QUESTION PAPER CODE 65/2/C

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

		Marks
1.	$e^{2x} \sin 2x$	1 m
2.	$y = mx, \dfrac{dy}{dx} = \dfrac{y}{x}$	½ + ½ m
3.	$\dfrac{dy}{dx} + \dfrac{1}{x \log x} y = \dfrac{2}{x}$, Integrating factor $= \log x$	½ + ½ m
4.	$\vec{a} \times \vec{b} = -17\hat{i} + 13\hat{j} + 7\hat{k}, \left\| \vec{a} \times \vec{b} \right\| = \sqrt{507}$	½+½ m
5.	$\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{\|\vec{a}\|\|\vec{b}\|}, \ \theta = \dfrac{2\pi}{3}$	½ + ½ m
6.	$d = \left\| \dfrac{\vec{a} \cdot \vec{n} - p}{\|\vec{n}\|} \right\|$, distance $= \dfrac{13}{7}$	½ + ½ m

SECTION - B

7.	$\overrightarrow{BA} = \hat{i} + (x-1)\hat{j} + 4\hat{k}, \overrightarrow{CA} = \hat{i} - 3\hat{k}, \ \overrightarrow{DA}\ 3\hat{i} + 3\hat{j} - 2\hat{k}$	1½ m
	$\left[\overrightarrow{BA}, \ \overrightarrow{CA}, \ \overrightarrow{DA} \right] = 0$	1 m
	$\begin{vmatrix} 1 & x-1 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$	1 m
	$x = 4$	½ m

8. $\vec{r} = (4\hat{i}+2\hat{j}+2\hat{k})+\lambda(2\hat{i}+3\hat{j}+6\hat{k})$ $\vec{a}+\lambda\vec{b}$ 1 m

Let L be the foot of perpendicular

Position vector of L is $(2\lambda+4)\hat{i}+(3\lambda+2)\hat{j}+(6\lambda+2)\hat{k}$ ½ m

$\vec{PL} = (2\lambda+3)\hat{i}+3\lambda\hat{j}+(6\lambda-1)\hat{k}$ ½ m

$\vec{PL}\cdot\vec{b} = 2(2\lambda+3)+3(3\lambda)+6(6\lambda-1) = 0$

$\Rightarrow \lambda = 0$ 1 m

$\vec{PL} = 3\hat{i}-\hat{k}$

$|\vec{PL}| = \sqrt{10}$ units 1 m

9. $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$(1-x) = \sin\left(\frac{\pi}{2}+2\sin^{-1}x\right)$ 1 m

$1-x = \cos(2\sin^{-1}x)$ 1 m

$1-x = 1-2x^2$ 1 m

$\Rightarrow x = 0, \frac{1}{2}$ ½ + ½ m

$x = \frac{1}{2}$ is rejected

OR

L.H.S $= 2\sin^{-1}\frac{3}{5} - \tan^{-1}\frac{17}{31}$

$= 2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31}$ 1 m

$= \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$ 1 m

$= \tan^{-1}\left(\frac{\frac{24}{7} - \frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right)$ 1 m

$= \tan^{-1}\left(\frac{625}{625}\right) = \frac{\pi}{4}$ 1 m

10. $A^2 = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$ 1½ m

$A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} + \begin{bmatrix} -4 & -8 & -8 \\ -8 & -4 & -8 \\ -8 & -8 & -4 \end{bmatrix} + \begin{bmatrix} -5 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & -5 \end{bmatrix} = O$ 1 m

$A^2 - 4A - 5I = O \Rightarrow A^{-1} = \frac{1}{5}(A - 4I)$ 1 m

$A^{-1} = \frac{1}{5}\left\{\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}\right\} = \frac{1}{5}\begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix}$ ½ m

OR

$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1 m

Using elementary row operations to reach at

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$ 2 m

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$ 1 m

11. $$\begin{vmatrix} x+2 & x+6 & x-1 \\ x+6 & x-1 & x+2 \\ x-1 & x+2 & x+6 \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} 3x+7 & x+6 & x-1 \\ 3x+7 & x-1 & x+2 \\ 3x+7 & x+2 & x+6 \end{vmatrix} = 0$$ 1 m

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - R_1$

$$\begin{vmatrix} 3x+7 & x+6 & x-1 \\ 1 & -7 & 3 \\ 1 & -4 & 7 \end{vmatrix} = 0$$ 2 m

$(3x+7)(-37) = 0 \Rightarrow x = \frac{-7}{3}$ 1 m

12. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx \Rightarrow 2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx$ 1 m

$$2I = \int_0^{\pi/2} \frac{\sec^2 x/2}{2\tan\frac{x}{2} + 1 - \tan^2 x/2} dx$$

$I = -\int_0^1 \frac{1}{(t-1)^2 - (\sqrt{2})^2} dt$, where $\tan\frac{x}{2} = t$ 1½ m

$$I = \left[-\frac{1}{2\sqrt{2}} \log\left| \frac{t-1-\sqrt{2}}{t-1+\sqrt{2}} \right| \right]_0^1$$ 1 m

$I = \frac{1}{2\sqrt{2}} \log \left| \frac{1+\sqrt{2}}{1-\sqrt{2}} \right|$ ½ m

OR

$\int_{-1}^{2} \left(e^{3x} + 7x - 5\right) dx$ here $h = \frac{3}{n}$ ½ m

$= \lim_{h \to 0} h \left[f(-1) + f(-1+h) + \dots\dots\dots \right]$

$= \lim_{h \to 0} h \left[\left(e^{-3} - 12\right) + \left(e^{-3+3h} + 7h - 12\right) + \dots\dots + \left(e^{-3+\overline{n-1}\,h} + 7(n-1)h - 12\right) \right]$ 1 m

$= \lim_{h \to 0} h \left[e^{-3}\left(1 + e^{3h} + e^{6h} + \dots\dots + e^{3(n-1)h}\right) + 7h\left(1 + 2 + 3 + \dots.\overline{n-1}\right) - 12\,nh \right]$ 1 m

$= \lim_{h \to 0} h \left[\frac{e^{-3}\left(e^{3nh} - 1\right)h}{e^{3h} - 1} + \frac{7(nh)(nh - h)}{2} - 12nh \right]$ 1 m

$= \frac{e^{-3}\left(e^{9} - 1\right)}{3} + \frac{63}{2} - 36 = \frac{e^{9} - 1}{3e^{3}} - \frac{9}{2}$ ½ m

13. $\int \frac{x^2}{x^4 + x^2 - 2} dx$

$\int \frac{x^2}{x^4 + x^2 - 2} = \frac{t}{t^2 + t - 2} = \frac{t}{(t+2)(t-1)}$ where $x^2 = t$ 1½ m

$= \frac{2}{3(t+2)} + \frac{1}{3(t-1)}$ 1½ m

$\int \frac{x^2}{x^4 + x^2 - 2} dx = \int \frac{2}{3\left(x^2 + 2\right)} dx + \int \frac{1}{3\left(x^2 - 1\right)} dx$

$= \frac{2}{3\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{1}{6} \log \left| \frac{x-1}{x+1} \right| + c$ 1 m

14. Let E_1 : two headed coin is chosen

E_2 : unbiased coin is chosen

A : All 5 tosses are heads ½ m

$P(E_1) = \frac{1}{5},\ P(E_2) = \frac{4}{5},\ P\left(A/E_1\right) = 1,\ P\left(A/E_2\right) = \frac{1}{32}$ 2 m

$P\left(E_1/A\right) = \frac{P(E_1)P\left(A/E_1\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)P\left(A/E_2\right)}$ ½ m

$P\left(E_1/A\right) = \frac{\frac{1}{5}\times 1}{\frac{1}{5}\times 1 + \frac{4}{5}\cdot\frac{1}{32}} = \frac{8}{9}$ 1 m

OR

Let the coin is tossed n times

$1 - P(0) > \frac{80}{100}$ 1½ m

$P(0) < \frac{1}{5}$ ½ m

${}^nC_0 \left(\frac{1}{2}\right)^n \left(\frac{1}{2}\right)^0 < \frac{1}{5}$ 1 m

$\left(\frac{1}{2}\right)^n < \frac{1}{5} \Rightarrow n \geq 3$ 1 m

15. $\int \frac{x+3}{(x+5)^3}\ e^x\ dx$

$\int \frac{1}{(x+5)^2} - \frac{2}{(x+5)^3}\ e^x\ dx$ 1 m

$$\int \frac{1}{(x+5)^2}\ e^x\ dx - \int \frac{2}{(x+5)^3}\ e^x\ dx$$ ½ m

$$= \frac{1}{(x+5)^2}\ e^x + \int \frac{2}{(x+5)^3}\ e^x\ dx - \int \frac{2}{(x+5)^3}\ e^x\ dx$$ 2 m

$$= \frac{e^x}{(x+5)^2} + c$$ ½ m

16.
$$\begin{matrix} & F & M & T \\ x \\ y \\ z \end{matrix} \begin{pmatrix} 30 & 12 & 70 \\ 40 & 15 & 55 \\ 35 & 20 & 75 \end{pmatrix} \begin{pmatrix} 25 \\ 100 \\ 50 \end{pmatrix} = \begin{pmatrix} 5450 \\ 5250 \\ 6625 \end{pmatrix}$$ 1½ m

Funds collected by school x : ₹ 5450, school y = ₹ 5250

school z = ₹ 6625 1

Total collected funds = ₹ 17325 ½ m

For writing any value 1 m

17. $y = e^{ax} \cos bx$

$y_1 = ae^{ax} \cos bx - b\, e^{ax} \sin b\, x$ 1 m

$y_1 = ay - b\, e^{ax} \sin b\, x$ 1 m

$y_2 = ay_1 - b\, [ae^{ax} \sin bx + b\, e^{ax} \cos b\, x]$ 1 m

$y_2 = ay_1 - a\, be^{ax} \sin bx - b^2 e^{ax} \cos b\, x$

$y_2 = a\, y_1 - a\, (ay - y_1) - b^2\, y$

$y_2 - 2\, a\, y_1 + (a^2 + b^2)\, y = 0$ 1 m

18. $x^x + x^y + y^x = a^b$

Let $u = x^x, v = x^y, \quad w = y^x, \quad \frac{du}{dx} + \frac{dv}{dx} + \frac{dw}{dx} = 0$ ½ m

$\frac{du}{dx} = x^x \,(1 + \log x)$ 1 m

$\frac{dv}{dx} = x^y \left(\frac{y}{x} + \frac{dy}{dx} \log x\right)$ 1 m

$\frac{dw}{dx} = y^x \left(\frac{x}{y} \cdot \frac{dy}{dx} + \log y\right)$ 1 m

$\frac{dy}{dx} = -\left(\frac{x^x (1 + \log x) + y\, x^{y-1} + y^x \log y}{x^y \log x + x\, y^{x-1}}\right)$ ½ m

19. $\frac{dx}{at} = a\,[\sin 2t\,(-2 \sin 2t) + (1 + \cos 2t)(2 \cos 2t)]$ 1 m

$\frac{dy}{dt} = b\,[2 \sin 2t \cos 2t - 2 \sin 2t\,(1 - \cos 2t)]$ 1 m

$\frac{dy}{dx} = \frac{b\,[2 \sin 2t \cos 2t - 2 \sin 2t\,(1 - \cos 2t)]}{a\,[\sin t\,(-2 \sin 2t) + (1 + \cos 2t)(2 \cos 2t)]}$ 1 m

$= \frac{4\,b \cos 3t \sin t}{4\,a \cos 3t \cos t} = \frac{b}{a} \tan t = \frac{b}{2} \times 1 = \frac{b}{a}$ 1 m

SECTION - C

20. $f(x) = \sin^2 x - \cos x$

$f'(x) = \sin x\,(2 \cos x + 1)$ 1 m

$f'(x) = 0 \Rightarrow \sin x = 0$ and $2 \cos x + 1 = 0 \Rightarrow x = 0,\ 2\frac{\pi}{3},\ \pi$ 2½ m

$f(0) = -1,\ f\left(\frac{2\pi}{3}\right) = \frac{5}{4},\ f(\pi)=1$ 1½ m

Absolute maximum value is $\frac{5}{4}$ ½ m

Absolute minimum value is – 1 ½ m

21. Two lines $\vec{r} = \vec{a_1} + \lambda\,\vec{b_1}$ and $r = \vec{a_2} + \mu\,\vec{b_2}$ are coplanar

if $\left(\vec{a_2} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = 0$ 1 m

Here $\left(-\hat{i} + 3\hat{j} + \hat{k}\right)\cdot\left[\left(\hat{i} - \hat{j} + \hat{k}\right)\times\left(2\hat{i} - \hat{j} + 3\hat{k}\right)\right] = 0$ 2 m

Equation of plane is

$$\left(\vec{r} - \vec{a_1}\right)\cdot\left(\vec{b_1} \times \vec{b_2}\right) = 0$$ 1 m

$$\left[\vec{r} - \left(\hat{i} + \hat{j} + \hat{k}\right)\right]\cdot\left[\left(\hat{i} - \hat{j} + \hat{k}\right)\times\left(2\hat{i} - \hat{j} + 3\hat{k}\right)\right] = 0$$

$\vec{r}\cdot\left(-2\hat{i} - \hat{j} + \hat{k}\right) + 2 = 0$ 2 m

22. (i) Let $(e,\ e')$ be the identity element in A

$(a, b) * (e,\ e') = (a, b) = (e,\ e') * (a, b)$

$(a\,e,\ b + a\,e') = (a, b)$

$$\left.\begin{array}{r} ae = a \Rightarrow e = 1 \\ b + a\,e' = b \Rightarrow e' = 0 \end{array}\right] \Rightarrow \text{identity} : (1, 0)$$ 2 ½ m

(ii) Let (x, y) is inverse of $(a, b) \in A$

$(a, b) * (x, y) = (1, 0) = (x, y) * (a, b)$

$(a\,x,\ b + a\,y) = (1, 0)$

$$\left.\begin{array}{l} ax = 1 \Rightarrow x = \dfrac{1}{a} \\ b + a y = 0 \Rightarrow y = \dfrac{-b}{a} \end{array}\right] \Rightarrow \text{inverse of } (a, b) = \left(\dfrac{1}{a}, \dfrac{-b}{a}\right)$$ 2 ½ m

Inverse of $(5, 3) = \left(\dfrac{1}{5}, \dfrac{-3}{5}\right)$ ½ m

Inverse of $\left(\dfrac{1}{2}, 4\right) = (2, -8)$ ½ m

OR

One – One : - Case I : when x and y are even

$$f(x) = f(y) \Rightarrow x + 1 = y + 1 \Rightarrow x = y$$

Case II : when x and y are odd

$$f(x) = f(y) \Rightarrow x - 1 = y - 1 \Rightarrow x = y$$

Case III : one of them is even and one of them is odd

$$f(x) \neq f(y) \Rightarrow x + 1 \neq y - 1 \Rightarrow x \neq y$$ 2 ½ m

Onto : Let $y \in W$

$f(y - 1) = y$ if y is odd

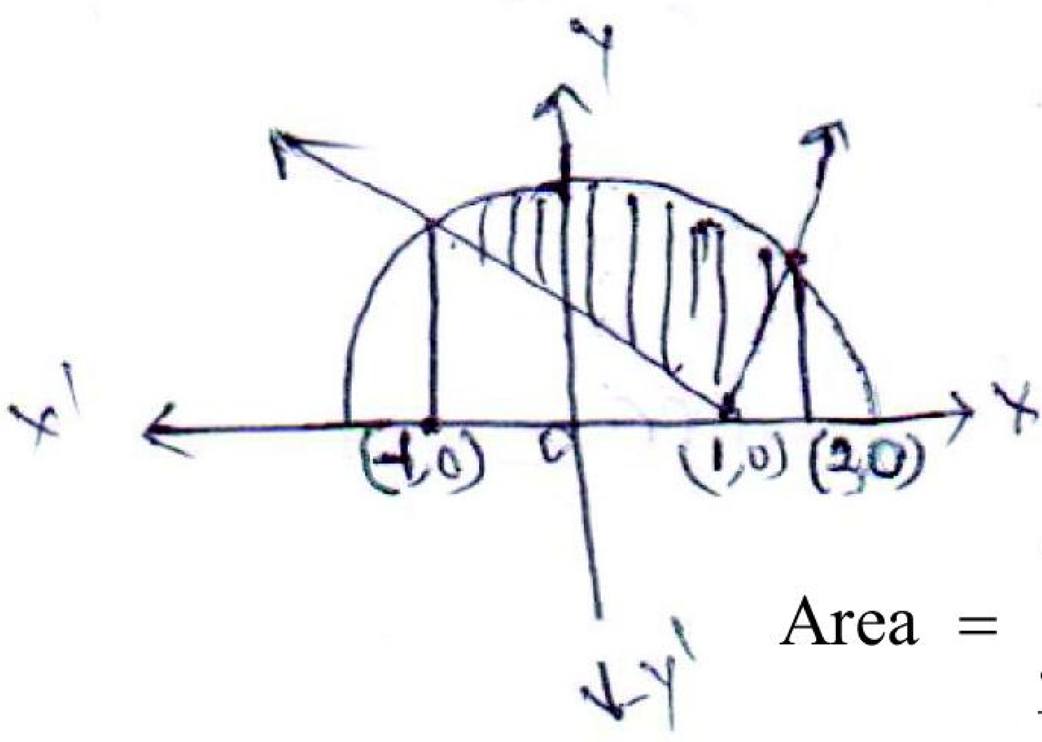
$f(y + 1) = y$ if y is even

So $\forall\ y \in W$, there exist some element in domain of f

$\Rightarrow$ f is invertible 2½ m

$$f^{-1}(x) = \begin{cases} x - 1, & x \text{ is odd} \\ x + 1, & x \text{ is even} \end{cases}$$ 1 m

23. Figure 1 m



For finding (– 1, 0) , (1, 0) (2, 0) 1½ m

$$\text{Area} = \int_{-1}^{2} \sqrt{5-x^2}\, dx - \int_{-1}^{1} -(x-1)\, dx - \int_{-1}^{2} (x-1)\, dx$$ 1½ m

$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} + \left[\frac{(x-1)^2}{2}\right]_{-1}^{1} - \left[\frac{(x-1)^2}{2}\right]_{1}^{2}$$ 1 ½ m

$$= \left(1+\frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right) + \left(1+\frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right) - \frac{1}{2}\times 4 - \frac{1}{2}\times 1$$

$$= \frac{5}{2}\left(\sin^{-1}\frac{2}{\sqrt{5}} + \sin^{-1}\frac{1}{\sqrt{5}}\right) - \frac{1}{2} \text{ sq. units}$$ ½ m

24. $x^2\, dy = (2\, xy + y^2)\, dx$

$$\frac{dy}{dx} = \frac{2\, xy + y^2}{x^2}$$ ½ m

$$y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$$ 1 m

$$v + x\frac{dv}{dx} = 2v + v^2 \Rightarrow \int \frac{1}{v^2+v}\, dv = \int \frac{1}{x}\, dx$$ 2 m

$$\Rightarrow \log\left|\frac{v}{v+1}\right| = \log x + \log c$$ 1 m

$$\Rightarrow \log\left|\frac{y}{y+x}\right| = \log cx \Rightarrow \frac{y}{y+x} = cx$$ 1 m

$$x = 1,\ y = 1 \Rightarrow c = \frac{1}{2}$$

$$x^2 + xy - 2\, y = 0$$ ½ m

OR

Given differential equation can be written as

$$\frac{dy}{dx} + \frac{1}{1+x^2}\, y = \frac{e^{m \tan^{-1}x}}{1+x^2}$$ 1 m

Integrating factor is $e^{\tan^{-1}x}$ 1 m

Solution is $y \cdot e^{\tan^{-1}x} = \int \frac{e^{m \tan^{-1}x}}{1+x^2} \cdot e^{\tan^{-1}x}\, dx$ 1½ m

$\Rightarrow\ y\, e^{\tan^{-1}x} = \int e^{(m+1)t} dt$, where $\tan^{-1}x = t$ 1 m

$$= \frac{e^{(m+1)t}}{m+1} = \frac{e^{(m+1)\tan^{-1}x}}{m+1} + c$$ 1 m

$y = 1, x = 0 \Rightarrow c = \frac{m}{m+1}$

$$y\, e^{\tan^{-1}x} = \frac{e^{(m+1)\tan^{-1}x}}{m+1} + \frac{m}{m+1}$$ ½ m

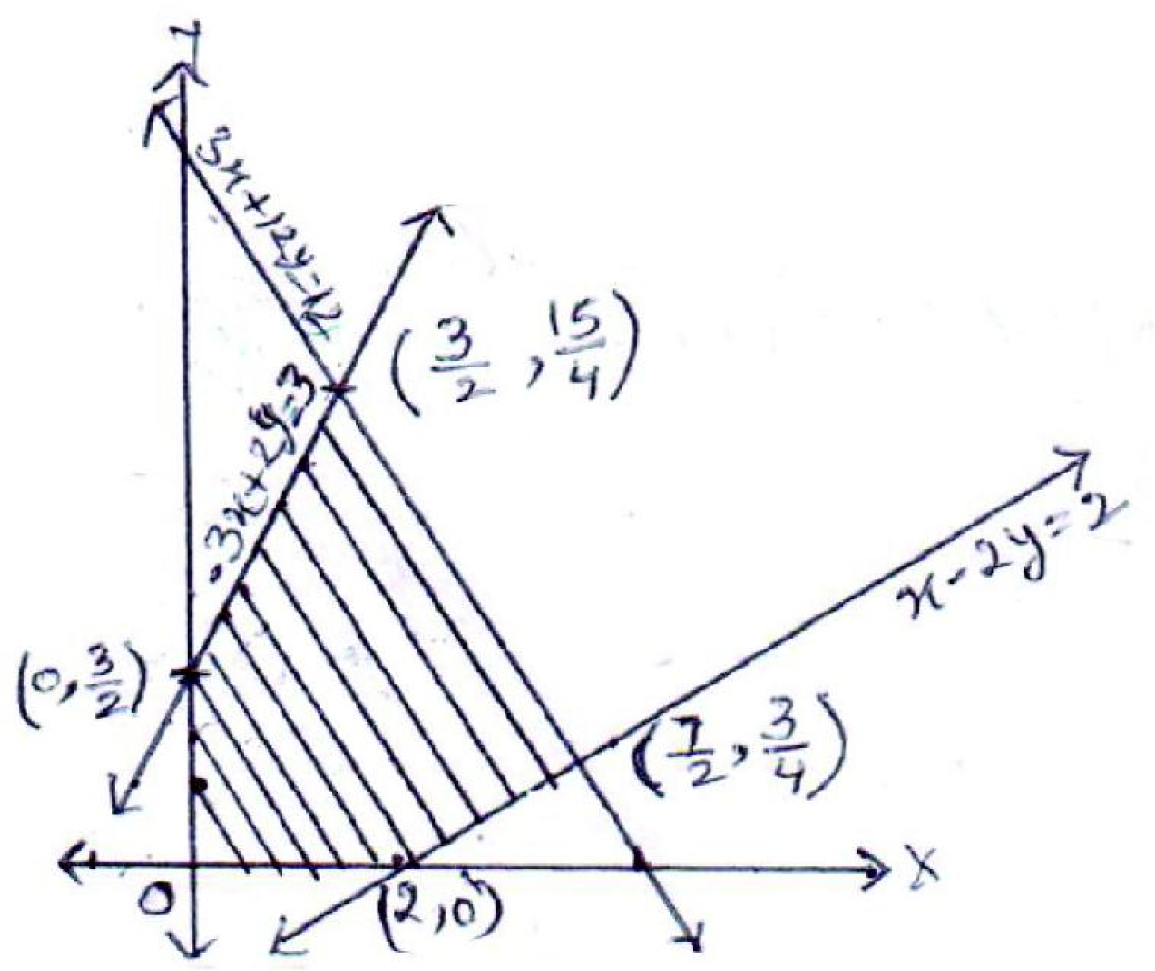
25.

x :	2	3	4	5	6	1 m
P (x) :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$	2 m
x . P (x) :	$\frac{2}{15}$	$\frac{6}{15}$	$\frac{12}{15}$	$\frac{20}{15}$	$\frac{30}{15}$	½ m
x^2 P (x) :	$\frac{4}{15}$	$\frac{18}{15}$	$\frac{48}{15}$	$\frac{100}{15}$	$\frac{180}{15}$	½ m

Mean $= \sum x \cdot P(x) = \frac{70}{15} = \frac{14}{3}$ 1 m

Variance $= \sum x^2\, P(x) = (\text{Mean})^2 = \frac{350}{15} - \frac{196}{9} = \frac{14}{9}$ 1 m

26.



Correct graph of three lines — 1×3 m

correct shading of feasible region — 1 m

vertices are $\left(0,\frac{3}{2}\right),\left(\frac{3}{2},\frac{15}{4}\right),$

$\left(\frac{7}{2},\frac{3}{4}\right),(2,0)$ — 1 m

$z=5x+2y$ is maximum

at $\left(\frac{7}{2},\frac{3}{4}\right) = 19$ and

minimum at $\left(0,\frac{3}{2}\right) = 3$ — 1 m