65/1/1

QUESTION PAPER CODE 65/1/1
EXPECTED ANSWER/VALUE POINTS
PART A
SECTION-I

1. IF A is a square matrix of order 3 such that $A(\text{adj } A) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then find |A|.

Ans. $A \cdot (\text{adj } A) = -2I$ $\frac{1}{2}$

$\therefore |A| = -2$ $\frac{1}{2}$

2. (a) Find the order of the matrix A such that

$$\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 \\ 1 & -2 \\ 9 & 22 \end{bmatrix}$$

Ans. Order of matrix A is 2×2 1

OR

(b) If $B = \begin{bmatrix} 1 & -5 \\ 0 & -3 \end{bmatrix}$ and $A + 2B = \begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix}$, find the matrix A.

Ans. $A = \begin{bmatrix} 0 & 4 \\ -7 & 5 \end{bmatrix} - \begin{bmatrix} 2 & -10 \\ 0 & -6 \end{bmatrix}$ $\frac{1}{2}$

$A = \begin{bmatrix} -2 & 14 \\ -7 & 11 \end{bmatrix}$ $\frac{1}{2}$

3. Write the smallest reflexive relation on set A = {a, b, c}.

Ans. The smallest reflexive relation is {(a, a), (b, b), (c, c)} 1

4. (a) Find:

$$\int e^x \left(\log \sqrt{x} + \frac{1}{2x} \right) dx$$

Ans. Here $f(x) = \log \sqrt{x}, f'(x) = \frac{1}{2x}$

$$\therefore \int e^x (\log\sqrt{x} + \frac{1}{2x})dx = e^x.\log\sqrt{x} + c$$ 1

OR

(b) Find:

$$\int e^{2\log x}\, dx$$

Ans. $\int e^{2\log x}\, dx = \int x^2\, dx + c$ $\frac{1}{2}$

$$= \frac{x^3}{3} + c$$ $\frac{1}{2}$

5. (a) Find the angle between the vectors $\hat{i} - \hat{j}$ and $\hat{j} - \hat{k}$.

Ans. Let θ be the angle,

$$\therefore \cos\theta = \frac{(\hat{i}-\hat{j})\cdot(\hat{j}-\hat{k})}{|\hat{i}-\hat{j}||\hat{j}-\hat{k}|} = -\frac{1}{2}$$ $\frac{1}{2}$

$$\Rightarrow \theta = \frac{2\pi}{3}$$ $\frac{1}{2}$

OR

(b) Write the projection of the vector $\vec{r} = 3\hat{i} - 4\hat{j} + 12\hat{k}$ on (i) x-axis, and (ii) y-axis.

Ans. (i) Projection of the vector $\vec{r}$ on x-axis = 3 $\frac{1}{2}$

(ii) Projection of the vector $\vec{r}$ on y-axis = 4 $\frac{1}{2}$

6. If $\vec{a} = \alpha\hat{i} + 3\hat{j} - 6\hat{k}$ and $\vec{b} = 2\hat{i} - \hat{j} - \beta\hat{k}$, find the value of α and β so that $\vec{a}$ and $\vec{b}$ may be collinear.

Ans. $\vec{a}$ and $\vec{b}$ are collinear.

$$\therefore \frac{\alpha}{2} = \frac{3}{-1} = \frac{-6}{-\beta} \Rightarrow \alpha = -6, \beta = -2$$ $\frac{1}{2} + \frac{1}{2}$

7. If f = {(1, 2), (2, 4), (3, 1), (4, k)} is a one-one function from set A to A, where A = {1, 2, 3, 4}, then find the value of k.

Ans. k = 3 1

8. (a) Check whether the relation R defind on the set {1, 2, 3, 4} as R = {(a, b) : b = a + 1} is transitive. Justify your answer.

Ans. (1, 2), (2, 3) $\in$ R but (1, 3) $\notin$ R $\therefore$ R is not transitive. 1

(Note: Any Similar other pair can be taken to show that R is not a transitive relation.)

OR

(b) If the relation R on the set A = {x : 0 $\leq$ x $\leq$ 12} given by R = {(a, b) : a = b} is an equivalenc relation, then find the set of all elements related to 1.

Ans. Set of all elements related to 1 is {1}. 1

9. If A = [1 0 4] and B = $\begin{bmatrix} 2 \\ 5 \\ 6 \end{bmatrix}$, find AB.

Ans. AB = [26] 1

10. (a) Write the order and degree of the differential equation:

$$\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$$

Ans. Order = 2 $\frac{1}{2}$

Degree is not defined. $\frac{1}{2}$

OR

(b) Find the general solution of the differential equation $\frac{dy}{dx} = a$, where a is an ab\rbitrary constant.

Ans. $\frac{dy}{dx} = a, \quad \int dy = \int a.dx$ $\frac{1}{2}$

$\Rightarrow y = ax + c$ $\frac{1}{2}$

11. Show that the function $f(x) = \frac{3}{x} + 7$ is strictly decreasing for x $\in$ R – {0}.

Ans. $f'(x) = -\frac{3}{x^2} < 0$ for all x $\in$ R – {0} 1

$\therefore$ f(x) is strictly decreasing.

12. Find the magnitude of vector $\vec{a}$ given by $\vec{b} = (\hat{i} + 3\hat{j} - 2\hat{k}) \times (-\hat{i} + 3\hat{k})$.

Ans. $\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix} = 9\hat{i} - \hat{j} + 3\hat{k}$ $\frac{1}{2}$

$\therefore\ |\vec{a}| = \sqrt{91}$ $\frac{1}{2}$

13. Write the equation of the plane that cuts the coordinate axes at (2, 0, 0), (0, 4, 0) and (0, 0, 7).

Ans. Equation of the plane is $\left.\begin{array}{c} \frac{x}{2} + \frac{y}{4} + \frac{z}{7} = 1 \\ \text{OR} \\ 14x + 7y + 4z = 28 \end{array}\right\}$ 1

14. Find the distance between the two parallel planes 3x + 5y + 7z = 3 and 9x + 15y + 21z = 12.

Ans. Distance between the two parallel planes $= \left|\frac{4-3}{\sqrt{9+25+49}}\right| = \frac{1}{\sqrt{83}}$ 1

15. If A and B are two independent events and $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$, find $P(\bar{A} \mid \bar{B})$.

Ans. A and B are independent $\Rightarrow$ $\bar{A}$ and $\bar{B}$ are independent

$\therefore\ P(\bar{A} / \bar{B}) = P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3}$ 1

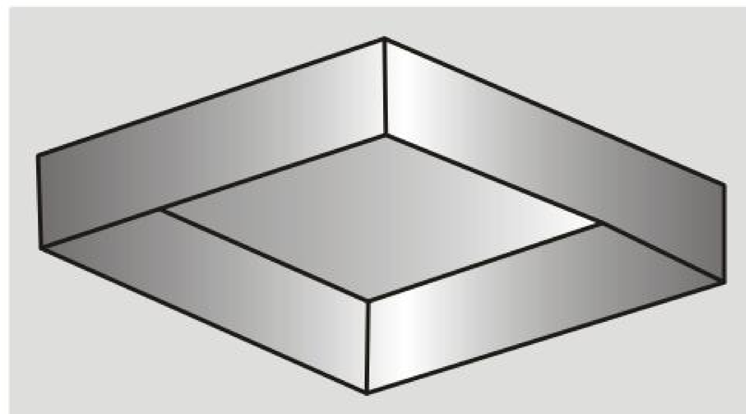
16. A coin is tossed once. If head comes up, a die is thrown, but if tail comes up, the coin is tossed again. Find the probability of obtaining head and number 6.

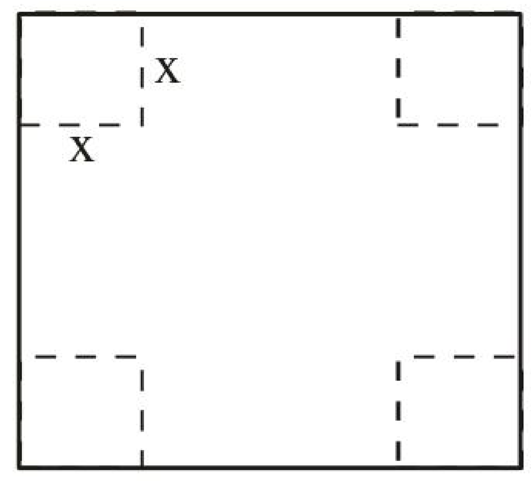
Ans. Probability of obtaining head and number $6 = \frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$ 1

SECTION-II

Both the case study based question (17 & 18) are compulsory. Attempt any 4 subparts out of 5 from each of question number 17 and 18. Each subpart carries 1 mark.

17. A factory makes an open cardboard box for a jewellery shop from a square sheet of side 18 cm by cutting off squares from each corner and folding up the flaps.





Based on the above information, answer any four of the following five question, if x is the length of each square cut from corners.

(i) The volume of the open box is:

(A) $4x\ (x^2 - 18x + 81)$

(B) $2x(2x^2 + 36x + 162)$

(C) $2x(2x^2 + 36x - 162)$

(D) $4x(x^2 + 18x + 81)$

Ans. (A) $4x\ (x^2 - 18x + 81)$ 1

(ii) The condition for the volume (V) to be maximum is:

(A) $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} < 0$

(B) $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} > 0$

(C) $\frac{dV}{dx} > 0$ and $\frac{d^2V}{dx^2} = 0$

(D) $\frac{dV}{dx} < 0$ and $\frac{d^2V}{dx^2} = 0$

Ans. (A) $\frac{dV}{dx} = 0$ and $\frac{d^2V}{dx^2} < 0$ 1

(iii) What should be the side of square to be cut off so that the volume is maximum?

(A) 6 cm

(B) 9 cm

(C) 3 cm

(D) 4 cm

Ans. (C) 3 cm 1

(iv) Maximum volume of the open box is:

(A) 423 cm^3

(B) 432 cm^3

(C) 400 cm^3

(D) 216 cm^3

Ans. (B) 432 cm^3 1

(v) The total area of the removed squares is:

(A) 324 cm^2

(B) 144 cm^2

(C) 36 cm^2

(D) 64 cm^2

Ans. (C) 36 cm^2 1

18. In answering a multiple choice test for class XII, a student either knows or guesses or copies the answer to a multiple choice question with four choices. The probability that he makes a guess is $\frac{1}{3}$ and the probability that he copies the answer is $\frac{1}{6}$. The probability that his answer is correct given that he copied is $\frac{1}{8}$.

Let E_1, E_2, E_3 be the events that the student guesses, copies or knows the answer respectively and A is the event that the student answrs correctly.

Based on the above information, answer any four of the following five questions:

(i) What is the probability that the student knows the answer?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{2}{3}$

(D) $\frac{1}{4}$

Ans. (B) $\frac{1}{2}$ 1

(ii) What is the probability that he answers correctly given that he knew the answer?

(A) 1

(B) 0

(C) $\frac{1}{4}$

(D) $\frac{1}{8}$

Ans. (A) 1 1

(iii) What is the probability that he answers correctly given that he had made a guess?

(A) $\frac{1}{4}$

(B) 0

(C) 1

(D) $\frac{1}{8}$

Ans. (A) $\frac{1}{4}$ 1

(iv) What is the probability that he knew the answer to the question, given that he answered it correctly?

(A) $\frac{24}{29}$

(B) $\frac{4}{29}$

(C) $\frac{1}{29}$

(D) $\frac{3}{29}$

Ans. (A) $\frac{24}{29}$ 1

(v) $\sum_{k=1}^{3} P(E_k \mid A)$ is:

(A) 0

(B) $\frac{1}{3}$

(C) 1

(D) $\frac{11}{8}$

Ans. (C) 1 1

PART B

SECTION-III

Question numbers 19 to 28 carry 2 marks each.

19. A random variable X has the probability distribution:

X	0	1	2	3	4
P(X):	0	K	4K	3K	2K

Find the value of K and $P(X \le 2)$.

Ans. $0 + K + 4K + 3K + 2K = 1 \Rightarrow K = \frac{1}{10}$ 1

$P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2) = 5K = \frac{1}{2}$ 1

20. Simplify $\sec^{-1}\left(\frac{1}{2x^2-1}\right), 0 < x < \frac{1}{\sqrt{2}}$.

Ans. Let $x = \cos\theta \quad \therefore \theta = \cos^{-1} x$ $\frac{1}{2}$

$\sec^{-1}\left(\frac{1}{2x^2-1}\right) = \sec^{-1}\left(\frac{1}{2\cos^2\theta - 1}\right) = \sec^{-1}\left(\frac{1}{\cos 2\theta}\right) = \sec^{-1}(\sec 2\theta)$ $\frac{1}{2}$

$= 2\theta$ $\frac{1}{2}$

$= 2\cos^{-1} x$ $\frac{1}{2}$

21. If the matrix $A = \begin{bmatrix} 0 & 6-5x \\ x^2 & x+3 \end{bmatrix}$ is symmetric, find the values of x.

Ans. Matrix A is symmetric $\Rightarrow x^2 = 6 - 5x$ 1

$\therefore\ x^2 + 5x - 6 = 0 \Rightarrow (x + 6)(x - 1) = 0 \qquad \therefore\ x = -6, 1$ 1

22. (a) Find the relationship between a and b so that the function f defined by

$$f(x) = \begin{cases} ax + 1 & \text{if} \quad x \leq 3 \\ bx + 3 & \text{if} \quad x > 3 \end{cases}$$

is continuous at x = 3.

Ans. f is continuous at x = 3 $\Rightarrow \lim_{x\to 3^-}(ax+1) = \lim_{x\to 3^+}(bx+3)$ 1

$\Rightarrow 3a + 1 = 3b + 3$ $\frac{1}{2}$

$\Rightarrow 3a - 3b = 2\left(\text{or } a - b = \frac{2}{3}\right)$ $\frac{1}{2}$

OR

(b) Check the differentiability of f(x) = |x – 3| at x = 3.

Ans. L.H.D. (x = 3) = $\lim_{x\to 3^-}\frac{|x-3|-0}{x-3} = \lim_{x\to 3^-}\frac{-(x-3)}{x-3} = -1$ 1

R.H.D (x = 3) = $\lim_{x\to 3^+}\frac{|x-3|-0}{x-3} = \lim_{x\to 3^+}\frac{x-3}{x-3} = 1$ $\frac{1}{2}$

L.H.D. ≠ R.H.D, $\quad\therefore$ f(x) is not differentiable at x = 3 $\frac{1}{2}$

23. Find:

$$\int \frac{x^2+2}{x^2+1}\,dx$$

Ans. $\int \frac{x^2+2}{x^2+1}\,dx = \int\left(1+\frac{1}{x^2+1}\right)dx = x + \tan^{-1}x + c$ 1+1

24. (a) Evaluate:

$$\int_{-1}^{1}\frac{|x|}{x}\,dx$$

Ans. Let $f(x)=\frac{|x|}{x}$, $\therefore f(-x)=\frac{|-x|}{-x}=-\frac{|x|}{x}=-f(x)$ 1

$\therefore \int_{-1}^{1}\frac{|x|}{x}=0$ 1

OR

(b) Evaluate:

$$\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$$

Ans. Let $I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\right)dx$ ...(i)

$\therefore I=\int_0^{\pi/2}\log\left(\frac{4+3\sin(\pi/2-x)}{4+3\cos(\pi/2-x)}\right)dx$

$=\int_0^{\pi/2}\log\left(\frac{4+3\cos x}{4+3\sin x}\right)dx$ ...(ii) $\frac{1}{2}$

Adding (i) and (ii), we get

$2I=\int_0^{\pi/2}\log\left(\frac{4+3\sin x}{4+3\cos x}\times\frac{4+3\cos x}{4+3\sin x}\right)dx=\int_0^{\pi/2}\log 1dx=0$ $\frac{1}{2}+\frac{1}{2}$

$\therefore I=0$ $\frac{1}{2}$

25. Find the integrating factor of $x\frac{dy}{dx}+(1+x\cot x)y=x$.

Ans. The differential equation can be written as: $\frac{dy}{dx}+\left(\frac{1}{x+\cot x}\right)y=1$ $\frac{1}{2}$

Integrating factor $=e^{\int\left(\frac{1}{x}+\cot x\right)dx}=e^{(\log x+\log\sin x)}=e^{\log(x\sin x)}=x.\sin x$ $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

26. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three mutually perpendicular unit vectors, find the value of $|\vec{a}+2\vec{b}+3\vec{c}|$.

Ans. $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$ and $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$ $\frac{1}{2}$

$|\vec{a}+2\vec{b}+3\vec{c}|^2=\vec{a}^2+4\vec{b}^2+9\vec{c}^2=1+4+9=14$ 1

$\therefore$ $|\vec{a}+2\vec{b}+3\vec{c}|=\sqrt{14}$ $\frac{1}{2}$

27. If the side AB and BC of a parallelogram ABCD are represented as vectors $\overrightarrow{AB}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\overrightarrow{BC}=\hat{i}+2\hat{j}+3\hat{k}$, then find the unit vector along diagonal AC.

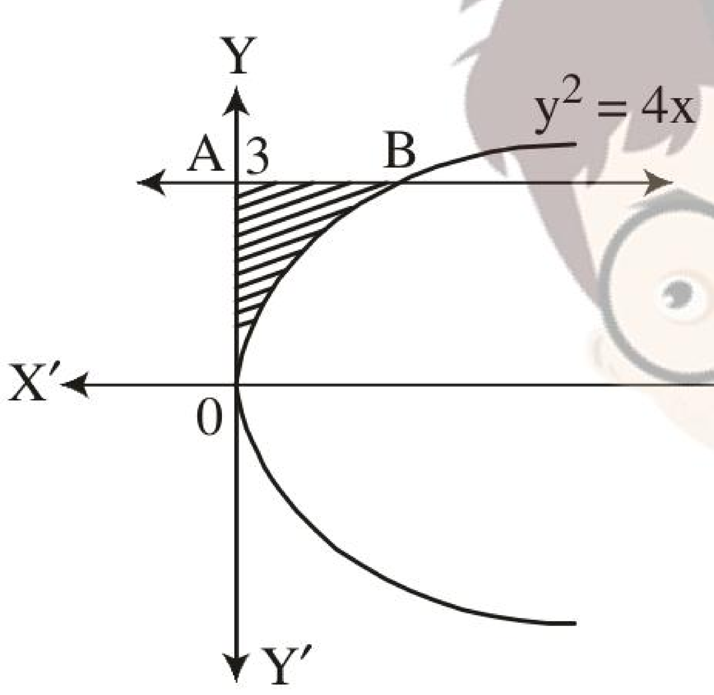
Ans. $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=(2\hat{i}+4\hat{j}-5\hat{k})+(\hat{i}+2\hat{j}+3\hat{k})$

$\therefore$ $\overrightarrow{AC}=3\hat{i}+46-2\hat{k}$ 1

Unit vector along AC = $\dfrac{3\hat{i}+6\hat{j}-2\hat{k}}{|3\hat{i}+6\hat{j}-2\hat{k}|}=\dfrac{3}{7}\hat{i}+\dfrac{6}{7}\hat{j}-\dfrac{2}{7}\hat{k}$ 1

28. (a) Using integration, find the area bounded by the curve $y^2=4x$, y-axis and $y=3$.

Ans.



Correct Fig. $\frac{1}{2}$

Required area = ar(OAB)

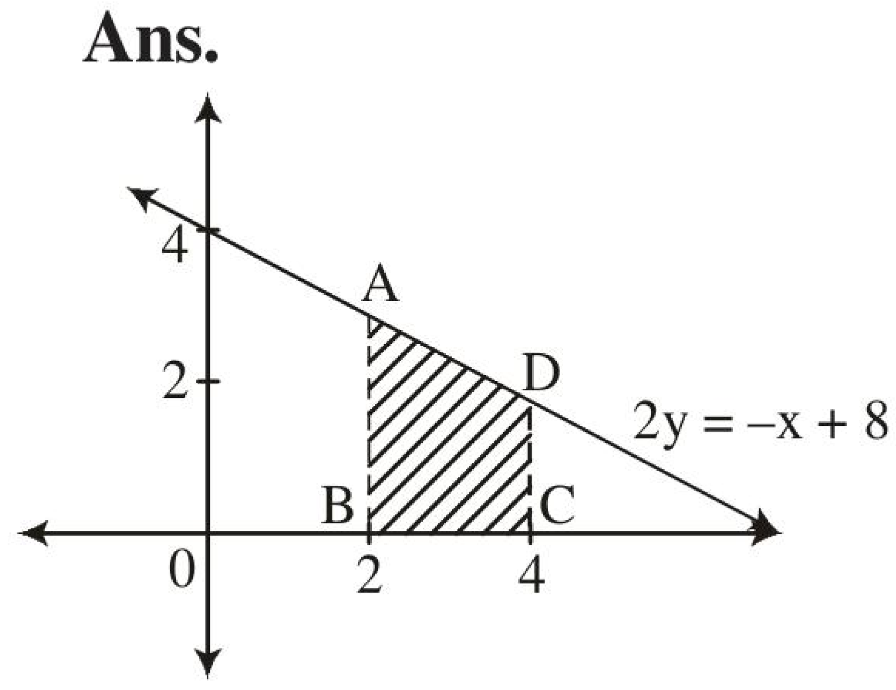
$= \dfrac{1}{4}\displaystyle\int_0^3 y^2\,dy$ $\frac{1}{2}$

$= \dfrac{1}{12}\cdot y^3\Big]_0^3 = \dfrac{9}{4}$ sq. units $\frac{1}{2}+\frac{1}{2}$

OR

(b) Using integration, find the area of the region bounded by the line $2y=-x+8$, x-axis, $x=2$ and $x=4$.

Ans.



Correct Graph $\frac{1}{2}$

Required area = ar(ABCD)

$= \displaystyle\int_2^4 \frac{8-x}{2}\,dx$ $\frac{1}{2}$

or 1 if graph is not drawn.

$$= -\frac{1}{4}(8-x)^2\Big]_2^4 \qquad \frac{1}{2}$$

$$= -\frac{1}{4}(4^2 - 6^2) = 5 \text{ sq. units} \qquad \frac{1}{2}$$

SECTION-IV

Question number 29 to 35 carry 3 marks each.

29. Show that the function $f : R - \{-1\} \to R - \{1\}$ given by $f(x) = \frac{x}{x+1}$ is bijective.

Ans. One-One:

$$\left.\begin{array}{l} \text{Let } f(x_1) = f(x_2),\ x_1, x_2 \in R - \{-1\} \\ \Rightarrow \frac{x_1}{x_1+1} = \frac{x_2}{x_2+1} \\ \Rightarrow x_1x_2 + x_1 = x_1x_2 + x_2 \\ \Rightarrow x_1 = x_2 \end{array}\right\} \qquad 1$$

$\therefore$ f is one-one.

Onto: Let any $y \in R - \{1\}$ such that $y = f(x)$

$$\Rightarrow y = \frac{x}{x+1} \Rightarrow x = \frac{y}{1-y} \qquad 1$$

$\therefore$ For each $y \in R - \{1\}$ there exists $x \in R - \{-1\}$. Such that $y = f(x)$ $\frac{1}{2}$

$\therefore$ 'f' is an onto function.

$\therefore$ f is a bijective function. $\frac{1}{2}$

30. (a) If x = a cos θ + b sin θ, y = a sin θ – b cos θ, then show that $\frac{dy}{dx} = -\frac{x}{y}$ and hene show that

$$y^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$$

Ans. x = a cos θ + b sin θ, y = a sin θ – b cos θ

$$\frac{dx}{d\theta} = -a\sin\theta + b\sin\theta,\ \frac{dy}{d\theta} = a\cos\theta + \sin\theta \qquad \frac{1}{2}+\frac{1}{2}$$

$$\frac{dy}{dx}=\frac{dy/d\theta}{dx/d\theta}=\frac{a\cos\theta+b\sin\theta}{-a\sin\theta+b\cos\theta}=-\frac{x}{y}$$ $\frac{1}{2}$

Differentiate $\frac{dy}{dx}=-\frac{x}{y}$ with respect to 'x'

$$\Rightarrow \frac{d^2y}{dx^2}=\frac{1.y-x\frac{dy}{dx}}{y^2}$$ 1

$$\Rightarrow y^2\frac{d^2y}{dx^2}=-y+x\frac{dy}{dx}$$

$$\Rightarrow y^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$$ $\frac{1}{2}$

OR

(b) If $e^{y-x}=y^x$, prove that $\frac{dy}{dx}=\frac{y(1+\log y)}{x\log y}$

Ans. Taking log on both side of $e^{y-x}=y^x$

$y-x=x\log y$ ...(i) $\frac{1}{2}$

Differentioate with respect to x

$$\frac{dy}{dx}-1=\log y+\frac{x}{y}\frac{dy}{dx}$$ $1\frac{1}{2}$

$$\Rightarrow \frac{dy}{dx}=\frac{y(1+\log y)}{y-x}=\frac{y(1+\log y)}{x\log y}$$, using (i) 1

31. Differentiate $\sin^2 x$ w.r.t $e^{\cos x}$.

Ans. Let $u=\sin^2 x$, $v=e^{\cos x}$

$$\frac{du}{dv}=\frac{du/dv}{dv/dx}=\frac{2\sin x.\cos x}{e^{\cos x}.(-\sin x)}=-\frac{2\cos x}{e^{\cos x}}$$ $1+1\frac{1}{2}+\frac{1}{2}$

32. (a) Find the equation of the normal to the curve $y^2=4ax$ at $\left(\frac{a}{m^2},\frac{2a}{m}\right)$.

Ans. $y^2=4ax$, differentiating with respect to 'x'

$$2y\frac{dy}{dx} = 4a \Rightarrow \frac{dy}{dx} = \frac{2a}{y} \Rightarrow \left.\frac{dy}{dx}\right]_{\left(\frac{a}{m^2}, \frac{2a}{m}\right)} = m$$ 1+1

Equation of normal.

$$y - \frac{2a}{m} = -\frac{1}{m}\left(x - \frac{a}{m^2}\right)$$ 1

or $m^2x + m^3y - 2am^2 - a = 0$

OR

(b) Find the equation of the tangent to the curve $y(1 + x^2) = 2 - x$, where it crosses x-axis.

Ans. The point where the curve crosses x-axis is (2, 0) $\frac{1}{2}$

Differentiating, $y(1 + x^2) = 2 - x$ with respect to 'x'

$\frac{dy}{dx} = \frac{-2xy-1}{1+x^2}$, slope of tangent at (2, 0) = $-\frac{1}{5}$ $1+\frac{1}{2}$

Equation of tangentt: $y - 0 = -\frac{1}{5}(x-2)$ 1

or $x + 5y - 2 = 0$

33. Find:

$$\int \frac{x^2}{(x-1)(x+1)^2} dx$$

Ans. $\int \frac{x^2}{(x-1)(x+1)^2} dx = \frac{1}{4}\int \frac{1}{x-1} dx + \frac{3}{4}\int \frac{1}{x+1} dx - \frac{1}{2}\int \frac{1}{(x+1)^2} dx$ $1\frac{1}{2}$

$= \frac{1}{4}\log|x-1) + \frac{3}{4}\log|x+1| + \frac{1}{2(x+1)} + c$ $1\frac{1}{2}$

34. If the solution of the differential equation $\frac{dy}{dx} = \frac{2xy - y^2}{2x^2}$ is $\frac{ax}{y} = b\log|x| + C$, find the value of a and b.

Ans. The given differential equation can be written as.

$$\frac{dy}{dx} = \frac{y}{x} - \frac{1}{2}\left(\frac{y}{x}\right)^2$$

Put $y = vx, \ \frac{dy}{dx} = v + x\frac{dv}{dx}$, we get $\frac{1}{2}$

$$v + x\frac{dv}{dx} = v - \frac{v^2}{2}$$

$\Rightarrow \quad \frac{1}{v^2}dv = \left(-\frac{1}{2}\right)\frac{1}{x}dx$, integrating both sides $\frac{1}{2}$

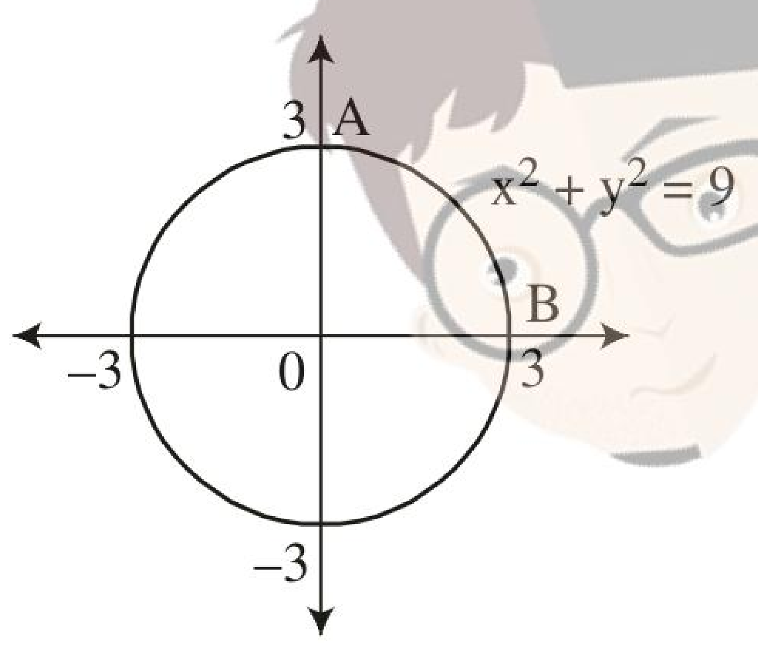
$\Rightarrow \quad -\frac{1}{v} = -\frac{1}{2}\log|x| + c$ $\frac{1}{2}$

$\Rightarrow \quad -\frac{x}{y} = -\frac{1}{2}\log|x| + c, \ \ a = -1, \ \ b = -\frac{1}{2}$ $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

or $\quad \frac{x}{y} = \frac{1}{2}\log|x| + c, a = 1, b = \frac{1}{2}$

35. Using integration, find the area bounded by the circle $x^2 + y^2 = 9$.

Ans.



Area of circle $= 4.\text{ar}(AOB) = 4\int_0^3 y\,dx$ 1

$= 4\int_0^3 \sqrt{3^2 - x^2}\,dx$ $\frac{1}{2}$

$= 4\left\{\int \frac{x}{2}\sqrt{9 - x^2} + \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right)\right\}\Bigg]_0^3$ 1

$= 4\left\{0 + \frac{9}{2}\times\frac{\pi}{2}\right\} - 0\Bigg]_0^3 = 9\pi$ sq. units $\frac{1}{2}$

SECTION-V

Question number 36 to 38 carry 5 marks each.

36. (a) If $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 2 & -3 \\ 1 & -2 & 6 \end{bmatrix}$, find A^{-1}.

Hence, soslve the following system of equations:

$3x + 4y + 2z = 8$

$2y - 3z = 3$

$x - 2y + 6z = -2$

Ans. $|A| = 2.$ $\frac{1}{2}$

co-factors of the elements of the matrix.

$$\left.\begin{array}{l} A_{11}=6 \;\; A_{12}=-3 \;\; A_{13}=-2 \\ A_{21}=-28 \;\; A_{22}=16 \;\; A_{23}=10 \\ A_{31}=-16 \;\; A_{32}=9 \;\; A_{33}=6 \end{array}\right\}$$ 2

(1 mark for any 4 correct co-factors)

$$\therefore \;\; A^{-1}=\frac{1}{|A|}\cdot \text{adj}(A)=\frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}$$ 1

The given system of equations can be written as

$A \cdot X = B$ $\frac{1}{2}$

where, $X = A^{-1}.B \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\frac{1}{2}\begin{bmatrix} 6 & -28 & -16 \\ -3 & 16 & 9 \\ -2 & 10 & 6 \end{bmatrix}\begin{bmatrix} 8 \\ 3 \\ -2 \end{bmatrix}=\begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$

$\therefore \; x = -2, \; y = 3, \; z = 1$ 1

OR

(b) If $A^{-1}=\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

Ans. $|B| = 1, \text{ adj }(B) = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ $\frac{1}{2}+2$

$$\therefore \quad B^{-1}=\frac{1}{|B|}\text{adj}(B)=\begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$$ 1

$$(AB)^{-1} = B^{-1}\cdot A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$ $\frac{1}{2}$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$ 1

37. (a) Find the shortest distance between the following lines:

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1} \text{ and } \frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$$

Ans. Vector equation of the lines:

$\vec{a}_1 = -\hat{i} - \hat{j} - \hat{k}, \vec{b}_1 = (7\hat{i} - 6\hat{j} + \hat{k})$

and $\vec{a}_2 = 3\hat{i} + 5\hat{j} + 7\hat{k}, \vec{b}_2 = (\hat{i} - 2\hat{j} + \hat{k})$ 1

$$\vec{a}_2 - \vec{a}_1 = 4\hat{i} + 6\hat{j} + 8\hat{k}, \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix} = -4\hat{i} - 6\hat{k} - 8\hat{k}$$ $1+1\frac{1}{2}$

$$\text{Shortest distance} = \left| \frac{(4\hat{i} + 6\hat{j} + 8\hat{k}) \cdot (-4\hat{i} - 6\hat{j} - 8\hat{k})}{|-4\hat{i} - 6\hat{j} - 8\hat{k}|} \right| = \sqrt{116}$$ $1\frac{1}{2}$

OR

(b) Find the distance of the point (–1, –5, –10) from the point of intersection of the line

$\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r} \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$.

Ans. General point on the line is: $\vec{r} = (2+3\lambda)\hat{i} + (-1+4\lambda)\hat{j} + (2+2\lambda)\hat{k}$ $1\frac{1}{2}$

For the point of intersection of the line with plane:

$1(2+3\lambda) - 1(-1+4\lambda) + 1(2+2\lambda) = 5 \Rightarrow \lambda = 0$ $1\frac{1}{2}$

$\therefore$ Point of intersection is : (2, –1, 2) 1

Distance = $\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = \sqrt{169} = 13$ 1

38. (a) Solve the following linear programming problem graphically:

Maximise $z = 3x + 9y$

subject to constraints

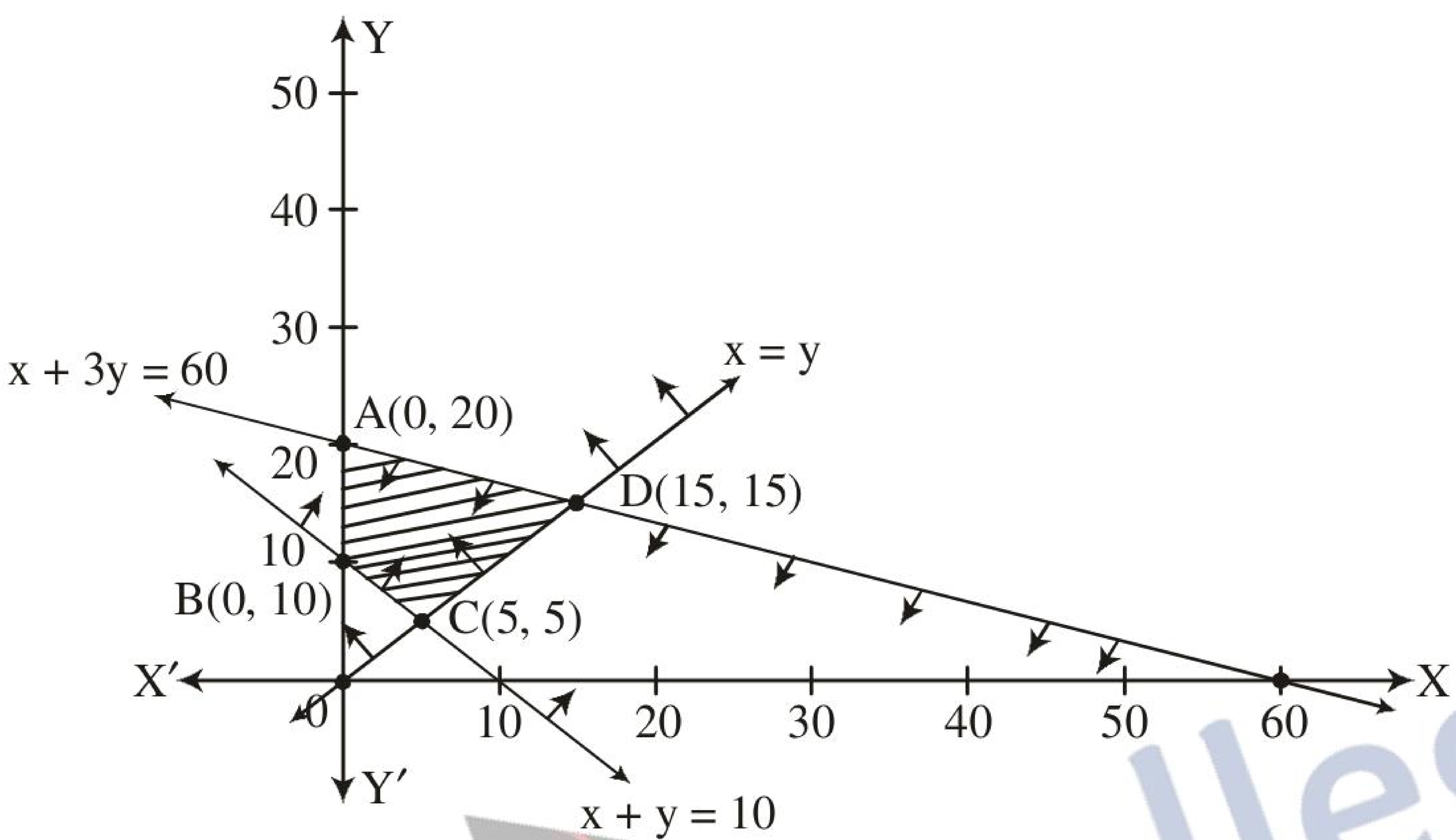
$x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x, y \geq 0$

Ans.



Correct graph 3

Value of z at corner points

$z(A) = 3(0) + 9(20) = 180$

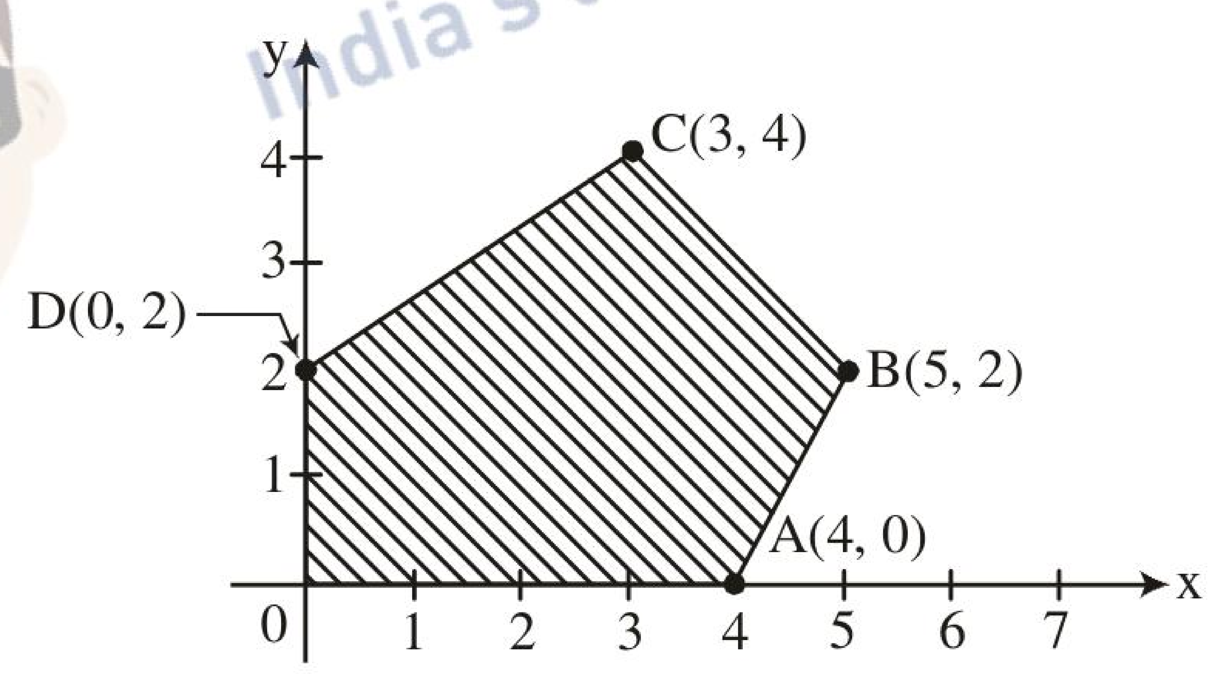
$z(B) = 0 + 90 = 90$

$z(C) = 15 + 45 = 60$ 1

$z(D) = 45 + 45 = 180$

Max(z) = 180 at any point on AD. 1

(b) The corner point of the feasible region determined by the system of linear inequations are as shown below:



Answer each of the following:

(i) Let $z = 13x - 15y$ be the objective function. Find the maximum and minimum values of z and also the corresponding points at which the maximum and minimum values occur.

Ans. $z(A) = 13(4) - 15(0) = 52$

$z(B) = 13(5) - 15(2) = 35$

$z(C) = 13(3) - 15(4) = -21$ 3

$z(D) = 13(0) - 15(2) = -30$

$z(0) = 0$ $\frac{1}{2}+\frac{1}{2}$

$\therefore$ Max(z) = 52 at A(4, 0), Min(z) = –30 at (0, 2)

(ii) Let z = kx + y be the objective function. Find k, if the value of z at A is same as the value of z at B.

Ans. $z(A) = z(B) \Rightarrow 4k + 0 = 5k + 2 \Rightarrow k = -2$ 1