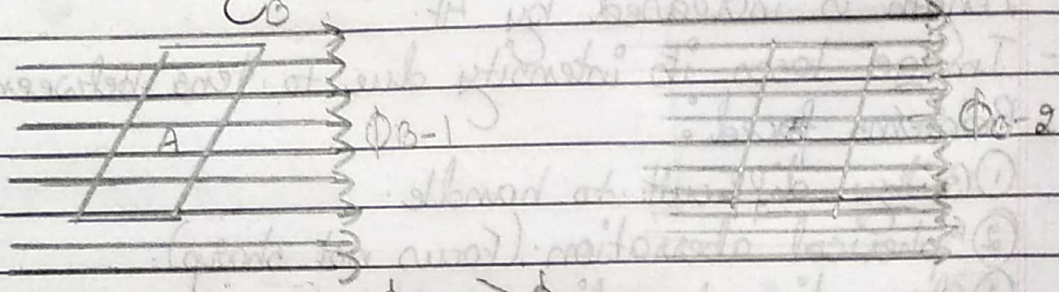
Chp 6 - Electromagnetic Induction

* Faraday's law of electromagnetic induction (EMI):

- Magnetic flux:

(1) Magnetic flux linked with an area 'A' $= \phi_B$
$= \vec{B}\,\vec{A}$ or
$\vec{B} = \dfrac{\phi}{A}$.

(2) Unit of magnetic flux is weber.



$\phi_{B-2} > \phi_{B-1}$

$\phi_B = \vec{B}\cdot\vec{A} = B \cdot A \cos\theta$

- Laws of Faraday:

Whenever there is change in magnetic flux linked to a coil, an E.M.F is linked ~~to a coil~~, across it and if circuit is closed a current flows through it.

Note:
Flux linked with a surface $\phi = \int_S \vec{B}\cdot\vec{dA}$.
Flux linked with a closed surface $\phi_B = \oint \vec{B}\cdot\vec{dA}$.
Flux linked for n turns $= \phi_B = nBA$.

- Lenz law -

The direction of induced current is such as required to oppose the change in magnetic flux linked to it.

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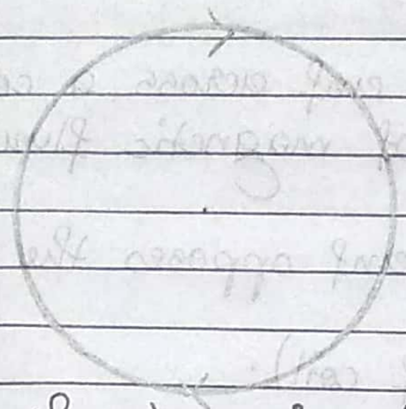
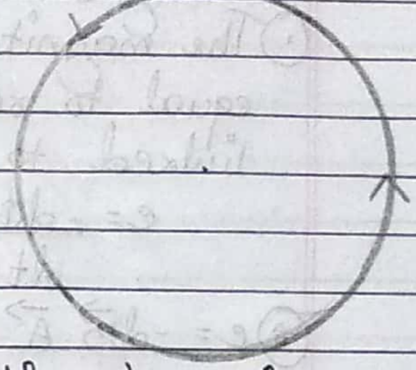
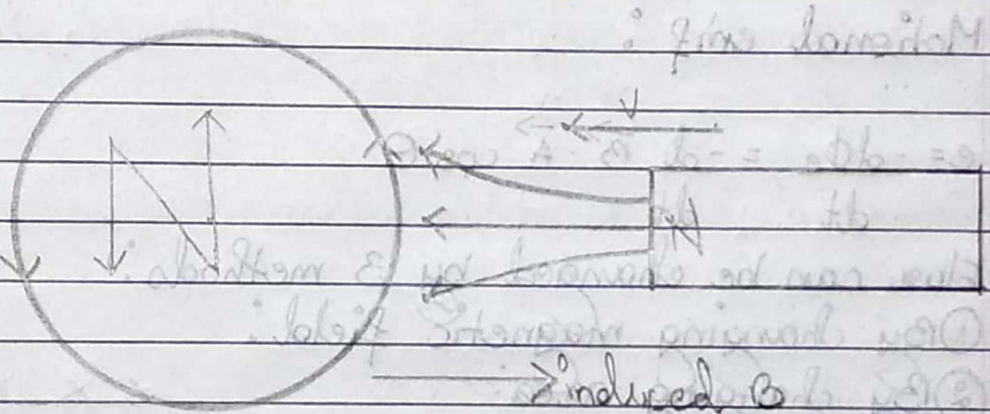
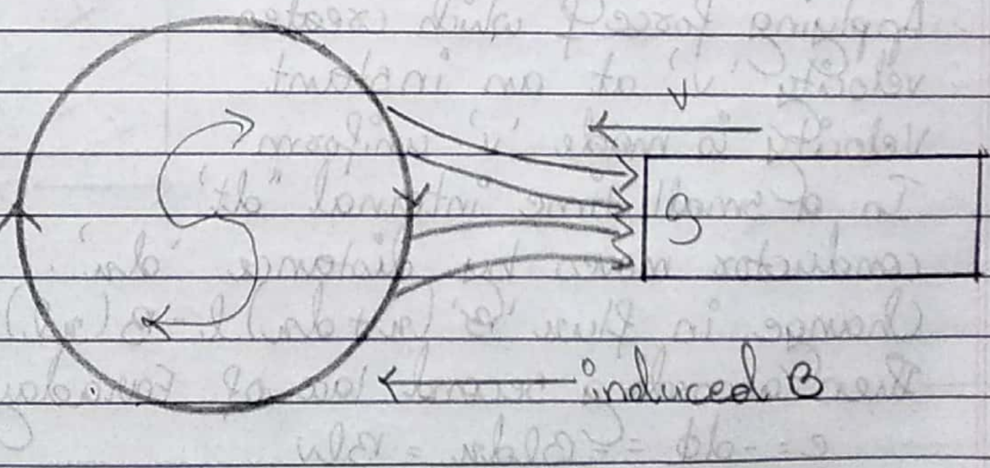


South pole moving in "B direction outcoming increasing".
∴ B produced by coil will be increased or i → clockwise.



South pole moving away from coil "B outward decreases".
∴ B produced is outward i → anticlockwise.





- Faraday's second law:

① The magnitude of induced emf across a coil is equal to rate of change of magnetic flux linked to it.

$$e = \frac{-d\phi}{dt}$$ (-ve sign shows emf opposes the change in flux).

② $e = -\frac{d\vec{B}\cdot\vec{A}}{dt}$ (For one turn of coil).

③ If there are 'n' turns then, $e = \frac{-nd\phi}{dt}$

$$i = \frac{e}{R} = \frac{-nd\phi}{R \cdot dt}$$

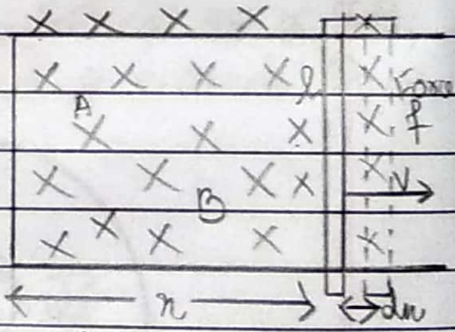
* Motional emf:

- $e = \frac{-d\phi_e}{dt} = \frac{-d}{dt}\vec{B}\cdot\vec{A}\cos\theta$

- Flux can be changed by 3 methods:

① By changing magnetic field.

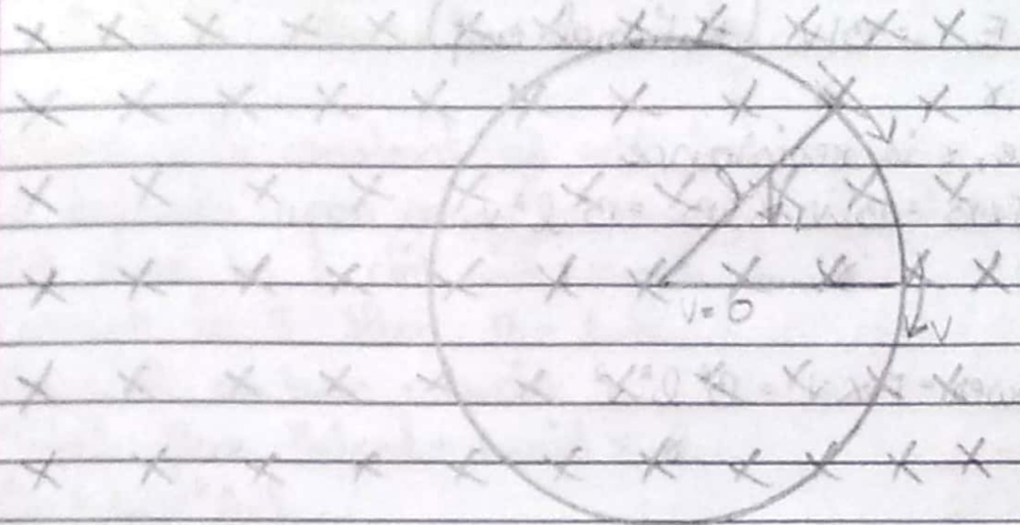
② By changing area.



Flux in rectangular coil = BA.
Applying force f which creates velocity 'v' at an instant
Velocity is made 'v' uniform.
In a small time interval "dt" conductor moves by distance 'dx'.
Change in flux 'B' $(x + dx)l - B(xl) = B \cdot l \cdot dx$.
Then according second law of Faraday

$$e = \frac{-d\phi}{dt} = \frac{-Bldx}{dt} = Blv$$

* Emf across rotating rod:



A rod length 'l' is rotating with angular velocity (ω).

Emf induced across rod, $e = \frac{Blv}{2}$

Average velocity of rod $= \frac{0+v}{2} = \frac{v}{2}$

In terms of angular velocity, $\varepsilon = \frac{1}{2} Bl^2 \omega$

Alternate method:

$$\varepsilon = \frac{-d\phi}{dt} = \frac{dB \cdot A}{dt} = \frac{B \times l^2}{T}$$

$$= \frac{B \times l^2}{\frac{2\pi}{\omega}} = \frac{1}{2} Bl^2 \omega$$

Note:
T = Change in time

* Energy conservation in motional emf:

$$I = \frac{E}{r} = \frac{Blv}{r} \text{ (Motional emf)}$$

where, r is resistance.

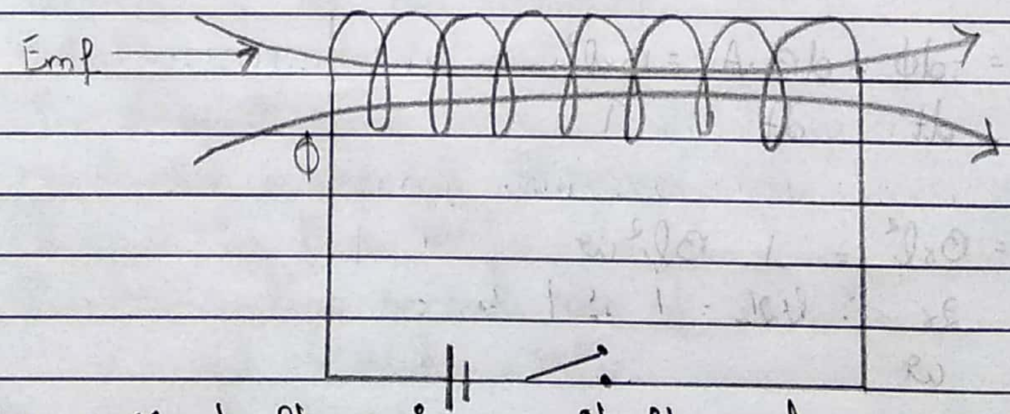
$$F = IlB = \frac{Blv}{r} \times lB = \frac{B^2 l^2 v}{r}$$

$$\text{Power} = F \times v = \frac{B^2 l^2 v^2}{r}$$

* Eddy current:

- Eddy current always opposes motion of that body which causes change in flux.
- Applications of eddy current:
  1. Damping or to make dead beat galvanometer.
  2. Electromaganetic brakes.
  3. Inductance furnance.
  4. Inductance motor

* Self inductance:

- 

When current flows in a coil, it produces a magnetic field. This magnetic field links itself with same coil and due to EMI induced, an emf in opposite direction. This phenomena is called self inductance.

$\phi_B \propto I \therefore \phi_B = L \cdot I$ or $L = \frac{\phi_B}{I}$.

- Where, L is constant of self inductance.
- L depends upon geometry and construction of coil.

Let there be a coil with length 'l', 'n' $\therefore N = nl$. If current is I then, $B = \mu_0 n i$.

ϕ_B with on turn $= \mu_0 n i A$.

Total flux linked $= \mu_0 n i A \times nl$

$\phi_B = \mu_0 n^2 A i l$

Now, $L = \frac{\phi}{l}$ $\therefore L = \frac{\mu_0 n^2 A i l}{l} = \mu_0 n^2 A l$

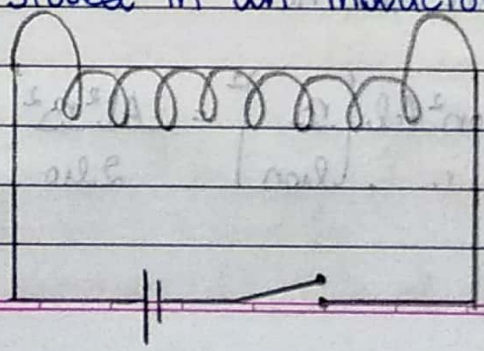
- When current changes through a conductor at rate of $\frac{dI}{dt}$ then 'B' also changes, ϕ linked also changes and this induces opposing emf in coil.

$\therefore e = -\frac{d\phi}{dt}$ or $e = -\frac{dLI}{dt} \Rightarrow e = -L\frac{dI}{dt}$

or $L = \frac{e}{(dI/dt)}$ where (dI/dt) is rate of change of current.

Note:
Self inductance of a coil is emf induced when rate of change is 1amp/second.

- Energy stored in an inductor:



$$W = q \cdot V$$

$$W = \frac{q}{t} V \times t$$

$$= i \cdot V \cdot t$$

$$\therefore dW = \varepsilon i dt = L \cdot \frac{di}{dt} \times i \times dt$$

$$\therefore dW = L i di \text{ or } W = L \int_0^i i di$$

$$W = \frac{1}{2} L i^2$$

This work converts to P.E stored in inductor. It is stored in magnetic field.

Note:

Energy formula:

① Energy in resistor $= RI^2$

② Energy in capacitor $= \frac{1}{2} CV^2$

③ Energy in ~~id~~ inductor $= \frac{1}{2} LI^2$.

Note:

U = Inductor energy $= \frac{1}{2} LI^2$.

~~u = Energy per~~

u = Energy per unit volume.

$$\therefore U = \frac{1}{2} LI^2 = \frac{1}{2} \mu_0 n^2 A l \left[\frac{B}{\mu_0 n}\right]^2 = \frac{1 A l^2 B^2}{2\mu_0}$$

$$= \frac{1}{2} B^2 A l \quad \therefore u = \frac{B^2 A l}{2\mu_0 \times V} = \frac{B^2 A l}{2\mu_0 A l}$$

$$u = \frac{1}{2\mu_0} B^2$$

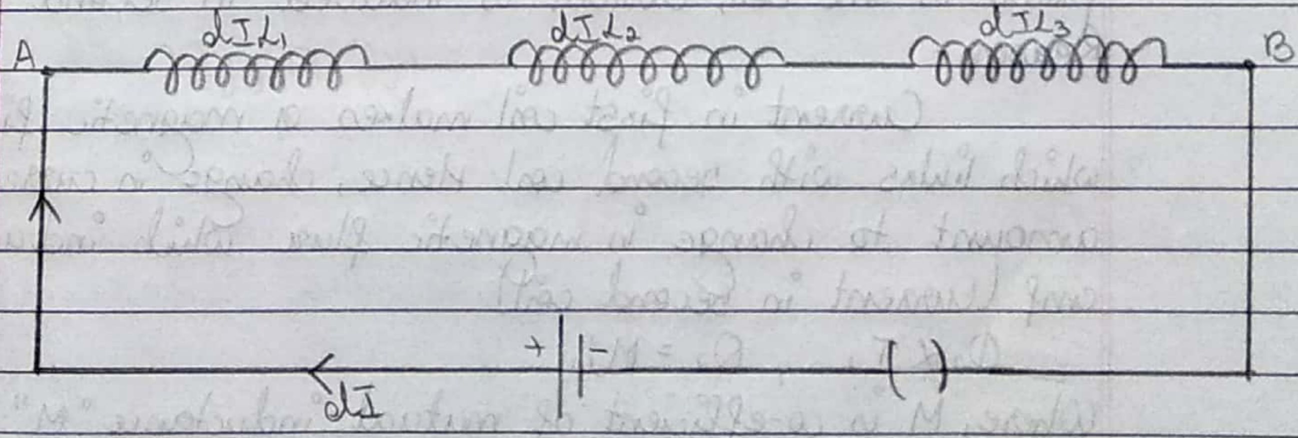
Note:
Per unit volume energy of capacitor $U_E = \frac{1}{2} \varepsilon_0 E^2$

Per unit volume energy of inductor $U_B = \frac{1}{2} \mu_0 B^2$

- Unit of co-efficient of self inductance 'L' is Henry 'H'.
- Henry :- 1 Henry inductance is the inductance of coil in which an emf of 1 volt is induced when rate of change of current is made amp/second.

* Combination of inductor:

- Series:



V divided $V = V_1 + V_2 + V_3$
Current common $= dI$ (Rate of change of current)

To find L_{AB} (Equivalent $V = V_1 + V_2 + V_3$)

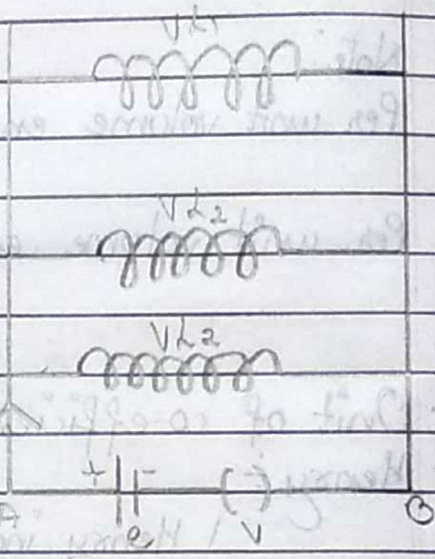
$$L_{AB} \times \frac{dI}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$\therefore L_{AB} = L_1 + L_2 + L_3$$

- Parallel:

Potential difference is same,

$dI = dI_1 + dI_2 + dI_3$



$$V = L \frac{dI}{dt} \quad \therefore \frac{V}{L} = \frac{dI}{dt}$$

$$\frac{V}{L_e} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}$$

$$\therefore \frac{1}{L_c} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

* Mutual inductance:

- In two neighbouring coil when current change takes place in one coil, current is induced in second coil.
- Reason -

Current in first coil makes a magnetic field which links with second coil. Hence, change in current amount to change in magnetic flux which induced emf (current in second coil)

$\therefore \phi_2 \propto I_1$, $\phi_2 = M i_1$

Where, M is co-efficient of mutual inductance "M".

$$M = \frac{\phi_2}{I}$$

- In coil 1, current is I_1 and flux linked to coil 2 is Φ_2.
∴ Emf induced in 2nd coil

$$e = -\frac{d\Phi_2}{dt} = -\frac{d}{dt} M \cdot I_1$$

$$e_2 = -M\frac{dI}{dt} \qquad M_{21} = \frac{e_2}{dI_1/dt}$$

- Co-efficient of mutual inductance in between two coils is numerically equal to emf induced in coil No. 2 when rate of change of current in coil No. 1 is 1 amp/second. It depends upon geometry/construction of coil.
- Calculations:

Calculation of 'M' between two coils.
B made by coil 1 is $B_1 = \mu_0 n_1 i_1$.
Flux with one turn of coil 2 is $\Phi_2 = \mu_0 n_1 I_1 a_2$.
Flux with n turns of coil 2 is $\Phi_2 = \mu_0 n_1 i_1 a_2 \times n_2 l$

$$\therefore M_{21} = \frac{\Phi_2}{I_1} = \mu_0 n_1 n_2 a_2 l.$$

$$M_{12} = \frac{\Phi_1}{I_2} = \mu_0 n_2 n_1 a_2 l.$$

$$\therefore M_{21} = M_{12} = M = \mu_0 n_1 n_2 a_2 l.$$

- Orientation -

In two coils decide the effectiveness of mutual inductance. This phenomena is called coupling.
- Co-efficient of coupling: