1. A set A has 17 elements and another set B has 38 elements. The number of elements of the set $A \cup B$ is 52. Then the number of elements in the set $A - B$ is

   (A) 3. (B) 5. (C) 12. (D) 14.

2. Let $P(x)$ be a quadratic polynomial such that $P(1) = P(-2)$. If -1 is a root of the equation $P(x) = 0$, then the other root is

   (A) $\frac{8}{5}$. (B) 1. (C) 0. (D) $\frac{4}{5}$.

3. Let $X = (2, 1, 3, 5)^t$ and $Y = (1, 5, 3, 2)^t$ where Z^t is the transpose of any row vector Z. For which of the following matrices P, $X = PY$ holds ?

   (A) $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$ (B) $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

   (C) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ (D) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

4. The radius of an expanding circle at the instance when the area of the circle is increasing twice as fast as that of its radius is

   (A) $\frac{1}{4\pi}$. (B) $\frac{1}{4}$. (C) $\frac{1}{\pi}$. (D) 1.

5. The region bounded by the x-axis, the curve $y = \cos x$ and the vertical lines $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two parts by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \le x \le k$ is three times the area of the region for $k \le x \le \frac{\pi}{2}$, then k is equal to

   (A) $\frac{\pi}{6}$. (B) $\frac{\pi}{4}$. (C) $\frac{\pi}{3}$. (D) arcsin(1/4).

6. Let f and g be two real-valued differentiable functions defined for all real x. Assume that $f'(x) > g'(x)$ for all x. Then the graph of $y = f(x)$ and the graph of $y = g(x)$

   (A) intersect exactly once.

   (B) intersect no more than once.

   (C) do not intersect.

   (D) have a common tangent at each point of intersection.

7. Consider the function

$$F(x) = \begin{cases} -1, & \text{if } x \leq 0 \\ 1, & \text{if } x > 0. \end{cases}$$

   Then the function $g(x) = \int_{-2}^{x} F(t)dt$

   (A) is discontinuous for some value of x.

   (B) is continuous everywhere, but not differentiable at some point.

   (C) is differentiable everywhere and $g'(x) = F(x)$.

   (D) is differentiable everywhere but $g'(x) \neq F(x)$.

8. Let A be the set of all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x) - f(y) = x - y; \quad x, y \in \mathbb{R}.$$

   Then

   (A) $A = \{f(x) = mx + c \mid m, c \in \mathbb{R}\}$.

   (B) $A = \{f(x) = x + c \mid c \in \mathbb{R}\}$.

   (C) $A = \{f(x) = mx \mid m > 0\}$.

   (D) $A = \{f(x) = mx + c \mid m = \pm 1, c > 0\}$.

9. Two runners, A and B, are running along a circular track in opposite directions. They start from a common point P. If A completes 3 laps in every 5 minutes and B completes 5 laps in every 3 minutes, then they will meet again at P, for the first time, exactly after

   (A) 3 minutes. (B) 5 minutes. (C) 8 minutes. (D) 15 minutes.

10. A garden is to be constructed in the shape of a right angled triangle. It must have an area of 10 square metres. If the length of one of the sides containing the right angle is x metres, then its perimeter (in metre) is given by

(A) $\sqrt{x^2+\frac{100}{x^2}}$.

(B) $x+\frac{20}{x}+\sqrt{x^2+\frac{400}{x^2}}$.

(C) $\sqrt{x^2+\frac{400}{x^2}}$.

(D) $x+\frac{20}{x}+\sqrt{x^2+\frac{100}{x^2}}$.

11. If $3x^2+2xy+y^2=2$, then the value of $\frac{dy}{dx}$ at $x=1$ is

(A) -2. (B) 0. (C) 4. (D) not defined.

12. The sum of the series $7+77+777+\cdots$ up to 10 terms, is

(A) $\frac{700}{81}(10^9-1)$. (B) $\frac{350}{9}(10^9-1)$.

(C) $\frac{350}{9}(10^7-1)$. (D) $\frac{700}{9}(10^7-1)$.

13. A ray of light passing through the point (1, 2) is reflected on the x-axis at a point P and the reflected ray passes through the point (5, 3). The distance of P from the origin is

(A) $\frac{13}{5}$. (B) $\frac{5}{13}$. (C) $\frac{7}{13}$. (D) $\frac{19}{13}$.

14. Suppose the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}})^n$ is $\sqrt{6}:1$. Then the value of n is

(A) 9. (B) 11. (C) 10. (D) 7.

15. The number of real roots of the equation $(x-17)^3+(x-23)^3=0$ is

(A) 3. (B) 2. (C) 1. (D) 0.

16. Let F be the point $(3,-2)$ and G be the point $(3,k)$. How many different squares can be formed with FG as a side if k can take any value in $\{5,6,...,40\}$?

(A) 35 (B) 36 (C) 70 (D) 72

17. The interval in which the function $f(x) = \log_{10}(x^2 - 2x - 3)$ is monotonically increasing is

(A) $(-\infty, -1)$. (B) $(-\infty, 1)$. (C) $(1, \infty)$. (D) $(3, \infty)$.

18. Consider a right-angled triangle with base, perpendicular and hypotenuse of lenghts a, b and c, respectively. Then $(a^3 + b^3)$

(A) is always greater than c^3.

(B) is always less than c^3.

(C) is always equal to c^3.

(D) can be greater than c^3 or less than c^3, depending on the value of a, b and c.

19. Consider the function $f(x) = \sin x + \cos x$ for all real x. Then the maximum value of $f(x)$ is

(A) 1. (B) 2. (C) $\sqrt{2}$. (D) $\frac{\sqrt{3}+1}{2}$.

20. Let $f(x) = |x - a| + |x - b|$ for $x \in \mathbb{R}$, where a and b are two distinct real constants. Then $f(x)$ is

(A) differentiable at every points.

(B) differentiable at every points except $x = a$ and $x = b$.

(C) discontinuous at $x = a$ and $x = b$.

(D) not differentiable for all x between a and b.

21. The minimum value of $f(x, y) = x^2 + 2y^2 - 4x + 6y + 10$, $x, y \in \mathbb{R}$, is

(A) $\frac{3}{2}$. (B) 10. (C) $-\frac{3}{2}$. (D) $\frac{37}{2}$.

22. Let F be a function defined by $F(x) = \int_0^x \frac{t^2}{(1+t^3)^2} dt$. Then $\lim\limits_{x \to \infty} F(x)$

(A) is $\frac{1}{3}$. (B) is $\frac{2}{3}$. (C) is 1. (D) does not exist.

23. An urn contains n balls of which some are red and the rest are black. The balls are being counted one by one to find how many are red and how many are black. In this counting, 49 out of first 50 were red. Thereafter, 7 out of every 8 were found to be red. It turns out, in all, 90% or more were counted to be red. Then the maximum value of n is

(A) 150. (B) 210. (C) 300. (D) 420.

24. The values of k for which the system of equations

$$\begin{array}{ccccccc} x & + & y & + & z & = & kx \\ x & + & y & + & z & = & ky \\ x & + & y & + & z & = & kz \end{array}$$

has non-trivial solutions are

(A) 0, 3. (B) 0, -3. (C) -3, 3. (D) -3, 3, 0.

25. The equation $x^2 + 4xy + y^2 - 2x + 2y + 6 = 0$ represents

(A) pair of intersecting straight lines.

(B) circle.

(C) ellipse.

(D) hyperbola.

26. The sum of two numbers is 20. The numbers for which the product of one and the cube of the other is maximum are

(A) 5, 15. (B) 10, 10. (C) 4, 16. (D) 8, 12.

27. The equation $x^4 + 2x^2 + 3x - 1 = 0$ has

(A) one real positive, one real negative and two complex roots.

(B) two real positive and two complex roots.

(C) two real negative and two complex roots.

(D) no real roots.

28. Let M be a non-zero $n \times n$ matrix with real entries such that $M^t = -M$, where M^t denotes the transpose of M. Which of the following statements is true?

(A) The rank of $M < n$ implies n is even.

(B) The rank of $M < n$ implies n is odd.

(C) The rank of $M = n$ implies n is even.

(D) The rank of $M = n$ implies n is odd.

29. Let $F : \mathbb{R} \to \mathbb{R}$ be an even function and is differentiable 4 times. Let F_n be its Maclaurin series upto n^{th} order for $n = 1, 2, 3, 4$. Then

(A) F_1, F_2, F_3, F_4 are all even.

(B) F_1, F_3 are even, but not necessarily F_2, F_4.

(C) F_2, F_4 are even, but not necessarily F_1, F_3.

(D) F_1 must be odd.

30. Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = |x-1| + |x-2| + |x-3|$. The value of $\int_1^3 f(x)dx$ is

(A) 5. (B) $\frac{5}{2}$. (C) 7. (D) $\frac{7}{2}$.