=0, y=0$ $\quad 0 = \frac{e^0}{2}(\sin 0 - \cos 0) + c$

$$0 = \frac{1}{2}(0-1) + c$$
$$c = \frac{1}{2}$$

∴ Curve : $y = \frac{e^x}{2}(\sin x - \cos x) + \frac{1}{2}$

$$\Rightarrow 2y = e^x(\sin x - \cos x) + 1$$

16] For the DE, $xy\frac{dy}{dx} = (x+2)(y+2)$, find the solution curve passing through (1,−1)

Sol] Gn: $xy\frac{dy}{dx} = (x+2)(y+2)$ $\quad$ pt = (1,−1)

$$\int \frac{y\,dy}{y+2} = \int \frac{(x+2)\,dx}{x}$$
$$\int \frac{(y+2)-2}{y+2}\,dy = \int \left(1 + \frac{2}{x}\right)dx$$

$$\int 1 - \frac{2}{y+2}\,dy = \int 1 + \frac{2}{x}\,dx$$

$$y - 2\log|y+2| = x + 2\log|x| + C$$

$x = 1,\ y = -1$

$$-1 - 2\log(1) = 1 + 2\log(1) + C$$

$$C = -2$$

$\therefore$ Curve : $y - 2\log|y+2| = x + 2\log|x| - 2$

$$y = x + \log(x(y+2))^2 - 2$$

23] The GS of DE $\frac{dy}{dx} = e^{x+y}$ is

(a) $e^x + e^{-y} = C$ (c) $e^{-x} + e^{y} = C$

(b) $e^x + e^y = C$ (d) $e^x + e^{-y} + C$

$$\frac{dy}{dx} = e^{x+y}$$

$$\frac{dy}{dx} = e^x e^y$$

$$\int e^{-y}\,dy = \int e^x\,dx$$

$$-e^{-y} = e^x + C$$

$$C = e^x + e^{-y}$$

*** imp

EXERCISE 9.6

Find GS for the following DEs

1] $\frac{dy}{dx} + 2y = \sin x$

Form : $\frac{dy}{dx} + Py = Q$; $P = 2$, $Q = \sin x$

$$IF = e^{\int P\,dx} = e^{\int 2\,dx} = e^{2x}$$

GS : $y(IF) = \int Q(IF)\,dx$

$$ye^{2x} = \int \sin x \cdot e^{2x}\,dx \longrightarrow (1)$$

$$I = \int e^{2x}\sin x\,dx$$
$$= -e^{2x}\cos x + \int \cos x \cdot 2e^{2x}\,dx$$
$$= -e^{2x}\cos x + 2\left[e^{2x}\sin x - \int \sin x \cdot e^{2x}\cdot 2\,dx\right]$$
$$= -e^{2x}\cos x + 2e^{2x}\sin x - 4\int \sin x \cdot e^{2x}\,dx$$
$$5I = e^{2x}(2\sin x - \cos x)$$
$$I = \frac{e^{2x}}{5}(2\sin x - \cos x)$$

(1) $\Rightarrow$ $ye^{2x} = \frac{e^{2x}}{5}(2\sin x - \cos x) + C$

2] $\frac{dy}{dx} + 3y = e^{-2x}$

Form: $\frac{dy}{dx} + Py = Q$; $P = 3$, $Q = e^{-2x}$

$IF = e^{\int P dx} = e^{\int 3 dx} = e^{3x}$

GS : $y(IF) = \int Q(IF)\,dx$

$ye^{3x} = \int e^{-2x}e^{3x}\,dx = \int e^{x}\,dx$

$ye^{3x} = e^{x} + C$

3] $\frac{dy}{dx} + \frac{y}{x} = x^2$

Form : $\frac{dy}{dx} + Py = Q$; $P = \frac{1}{x}$, $Q = x^2$

IF : $e^{\int P dx} = e^{\int \frac{1}{x} dx} = e^{\log x} = x$

GS : $y(IF) = \int Q(IF)\,dx$

$yx = \int x^2 \cdot x\,dx = \int x^3\,dx$

$xy = \frac{x^4}{4} + C$

4] $\frac{dy}{dx} + \sec x \cdot y = \tan x$

Form : $\frac{dy}{dx} + Py = Q$; $P = \sec x$, $Q = \tan x$

$IF = e^{\int P dx} = e^{\int \sec x\, dx} = e^{\log(\sec x + \tan x)} = \sec x + \tan x$

$GS:\ y(IF) = \int Q(IF)\,dx$

$y(\sec x + \tan x) = \int \tan x(\sec x + \tan x)\,dx$

$y(\sec x + \tan x) = \int \tan x \sec x + \tan^2 x\, dx$

$y(\sec x + \tan x) = \int(\sec x \tan x + \sec^2 x - 1)\,dx$

$y(\sec x + \tan x) = \sec x + \tan x - x + C$

**

5] $\cos^2 x \frac{dy}{dx} + y = \tan x$

$\div$ by $\cos^2 x$

$\frac{dy}{dx} + \sec^2 x\, y = \tan x \sec^2 x$

Form : $\frac{dy}{dx} + Py = Q$; $P = \sec^2 x$, $Q = \tan x \sec^2 x$

$IF : e^{\int P dx} = e^{\int \sec^2 x\, dx} = e^{\tan x}$

$GS : y e^{\tan x} = \int \tan x \sec^2 x\, e^{\tan x}\, dx$ | $\tan x = t$

$y e^{\tan x} = \int t e^t\, dt$ | $\sec^2 x\, dx = dt$

$= te^t - \int e^t\, dt$

$y e^{\tan x} = \tan x \cdot e^{\tan x} - e^{\tan x} + C$

$\div$ by $e^{\tan x}$

$y = \tan x - 1 + Ce^{-\tan x}$

6] $x\frac{dy}{dx} + 2y = x^2 \log x$

$\div$ by x

$\frac{dy}{dx} + \frac{2}{x} y = x \log x$

Form : $\frac{dy}{dx} + Py = Q$; $P = \frac{2}{x}$, $Q = x\log x$

$IF = e^{\int \frac{2}{x} dx} = e^{2\log x} = x^2$

$GS : y \cdot x^2 = \int x(\log x)\, x^2\, dx = \int x^3 \log x\, dx$

$x^2 y = \log x \frac{x^4}{4} - \int \frac{x^{4}}{4} \cdot \frac{1}{x}\, dx$

$x^2 y = \frac{x^4}{4} \log x - \frac{1}{16} x^4 + C$

7] $x\log x\dfrac{dy}{dx} + y = \dfrac{2}{x}\log x$ $\quad \div\ x\cdot\log x$

$\dfrac{dy}{dx} + \dfrac{1}{x\log x}y = \dfrac{2}{x^2}$

$P = \dfrac{1}{x\log x}$; $Q = \dfrac{2}{x^2}$

$IF = e^{\int P dx} = e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$

$GS: y(IF) = \int Q(IF)dx$

$y\log x = \int \dfrac{2}{x^2}\log x\, dx$

$= \log x\left(\dfrac{-2}{x}\right) + \int \dfrac{2}{x}\cdot\dfrac{1}{x}dx$

$y\log x = -\dfrac{2\log x}{x} - \dfrac{2}{x} + C$

$y\log x + \dfrac{2\log x}{x} + \dfrac{2}{x} = C$

8] $(1+x^2)dy + 2xydx = \cot x\, dx$

$\dfrac{dy}{dx} + \dfrac{2x}{1+x^2}y = \dfrac{\cot x}{1+x^2}$ $\quad \div$ by $(1+x^2)dx$

Form : $\dfrac{dy}{dx} + Py = Q$ $\Rightarrow$ $P = \dfrac{2x}{1+x^2}$; $Q = \dfrac{\cot x}{1+x^2}$

$IF = e^{\int P dx} = e^{\int \frac{2x}{1+x^2}} = e^{\log(1+x^2)} = 1+x^2$

$GS: y(IF) = \int Q(IF)dx$

$y(1+x^2) = \int \dfrac{\cot x}{1+x^2}(1+x^2)dx$

$y(1+x^2) = \log|\sin x| + C$

9] $x\dfrac{dy}{dx} + y - x + xy\cot x = 0$

$\div$ by x

$$\frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$$

Form : $P = \frac{1}{x} + \cot x$; $Q = 1$

IF : $e^{\int P dx} = e^{\int \frac{1}{x} + \cot x} = e^{\log x + \log \sin x} = x \sin x$

GS : $y(IF) = \int Q(IF)\, dx$

$y \cdot x \cdot \sin x = \int 1 \cdot x \cdot \sin x \, dx$

$xy \sin x = x(-)\cos x + \int \cos x \, dx$

$xy \sin x + x \cos x = \sin x + C$

** 10] $(x+y)\frac{dy}{dx} = 1$

$\cancel{x \frac{dy}{dx} + y \frac{dy}{dx} = 1}$

$x + y = \frac{dx}{dy}$

$\Rightarrow \frac{dx}{dy} - x = y$

Form : $\frac{dx}{dy} + Px = Q$; $P = -1$, $Q = y$

IF : $e^{\int P dy} = e^{\int -1 dy} = e^{-y}$

GS : $x(IF) = \int Q(IF)\, dy$

$xe^{-y} = \int y \cdot e^{-y} dy$

$= y(-e^{-y}) + \int e^{-y} dy$

$xe^{-y} + e^{-y}y + e^{-y} = C$

$\div$ by e^{-y}

$x + y + 1 = \frac{C}{e^{-y}}$

** 11] $y\,dx + (x - y^2)\,dy = 0$

$y\,dx = (y^2 - x)\,dy$ $\div$ by $y\,dy$

$\frac{dx}{dy} + \frac{x}{y} = y$

Form : $\frac{dx}{dy} + Px = Q \Rightarrow P = \frac{1}{y}$; $Q = y$

IF : $e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$

GS : $x(IF) = \int Q(IF)dy$

$xy = \int y^2 dy$

$xy = \frac{y^3}{3} + C$

** 12] $(x+3y^2)\frac{dy}{dx} = y$

$(x+3y^2) = y\frac{dx}{dy}$ $\div$ by y

$\frac{dx}{dy} = \frac{x}{y} + 3y$

$\frac{dx}{dy} - x\left(\frac{1}{y}\right) = 3y$

Form : $P = -\frac{1}{y}$; $Q = 3y$

$IF = e^{\int P dy} = e^{\int -\frac{1}{y}dy} = e^{-\log y} = \frac{1}{y}$

GS : $x(IF) = \int Q(IF)dy$

$x\left(\frac{1}{y}\right) = \int 3y\left(\frac{1}{y}\right)dy$

$\frac{x}{y} = 3y + C$

For each of the following find PS, satisfying the condition.

13] $\frac{dy}{dx} + 2y\tan x = \sin x$; $y=0, x=\frac{\pi}{3}$

Form : $\frac{dy}{dx} + Py = Q$ $\Rightarrow P = 2\tan x$; $Q = \sin x$

$IF : e^{\int P dx} = e^{\int 2\tan x\, dx} = e^{\int 2\log\sec x} = e^{\log\sec^2 x} = \sec^2 x$

GS : $y(IF) = \int Q\cdot(IF)dx$

$y\sec^2 x = \int \sin x\cdot\sec^2 x\, dx = \int\frac{\sin x}{\cos x}\sec x\, dx$

$y\sec^2 x = \int \tan x\sec x\, dx$

GS : $y\sec^2 x = \sec x + C$

$x = \frac{\pi}{3}, y = 0$

$0 = 2 + C$ $\Rightarrow C = -2$

$\Rightarrow$ PS : $y\sec^2 x = \sec x - 2$

14] $(1+x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$; $y=0, x=1$

$\div$ by $(1+x^2)$

$\frac{dy}{dx} + y\left(\frac{2x}{1+x^2}\right) = \frac{1}{(1+x^2)^2}$

Form : $P = \frac{2x}{1+x^2}$, $Q = \frac{1}{(1+x^2)^2}$

IF : $e^{\int \frac{2x}{1+x^2}dx} = e^{\log(1+x^2)} = 1+x^2$

GS : $y(1+x^2) = \int \frac{1}{(1+x^2)^{\cancel{2}}}\cancel{(1+x^2)}\, dx$

$(1+x^2)y = \int \frac{1}{1+x^2}\, dx$

GS : $(1+x^2)y = \tan^{-1}x + C$

$x=1, y=0$

$0 = \frac{\pi}{4} + C \Rightarrow C = -\frac{\pi}{4}$

PS : $(1+x^2)y = \tan^{-1}x - \frac{\pi}{4}$

** 15] $\frac{dy}{dx} - 3y\cot x = \sin 2x$ $y=2, x=\frac{\pi}{2}$

Form : $P = -3\cot x$, $Q = \sin 2x$

IF $= e^{\int -3\cot x\, dx} = e^{-3\log(\sin x)} = e^{\log(\mathrm{cosec}^3 x)} = \mathrm{cosec}^3 x$

GS : $y\cdot \mathrm{cosec}^3 x = \int 2\sin x\cos x \cdot \mathrm{cosec}^3 x\, dx$

$= \int \frac{2\sin x\cos x}{\sin x \cdot \sin^2 x}\, dx = \int 2\,\mathrm{cosec}\, x\cot x\, dx$

GS : $y\,\mathrm{cosec}^3 x = -2\,\mathrm{cosec}\, x + C$

$y = 2, x = \frac{\pi}{2}$

$2(1) = -2(1) + C \Rightarrow C = 4$

$\therefore$ PS : $y\,\mathrm{cosec}^3 x + 2\,\mathrm{cosec}\, x = 4$

18] The IF of the DE : $x\frac{dy}{dx} - y = 2x^2$ is

$\div$ by x $\quad \frac{dy}{dx} - \left(\frac{1}{x}\right)y = 2x$

$P = -\frac{1}{x}$, $Q = 2x$

IF : $e^{\int -\frac{1}{x}dx} = e^{\log\left(\frac{1}{x}\right)} = \frac{1}{x}$

19] IF of the DE $(1-y^2)\frac{dx}{dy} + yx = ay$ $\quad \div$ by $(1-y^2)$

$\frac{dx}{dy} + x\left(\frac{y}{1-y^2}\right) = \frac{ay}{1-y^2}$

$P = \frac{xy}{1-y^2}$; $Q = \frac{ay}{1-y^2}$

IF : $e^{\int P dy} = e^{\int \frac{y}{1-y^2}dy} = e^{\frac{1}{2}\int \frac{+2y}{1-y^2}dy} = e^{-\frac{1}{2}\log(1-y^2)} = \frac{1}{\sqrt{1-y^2}}$

16] Find the eq. of a curve passing through origin given that slope of the tangent to the curve at any point (x,y) is equal to the sum of the coordinates of the point.

Gn : pt $= (0,0)$

m of T $= \frac{dy}{dx}$ = sum of coordinates

$\frac{dy}{dx} = x + y$

$\frac{dy}{dx} - y = x$

Form : $P = -1$, $Q = x$

IF : $e^{\int -1 dx} = e^{-x}$

GS : $ye^{-x} = \int x \cdot e^{-x} dx$

$= -xe^{-x} + \int e^{-x}(-1)dx$

$= -xe^{-x} - e^{-x} + c$

$ye^{-x} = -xe^{-x} - e^{-x} + ce^{x}$ $\quad \div$ by e^{-x}

$y + x = -1 + ce^{x}$

At pt $= (0,0)$ $\quad 0 + 0 = -1 + ce^{0} \Rightarrow c = 1$

$\therefore$ The curve is $\boxed{x + y + 1 = e^{x}}$

17] Find the eq. of the curve passing through (0, 2), given that sum of coordinates of any point on the curve exceeds the magnitude of m of T to the curve at that point by 5.

Gn: pt = (0,2)

m of T + 5 = sum of coordinates

$$\frac{dy}{dx} + 5 = x + y$$

$$\frac{dy}{dx} - y = x - 5$$

Form: $P = -1$, $Q = x - 5$

$$IF = e^{\int -1 dx} = e^{-x}$$

GS: $ye^{-x} = \int (x-5)e^{-x} dx = \int (xe^{-x} - 5e^{-x}) dx$

$$= \left[-xe^{-x} + \int e^{-x} \cdot 1 dx\right] - \int 5e^{-x}$$

$$ye^{-x} = -xe^{-x} - e^{-x} + 5e^{-x} + C \qquad \div \text{ by } e^{-x}$$

$$y = -x - 1 + 5 + Ce^{x}$$

pt = (0,2)

$$2 = 0 - 1 + 5 + Ce^{0} \Rightarrow C = -2$$

∴ The curve is $\boxed{y = 4 - x - 2e^{x}}$

Homogeneous DE:

EXERCISE 9.5

NOTE: * If the variables can't be seperated, then check for homogenety.

* If $F(\lambda x, \lambda y) = \lambda^{\circ} F(x,y)$

Then the given DE is homogeneous, λ representing the degree of components of DE.

** Eg 15] ST the DE $(x-y)\frac{dy}{dx} = x + 2y$ is homogeneous and hence solve it

Gn: $(x-y)\frac{dy}{dx} = x + 2y$

$$\frac{dy}{dx} = \frac{x+2y}{x-y} \begin{matrix} \rightarrow \text{Degree} = 1 \\ \rightarrow \text{Degree} = 1 \end{matrix}$$

$$F(x,y) = \frac{x+2y}{x-y}$$

$$F(\lambda x, \lambda y) = \frac{\lambda(x+2y)}{\lambda(x-y)} = \lambda^{\circ} F(x,y)$$

$\Rightarrow$ Gn DE is homogeneous.

$$\frac{dy}{dx} = \frac{x+2y}{x-y}$$

put $y = vx \Rightarrow v = \frac{y}{x}$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{x+2vx}{x-vx}$$

$$x\frac{dv}{dx} = \frac{1+2v}{1-v} - v = \frac{1+2v-v+v^2}{1-v}$$

$$x\frac{dv}{dx} = \frac{v^2+v+1}{1-v}$$

$$\int \frac{1-v}{v^2+v+1}\,dv = \int \frac{dx}{x}$$

$Nu = A\frac{d}{dx}(de) + B$

$1 - v = A(2v+1) + B$

coeffs v : $-1 = 2A \Rightarrow A = -\frac{1}{2}$

constants : $1 = A + B \Rightarrow B = \frac{3}{2}$

$$\int \frac{1-v}{v^2+v+1}\,dv = \frac{1}{2}\int \frac{-1(2v+1)+\frac{3}{2}}{v^2+v+1}\,dv = \frac{-1}{2}\int\frac{2v+1\,dv}{v^2+v+1} + \frac{3}{2}\int\frac{dv}{v^2+v+1}$$

$v^2+v+1 = t$

$(2v+1)\,dv = dt$

v^2+v+1

$(v+\frac{1}{2})^2 + 1 - \frac{1}{4} = (v+\frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2$

$$\int \frac{1-v}{v^2+v+1}\,dv = \frac{-1}{2}\int\frac{dt}{t} + \frac{3}{2}\int\frac{dt}{(v+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}$$

$$= -\frac{1}{2}\log(v^2+v+1) + \frac{3}{2}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{v+\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right]$$

$$= -\log\sqrt{v^2+v+1} + \sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right)$$

$$-\log\sqrt{v^2+v+1} + \sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right) = \log x + C$$

$$\sqrt{3}\tan^{-1}\left(\frac{2\frac{y}{x}+1}{\sqrt{3}}\right) = \log\left[x\sqrt{\frac{y^2}{x^2}+\frac{y}{x}+1}\right] + C$$

1] $(x^2+xy)\,dy = (x^2+y^2)\,dx$

$$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy} = F(x,y)$$

$$F(\lambda x,\lambda y) = \frac{\lambda(x^2+y^2)}{\lambda(x^2+xy)} = \lambda^0 F(x,y)$$

$\Rightarrow$ Gn DE is homogeneous

$$\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{x^2+v^2x^2}{x^2+x^2v} = \frac{x^2(1+v^2)}{x^2(1+v)}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{1+v} - v = \frac{1+v^2-v-v^2}{1+v}$$

$$x\frac{dv}{dx} = \frac{1-v}{1+v}$$

$$\int\left(\frac{1+v}{1-v}\right)dv = \int\frac{dx}{x}$$

$$\cancel{\int \frac{1+v+v-v}{1-v}\,dv = \log x + c}$$

$$-\int \frac{1+v}{v-1}\,dv = \int\frac{dx}{x}$$

$$-\int \frac{1+(v-1)+1}{v-1}\,dv = \log x + c$$

$$-\int 1 + \frac{2}{v-1}\,dv = \log x + c$$

$$-\left[v + 2\log(v-1)\right] = \log x + c$$

$$\frac{y}{x} + 2\log\left[\frac{y}{x}-1\right] + \log x = C$$

2] $\frac{dy}{dx} = \frac{x+y}{x}$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{x+vx}{x} = 1+v$

$x\frac{dv}{dx} = 1+\cancel{v}-\cancel{v}$

$\int dv = \int \frac{dx}{x}$

$v = \log x + C$

$\frac{y}{x} = \log x + C$

3] $(x-y)dy - (x+y)dx = 0$

$\frac{dy}{dx} = \frac{x+y}{x-y} = F(x,y)$

$F(\lambda x, \lambda y) = \frac{\lambda(x+y)}{\lambda(x-y)} = \lambda^0 F(x,y)$

$\Rightarrow$ Gn DE is homogeneous

$\frac{dy}{dx} = \frac{x+y}{x-y}$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{x+vx}{x-vx}$

$v + x\frac{dv}{dx} = \frac{\cancel{x}(1+v)}{\cancel{x}(1-v)}$

$x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+\cancel{v}-\cancel{v}+v^2}{1-v}$

$x\frac{dv}{dx} = \frac{1+v^2}{1-v}$

$\int \frac{1-v}{1+v^2}\,dv = \int \frac{dx}{x}$

$\int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \log c + C$

$\tan^{-1} v - \frac{1}{2}\log|1+v^2| = \log x + C$

$\tan^{-1}\frac{y}{x} = \frac{1}{2}\log\left|1+\frac{y^2}{x^2}\right| + \log x + C$

$\tan^{-1}\frac{y}{x} = \log|\sqrt{x^2+y^2}| + C$

8] $x\frac{dy}{dx} - y + x\sin\left(\frac{y}{x}\right) = 0$ $\quad \div$ by x

$\frac{dy}{dx} - \frac{y}{x} + \sin\left(\frac{y}{x}\right) = 0$

$\frac{dy}{dx} = \frac{y}{x} - \sin\left(\frac{y}{x}\right)$

$y = xv$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx}{x} - \sin\left(\frac{vx}{x}\right)$

$\cancel{v} + x\frac{dv}{dx} = \cancel{v} - \sin v$

$\int \frac{dv}{\sin v} = -\int \frac{dx}{x}$

$\int \operatorname{cosec} v \, dv = -\int \frac{dx}{x}$

$\log|\operatorname{cosec} v - \cot v| = -\log x + \log c$

$(\operatorname{cosec} v - \cot v)x = C$

$\left(\operatorname{cosec}\frac{y}{x} - \cot\frac{y}{x}\right)x = C$

$\left(\frac{1}{\sin\frac{y}{x}} - \frac{\cos\frac{y}{x}}{\sin\frac{y}{x}}\right)x = C$

$\left(1 - \cos\frac{y}{x}\right)x = C\sin\left(\frac{y}{x}\right)$

9] $y\,dx + x\log\left(\frac{y}{x}\right)dy - 2x\,dy = 0$

$y\,dx + dy\left[x\log\left(\frac{y}{x}\right) - 2x\right] = 0$

$\frac{dy}{dx} = \frac{-y}{x\log\left(\frac{y}{x}\right) - 2x} = \frac{-vx}{x\log v - 2x} = \frac{-v}{\log v - 2}$

$v + x\frac{dv}{dx} = \frac{v}{2 - \log v}$

$x\frac{dv}{dx} = \frac{v}{2 - \log v} - v$

$x\frac{dv}{dx} = \frac{v - 2v + v\log v}{2 - \log v} = \frac{v\log v - v}{2 - \log v} = \frac{v(\log v - 1)}{2 - \log v}$

$$\int \frac{2-\log v}{v(\log v-1)}\,dv = \int \frac{dx}{x}$$

$$\int \frac{1-(\log v-1)}{v(\log v-1)}\,dv = \int \frac{dx}{x}$$

$$\int \left(\frac{1}{v(\log v-1)} - \frac{1}{v}\right) dv = \int \frac{dx}{x}$$

$$\log|\log v-1| - \log|v| = \log|x| + \log c$$

$$\frac{\log \frac{y}{x}-1}{\frac{y}{x}} = xC$$

$$\left(\log \frac{y}{x}-1\right)\frac{x}{y} = xC$$

$$\log \frac{y}{x} - 1 = yC$$

14] $\frac{dy}{dx} - \frac{y}{x} + \operatorname{cosec}\left(\frac{y}{x}\right) = 0$; $y=0$, $x=1$

$$\frac{dy}{dx} = \frac{y}{x} - \operatorname{cosec}\left(\frac{y}{x}\right)$$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{vx}{x} - \operatorname{cosec}\left(\frac{vx}{x}\right)$$

$$x\frac{dv}{dx} = -\operatorname{cosec} v$$

$$\int \frac{dv}{\operatorname{cosec} v} = \int -\frac{dx}{x}$$

$$-\cos v = -\log x + C$$

GS: $\log x - \cos \frac{y}{x} = C$

$x=1, y=0$

$$\log 1 - \cos 0 = C$$

$$-1 = C$$

PS: $\log x - \cos \frac{y}{x} = -1$

$$\log x + \log e = \cos \frac{y}{x}$$

$$\log|xe| = \cos \frac{y}{x}$$

15] $2xy + y^2 - 2x^2\frac{dy}{dx} = 0$; $y = 2, x = 1$

$$2xy + y^2 = 2x^2\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{2xvx + x^2v^2}{2x^2} = \frac{x^2(2v + v^2)}{2x^2}$$

$$\cancel{v} + x\frac{dv}{dx} = \cancel{v} + \frac{v^2}{2}$$

$$\int \frac{2}{v^2}dv = \int \frac{dx}{x}$$

$$\frac{-2}{v} = \log x + C$$

$$\frac{-2}{\frac{y}{x}} = \log x + C$$

GS: $-\frac{2x}{y} = \log x + C$

$y = 2, x = 1$:

$$-1 = C$$

PS: $\log x + \frac{2x}{y} = +1$

$$-\frac{2x}{y} = \log x - 1$$

$$y = \frac{-2x}{\log x - 1}$$

PS: $y = \frac{2x}{1 - \log x}$

6] $x\,dy - y\,dx = \sqrt{x^2+y^2}\,dx$

$x\,dy = \left(\sqrt{x^2+y^2}+y\right)dx$

$\frac{dy}{dx} = \frac{\sqrt{x^2+y^2}+y}{x}$

$y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\cancel{v} + x\frac{dv}{dx} = \frac{\sqrt{x^2+v^2x^2}+vx}{x} = \sqrt{1+v^2}+\cancel{v}$

$x\frac{dv}{dx} = \sqrt{1+v^2}$

$\int\frac{dv}{\sqrt{1+v^2}} = \int\frac{dx}{x}$

$\log\left|v+\sqrt{1+v^2}\right| = \log x + \log c$

$\frac{y}{x} + \frac{\sqrt{x^2+y^2}}{x} = cx$

$y + \sqrt{x^2+y^2} = Cx^2$

10] $\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$

($\frac{x}{y}$ is dominative)

$\left(1+e^{x/y}\right)dx = e^{x/y}\left(\frac{x}{y}-1\right)dy$

put $x = vy$

$\frac{dx}{dy} = v + y\frac{dv}{dy}$

$\frac{dy}{dx} = \frac{\left(1+e^{x/y}\right)}{e^{x/y}\left(\frac{x}{y}-1\right)}$

$\frac{dx}{dy} = \frac{e^{x/y}\left(\frac{x}{y}-1\right)}{\left(1+e^{x/y}\right)}$

$v + y\frac{dv}{dy} = \frac{e^v(v-1)}{(1+e^v)}$

$y\frac{dv}{dy} = \frac{\cancel{ve^v} - e^v - v - \cancel{ve^v}}{1+e^v}$

$\int\frac{1+e^v}{v+e^v}\,dv = -\int\frac{dy}{y}$

$\log\left|v+e^v\right| + \log y = \log c$

$\left(\frac{x}{y}+e^{x/y}\right)\cdot y = c$

11] $(x+y)dy + (x-y)dx = 0$ $y=1,\ x=1$

$\frac{dy}{dx} = \frac{y-x}{x+y}$ $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{vx - x}{x + vx}$

$x\frac{dv}{dx} = \left(\frac{v-1}{1+v}\right) - v$

$x\frac{dv}{dx} = \frac{\cancel{v} - 1 - \cancel{v} - v^2}{1+v}$

$x\frac{dv}{dx} = \frac{-1-v^2}{1+v}$

$\int \frac{1+v}{-1-v^2}dv = \int \frac{dx}{x}$

$-\int \frac{dv}{(1+v^2)} - \frac{1}{2}\int \frac{2v}{(1+v^2)} = \int \frac{dx}{x}$

$-\tan^{-1} v - \frac{1}{2}\log(1+v^2) = \log x + C$

$\log x + \tan^{-1} v + \log\sqrt{1+v^2} = C$

$\log x + \tan^{-1}\left(\frac{y}{x}\right) + \log\sqrt{\frac{x^2+y^2}{x^2}} = C$

GS : $\log\sqrt{x^2+y^2} + \tan^{-1}\left(\frac{y}{x}\right) = C$

$x=1,\ y=1$

$\log\sqrt{2} + \frac{\pi}{4} = C$

PS : $\log\sqrt{x^2+y^2} + \tan^{-1}\frac{y}{x} = \frac{\pi}{4} + \log\sqrt{2}$

12] $x^2dy + (xy + y^2)dx = 0$ $x=1,\ y=1$

$\frac{dy}{dx} = \frac{-(xy+y^2)}{x^2}$ $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{-(x^2v + x^2v^2)}{x^2}$

$v + x\frac{dv}{dx} = -(v + v^2)$

$x\frac{dv}{dx} = -v - v^2 - v$

$$\int \frac{dv}{-2v-v^2} = \int \frac{dx}{x}$$

$$\int \frac{dv}{2v+v^2} = -\int \frac{dx}{x}$$

$$\int \frac{dv}{(v+1)^2-1} = -\log x + \log c$$

$$\frac{1}{2}\log\left|\frac{v+1-1}{v+1+1}\right| + \log x = \log C \quad x^{le} \text{ by } 2$$

$$\log x^2\left[\frac{\frac{y}{x}}{\frac{y}{x}+2}\right] = C^2$$

$$\left(\frac{x^2y}{2x+y}\right) = C^2$$

GS : $x^2y = C^2(2x+y)$

$x=1, y=1 \Rightarrow 1 = C^2(3) \Rightarrow C = \pm\frac{1}{\sqrt{3}} \Rightarrow C^2 = \frac{1}{3}$

PS : $x^2y = \frac{1}{3}(2x+y)$

$3x^2y = 2x+y$

16] A homogeneous DE of the form $\frac{dx}{dy} = h\frac{x}{y}$ can be solved by the substitution

Ans] $x = vy$

**

17] Which of the following is homogeneous DE

(a) $(4x+6y+5)\,dy - (3y+2x+4)\,dx = 0$

(b) $(xy)\,dx - (x^3+y^3)\,dy = 0$

(c) $(x^2+2y^2)\,dx + 2xy\,dy = 0$

(d) $y^2dx + (x^2-xy-y^2)\,dy = 0$