

DETAILED PG SYLLABUS

MAT501: GROUP THEORY

**Unit 1:** Isomorphism theorems for groups, Symmetric groups, Alternating groups, Dihedral groups, Matrix groups, Isometry groups of  $\mathbb{R}^2$  and  $\mathbb{R}^3$ , Internal and External direct product and their relationship, Indecomposable groups.

**Unit 2:** Subnormal and normal series, Zassenhaus' lemma, Schreier's refinement theorem, Composition series, Jordan-Hölder's theorem, Chain conditions.

**Unit 3:** Action of a group  $G$  on a set, Stabilizer subgroups and Orbit decomposition, Class equation of an action, Burnside's theorem, Transitive and effective actions, Equivalence of actions, core of a subgroup.

**Unit 4:** Sylow subgroups, Sylow's Theorem I, II and III,  $p$ -groups, Examples and applications, Groups of order  $pq$ , Direct and inverse images of Sylow subgroups.

~~**Unit 5:** Commutator subgroup and commutator series of a group, Solvable groups, Solvability of subgroups and factor groups and of finite  $p$ -groups, Examples, Lower and upper central series, Nilpotent groups and their equivalent characterizations.~~

**Books Recommended:**

1. D. S. Dummit and R.M. Foote, Abstract Algebra, John Wiley, N.Y., 2003.
2. N. S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.
3. J. A. Gallian, Contemporary Abstract Algebra, 4<sup>th</sup> Edition, Narosa Publ. House, 1998.

**Further Reading:**

1. T. W. Hungerford, Algebra, Springer (India) Pvt. Ltd., New Delhi, 2004.
2. J. B. Fraleigh, A first Course in Abstract Algebra, Pearson Edu. Inc., 2002.
3. Ramji Lal, Algebra, Vols. I & II, Shail Publications, Allahabad, 2002.

MK  
26-9-2020  
अध्यक्ष  
Head  
गणित विभाग  
Department of Mathematics  
इलाहाबाद विश्वविद्यालय  
University of Allahabad  
इलाहाबाद/Allahabad-211002

MK  
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Satyam  
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A. Chandra

26/9/16

## MAT503: COMPLEX ANALYSIS

**UNIT ONE:** Complex differentiability, Cauchy-Riemann equations, analytic functions, harmonic functions, harmonic conjugates, analyticity of functions defined by power series, the exponential function and its properties.

**UNIT TWO:** Branch of logarithm, power of a complex number, basic properties of contour integration, M-L inequality, fundamental theorem of contour integration, Cauchy's integral theorem, Cauchy-Goursat theorem (statement only), Cauchy's integral formula, Cauchy's integral formula for higher derivatives (statement only), Morera's theorem.

**UNIT THREE:** Maximum modulus theorem, Schwarz lemma, Taylor's theorem, Cauchy's estimate, zeros of an analytic functions, the identity theorem for analytic functions, Liouville's theorem, the fundamental theorem of algebra, Laurent's theorem.

**UNIT FOUR:** Singularities of functions, removable singularity, poles and essential singularities, Casoratti-Weierstrass theorem, residues, Cauchy's residue theorem, evaluation of simple definite integrals using contour integration, meromorphic functions, argument principle, Rouché's theorem, open mapping theorem, singularity and residue at  $\infty$ .

~~**UNIT FIVE:** Conformality, Möbius transformations, the group of Möbius transformations, cross ratio, invariance of circles, determination of Möbius transformations mapping real line onto itself, upper half plane onto itself, upper half plane onto open disc and an open disc onto an open disc.~~

### Books Recommended:

1. J. B. Conway, Functions of One Complex Variable, Narosa Publ. House, New Delhi, 2002.
2. S. Ponnusamy and H. Silverman, Complex Variables, Birkhäuser, Inc., Boston, MA, 2006.
3. J. Bak, Complex Analysis, Springer, 1996.
4. V. Ahlfors, Complex Analysis (Third Edition), McGraw-Hill, 1979.
5. A. R. Shastri, An Introduction to Complex Analysis, Macmillan India Ltd., 1999.

*M. Shrivastava*  
26/9/16

*R. S. N.*

*M. K. L.*  
26.9.16

*M. S. D.*  
26/9/16

*R. P. S.*

*S. S. S.*  
26/9/16

*M. K. L.*  
25.10.2020

अध्यक्ष

Head

गणित विभाग

Department of Mathematics

इलाहाबाद विश्वविद्यालय

University of Allahabad

इलाहाबाद/Allahabad-211002

*A. S. S.*  
26/9/16

*S. S. S.*

*A. S. S.*

MAT505: POINT-SET TOPOLOGY

**Unit 1:** Topological Spaces, metric topology, ordered topology, open sets, closed sets; interior, exterior, boundary and closure of a set, limit points of a set, characterization of closed sets and dense sets, separable spaces, basis and subbasis of a topology, first countable and second countable spaces.

**Unit 2:** Sequences in a metric space, convergence of a sequence, complete metric spaces, nets and filters, continuous maps and their characterization, open maps, closed maps, homeomorphisms, topological invariants.

**Unit 3:** Product topology, Quotient topology and identification spaces (torus, projective spaces  $P^n$ , Moebius strip and Klein bottle), connected spaces, locally connected spaces, path connected and locally path connected spaces.

~~**Unit 4:** Separation Axioms:  $T_0$  spaces,  $T_1$  spaces,  $T_2$  spaces, regular spaces,  $T_3$  spaces, completely regular spaces, normal spaces, Tychonoff spaces,  $T_4$  spaces, characterization of these spaces, Urysohn's lemma, Tietze's extension theorem, Urysohn's embedding and metrization theorem.~~

**Unit 5:** Compact spaces and their characterizations, compactness in metric spaces and their characterisation (limit point compactness, sequential compactness, complete and total boundeness), locally compact spaces, Tychonoff's theorem, one point compactification.

**Books Recommended:**

1. J. L. Kelley, General Topology, Van Nostrand, 1995.
2. K. D. Joshi, Introduction to General Topology, Wiley Eastern, 1983.
3. James R. Munkres, Topology, 2<sup>nd</sup> Edition, Pearson International, 2000.
4. J. Dugundji, Topology, Prentice-Hall of India, 1966.
5. George F. Simmons, Introduction to Topology and Modern Analysis, McGraw-Hill, 1963.
6. S. Willard, General Topology, Addison-Wesley, 1970.

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 Sutyar 26.9.16  
 Mudi 26/9/16  
 RBSU  
 CA. Chatterjee  
 26/9/16  
 ABHMEER

MS  
 26/9/16

Mkl 15.10.2020  
 अध्यक्ष  
 Head  
 गणित विभाग  
 Department of Mathematics  
 इलाहाबाद विश्वविद्यालय  
 University of Allahabad  
 इलाहाबाद/Allahabad

# MAT507: DIFFERENTIAL GEOMETRY I

**UNIT ONE:** Curves in space  $R^3$ , parameterized curves, regular curves, helices, arc length, reparametrization (by arc length), tangent, principal normal, binormal, osculating plane, normal plane, rectifying plane, curvature and torsion of smooth curves, Frenet-Serret formulae, Frenet approximation of a space curve.

**UNIT TWO:** Osculating circle, osculating sphere, spherical indicatrices, involutes and evolutes, intrinsic equations of space curves, isometries of  $R^3$ , fundamental theorem of space curves, surfaces in  $R^3$ , regular surfaces, co-ordinate neighborhoods, parameterized surfaces, change of parameters, level sets of smooth functions on  $R^3$ , surfaces of revolution, tangent vectors, tangent plane, differential of a map.

**UNIT THREE:** Normal fields and orientability of surfaces, angle between two intersecting curves on a surface, Gauss map and its properties, Weingarten map, second and third fundamental forms, classification of points on a surface.

**UNIT FOUR:** Curvature of curves on surfaces, normal curvature, Meusnier theorem, principal curvatures, geometric interpretation of principal curvatures, Euler theorem, mean curvature, lines of curvature, umbilical points, minimal surfaces, definition and examples, Gaussian curvature, intrinsic formulae for the Gaussian curvature, isometries of surfaces, Gauss Theorem Egregium (statement only).

~~**UNIT FIVE:** Christoffel symbols, Gauss formulae, Weingarten formulae, Gauss equations, Codazzi-Mainardi equations, curvature tensor, geodesics, geodesics on a surface of evolution, geodesic curvature of a curve, Gauss-Bonnet Theorem (statement only).~~

## Books Recommended:

1. M. P. Do Carmo, Differential Geometry of Curves and Surfaces, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976.
2. B. O' Neill, Elementary Differential Geometry, Academic Press, 1997.
3. A. Gray, Differential Geometry of Curves and Surfaces, CRC Press, 1998.
4. A. Pressley, Elementary Differential Geometry, Springer (Undergraduate Mathematics Series), 2001.
5. J. A. Thorpe, Elementary Topics in Differential Geometry, Springer (Undergraduate Texts in Mathematics), 1979.
6. D. Somasundaram, Differential Geometry, A First Course, Narosa Publishing House, New Delhi, 2005.
7. L. P. Eisenhart, A Treatise on the Differential Geometry of Curves and Surfaces, Ginn and Company, Boston, 1909.

प्र.प्र.  
Mkl  
15-10-2020  
अध्यक्ष  
Head  
गणित विभाग  
Department of Mathematics  
इलाहाबाद विश्वविद्यालय  
University of Allahabad  
इलाहाबाद/Allahabad-201002  
A. K. Prasad

M. Srinivasan  
26/9/16

Mkl  
26.9.16

M. Srivastava  
26/9/16

RPS

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Satyajit  
27/9/16

Abhinav  
28/9/16

CA. Chatterjee

## MAT509: CLASSICAL MECHANICS

**UNIT ONE:** The momentum of a system of particles, the linear and the angular momentum, rate of change of momentum and the equations of motion for a system of particles, principles of linear and angular momentum, motion of the centre of mass of a system, theorems on the rate of change of angular momentum about different points, with special reference to the centre of mass, the kinetic energy of a system of particles in terms of the motion relative to the centre of mass of the system.

Rigid bodies as systems of particles, general displacement of a rigid body, the displacement of a rigid body about one of its points and the concept of angular velocity, computation of the angular velocity of a rigid body in terms of the velocities of two particles of the system chosen appropriately, kinematical examples.

**UNIT TWO:** The angular momentum and the kinetic energy of a rigid body in terms of inertia constants, equations of motion, examples on the motion of a sphere on horizontal and on inclined planes.

Euler's dynamical equations of motion, motion under no forces, the invariable line and the invariable cone, Eulerian angles and the geometrical equations of Euler.

**UNIT THREE:** Generalized co-ordinates, Geometrical equations, holonomic and non-holonomic systems, Configuration space, Lagrange's equations using D'Alembert's Principle for a holonomic conservative system, deduction of equation of energy when the geometrical equations do not contain time  $t$  explicitly, Lagrange's multipliers case.

**UNIT FOUR:** Deduction of Euler's dynamical equations from Lagrange's equations, Theory of small oscillations, Lagrange's method, normal (principal) co-ordinates and the normal modes of oscillation, small oscillations under holonomic constraints, Lagrange equations for impulsive motion.

~~**UNIT FIVE:** Generalized momentum and the Hamiltonian for a dynamical system, Hamilton's canonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, Phase space and Hamilton's variational principle, the principle of least action, Canonical transformations, Poisson-Brackets, Poisson-Jacobi identity, Hamilton-Jacobi theory (outline only).~~

### Books Recommended:

1. E. A. Milne, Vectorial Mechanics, Methuen & Co. Ltd., London, 1965.
2. A. S. Ramsey, Dynamics, Part II, CBS Publishers & Distributors, Delhi, 1985.
3. H. Goldstein, Classical Mechanics, Addison-Wesley Publishing Company, London, 1969.
4. N. Kumar, Generalized Motion of Rigid Body, Narosa Publishing House, New Delhi, 2004.

Mk  
15.10.2020  
अध्यक्ष  
Head  
गणित विभाग  
Department of Mathematics  
इलाहाबाद विश्वविद्यालय  
University of Allahabad  
इलाहाबाद/Allahabad  
A. Ahmad

Mk  
26/9/16  
RBSM  
Sanyal  
26/9/16  
A. Chatterjee