25/01/2023
Morning

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Memory Based
Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2023 (Online) Phase-1

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

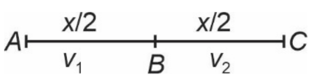
1. A car moving on a straight line travels in same direction half of the distance with uniform velocity v_1 and other half of the distance with uniform velocity v_2. Average velocity of the car is equal to

(1) $\frac{2v_1v_2}{v_1+v_2}$ (2) $\frac{v_1+v_2}{2}$

(3) $v_1 + v_2$ (4) $\sqrt{v_1v_2}$

Answer (1)

Sol.



$$t_1 = \frac{x}{2v_1},\ t_2 = \frac{x}{2v_2}$$

$$\text{So } v_{av} = \frac{\text{Total distance}}{\text{Total time}}$$

$$= \frac{x}{t_1+t_2}$$

$$= \frac{x}{\frac{x}{2v_1}+\frac{x}{2v_2}}$$

$$= \frac{2v_1v_2}{v_1+v_2}$$

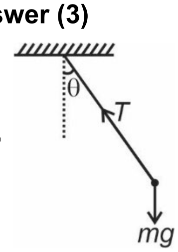
2. A car is moving with a constant speed of 2 m/s in circle having radius R. A pendulum is suspended from the ceiling of the car. Find the angle made by the pendulum with the vertical. Take $R = \frac{8}{15}$ m and g = 10 m/s^2

(1) 30° (2) 53°

(3) 37° (4) 60°

Answer (3)

Sol.



$$T\cos\theta = mg \qquad \ldots(1)$$

$$T\sin\theta = \frac{mv^2}{R} \qquad \ldots(2)$$

$$\Rightarrow \tan\theta = \frac{v^2}{Rg}$$

$$= \frac{4}{\frac{8}{15}\times 10} = \frac{3}{4}$$

$$\Rightarrow \theta = 37°$$

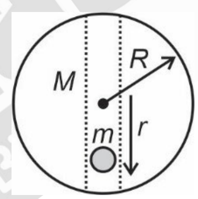
3. A particle is droped inside tunnel of earth about any diameter. Particle starts oscillating, with time period T. (R = radius of earth, g = acceleration due to gravity on earth's surface). Then find T.

(1) $T = 2\pi\sqrt{\frac{R}{g}}$ (2) $T = \pi\sqrt{\frac{R}{g}}$

(3) $T = 2\pi\sqrt{\frac{2R}{g}}$ (4) $T = 2\pi\sqrt{\frac{3R}{g}}$

Answer (1)

Sol.



$F = mg$ (towards centre)

$$\frac{mdv}{dt} = -\left(\frac{GMm}{R^3}\right)r$$

$$\frac{dv}{dt} = -\left(\frac{GM}{R^3}\right)r$$

$$g = \frac{GM}{R^2}$$

$$\frac{dv}{dt} = -\left(\frac{g}{R}\right)r$$

$$\omega^2 = \left(\frac{g}{R}\right)$$

$$\omega = \sqrt{\frac{g}{R}}$$

$$\boxed{T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{R}{g}}}$$

4. If T is the temperature of a gas then RMS velocity of the gas molecules is proportional to

(1) $T^{1/2}$ (2) $T^{-1/2}$

(3) T (4) T^2

Answer (1)

Sol. $v_{rms} = \sqrt{\frac{3RT}{M_0}}$

So $v_{rms} \propto \sqrt{T}$

5. Time period of a pendulum at earth's surface is T. Find the time period of the pendulum at distance (from centre) which is twice the radius of earth.

(1) $\frac{T}{4}$ (2) $4T$

(3) $\frac{T}{2}$ (4) $2T$

Answer (4)

Sol. We know that $T = 2\pi\sqrt{\frac{l}{g}}$

$\Rightarrow \quad T = 2\pi\sqrt{\frac{l}{\frac{GM}{R^2}}}$ ...(i)

Also, $T' = 2\pi\sqrt{\frac{l}{\frac{GM}{(2R)^2}}}$ ...(ii)

$\Rightarrow \quad \frac{T'}{T} = \frac{2}{1}$

$\Rightarrow \quad T' = 2T$

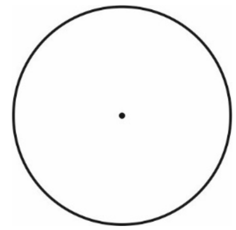
6. Let I_{cm} be the moment of Inertia of disc passing through center and perpendicular to its plane. I_{AB} be the moment of Inertia about axis AB that is in the plane of disc and $\frac{2r}{3}$ distance from centre, Find $\frac{I_{cm}}{I_{AB}}$?

(1) $\frac{1}{4}$ (2) $\frac{18}{25}$

(3) $\frac{9}{17}$ (4) $\frac{1}{2}$

Answer (2)

Sol.



$I_{cm} = \frac{1}{2}MR^2$ (Perpendicular to plane)

$I_{cm}(\text{in plane}) = \frac{1}{4}MR^2$

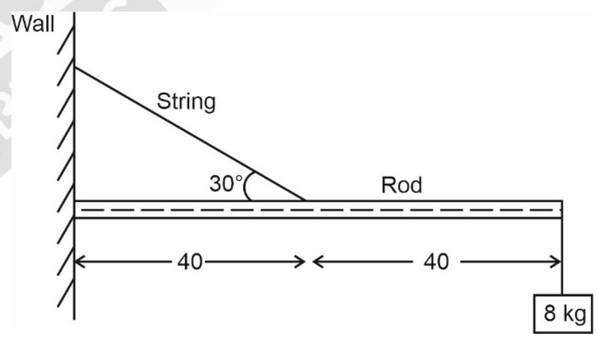
$I_{AB} = \frac{1}{4}MR^2 + M\left(\frac{2}{3}R\right)^2$

$= \frac{1}{4}MR^2 + \frac{4}{9}MR^2$

$= \frac{(9+16)MR^2}{36} = \frac{25}{36}MR^2$

$\frac{I_{cm}(\text{Perpendiular})}{I_{AB}} = \frac{\frac{1}{2}MR^2}{\frac{25}{36}MR^2} = \left(\frac{18}{25}\right)$

7. A massless rod is arranged as shown:



Find the tension in the string

(1) 320 N

(2) 640 N

(3) 160 N

(4) 480 N

Answer (1)

Sol. Balancing the torque on the rod about the point of contact with the wall:

$(T\sin 30^\circ) \times 40 = (mg) \times (40 + 40)$

$\Rightarrow \quad T = 320$ N

8. A carnot engine working between a source and sink at 200 K has efficiency of 50%. Another carnot engine working between the same source and another sink with unknown temperature *T* has efficiency of 75%. The value of *T* is equal to

(1) 400 K (2) 300 K
(3) 200 K (4) 100 K

Answer (4)

Sol. $\frac{50}{100} = 1 - \frac{200}{T}$

$\Rightarrow T = 400$ K

$T' = 100$ K

9. Mark the option correctly matching the following columns with appropriate dimensions

Column-I	Column-II
(A) Surface tension	(P) $[ML^{-1}T^{-2}]$
(B) Pressure	(Q) $[MT^{-2}]$
(C) Viscosity	(R) $[MLT^{-1}]$
(D) Impulse	(S) $ML–1T^{-1}$

(1) A(Q), B(P), C(R), D(S)
(2) A(Q), B(P), C(S), D(R)
(3) A(S), B(Q), C(P), D(R)
(4) A(R), B(P), C(Q), D(S)

Answer (2)

Sol. For surface tension

$F = SL$

$$[S] = \frac{[F]}{[L]} = [MT^{-2}]$$

For pressure

$$P = \frac{F}{A}$$

$$[P] = \frac{[F]}{[A]} = [ML^{-1}T^{-2}]$$

For viscosity coefficient

$$F = A\left(\frac{\Delta v}{\Delta z}\right)\eta$$

$$[\eta] = \frac{[F]}{[A]\left[\frac{\Delta v}{\Delta z}\right]} = [ML^{-1}T^{-1}]$$

For Impulse

$I = \Delta p$

$[I] = [\Delta p] = [MLT^{-1}]$

10. Assertion (A): Reverse biased diode is used in photodiode.

Reason (R): Forward biased current is more than reverse bias current.

(1) A & R are correct and R is correct explanation of A
(2) A & R are correct, R is not correct explanation of A
(3) A is incorrect and R is correct
(4) A is correct and R is incorrect

Answer (??)

Sol. (NCERT) It is easier to observe small changes in current due to intensity, when diode is in reverse bias.

11. Temperature of hot soup in a bowl goes from 98°C to 86°C in 2 minutes. The temperature of surroundings is 22°C. Find the time taken for the temperature of soup to go from 75°C to 69°C. [Assume Newton's law of cooling is valid]

(1) 1 minute
(2) 1.4 minute
(3) 2 minute
(4) 3.2 minute

Answer (2)

Sol. By Newton's law of cooling:

Rate of cooling (R) $\propto$ temperature difference

$\Rightarrow R_1 = kx\,(92°C – 22°C)$ …(i)

and $R_2 = kx\,(72°C – 22°C)$ …(ii)

$$\Rightarrow \frac{R_1}{R_2} = \frac{70}{50} = \frac{7}{5}$$

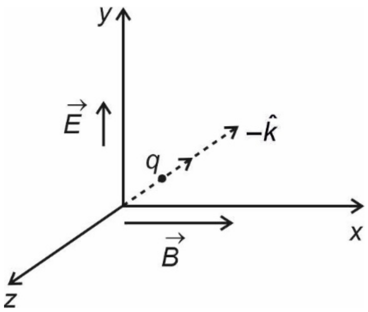
$\Rightarrow \Delta t_2 = 1.4$ minute

12. Electric field is applied along $+y$ direction. A charged particle is travelling along $-\hat{k}$, undeflected. Then magnetic field in the region will be along?

(1) $\hat{i}$ (2) $-\hat{i}$
(3) $\hat{j}$ (4) $-\hat{k}$

Answer (1)

Sol.



$$q(\vec{E}+\vec{V}\times\vec{B})=0$$

$$\vec{V}\times\vec{B}=-\vec{E}$$

$$\Rightarrow \quad V_0(-\hat{k})\times\vec{B}=-E_0\hat{j}$$

$\vec{B}$ should be in $\hat{i}$ direction to balance the electrostatic force on the charge particle.

13. When an electron is accelerated by 20 kV, its de-Broglie wavelength is λ_0. If the electron is accelerated by 40 kV, find its de-Broglie wavelength.

(1) $2\lambda_0$ (2) $\frac{\lambda_0}{2}$

(3) $\sqrt{2}\lambda_0$ (4) $\frac{\lambda_0}{\sqrt{2}}$

Answer (4)

Sol. We know $\lambda_0=\frac{h}{p}$

$$\Rightarrow \quad \lambda_0=\frac{h}{\sqrt{2mK}}$$

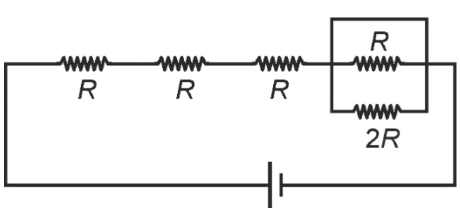
$$=\frac{h}{\sqrt{2meV}}$$

Since V doubles

$$\Rightarrow \quad \frac{\lambda'}{\lambda_0}=\sqrt{\frac{V}{2V}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \lambda'=\frac{\lambda_0}{\sqrt{2}}$$

14.



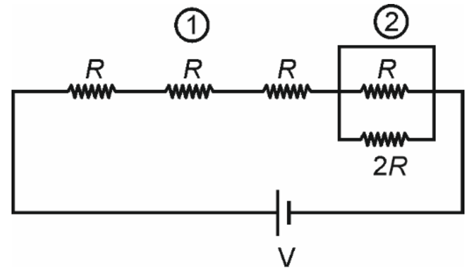
Find the equivalent resistance of the shown circuit across the terminals of ideal battery.

(1) $2R$ (2) $3R$

(3) $4R$ (4) $5R$

Answer (2)

Sol. In 2nd part of diagram a connecting wire is nullifying the resistance of parallel resistance thus their net resistance is zero. So net resistance of circuit is $3R$.



15. For an AM signal, it is given that

f_{carrier} = 10 MHz

f_{signal} = 5 kHz

Find the bandwidth of the transmitted signal.

(1) 5 kHz

(2) 10 kHz

(3) 2.5 kHz

(4) 20 MHz

Answer (2)

Sol. We know bandwidth = 2 fm

$\Rightarrow$ bandwidth = 10 kHz

16. Let nuclear densities of ${}^{4}_{2}\text{He}$ and ${}^{40}_{20}\text{Ca}$ be ρ_1 and ρ_2 respectively. Find the ratio $\frac{\rho_1}{\rho_2}$.

(1) 1 : 10

(2) 10 : 1

(3) 1 : 1

(4) 1 : 2

Answer (3)

Sol. We know radius $R=R_0A^{\frac{1}{3}}$

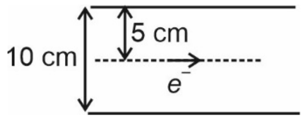
$$\Rightarrow \quad \text{Density}=\frac{\text{Mass}}{\text{Volume}}=\frac{A}{\frac{4}{3}\pi\left(R_0A^{\frac{1}{3}}\right)^3}$$

$$=\frac{1}{\frac{4}{3}\pi R_0^3}$$

$\Rightarrow$ Density is independent of A.

$$\Rightarrow \quad \frac{\rho_1}{\rho_2}=1$$

17. A particle is projected with 0.5 eV kinetic energy in an uniform electric field $\vec{E} = -10\text{ N/C }\hat{j}$, as shown in the figure. Find the angle made by the particle from the x-axis when it leaves $\vec{E}$.

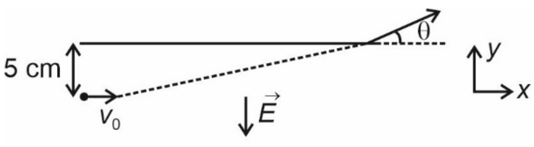


(1) θ = 45° (2) θ = 60°

(3) θ = 30° (4) θ = 37°

Answer (1)

Sol.



$v_x = v_0$

$a_y = \left(\frac{eE}{m_e}\right)$

$S_y = 5 \times 10^{-2}$ m

$v_y^2 = 2a_yS_y$

$v_y = \sqrt{\frac{2eE}{m_e}S_y}$

$\tan\theta = \left(\frac{v_y}{v_x}\right)$

$K_i = 0.5\text{ eV} = \frac{1}{2}\frac{m_ev_x^2}{e}$

$v_x = \sqrt{\frac{0.5 \times 2e}{m_e}} = \sqrt{\frac{e}{m_e}}$

$\tan\theta = \frac{\sqrt{\frac{2eE}{m_e} \times S_y}}{\sqrt{\frac{e}{m_e}}} = \sqrt{2ES_y} = \sqrt{2 \times 10 \times 5 \times 10^{-2}}$

$= \sqrt{1}$

tanθ =1

θ = 45°

18. ??

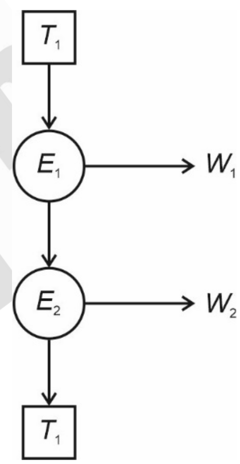
19. ??

20. ??

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. In the series sequence of two engines E_1 and E_2 as shown T_1 = 600 K and T_2 = 300 K. It is given that both the engines working on carnot principle have same efficiency, then temperature T at which exhaust of E_1 is fed into E_2 is equal to $300\sqrt{n}$ K. Value of n is equal to



Answer (02.00)

Sol. $\eta_1 = 1 - \frac{T}{600}$

$\eta_2 = 1 - \frac{300}{T}$

As efficiency is same

$\eta_1 = \eta_2$

$\frac{T}{600} = \frac{300}{T}$

$\Rightarrow T = \sqrt{180000}$

$= 300\sqrt{2}$ K.

So n = 2

22. A solenoid of length 2 m, has 1200 turns. The magnetic field inside the solenoid when 2 A current is passed through it is $N\pi \times 10^{-5}$ T. Find the value of N. (Diameter of solenoid is 0.5 m)

Answer (48.00)

Sol. 

$B_{inside} = \mu_0 n i$

N = Number of turns per unit length

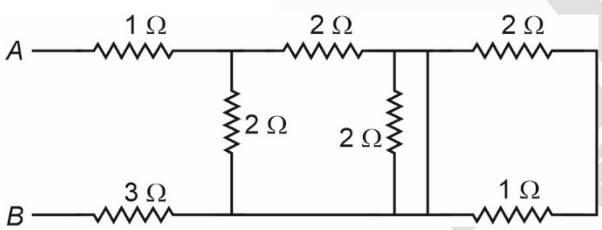
$$= \frac{1200}{2} = 600$$

i = current in a turn = 2 A

$B = 4\pi \times 10^{-7} \times 600 \times 2$

$= 48\pi \times 10^{-5}$ T

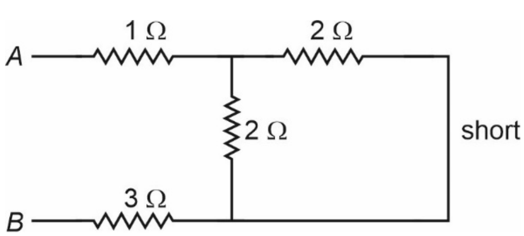
23. Consider a network of resistors as shown:



Find the effective resistance (in Ω) across A and B.

Answer (05.00)

Sol. Effectively, the network is:



$$\Rightarrow R_{AB} = 1\Omega + \frac{2 \times 2}{2+2}\Omega + 3\Omega$$

= 5 Ω

24. Find the ratio of density of oxygen $\left({}^{16}_{8}\text{O}\right)$ to the density of Helium $\left({}^{4}_{2}\text{He}\right)$ at STP.

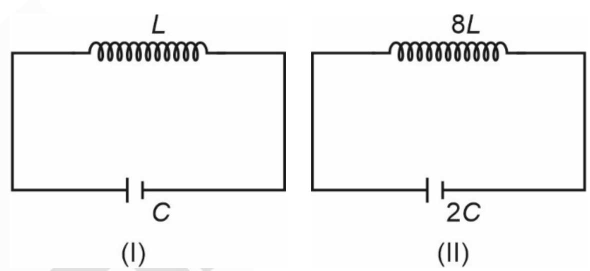
Answer (08.00)

Sol. $\frac{P}{\rho} = \frac{RT}{M_0}$

$$\Rightarrow \frac{\rho_1}{\rho_2} = \frac{M_1}{M_2}$$

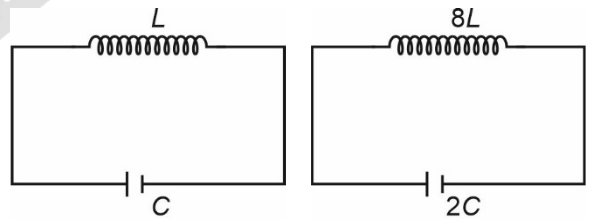
$$\frac{\rho_1}{\rho_2} = \frac{32}{4} = 8$$

25. Consider the following two LC circuits.



Then find $\frac{\omega_I}{\omega_{II}}$, where ω_I and ω_{II} are resonance frequencies of the Circuit I and Circuit II respectively.

Answer (04.00)

Sol. 

$$\omega_1 = \frac{1}{\sqrt{LC}}, \quad \omega_2 = \frac{1}{\sqrt{(8L \times 2C)}} = \frac{1}{4\sqrt{LC}}$$

$$\frac{\omega_1}{\omega_2} = \frac{4}{1}$$

26. ??

27. ??

28. ??

29. ??

30. ??

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Radius of 2nd orbit of Li^{2+} ion is x, radius of 3rd orbit of Be^{3+} will be

(1) $\frac{27x}{16}$ (2) $\frac{16x}{27}$

(3) $\frac{4}{3}x$ (4) $\frac{3}{4}x$

Answer (1)

Sol. $r_{Li^{2+}} = r_0 \times \frac{2^2}{3} = \frac{4r_0}{3} = x$

$\Rightarrow r_0 = \frac{3x}{4}$

$r_{Be^{3+}} = r_0 \times \frac{3^2}{4} = \frac{9r_0}{4} = \frac{9 \times 3 \times x}{4 \times 4}$

$r_{Be^{3+}} = \frac{27x}{16}$

2. If X-atoms are present at alternate corners and at body centre of a cube and Y-atoms are present at 1/3rd of face centres then what will be empirical formula?

(1) $X_{2.5}Y$

(2) X_5Y_2

(3) $X_{1.5}Y_2$

(4) X_3Y_2

Answer (4)

Sol. Number of X-atoms per unit cell = $1 + 4 \times \frac{1}{8}$

$= \frac{3}{2}$

Number of Y-atoms per unit cell = $2 \times \frac{1}{2} = 1$

$\therefore$ Empirical formula of the solid is X_3Y_2.

3. Thionyl chloride on reaction with white phosphorous gives compound A. A on hydrolysis gives compound B which is dibasic. Identify A and B.

(1) A– PCl_5, B–H_3PO_2 (2) A–P_4O_6, B–H_3PO_4

(3) A–$POCl_3$, B–H_3PO_4 (4) A–PCl_3, B–H_3PO_3

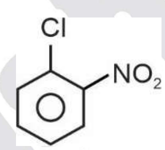
Answer (4)

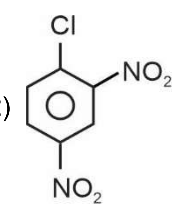
Sol. $P_4 + 8SOCl_2 \rightarrow \underset{(A)}{4\,PCl_3} + 4SO_2 + 2S_2Cl_2$

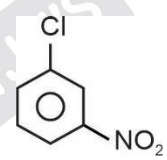
$\underset{(B)}{PCl_3 + H_2O \rightarrow H_3PO_3}$

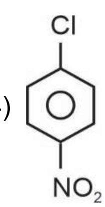
Correct answer is (4).

4. Which of the following shows least reactivity towards nucleophilic substitution reaction

(1) 

(2) 

(3) 

(4) 

Answer (3)

Sol. Aryl halides containing E.W.G at ortho or para position are more reactive than meta isomer towards nucleophilic substitution reaction.

5. The correct decreasing order of positive electron gain enthalpy for the following inert gases

He, Ne, Kr, Xe

(1) He > Ne > Kr > Xe

(2) He > Ne > Xe > Kr

(3) He > Xe > Ne > Kr

(4) Ne > Kr > Xe > He

Answer (4)

Sol. Correct order is Ne > Kr > Xe > He

6. Which of the following reaction is not involved in the extraction of copper metal?

(1) $CuFeS_2 \xrightarrow[\text{roasting}]{\text{partial}} Cu_2S + FeS + SO_2 + Cu_2O$

(2) $Cu_2S + 2Cu_2O \rightarrow 6Cu + SO_2$

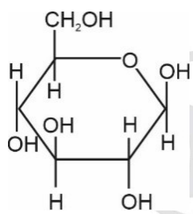
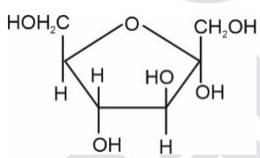
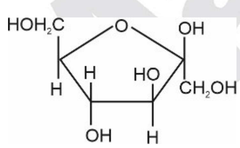
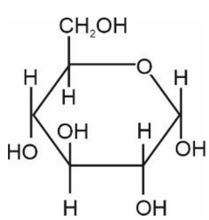
(3) $FeO + SiO_2 \rightarrow FeSiO_3$

(4) $2Fe_2O_3 + 3C \rightarrow 2Fe + 3CO_2$

Answer (4)

Sol. Option (4) contains the reaction involved in the reduction of hematite ore not in copper extraction.

7. Match the **List-I** and **List-II**.

List-I	List-II
(A) α-D-Glucopyranose	(1) 
(B) β-D-Glucopyranose	(2) 
(C) α-D-Fructofuranose	(3) 
(D) β-D-Fructofuranose	(4) 

(1) A → 4; B → 1; C → 2; D → 3

(2) A → 1; B → 4; C → 3; D → 2

(3) A → 2; B → 3; C → 4; D → 1

(4) A → 1; B → 3; C → 2; D → 4

Answer (1)

Sol. The correct options is (1).

8. Identify the correct sequence of reagents for the following conversion.

n-Heptane → → → → PhCOOH + $PhCH_2OH$

(1) Al_2O_3/Cr_2O_3, $CrO_2Cl_2 / H_3\overset{+}{O}$

Conc. NaOH, $H_3\overset{+}{O}$

(2) Al_2O_3/Cr_2O_3, $CrO_2Cl_2 / H_3\overset{+}{O}$

$H_3\overset{+}{O}$, Conc. NaOH

(3) CrO_2Cl_2, Al_2O_3,

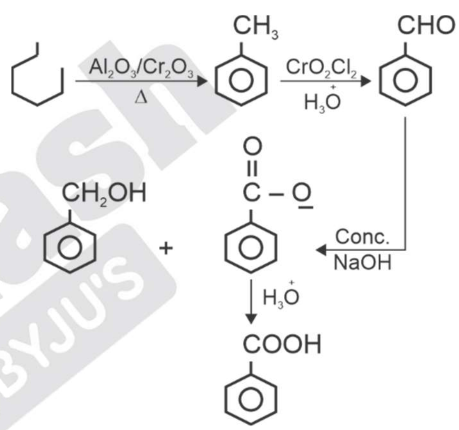
Conc. NaOH, $H_3\overset{+}{O}$

(4) Sn/HCl, NaOH

Conc. CrO_2Cl_2, HNO_3

Answer (1)

Sol.



9. Which of the following option contains the correct match?

Table-1 (Elements)	Table-2 (Flame colour)
(A) K	(P) Violet
(B) Ca	(Q) Brick red
(C) Sr	(R) Apple green
(D) Ba	(S) Crimson red

(1) (A) → P, (B) → Q, (C) → S, (D) → R

(2) (A) → Q, (B) → P, (C) → S, (D) → R

(3) (A) → R, (B) → S, (C) → P, (D) → Q

(4) (A) → S, (B) → R, (C) → Q, (D) → P

Answer (1)

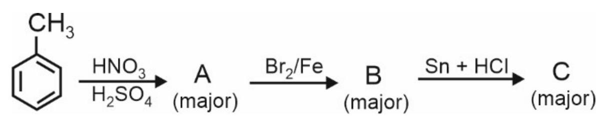
Sol. K → Violet

Ca → Brick red

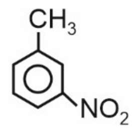
Sr → Crimson red

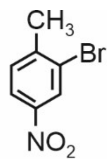
Ba → Apple green

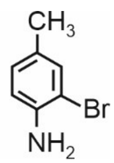
10. Consider the following sequence of reaction

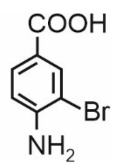


Which of the following options contains the correct structure?

(1) A is 

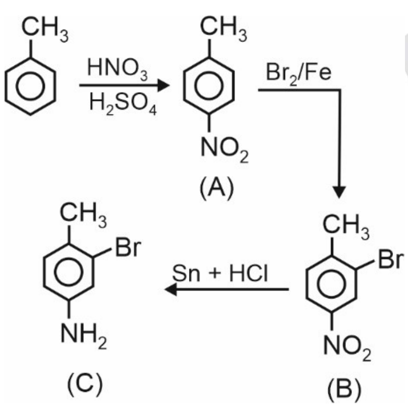
(2) B is 

(3) C is 

(4) C is 

Answer (2)

Sol.



11. Correct order of basic strength for

CH_3-NH_2 (1), $CH_3-\underset{|}{\overset{H}{N}}-CH_3$ (2)

$CH_3-N(CH_3)-CH_3$ (3), NH_3 (4)

is

(1) 2 > 1 > 3 > 4

(2) 3 > 2 > 1 > 4

(3) 4 > 2 > 1 > 3

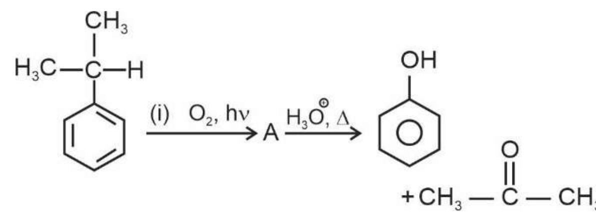
(4) 2 > 4 > 3 > 1

Answer (1)

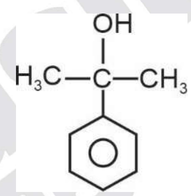
Sol. The correct order of basic strength in aqueous medium is

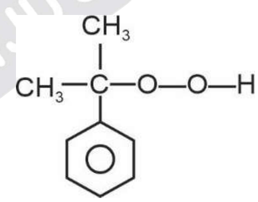
$$\underset{(2)}{CH_3-NH-CH_3} > \underset{(1)}{CH_3-NH_2} > \underset{(3)}{CH_3-N(CH_3)-CH_3} > \underset{(4)}{NH_3}$$

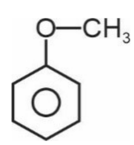
12. Consider the following conversion

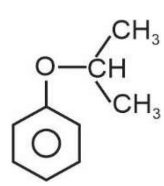


Which of the following option contains the correct structure of 'A'?

(1) 

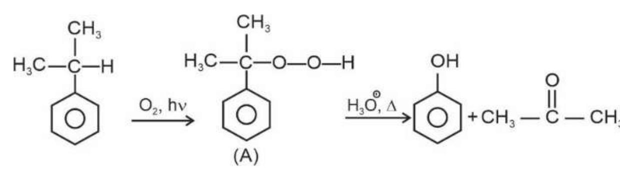
(2) 

(3) 

(4) 

Answer (2)

Sol.



13. Consider the following sequence of reactions

$$NO_2 \xrightarrow{H_2O} A + B$$

$$B + O_2 \longrightarrow O_3(g)$$

A is?

(1) N_2O (2) NO

(3) N_2O_3 (4) N_2

Answer (2)

Sol. $NO_2 \xrightarrow{H_2O} \underset{(A)}{\underset{(g)}{NO}} + \underset{(B)}{\underset{(g)}{O}}$

$$\underset{(B)}{O}(g) + O_2(g) \rightleftharpoons O_3(g)$$

14. Which one of the following complexes is paramagnetic in nature?

(1) $\left[Fe(NH_3)_2(CN)_4\right]^{2-}$

(2) $\left[Ni(CN)_4\right]^{2-}$

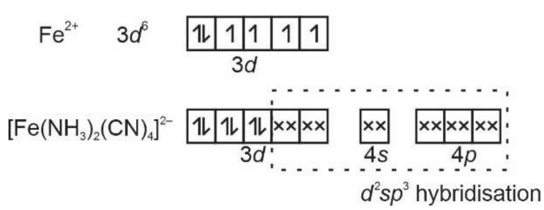
(3) $\left[Ni(H_2O)_6\right]^{2+}$

(4) $\left[Co(NH_3)_4Cl_2\right]^{+}$

Answer (3)

Sol.

(1) $\left[Fe(NH_3)_2(CN)_4\right]^{2-}$



Complex is diamagnetic

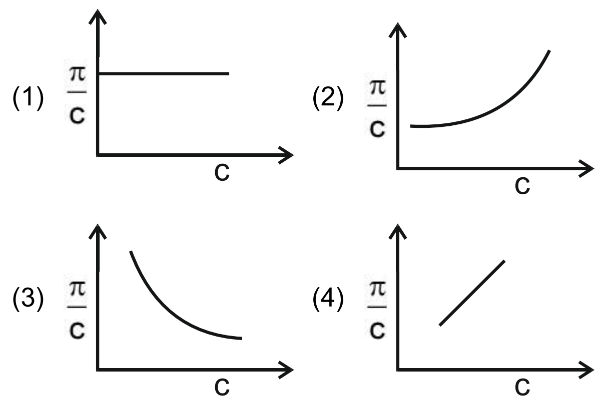
(2) $\left[Ni(CN)_4\right]^{2-}$ dsp^2 hybridisation, diamagnetic

(3) $\left[Ni(H_2O)_6\right]^{2+}$ sp^3d^2 hybridisation, paramagnetic

(4) $\left[Co(NH_3)_4Cl_2\right]^{+}$ d^2sp^3 hybridisations, diamagnetic

15. Which of the following options contains the correct graph between $\frac{\pi}{c}$ and c at constant temperature?

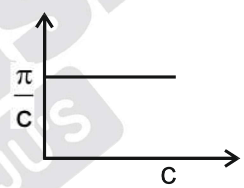
[where π is osmotic pressure and c is concentration of solute]



Answer (1)

Sol. π = cRT

$\therefore \frac{\pi}{c}$ = RT



$\therefore$ The value of $\frac{\pi}{c}$ is constant at constant temperature.

16. Which of the following is correct about antibiotics.

(1) Antibiotics are the substances that promote the growth of microorganism

(2) Penicillin has bacteriostatic effect

(3) Erythromycin has Bactericidal effect

(4) These are synthesized artificially

Answer (4)

Sol. Antibiotics are synthesized artificially.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. How many of the following complexe(s) is(are) paramagnetic:

$[Fe(CN)_6]^{3-}$, $[Fe(CN)_6]^{4-}$, $[NiCl_4]^{2-}$, $[Ni(CN)_4]^{2-}$, $[CuCl_4]^{2-}$, $[Cu(CN)_4]^{3-}$, $[Cu(H_2O)_4]^{2+}$

Answer (4)

Sol. $[Fe(CN)_6]^{3-} \rightarrow d^5$ paramagnetic

$[Fe(CN)_6]^{4-} \rightarrow d^6$ diamagnetic

$[NiCl_4]^{2-} \rightarrow d^8$ paramagnetic

$[Ni(CN)_4]^{2-} \rightarrow d^8$ diamagnetic

$[CuCl_4]^{2-} \rightarrow d^9$ paramagnetic

$[Cu(CN)_4]^{3-} \rightarrow d^{10}$ diamagnetic

$[Cu(H_2O)_4]^{2+} \rightarrow d^9$ paramagnetic

22. For a first order reaction $A \longrightarrow B$, $t_{1/2}$ is 30 min. Then find the time (in minutes) required for 75%. Completion of reaction

Answer (60.00)

Sol. $A \xrightarrow[50\%]{t_{1/2}} B$

$A \xrightarrow[75\%]{2t_{1/2}} B$

∴ In 75% completion, two $t_{1/2}$ will be required.

∴ Time required will be 60 minutes.

23. Consider the following cell representation:

$$Pt \mid H_2 \mid H^+ \parallel Fe^{3+} \mid Fe^{2+}$$
$$(1\ atm)(1\ M)$$

Then find the ratio of concentration of Fe^{2+} to Fe^{3+}

[Given: $E_{cell} = 0.712$ and $E^o_{Cell} = 0.771$]

Answer (10.00)

Sol. $$E_{cell} = E^o_{cell} - \frac{0.059}{2}\log\left[\frac{[Fe^{2+}][H^+]}{[Fe^{3+}]}\right]^2$$

$$0.712 = 0.771 - \frac{0.059}{2} \times 2\log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

$$-0.059 = -0.059 \log\frac{[Fe^{2+}]}{[Fe^{3+}]}$$

$$\therefore \frac{[Fe^{2+}]}{[Fe^{3+}]} = 10$$

24. How many of the following ions/elements has/have same value of spin magnetic moment?

V^{3+}, Cr^{3+}, Fe^{2+}, Ni^{2+}

Answer (2)

Sol. $V^{3+} = d^2 \rightarrow$ 2 unpaired electrons

$Cr^{3+} = d^3 \rightarrow$ 3 unpaired electrons

$Fe^{2+} = d^6 \rightarrow$ 4 unpaired electrons

$Ni^{2+} = d^8 \rightarrow$ 2 unpaired electrons

25. An athlete is given 100 g of glucose energy equivalent to 1560 kJ. He utilizes 50% of this gained energy in an event. Enthalpy of evaporation of H_2O is 44 kJ/mole. In order to avoid storage of energy in body, mass of water (in g) he would need to perspire is:

Answer (319)

Sol. $C_6H_{12}O_6 + 6O_2 \longrightarrow 6CO_2 + 6H_2O(l)$

$$h = \frac{100}{180}$$

∴ Energy needed to perspire water $= 1560 \times \frac{1}{2}$

$= 780$ kJ

∴ Moles of water evaporated $= \frac{780}{44}$ mole

∴ Weight of water evaporated $= \frac{780}{44} \times 18$

$= 319$ g

Assuming water is contained in the body.

26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$, $x\in[-1, 1]$.

Sum of all solutions is $\alpha-\frac{4}{\sqrt{3}}$ then α is

(1) 1 (2) 2

(3) –2 (4) $\sqrt{3}$

Answer (2)

Sol. $\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$

For $x < 0$,

$2\tan^{-1}x+2\tan^{-1}x+\pi=\frac{\pi}{3}$

$\Rightarrow \quad 4\tan^{-1}x=-\frac{2\pi}{3}$

$\Rightarrow \quad x=-\frac{1}{\sqrt{3}}$

For $x > 0$,

$4\tan^{-1}x=\frac{\pi}{3}$

$\Rightarrow \quad x=\tan\frac{\pi}{12}=2-\sqrt{3}$

Sum $=2-\sqrt{3}-\frac{1}{\sqrt{3}}=2-\frac{4}{\sqrt{3}}$

2. Mean of a data set is 10 and variance is 4. If one entry of data set changes from 8 to 12, then new mean becomes 10.2. Then new variance is

(1) 3.92 (2) 3.96

(3) 4.04 (4) 4.08

Answer (2)

Sol. Let number of observations is n

$(10.2)n = 10n - 8 + 12$

$\Rightarrow \quad (10.2)n = 10n + 4$

$\Rightarrow \quad \boxed{n = 20}$

For earlier observation set

$\frac{\sum x_i^2}{20}-(10)^2=4$

$\sum x_i^2=(104)(20)=2080$

After change

$\left(\sum x_i^2\right)_{\text{new}}=2080-8^2+12^2$

$= 2160$

New variance $=\frac{2160}{20}-(10.2)^2$

$= 108 - (10.2)^2$

$= 3.96$

3. If $y = (1 + x)(x^2 + 1)(x^4 + 1)(x^8 + 1)(x^{16} + 1)$, then $y'' - y'$ is, when $x = -1$

(1) 496 (2) 946

(3) –496 (4) –946

Answer (3)

Sol. $y = (x + 1)(x^2 + 1)(x^4 + 1)(x^8 + 1)(x^{16} + 1)$

Multiplying and dividing by $(x - 1)$ we get

$y=\frac{x^{32}-1}{x-1}$

at $x = -1$, $y = 0$

$y(x - 1) = x^{32} - 1$

Diff. on both side

$y'(x - 1) + y = 32x^{31}$ …(i)

at $x = -1$

$y'(-1) = 16$

Diff. (i) on both side

$y''(x - 1) + y' + y' = 32 \times 31x^{30}$

substitute $x = -1$

$y''(-1) = -480$

$y''(-1) - y'(-1) = -480 - 16$

$= -496$

4. The logical statement $(p \wedge \sim q) \rightarrow (p \rightarrow \sim q)$ is a

(1) Tautology

(2) Fallacy

(3) Equivalent to $p \vee \sim q$

(4) Equivalent to $p \wedge \sim q$

Answer (1)

Sol. $(p \wedge \sim q) \rightarrow (p \rightarrow \sim q)$

$= (p \wedge \sim q) \rightarrow (\sim p \vee \sim q)$

$= \sim(p \wedge \sim q) \vee (\sim p \vee \sim q)$

$= \sim p \vee q \vee (\sim p \vee \sim q)$

$= \sim p \vee T = T$ (Tautology)

5. If a_r is the coefficient of x^{10-r} in expansion of $(1 + x)^{10}$ then $\sum_{r=1}^{10} r^3\left(\frac{a_r}{a_{r-1}}\right)^2$ is

(1) 390

(2) 1210

(3) 485

(4) 220

Answer (2)

Sol. $a_r = {}^{10}C_{10-r}$

$$\sum_{r=1}^{10} r^3\left(\frac{^{10}C_{10-r}}{^{10}C_{11-r}}\right)^2 = \sum_{r=1}^{10} r^3\left(\frac{10!}{r!(10-r)!}\cdot\frac{(11-r)!(r-1)!}{10!}\right)^2$$

$$= \sum_{r=1}^{10} r^3\left(\frac{11-r}{r}\right)^2 = \sum_{r=1}^{10} r(11-r)^2$$

$$= \sum_{r=1}^{10} r^2(11-r)$$

$$= 11\sum_{r=1}^{10} r^2 - \sum_{r=1}^{10} r^3$$

$$= 11\left(\frac{10\cdot 11\cdot 21}{6}\right) - \left(\frac{10\cdot 11}{2}\right)^2$$

$= (11)^2\, 35 - (11)^2 \cdot 25$

$= (11)^2 \times 10 = 1210$

6. $\lim\limits_{n\to\infty} \dfrac{1+2-3+4+5-6+....(3n-2)+(3n-1)-3n}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$ is equal to

(1) $\frac{3}{2}\left(\sqrt{2}+1\right)$

(2) $\frac{2}{3}\left(\sqrt{2}+1\right)$

(3) $\frac{2}{3\sqrt{2}}$

(4) $2\sqrt{2}$

Answer (1)

Sol. $$\lim_{n\to\infty} \frac{\sum_{r=1}^{n}((3r-2)+(3r-1)-3r)}{\sqrt{2n^4+3n+1}-\sqrt{n^4+n+3}}$$

$$\lim_{n\to\infty} \frac{\sum_{r=1}^{n}3(r-1)}{\sqrt{2n^4+3n-1}-\sqrt{n^4+n+3}}$$

$$= \lim_{n\to\infty} \frac{3\frac{n(n-1)}{2}\left(\sqrt{2n^4+3n-1}+\sqrt{n^4+n+3}\right)}{\left(2n^4+3n-1\right)-\left(n^4+n+3\right)}$$

$$= \frac{3}{2}\left(\sqrt{2}+1\right)$$

7. If $|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$ when $z_1 = 2 + 3i$ and $z_2 = 3 + 4i$, then locus of z is

(1) Straight line with slope $-\frac{1}{2}$

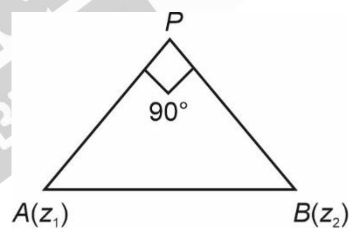
(2) Circle with radius $\frac{1}{\sqrt{2}}$

(3) Hyperbola with eccentricity $\sqrt{2}$

(4) Hyperbola with eccentricity $\frac{5}{2}$

Answer (2)

Sol.



So Locus of P is circle whose diameter is AB

$AB = \sqrt{2}$

Radius of circle $= \frac{1}{\sqrt{2}}$

8. $f(x) = \int \frac{2x}{(x^2+1)(x^2+3)}dx$ if $f(3) = \frac{1}{2}[\ln 5 - \ln 6]$, then $f(4)$ is

(1) $\frac{1}{2}[\ln 17 - \ln 19]$

(2) $\frac{1}{2}[\ln 19 - \ln 17]$

(3) $\ln 19 - \ln 17$

(4) $\ln 17 - \ln 19$

Answer (1)

Sol. $f(x)=\int \frac{2x}{(x^2+1)(x^2+3)}dx$

Let $x^2 = t$

$2xdx = dt$

$\int \frac{dt}{(t+1)(t+3)}$

$=\frac{1}{2}\int \frac{(t+3)-(t+1)}{(t+1)(t+3)}dt$

$=\frac{1}{2}[\ln|t+1|-\ln|t+3|]+\frac{C}{2}$

$=\frac{1}{2}\left[\ln|x^2+1|-\ln|x^2+3|\right]+\frac{C}{2}$

$\because\ f(3)=\frac{1}{2}[\ln 5-\ln 6]$

$\therefore\ \frac{1}{2}[\ln 5-\ln 6]=\frac{1}{2}[\ln 10-\ln 12]+\frac{C}{2}$

$\Rightarrow\ C = 0$

$\therefore\ f(x)=\frac{1}{2}\left[\ln|x^2+1|-\ln|x^2+3|\right]$

$f(4)=\frac{1}{2}[\ln 17-\ln 19]$

9. If $f(x)=\int_0^2 e^{|x-t|}\,dt$, then the minimum value of $f(x)$ is equal to

(1) $2(e-1)$ (2) $2(e+1)$

(3) $2e-1$ (4) $2e+1$

Answer (1)

Sol. For $x > 2$

$f(x)=\int_0^2 e^{x-t}dt$

$= e^x\left(-e^{-t}\right)\Big|_0^2$

$= e^x(1-e^{-2})$

For $x < 0$

$f(x)=\int_0^2 e^{t-x}dt = e^{-x}e^t|_0^2 = e^{-x}(e^2-1)$

For $0 \le x \le 2$

$f(x)=\int_0^x e^{x-t}\,dt+\int_x^2 e^{t-x}\,dt$

$= -e^x e^{-t}|_0^x + e^{-x}e^t|_x^2$

$\Rightarrow\ -e^x(e^{-x}-1)+e^{-x}(e^2-e^x)$

$\Rightarrow\ -1+e^x+e^{2-x}-1$

$= e^{2-x}+e^x-2$

$$f(x)=\begin{cases} e^x(1-e^{-2}); & x>2 \\ e^{2-x}+e^x-2; & 0\le x\le 2 \\ e^{-x}(e^2-1); & x<0 \end{cases}$$

For $x > 2$

$f(x)_{min} = e^2 - 1$

For $0 \le x \le 2$

$f'(x) = -e^{2-x}+e^x = 0 \Rightarrow e^x = e^{2-x} \Rightarrow e^{2x} = e^2 \Rightarrow x = 1$

$f(x) = 2e - 2 = 2(e-1)$

For $x < 0$

$f(x)_{min} = e^2 - 1$

10. If $f(x) = x^b + 3$, $g(x) = ax + c$. If $(g(fx))^{-1}=\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$ then fog(ac) + gof(b) is

(1) 189 (2) 195

(3) 194 (4) 89

Answer (1)

Sol. $g(fx) = a(x^b + 3) + c$.

$(g(f(x)))^{-1}=\left[\frac{x-3a-c}{a}\right]^{\frac{1}{b}}=\left(\frac{x-7}{2}\right)^{\frac{1}{3}}$

$\Rightarrow\ a = 2$

$b = 3$

$c = 1$

$g(x) = 2x + 1$

$f(x) = x^3 + 3$

Now fog(2) + gof(3)

= 128 + 61

= 189

11. Term independent of x in expansion of $\left(2x+\frac{1}{x^7}-7x^2\right)^5$ is

(1) 1372 (2) 2744

(3) –13720 (4) 13720

Answer (3)

Sol. $\dfrac{1}{x^{35}}\left(2x^8+1-7x^9\right)^5=\dfrac{1}{x^{35}}\left(1+x^8\left(2-7x\right)\right)^5$

Term independent of x = coefficient of x^{35} in $(1 + x^8(2 - 7x))^5$

= coefficient of x^{35} in ${}^5C_4\left(x^8\left(2-7x\right)\right)^4$

= ${}^5C_4$ coefficient of x^3 in $(2 - 7x)^4$

= ${}^5C_4 \cdot {}^4C_3\left(2^1\right)\left(-7\right)^3$

= –13720

12. The value of $A=\begin{bmatrix}1 & \log_x y & \log_x z\\ \log_y x & 2 & \log_y z\\ \log_z x & \log_z y & 3\end{bmatrix}$ then |adj (adj A^2)| is

(1) 6^4 (2) 4^8

(3) 4^5 (4) 2^8

Answer (4)

Sol. $A=\begin{bmatrix}1 & \log_x y & \log_x z\\ \log_y x & 2 & \log_y z\\ \log_z x & \log_z y & 3\end{bmatrix}$

$$|A|=\frac{1}{\log x\log y\log z}\begin{vmatrix}\log x & \log y & \log z\\ \log x & 2\log y & \log z\\ \log x & \log y & 3\log z\end{vmatrix}$$

$$|A|=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 1\\ 1 & 1 & 3\end{vmatrix}$$

$|A| = 2$

$|\text{adj}(\text{adj } A^2)| = |A|^8$

$= 2^8$

13. Sum of two positive integers is 66 and μ is the maximum value of their product

$S=\left\{x\in Z,\ x(66-x)\geq\frac{5\mu}{9}\right\},\ x\neq 0$ then probability of A when $A=\left\{x\in S;\ x=3k,\ x\in N\right\}$

(1) $\frac{1}{4}$ (2) $\frac{2}{3}$

(3) $\frac{1}{3}$ (4) $\frac{1}{2}$

Answer (3)

Sol. $\mu = 33 \times 33 = 1089$

$x(66-x) \geq 605$

$x^2 - 66x + 605 \leq 0$

$x \in [11, 55]$

Favourable set of values of x for event A

= {12, 15, 18, ...54}

$P(A)=\frac{15}{45}=\frac{1}{3}$

14. Let $L_1=\frac{x-3}{1}=\frac{y-2}{2}=\frac{z-1}{3}$ and

$L_2=\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$

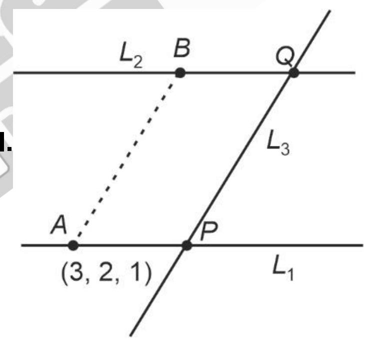
and direction ratios of line L_3 are <1, –1, 3>. P and Q are point of intersection of L_1 and L_3 and L_2 and L_3 respectively. Then distance between P and Q is

(1) $\frac{10}{3}\sqrt{6}$ (2) $\frac{8}{3}\sqrt{11}$

(3) $\frac{4}{3}\sqrt{11}$ (4) $\frac{11}{3}\sqrt{6}$

Answer (3)

Sol.



Let

$PQ = AB$

Let $A(3, 2, 1)$

Equation of line AB:

$\frac{x-3}{1}=\frac{y-2}{-1}=\frac{z-1}{3}=k \qquad (k\in R)$

$\Rightarrow \quad x = k + 3,\ y = -k + 2,\ z = 3k + 1$

Let coordinates of $B(k + 3, -k + 2, 3k + 1)$

B lies on L_2

$B(\lambda + 1, 2\lambda + 2, 3\lambda + 3)$

$k + 3 = \lambda + 1 \Rightarrow \lambda - k = 2$

$2 - k = 2\lambda + 2 \Rightarrow 2\lambda + k = 0$

$\Rightarrow \ k = -2\lambda$

$\Rightarrow \ 3\lambda = 2$

$\Rightarrow \ \lambda = \frac{2}{3}$

$B\left(\frac{5}{3}, \frac{10}{3}, 5\right)$

$AB = \sqrt{\left(\frac{4}{3}\right)^2 + \left(\frac{4}{3}\right)^2 + 16}$

$= \frac{4}{3}\sqrt{11} = PQ$

15. If $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$ is rotated by 90° about origin passing through y-axis. If new vector is $\vec{b}$ then projection of $\vec{b}$ on $\vec{c} = 5\hat{i} + 4\hat{j} + 3\hat{k}$ is equal to

(1) $\frac{6}{5}$

(2) $\frac{3}{5}$

(3) $\frac{6}{5\sqrt{3}}$

(4) $\frac{6\sqrt{3}}{5}$

Answer (1)

Sol. $\vec{b} = \lambda\vec{a} + \mu\hat{j}$

$= \left(\lambda(-\hat{i} + 2\hat{j} + \hat{k}) + \mu\hat{j}\right)$

$\vec{b} \cdot \vec{a} = 0$

$(\lambda\vec{a} + \mu\hat{j})\vec{a} = 0$

$6\lambda + 2\mu = 0$

$\mu = -3\lambda$

$\vec{b} = \lambda(\vec{a} - 3\hat{j}) = \lambda(-\hat{i} - \hat{j} + \hat{k})$

$\lambda = \pm\sqrt{2}$

Projection of $\vec{b}$ on $\vec{c}$ $= \left|\vec{b} \cdot \hat{c}\right|$

$= \left|\sqrt{2}(-\hat{i} - \hat{j} + \hat{k}) \frac{(5\hat{i} + 4\hat{j} + 3\hat{k})}{5\sqrt{2}}\right|$

$= \frac{6\sqrt{2}}{5\sqrt{2}} = \frac{6}{5}$

16. Given $\frac{dy}{dx} = \frac{y}{x}\left(1 + xy^2(1 + \ln x)\right)$. If $y(1) = 3$, then the value of $\frac{y^2(3)}{9}$ is

(1) $-\frac{1}{43 + 27\ln 3}$

(2) $\frac{1}{43 + 27\ln 3}$

(3) $\frac{9}{59 - 162(1 + \ln 3)}$

(4) $\frac{1}{27 - 43\ln 3}$

Answer (3)

Sol. $\frac{dy}{dx} - \frac{y}{x} = y^3(1 + \ln x)$

$\frac{1}{y^3}\frac{dy}{dx} - \frac{1}{x}\frac{1}{y^2} = (1 + \ln x)$

$\frac{1}{y^2} = t \Rightarrow \frac{-2}{y^3}\frac{dy}{dx} = \frac{dt}{dx}$

$\therefore \ \frac{-1}{2}\frac{dt}{dx} - \frac{t}{x} = (1 + \ln x)$

$\frac{dt}{dx} + \frac{2t}{x} = -2(1 + \ln x)$

IF $e^{\int \frac{2}{x}dx} = x^2$

$\therefore \ tx^2 = \int -2(1 + \ln x)x^2 dx$

$tx^2 = -2\left[(1 + \ln x)\frac{x^3}{3} - \int \frac{x^2}{3}dx\right] + c$

$\frac{x^2}{y^2} = -2\left[\frac{x^3}{3}(1 + \ln x) - \frac{x^3}{9}\right] + c \ \ldots\text{(i)}$

$y(1) = 3 \Rightarrow \frac{1}{9} = -2\left(\frac{1}{3} - \frac{1}{9}\right) + c$

$\therefore \ c = \frac{5}{9}$

Now putting $x = 3, c = \frac{5}{9}$ in (1)

$\frac{9}{y^2} = -2\left(9(1 + \ln 3) - 3\right) + \frac{5}{9}$

$= \frac{59}{9} - 18(1 + \ln 3)$

$\frac{y^2}{9} = \frac{9}{59 - 162(1 + \ln 3)}$

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Consider the set $S = \{1, 2, 3, 5, 7, 10, 11\}$. Number of subsets of S having sum of its elements equal to multiple of 3, is equal to.

Answer (44.00)

Sol. Out of the given numbers one is $3k$ type and 3 of $3k + 1$ type and remaining three are $3k + 2$ type.

Number of subsets with 0 elements = 1

[Considering the sum of elements of empty set equal to zero]

Number of subsets with 1 element = 1

1 of $3k$ type

Number of subsets with 2 elements

1 of $(3k + 1)$ type + 1 of $(3k + 2)$ type = 9

Number of subsets with 3 elements

1 of $3k$ type + 1 of $(3k + 1)$ type + 1 of $(3k + 2)$ type = 9

3 of $(3k + 1)$ type = 1

3 of $(3k + 2)$ type = 1

Number of subsets with 4 elements

1 of $3k$ type + 3 of $(3k + 1)$ type = 1

1 of $3k$ type + 3 of $(3k + 2)$ type = 1

2 of $(3k + 1)$ type + 2 of $(3k + 2)$ = 9

Number of subsets with 5 elements

1 of $3k$ type + 2 of $(3k + 1)$ type + 2 of $(3k + 2)$ type = 9

Number of subsets with 6 elements

3 of $3k + 1$ type + 3 of $3k + 2$ type = 1

The set itself = 1

Total = 44.

22. If $a, b \in [1, 25]$, $a, b \in N$ such that $a + b$ is multiple of 5. Find the number of ordered pair (a, b).

Answer (125)

Sol. Type	Numbers
$5k$	5, 10, 15, 20, 25
$5k + 1$	1, 6, 11, 16, 21
$5k + 2$	2, 7, 12, 17, 22
$5k + 3$	3, 8, 13, 18, 23
$5k + 4$	4, 9, 14, 19, 24

(a, b) can be selected as

I 1 of $(5k + 1)$ and 1 of $(5k + 4) = 2 \times 25 = 50$

II 1 of $(5k + 2)$ and 1 of $(5k + 3) = 2 \times 25 = 50$

III both of the type $5k = 25$

$\therefore$ Total = 125

23. If $\log_2\left(9^{2\alpha-4} + 13\right) - \log_2\left(3^{2\alpha-4} \cdot \frac{5}{2} + 1\right) = 2$.

Then maximum integral value of β for which equation.

$x^2 - \left(\left(\sum \alpha\right)^2 x\right) - \sum(\alpha + 1)^2 \beta$ has real roots is

Answer (06)

Sol. $\log_2\left(9^{2\alpha-4} + 13\right) - \log_2\left(3^{2x-4} \cdot \frac{5}{2} + 1\right) = 2$

$\therefore \dfrac{9^{2\alpha-4} + 13}{3^{2\alpha-4} \cdot \frac{5}{2} + 1} = 4$

Let $3^{2\alpha-4} = t$

$t^2 + 13 = 10t + 4$

$t^2 - 10t + 9 = 0$

$\therefore \quad t = 9, 1$

$\Rightarrow \quad \alpha = 3, 2$

Now equation will become

$x^2 - 25x + 25\beta = 0$

has real roots

$\therefore \quad D \geq 0$

$25^2 - 4.25\,\beta \geq 0$

$\Rightarrow \quad \beta \leq \frac{25}{4}$

Max integral value = 6

24.

25.

26.

27.

28.

29.

30.

❑ ❑ ❑