JEE MAINS-11-APRIL-2015
CHEMISTRY

1. $A + 2B + 3C \rightleftharpoons AB_2C_3$

Reaction of 6.0 g of A, 6.0×10^{23} atoms of B, and 0.036 mol of C yields 4.8 g of compound AB_2C_3 . If the atomic mass of A and C are 60 and 80 amu, respectively, the atomic mass of B is (Avogadro no. = 6×10^{23}):

(1) 70 amu

(2) 60 amu

(3) 50 amu

(4) 40 amu

2. When does a gas deviate the most from its most from its ideal behaviour?

(1) At low pressure and low temperature

(2) At low pressure and high temperature

(3) At high pressure and low temperature

(4) At high pressure and high temperature

3. At temperatuere T, the average kinetic energy of any particle is $\frac{3}{2}$kT. The de Broglie wavelength follows the order

(1) Thermal proton > Visible photon > Thermal electron

(2) Thermal proton > Thermal electron> Visible photon

(3) Visible photon > Thermal electron >Thermal neutron

(4) Visible photon > Thermal neutron > Thermal electron

4. Molecular AB has a bond length of 1.61Å and a dipole moment of 0.38D. The fractional charge on each atom (absolute magnitude) is :($e_0 = 4.802 \times 10^{-10}$ esu)

(1) 0

(2) 0.05

(3) 0.5

(4) 1.0

5. . For the equilibrium, $A(g) \rightleftharpoons B(g)$, ΔH is –40 kJ/mol. If the ratio of the activation energies of the forward (E_f) and reverse (E_b) reactions is $\frac{2}{3}$ then :

(1) $E_f = 60$ kJ/mol; $E_b = 100$ kJ/mol

(2) $E_f = 30$ kJ/mol; $E_b = 70$ kJ/mol

(3) $E_f = 80$ kJ/mol; $E_b = 120$ kJ/mol

(4) $E_f = 70$ kJ/mol; $E_b = 30$ kJ/mol

6. Determination of the molar mass of acetic acid in benzene using freezing point depression is affected by :

(1) dissociation

(2) association

(3) partial ionization

(4) complex formation

7. The increase of pressure on ice water system at constant temperature will lead to :

(1) no effect on the equilibrium

(2) a decrease in the entropy of the system

(3) a shift of the equilibrium in the forward direction

(4) an increase in the Gibbs energy of the system

8. At 298 K, the standard reduction potentials are 1.51 V for MnO_4^- |Mn^{2+}, 1.36 V for Cl_2 | Cl^- , 1.07 V for Br_2 |Br, and 0.54

V for $I_2|I^-$. At pH = 3, permanganate is expected to oxidize :

$\left(\frac{RT}{F} = 0.059\ V\right)$

(1) Cl^-, Br^- and I^-

(2) Cl^- and Br^-

(3) Br^- and I^-

(4) I^- only

9. $A + 2B \rightarrow C$, the rate equation for this reaction is given as

Rate = K[A][B].

If the concentration of A is kept the same but that of B is doubled what will happen to the rate itself?:

(1) halved

(2) the same

(3) doubled

(4) quadrupled

10. Under ambient conditions, which among the following surfactants will form micelles in aqueous solution at lowest molar concentration?

(1) $CH_3\ (CH_2)_{15}N^{\oplus}\ (CH_3\ )_3Br^-$

(2) $CH_3 –(CH_2)_{13}–OSO_3^- Na^+$

(3) $CH_3 –(CH_2)_8 –COO^- Na^+$

(4) $CH_3 (CH_2)_{11}N^{\oplus} (CH_3)_3Br^-$

11. Choose the incorrect formula out of the four compounds for an element X below

(1) X_2Cl_3

(2) X_2O_3

(3) $X_2(SO_4)_3$

(4) XPO_4

12. Calamine is an ore of:

(1) Aluminium

(2) Copper

(3) Iron

(4) Zinc

13. Which physical property of dihydrogen is wrong?

(1) Colourless gas

(2) Odourless gas

(3) Tasteless gas

(4) Non-inflammable gas

14. Which of the alkaline earth metal halides given below is essentially covalent in nature ? (1) (2) (3*) (4)

(1) $MgCl_2$

(2) $BeCl_2$

(3) $SrCl_2$

(4) $CaCl_2$

15. Which of the following compound has a P–P bond ?

(1) $H_4P_2O_5$

(2) $H_4P_2O_6$

(3) $H_4P_2O_7$

(4) $(HPO_3)_3$

16. Chlorine water on standing loses its colour and forms :

(1) HCl only

(2) HCl and $HClO_2$

(3) HCl and HOCl

(4) HOCl and $HOCl_2$

17. Which of the following statements is false?

(1) CrO_4^{2-} is tetrahedral in shape

(2) CrO_7^{2-} has a Cr–O–Cr bond

(3) $Na_2Cr_2O_7$ is primary standard in volumetry

(4) $Na_2Cr_2O_7$ is less soluble than $K_2Cr_2O_7$

18. When concentrated HCl is added to an aqueous solution of $CoCl_2$, its colour changes from reddish pink to deep blue. Which complex ion gives blue colour in this reaction ?

(1) $[CoCl_6]^{4-}$

(2) $[CoCl_6]^{3-}$

(3) $[CoCl_4]^{2-}$

(4) $[Co(H2O)_6]^{2+}$

19. Which of the following complex ions has electrons that are symmetrically filled in both t_{2g} and e_g orbitals ?

(1) $[CoF_6]^{3-}$

(2) $[Co(NH_3)_6]^{2+}$

(3) $[Mn(CN)_6]^{4-}$

(4) $[FeF_3]^{3-}$

20. Addition of phosophate fertilisers to water bodies causes:

(1) enhanced growth of algae

(2) increase in amount of dissolved oxygen in water

(3) deposition of calcium phosphate

(4) increase in fish population

21. Match the organic compounds in column-I with the Lassaigne's test results in column-II appropriately :

Column-I		**Column-II**	
(A)	Aniline	(i)	Red colour with $FeCl_3$
(B)	Benzene sulfonic acid	(ii)	Violet colour with sodium nitroprusside
(C)	Thiourea	(iii)	Blue colour with hot and acidic solution of $FeSO_4$

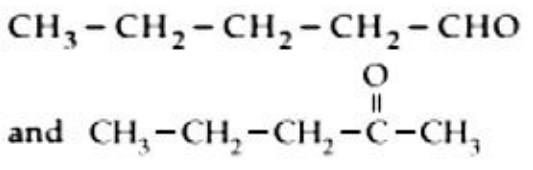
(1) (A)-(ii); (B)-(i); (C)-(iii)

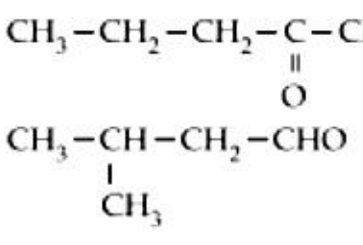
(2) (A)-(iii); (B)-(ii); (C)-(i)

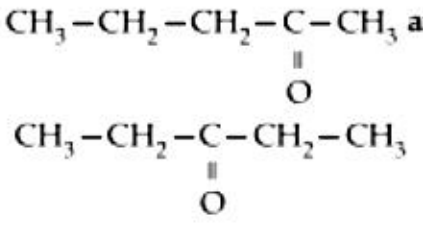
(3) (A)-(ii); (B)-(iii); (C)-(i)

(4) (A)-(iii); (B)-(i); (C)-(ii)

22. Which of the following pairs of compounds are positional isomers?

(1) 

(2) 

(3) 

(4) 

23. The number of structural isomers for C_6H_{14} is :

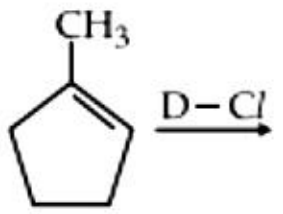
(1) 3

(2) 4

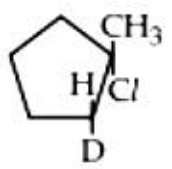
(3) 5

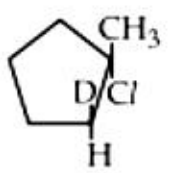
(4) 6

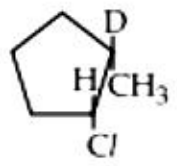
24. What is the major product expected from the following reaction?

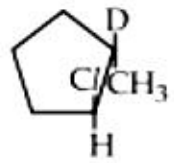


Where D is an isotope of Hydrogen.

(1) 

(2) 

(3) 

(4) 

25. In the reacdtion sequence; $2CH_3CHO \xrightarrow{OH^-} A \xrightarrow{\Delta} B$ the product B is :

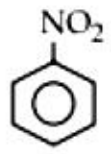
(1) $CH_3-CH_2-CH_2-CH_2-OH$

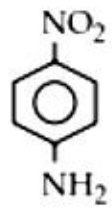
(2) $CH_3-CH=CH-CHO$

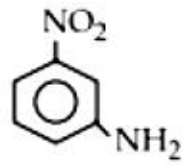
(3) $CH_3-\overset{\overset{O}{\|}}{C}-CH_3$

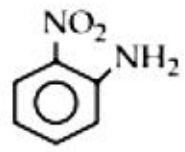
(4) $CH_3-CH_2-CH_2-CH_3$

26. Which compound exhibits maximum dipole moment among the following?

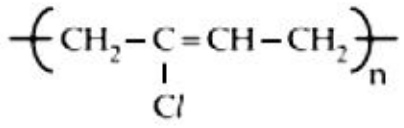
(1) 

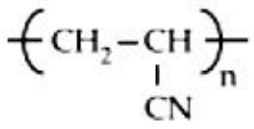
(2) 

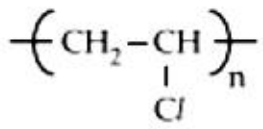
(3) 

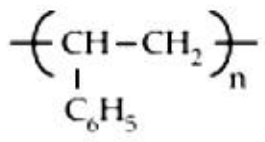
(4) 

27. Which one of the following structures represents the neoprene polymer?

(1) 

(2) 

(3) 

(4) 

28. Accumulation of which of the following molecules in the muscles occurs as a result of vigorous exercise?

(1) Glucose

(2) Glycogen

(3) L-lactic acid

(4) Pyruvic acid

29. Which artificial sweetener contains chlorine ?

(1) Aspartame

(2) Saccharin

(3) Sucralose

(4) Alitame

30. A pink coloured salt turns blue on heating. The presence of which cation is most likely?

(1) Cu^{2+}

(2) Fe^{2+}

(3) Zn^{2+}

(4) Co^{2+}

PART-2

1. Sol:

The number of moles of A is.

$$\text{Moles of A} = \frac{6.0}{60}$$
$$= 0.1$$

The number of moles of B is.

$$\text{Moles of B} = \frac{6.023 \times 10^{23}}{6.023 \times 10^{23}}$$
$$= 1$$

The number of moles of C is 0.036 mole. Therefore, C is the limiting reagent.

The number of moles of product formed is,

$$\text{Number of moles of product formed} = \frac{0.036}{3}$$
$$= 0.012$$

The expression for the molar mass is,

$$\text{Molar mass} = \frac{\text{Given mass}}{\text{Mole of product}}$$

Substitute the value in the above expression.

$$60 + 2x + 80 \times 3 = \frac{4.8}{0.012}$$

$$x = 50 \text{ amu}$$

2. Sol:

Due to low temperature, the kinetic energy of the gas decreases and due to high pressure, the intermolecular forces between the atoms of the gas is more. Thus, a gas deviates from its ideal behavior at high pressure and low temperature.

3. Sol:

From the expression of de Broglie wavelength,

$$\lambda \propto \frac{1}{\sqrt{m}}$$

Thus, de Broglie wavelength follows the order is

Visible photon > Thermal electron > Thermal neutron

4. Sol:

Unit Conversion,

$$1.617 \text{ Å} = 1.617 \times 10^{-8} \text{ cm}$$

Unit Conversion,

$$0.38\times10^{-18}\,\mathrm{D} = 0.38\times10^{-18}\,\text{esu cm}$$

The fractional charge on each atom is,

$$\delta = \frac{\text{Dipole moment}}{\text{Bond length}\times e_0}$$

$$= \frac{0.38\times10^{-18}\,\text{esu cm}}{1.617\times10^{-8}\,\text{cm}\times4.802\times10^{-10}\,\text{esu}}$$

$$= \frac{0.38\times10^{-18}}{7.765\times10^{-18}}$$

$$= 0.0489 \approx 0.05$$

5. Sol:

Let the values of (activation energies of the forward reaction) E_f and (activation energies of the forward reaction) $\mathrm{E_b}$ are $2x$ and $3x$ respectively.

The value of ΔH is calculated by the formula,

$$\Delta\mathrm{H} = \mathrm{E}_f - \mathrm{E_b}$$

$$-40\,\text{kJ/mol} = 2x - 3x$$

$$40\,\text{kJ/mol} = x$$

Thus, the values of E_f and E_b are,

$$\mathrm{E}_f = 2\times40\,\text{kJ/mol}$$

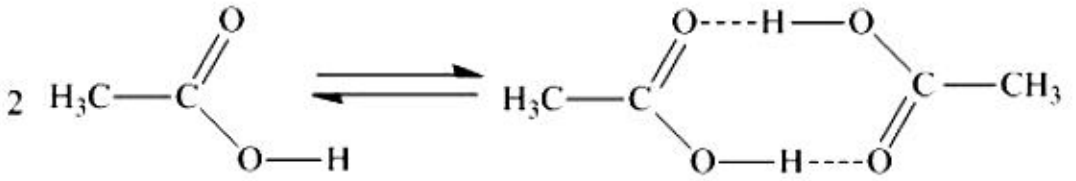
$$= 80\,\text{kJ/mol}$$

$$E_b = 3 \times 40 \text{ kJ/mol}$$
$$= 120 \text{ kJ/mol}$$

6. Sol:

Acetic acid and benzene are polar whereas benzene is non-polar solvents respectively.

Due to non-polar nature of benzene, the molecules of acetic acid stick to each other to form dimer as shown in the figure above.



Monomer Dimer

7. Sol:

The value of density of ice is 0.9340 g/cm^3.

The value of density of water is 0.997 g/cm^3.

Thus, the reaction will shift in forward direction because the density of water is higher than the density of ice.

8. Sol:

The ions Br^- and I^- are expected to oxidize easily because these ions possess less value of reduction potential. The species with low reduction potential tends to lose electrons easily.

Consider the following reaction.

$$MnO_4^- + 8H^+ + 5e^- \rightarrow Mn^{2+} + 4H_2O$$

The value of E is calculated by the formula.

$$E = 1.51 - \frac{0.059}{5}\log\frac{\left[Mn^{2+}\right]}{\left[MnO_4^-\right]\left[H^+\right]^8}$$

$$= 1.51 - \frac{0.059}{5} \times 8\log\frac{1}{\left[H^+\right]}$$

$$= 1.51 - \frac{0.059}{5} \times 8 \times 3$$

$$= 1.227\ V$$

Thus, MnO_4^- oxidise only Br^- and I^- because the standard reduction value of Cl^- is high than MnO_4^- / Mn^{2+}.

9. Sol:

The given rate equation for the reaction is first order with respect to B. Therefore, the rate will be doubled when the

concentration of A is same and the concentration of B is doubled.

10. Sol:

The more surfactants with long hydrocarbon chains, the lesser it has value of CMC and the lower the tendency of surfactants to form micelles.

11. Sol:

The incorrect formula out of the four components is X_2Cl_3.

The valency of X in X_2O_3, $X_2(SO_4)_3$ and XPO_4 is $+3$. Thus, the formula for chloride is XCl_3.

12. Sol:

Chemical formula of calamine is $ZnCO_3$. It is an ore of zinc.

13. Sol:

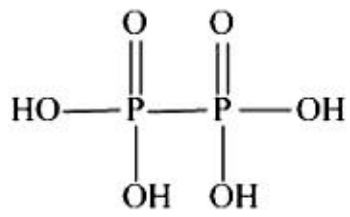
Dihydrogen(H_2) is highly inflammable gas.

14. Sol:

Beryllium chloride forms covalent bond due to more polarization that results in permanent displacement of electrons. Therefore, $BeCl_2$ prefers to share electrons rather than accepting or donating electrons.

15. Sol:

The compound $H_4P_2O_6$ has a $P-P$ bond as shown below.



16. Sol:

Chlorine water on standing loses its color because of the formation of HCl and $HOCl$ as shown in the chemical reaction below.

$$Cl_2 + H_2O \rightarrow HCl + HOCl$$

17. Sol:

The compound $Na_2Cr_2O_7$ is more soluble than $K_2Cr_2O_7$ due to small size of sodium.

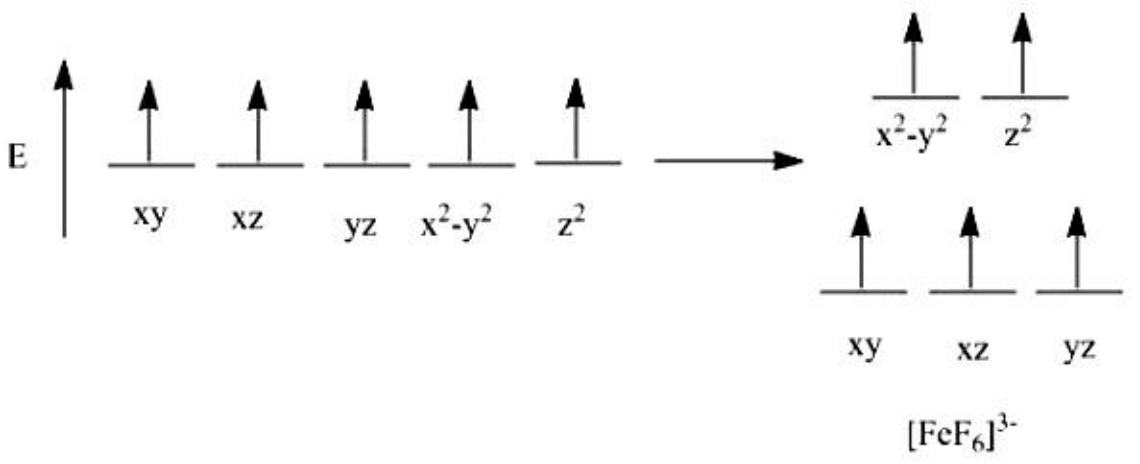
18. Sol:

When concentrated HCl is added to an aqueous solution of $CoCl_2$, its colour changes from reddish pink to deep blue due to the formation of blue colored complex $\left[CoCl_4\right]^{2-}$.

19. Sol:

$\left[FeF_6\right]^{3-}$ is high spin complex because of which t_{2g} and e_g orbitals are symmetrically filled.

In $\left[FeF_6\right]^{3-}$ complex Fe has +3 charge, which leads to five unpaired electrons in d-orbital.



20. Sol:

Addition of phosphate fertilisers to water bodies causes the enhanced growth of algae, which ultimately affects animals

live in water bodies by decreasing the amount of dissolved oxygen in water.

21. Sol:

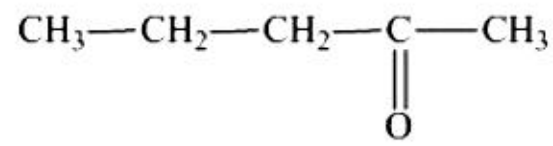
Aniline reacts with hot and acidic solution of $FeSO_4$ to form blue color.

In Lassaigne's test, sulphur from benzene sulphonic acid reacts with sodium nitroprusside to form violet colored compound.

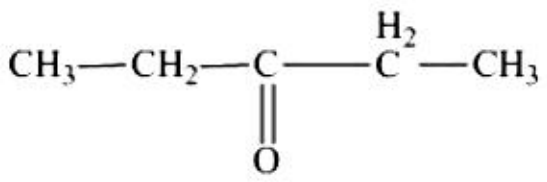
Thiourea in the presence of $FeCl_3$ produces red color.

22. Sol:

The pair of compound are positional isomers is shown below,



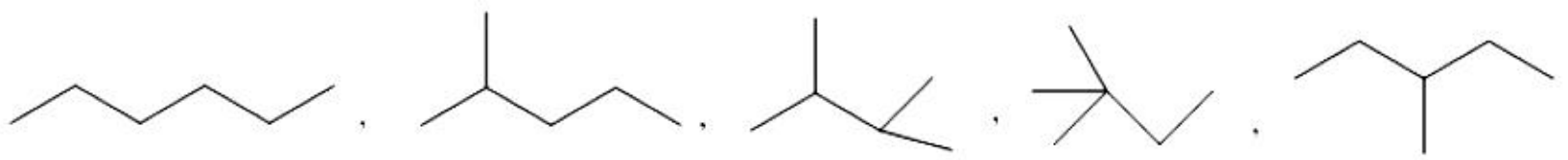
and



The names of both compounds are pentane – 2 – one and pentane – 3 – one. Both the compounds differ from each other due to position of $-C=O$ group.

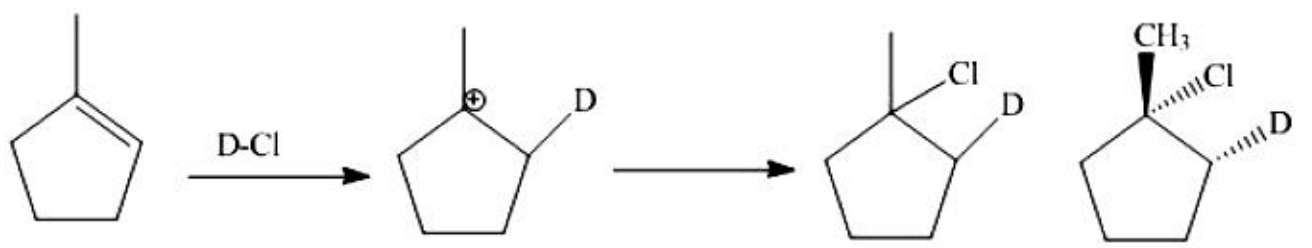
23. Sol:

The figure below represents the number of structural isomers of C_6H_{14}.



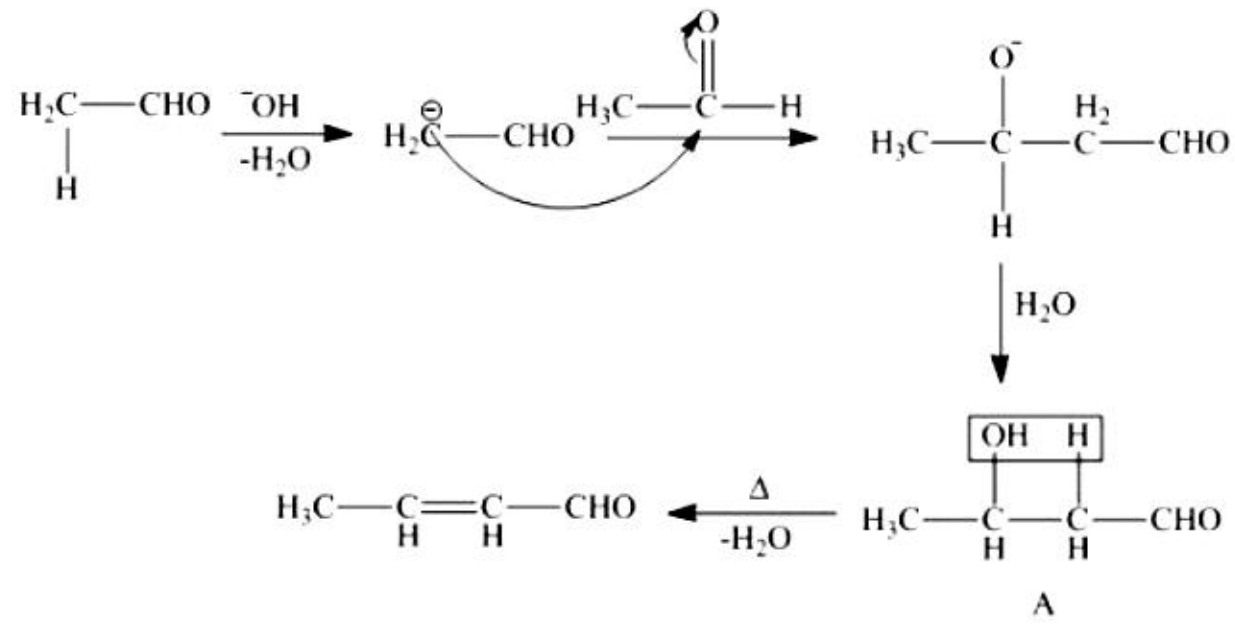
24. Sol:

The major product expected from the given reaction is,



25. Sol:

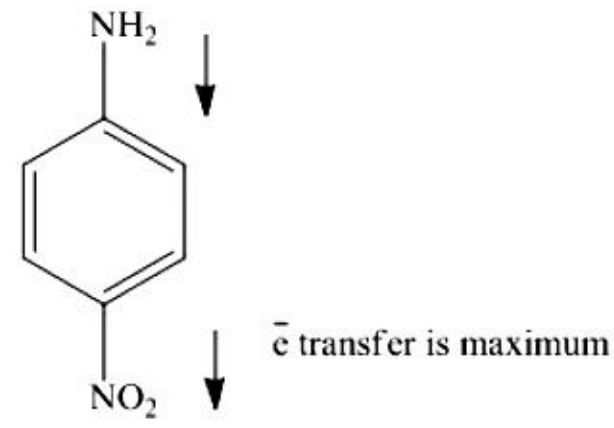
The formation of product A and B is shown in the given chemical reaction.



In this reaction, the product is formed by aldol condensation reaction.

26. Sol:

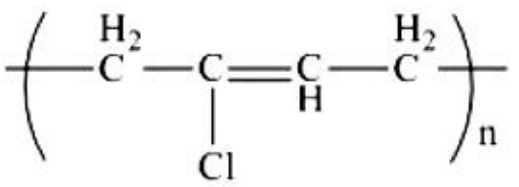
The figure below represents the compound exhibits maximum dipole moment.



The above structure has more dipole moment due to maximum transfer of electrons.

27. Sol:

The figure below represents the structure of neoprene polymer.



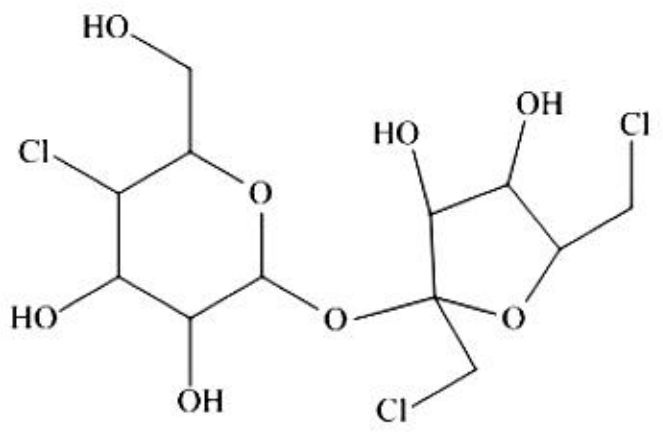
Neoprene polymer is also known as polychloroprene because it is madeup of monomer units of chloroprene.

28. Sol:

The accumulation of L-lactic acid occurs in the muscles as a result of vigorous exercise (due to anaerobic respiration).

29. Sol:

The figure below represents the structure of artificial sweetener which contains chlorine.



The above structure is of sucralose, which is an artificial sweetener.

30. Sol:

Fe^{2+} and Cu^{2+} ions generally shows pale green and blue color respectively whereas Zn^{2+} salts are white in color due to the absence of unpaired electrons in d-orbital. Therefore, Co^{2+} forms pink color in aqueous solution.

JEE MAINS-11-APRIL-2015
MATHEMATICS

1 Let $A=\{x_1,x_2,....,x_7\}$ and $B=\{y_1,y_2,y_3\}$ be two sets containing seven and three distinct elements respectively. Then the total number of functions $f:A\to B$ that are onto, if there exist exactly three elements x in A such that $f(x)=y_2$

(1) $14.^7C_3$

(2) $16.^7C_3$

(3) $12.^7C_2$

(4) $14.^7C_3$

2 If z is a non real complex number, then the minimum value of $\frac{\text{Im } z^5}{(\text{Im } z)^5}$ is:

(1) -1

(2) -2

(3) -4

(4) -5

3 If the two roots of the equation, $(a-1)\left(x^4+x^2+1\right)+(a+1)\left(x^2+x+1\right)^2=0$ are real and distinct, then the set of all values of 'a' is:

(1) $\left(-\frac{1}{2},0\right)$

(2) $(-\infty,-2)\cup(2,\infty)$

(3) $\left(-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right)$

(4) $\left(0,\frac{1}{2}\right)$

4. If A is a 3×3 matrix such that $|5.\text{adjA}|=5$, then $|A|$ is equal to:

1. $\pm\frac{1}{5}$

2. ±5

3. ±1

4. $\pm\frac{1}{25}$

5. If $\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix} = ax-12$, then 'a' is equal to

(1) 12

(2) 24

(3) -12

(4) -24

6. In a regular polygon the number of diagonals is 54, then the number of sides of this polygon is:

(1) 10

(2) 12

(3) 9

(4) 6

7. The term independent of x in the binomial expansion of $\left(1-\frac{1}{x}+3x^5\right)\left(2x^2-\frac{1}{x}\right)^8$ is:

(1) 400

(2) 496

(3) -400

(4) -496

8. The sum of the 3^{rd} and the 4^{th} terms of a G.P is 60 and the product of its first three terms is 1000. If the first term of this G.P is positive, then its 7^{th} term is:

(1) 7290

(2) 320

(3) 640

(4) 2430

9. If $\sum_{n=1}^{5} \frac{1}{n(n+1)(n+2)(n+3)} = \frac{k}{3}$, then k is equal to:

(1) $\frac{55}{36}$

(2) $\frac{17}{105}$

(3) $\frac{1}{6}$

(4) $\frac{19}{112}$

10. Let k be a non-zero real number. If

$$f(x)=\begin{cases}\dfrac{\left(e^{x}-1\right)^{2}}{\sin\left(\dfrac{x}{k}\right)\log\left(1+\dfrac{x}{4}\right)}, & x\neq 0\\ 12, & x=0\end{cases}$$

is a continuous function, then the value of k is:

(1) 1

(2) 2

(3) 3

(4) 4

11. The equation of a normal to the curve, $\sin y = x\sin\left(\dfrac{\pi}{3}+y\right)$ at x=0, is:

(1) $2x+\sqrt{3}y=0$

(2) $2y-\sqrt{3}x=0$

(3) $2y+\sqrt{3}x=0$

(4) $2x-\sqrt{3}y=0$

12. Let k and K be the minimum and the maximum values of the function $f(x)=\frac{(1+x)^{0.6}}{1+x^{0.6}}$ in [0, 1] respectively, then the ordered pair (k, K) is equal to:

(1) $\left(1, 2^{0.6}\right)$

(2) $\left(2^{-0.4}, 2^{0.6}\right)$

(3) $\left(2^{-0.6}, 1\right)$

(4) $\left(2^{-0.4}, 1\right)$

13. From the top of a 64 metres high tower, a stone is thrown upwards vertically with the velocity of 48 m/s. The greatest height (in metres) attained by the stone, assuming the value of the gravitational acceleration $g = 32\ m/s^2$

(1) 100

(2) 88

(3) 128

(4) 112

14. If $\int \frac{\log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}dt = \frac{1}{2}\left(g(t)\right)^2 + C$, where C is constant, then g(2) is equal to:

(1) $2\log\left(2+\sqrt{5}\right)$

(2) $\log\left(2+\sqrt{5}\right)$

(3) $\frac{1}{\sqrt{5}}\log\left(2+\sqrt{5}\right)$

(4) $\frac{1}{2}\log\left(2+2\sqrt{5}\right)$

15. Let $f:R\to R$ be a function such that $f(2-x)=f(2+x)$ and $f(4-x)=f(4+x)$, for all $x\in R$ and $\int_0^2 f(x)dx = 5$. Then the value $\int_{10}^{50} f(x)dx$ is:

(1) 80

(2) 100

(3) 125

(4) 200

16. Let $f:(-1,1)\to R$ be a continuous function. If $\int_0^{\sin x} f(t)dt=\frac{\sqrt{3}}{2}x$, then $f\left(\frac{\sqrt{3}}{2}\right)$ is equal to

(1) $\frac{\sqrt{3}}{2}$

(2) $\sqrt{3}$

(3) $\sqrt{\frac{3}{2}}$

(4) $\frac{1}{2}$

17. The solution of the differential equation $ydx-\left(x+2y^2\right)dy=0$ is $x=f(y)$. If $f(-1)=1$, then $f(1)$ is equal to:

(1) 4

(2) 3

(3) 2

(4) 1

18. A straight line L through the point (3, -2) is inclined at an angle of $60°$ to the line $\sqrt{3}x + y = 1$. If L also intersects the x-axis, then the equation of L is:

(1) $y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$

(2) $y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$

(3) $\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$

(4) $\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$

19. If the incentre of an equilateral triangle is (1, 1) and the equation of its one side is $3x + 4y + 3 = 0$, then the equation of the circumcircle of this triangle is:

(1) $x^2 + y^2 - 2x - 2y - 2 = 0$

(2) $x^2 + y^2 - 2x - 2y - 14 = 0$

(3) $x^2 + y^2 - 2x - 2y + 2 = 0$

(4) $x^2 + y^2 - 2x - 2y - 7 = 0$

20. If a circle passing through the point $(-1, 0)$ touches y-axis at (0, 2), then the lengths of the chord of the circle along the x-axis:

(1) $\frac{3}{2}$

(2) $\frac{5}{2}$

(3) 3

(4) 5

21. If the distance between the foci of an ellipse is half the length of its latus rectum, then the eccentricity of the ellipse is:

(1) $\frac{1}{2}$

(2) $\frac{2\sqrt{2}-1}{2}$

(3) $\sqrt{2}-1$

(4) $\frac{\sqrt{2}-1}{2}$

22. Let PQ be a double ordinate of the parabola, $y^2 = -4x$, where P lies in the second quadrant. If R divided PQ in the ratio 2:1, then the locus of R is :

(1) $9y^2 = 4x$

(2) $9y^2 = -4x$

(3) $3y^2 = 2x$

(4) $3y^2 = -2x$

23. The shortest distance between the z-axis and the line $x + y + 2z - 3 = 0 = 2x + 3y + 4z - 4$, is :

(1) 1

(2) 2

(3) 3

(4) 4

24. A plane containing the point (3, 2, 0) and the line $\frac{x-1}{1} = \frac{y-2}{5} = \frac{z-3}{4}$ also contains the point:

(1) (0,-3,1)

(2) (0,7,10)

(3) (0,7,-10)

(4) (0,3,1)

25. In a parallelogram ABCD, $\left|\overrightarrow{AB}\right| = a, \left|\overrightarrow{AD}\right| = b$ and $\left|\overrightarrow{AC}\right| = c$, then $\overrightarrow{DB} \cdot \overrightarrow{AB}$ has the value:

(1) $\frac{1}{2}\left(a^2-b^2+c^2\right)$

(2) $\frac{1}{4}\left(a^2+b^2-c^2\right)$

(3) $\frac{1}{3}\left(b^2+c^2-a^2\right)$

(5) $\frac{1}{2}\left(a^2+b^2+c^2\right)$

26. If the length of the sides of a triangle are decided by the three throws of a single fair die, then the probability that the triangle is of maximum area given tit is an isosceles triangle, is:

(1) $\frac{1}{26}$

(2) $\frac{1}{27}$

(3) $\frac{1}{21}$

(4) $\frac{1}{15}$

27. If the mean and the variance of a binomial variate X are 2 and 1 respectively, then the probability that X takes a value greter than or equal to one is:

(1) $\frac{1}{16}$

(2) $\frac{9}{16}$

(3) $\frac{3}{4}$

(4) $\frac{15}{16}$

28. If $\cos\alpha+\cos\beta=\frac{3}{2}$ and $\sin\alpha+\sin\beta=\frac{1}{2}$ and θ is the arithmetic mean of α and β, then $\sin 2\theta+\cos 2\theta$ is equal to :

(1) $\frac{3}{5}$

(2) $\frac{4}{5}$

(3) $\frac{7}{5}$

(4) $\frac{8}{5}$

29. Let 10 vertical poles standing at equal distances on a straight line, subtend the same angle of elevation α at a point O on this line and all the poles are on the poles are on the same side O. If the height of the longest pole is 'h' and the distance of the foot of the smallest pole from O is 'a'; then the distance beteen two consecutive poles, is :

(1) $\frac{h\sin\alpha + a\cos\alpha}{9\sin\alpha}$

(2) $\frac{h\cos\alpha - a\sin\alpha}{9\cos\alpha}$

(3) $\frac{h\cos\alpha - a\sin\alpha}{9\sin\alpha}$

(4) $\frac{h\sin\alpha + a\cos\alpha}{9\cos\alpha}$

30. Consider the following statemensts:

P: Suman is brilliant.

Q: Suman is rich.

R: Suman is honest.

The negation of the statement,

"Suman is brilliant and dishonest if and only if Suman is rich"

can be equivalently expressed as :

(1) $\sim Q \leftrightarrow \sim P \vee R$

(2) $\sim Q \leftrightarrow \sim P \vee R$

(3) $\sim Q \leftrightarrow \sim P \vee \sim R$

(4) $\sim Q \leftrightarrow \sim P \wedge \sim R$

Part-2

1. Sol:

The given two sets of the distinct elements is,

$$A = \{x_1, x_2, ..., x_7\}$$

And,

$$B = \{y_1, y_2, y_3\}$$

The exactly three element x in A such that $f(x)$ is,

$$f(x) = y_2$$

$$= {}^7C_3$$

$$= \frac{7!}{3! \times (7-3)!}$$

$$= 35$$

Out of remaining 4 elements in A, 2 elements are in B.

Therefore, the total number of onto function is,

$${}^7C_3 \times \left(2^4 - {}^2C_1 (2-1)^4\right) = 14.{}^7C_3$$

2. Sol:

Le the complex value be $z = re^{i\theta}$.

Here, z is a non-real complex number.

The minimum value of the given expression is obtained by substituting the value of z in the expression,

$$\frac{\mathrm{Im}\left(z^5\right)}{\left(\mathrm{Im}\left(z\right)\right)^5} = \frac{r^5\left(\sin 5\theta\right)}{r^5\left(\sin\theta\right)^5}$$

$$= \frac{\sin 5\theta}{\sin^5\theta}$$

$$= \frac{16\sin^5\theta - 20\sin^3\theta + 5\sin\theta}{\sin^5\theta}$$

$$\frac{\mathrm{Im}\left(z^5\right)}{\left(\mathrm{Im}\left(z\right)\right)^5} = 16 - 20\,\mathrm{cosec}^2\theta + 5\,\mathrm{cosec}^4\theta$$

Thus, the minimum value of $\frac{\mathrm{Im}\left(z^5\right)}{\left(\mathrm{Im}\left(z\right)\right)^5}$ is -4.

3. Sol:

By simplifying the given equation,

$$\left(\mathrm{a}-1\right)\left(x^2 - x + 1\right) + \left(\mathrm{a}+1\right)\left(x^2 + x + 1\right) = 0$$

$$\mathrm{a}x^2 + x + \mathrm{a} = 0$$

For the roots to be real and distinct,

$$D > 0$$

$$\left(1\right)^2 - \left(4 \times a \times a\right) > 0$$

Further, simplify.

$$1-4a^2>0$$

$$4a^2<1$$

$$a^2<\frac{1}{4} \qquad [a\neq 0]$$

$$a\in\left(-\frac{1}{2},0\right)\cup\left(0,\frac{1}{2}\right)$$

4. Sol:

By simplifying the given equation,

$$|5\cdot \text{adjA}|=5 \quad \left[\because |\text{kadjA}|=\text{k}^n|\text{A}|^{n-1} \text{ here n is order of the matrix}\right]$$

$$(5)^3|\text{A}|^2=5$$

$$|\text{A}|=\pm\frac{1}{5}$$

5. Sol:

The given determinant is,

$$\begin{vmatrix} x^2+x & x+1 & x-2 \\ 2x^2+3x-1 & 3x & 3x-3 \\ x^2+2x+3 & 2x-1 & 2x-1 \end{vmatrix}=ax-12$$

Substitute -1 for x in the above determinant,

$$\begin{vmatrix} 0 & 0 & -3 \\ -2 & -3 & 0 \\ 2 & -3 & -3 \end{vmatrix} = -a - 12$$

$$-36 = -a - 12$$

$$a = 24$$

6. Sol:

The number of diagonals of the regular polynomial is 54.

The formula for the number of sides of the polynomial is,

$$\frac{n(n-3)}{2} = 54$$

Here, n is number of sides of the polynomial.

Further, simplify

$$n^2 - 3n - 108 = 0$$

$$(n+9)(n-12) = 0$$

$$n = 12$$

7. Sol:

The given binomial expression is,

$$\left(1 - \frac{1}{x} + 3x^5\right)\left(2x^2 - \frac{1}{x}\right)^8$$

Simplify the given expression,

$$\left(1-\frac{1}{x}+3x^5\right)\cdot {}^8C_r\left(2x^2\right)^{8-r}\left(-\frac{1}{x}\right)^r$$

$$={}^8C_r\left(2x^2\right)^{8-r}\left(-\frac{1}{x}\right)^r-\frac{1}{x}{}^8C_r\left(2x^2\right)^{8-r}\left(-\frac{1}{x}\right)^r+3x^5{}^8C_r\left(2x^2\right)^{8-r}\left(-\frac{1}{x}\right)^r$$

$$={}^8C_r2^{8-r}\left(-1\right)^r x^{16-3r}-{}^8C_r2^{8-r}\left(-1\right)^r x^{15-3r}+3{}^8C_r2^{8-r}\left(-1\right)^r x^{21-3r}$$

For the term independent of x,

$16-3r=0$

$15-3r=0$

And,

$21-3r=0$

Only two possibilities

$r=5$

$r=7$

Thus,

$$-\left({}^8C_5 2^3\left(-1\right)-3{}^8C_r\cdot 2\right)=-\left(448-6\times 8\right)$$

$$=-\left(448-48\right)$$

$$=-400$$

8. Sol:

Consider the three terms $a,\ ar$ and ar^2 are in GP.

The given relation,

$$a(ar)(ar^2)=1000$$

$$ar=10$$

And,

$$ar^2+ar^3=60$$

$$10r+10r^2=60$$

$$r^2+r-6=0$$

$$r=2,-3$$

Hence,

$$a=5,-\frac{10}{3}$$

Hence, the value of the 7^{th} terms is,

$$T_7=ar^6$$

$$=5(2)^6$$

$$=320$$

9. Sol:

The given expression is,

$$\sum_{n=1}^{5}\left[\frac{1}{n(n+1)(n+2)}-\frac{1}{(n+1)(n+2)(n+3)}\right]=\frac{k}{3}$$

Simplify the given expression,

$$T_r=\frac{1}{3}\left[\frac{1}{n(n+1)(n+2)}-\frac{1}{(n+1)(n+2)(n+3)}\right]$$

Rewrite the above equation.

$$\sum_{r=1}^{5}T_r=\sum_{r=1}^{5}\frac{1}{3}\left[\frac{1}{n(n+1)(n+2)}-\frac{1}{(n+1)(n+2)(n+3)}\right]$$

$$\frac{k}{3}=\frac{1}{3}\left[\frac{1}{6}-\frac{1}{6\times7\times8}\right]$$

$$k=\frac{55}{336}$$

10. Sol:

The function is given as,

$$f(x)=\begin{cases}\dfrac{\left(e^x-1\right)^2}{\sin\left(\dfrac{x}{k}\right)\log\left(1+\dfrac{x}{4}\right)}, & x\neq 0\\ 12 & x=0\end{cases}$$

The given function is the continuous function, therefore,

$$f(12)=\lim_{x\to 0}f(12)$$

Simplify the expression,

$$\lim_{x\to 0}\frac{\left(e^x-1\right)^2}{\sin\left(\frac{x}{k}\right)\log\left(1+\frac{x}{4}\right)}=12$$

$$\lim_{x\to 0}\frac{\left(e^x-1\right)^2}{\left(\frac{x}{k}\right)\frac{\sin\left(\frac{x}{k}\right)}{\left(\frac{x}{k}\right)}\left(\frac{x}{4}\right)\frac{\log\left(1+\frac{x}{4}\right)}{\left(\frac{x}{4}\right)}}=12$$

$$\lim_{x\to 0}\frac{\left(e^x-1\right)^2}{\left(\frac{x}{k}\right)\left(\frac{x}{4}\right)}=12$$

$$\lim_{x\to 0}\frac{2\left(e^x-1\right)e^x}{\left(\frac{1}{k}\right)\left(\frac{x}{4}\right)+\left(\frac{x}{k}\right)\left(\frac{1}{4}\right)}=12$$

Further, simplify,

$$\lim_{x\to 0}\frac{2\left(e^x-1\right)e^x+2e^{2x}}{\left(\frac{1}{k}\right)\left(\frac{1}{4}\right)+\left(\frac{1}{k}\right)\left(\frac{1}{4}\right)}=12$$

$$\frac{2}{2\left(\frac{1}{4k}\right)}=12$$

$$4k=12$$

$$k=3$$

11. Sol:

The given equation of the curve is,

$$\sin y = x\sin\left(\frac{\pi}{3}+y\right)$$

Differentiate the given equation with respect to x,

$$\cos y\frac{dy}{dx} = \sin\left(\frac{\pi}{3}+y\right)+x\cos\left(\frac{\pi}{3}+y\right)\left(\frac{dy}{dx}\right)$$

At origin $(0,0)$,

$$\frac{dy}{dx} = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

Therefore, the equation of a normal to the curve is,

$$(y-0) = \left(-\frac{2}{\sqrt{3}}\right)(x-0)$$

$$y = -\frac{2}{\sqrt{3}}x$$

$$2x+\sqrt{3}y = 0$$

12. Sol:

The function is given as,

$$f(x) = \frac{(1+x)^{0.6}}{1+x^{0.6}} \qquad \forall x \in [0,1]$$

Differentiate the given function with respect to x,

$$f'(x)=\frac{(1+x)^{3/5}(3/5)(1+x)^{-(2/5)}-(3/5)(1+x)^{3/5}\left(x^{-2/5}\right)}{\left(1+x^{3/5}\right)^2}$$

$$=\frac{3}{5}\left[\left(1+x^{3/5}\right)(1+x)^{-(2/5)}-(1+x)^{3/5}\,x^{-2/5}\right]$$

$$<0$$

So,

$$f(0)=1$$

And,

$$f(1)=2^{-0.4}$$

Thus, $f(x)\in\left[2^{-0.4},1\right]$

13. Sol:

The value of velocity of stone is zero at maximum height,

$$v = 0$$

From the equation of the motion,

$$v^2 = u^2 - 2gh$$

$$0=48^2-2(32)h$$

$$h=36$$

Thus, the maximum height attained by the stone is,

$36+64=100\text{ m}$

14. Sol:

The given equation is,

$$\int \frac{\log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}dt=\frac{1}{2}\left(g(t)\right)^2+C$$

Consider the integral function is,

$$I=\int \frac{\log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}dt$$

Consider,

$$u=\log\left(t+\sqrt{1+t^2}\right)$$

Differentiate with respect to t,

$$du=\frac{1}{\sqrt{1+t^2}}dt$$

Substitute the value in the integral function I,

$$I = \int u du$$

$$= \frac{u^2}{2} + C$$

$$\frac{1}{2} g^2(t) + C = \frac{\log^2\left(t + \sqrt{1+t^2}\right)}{2} + C$$

$$g(t) = \log\left(t + \sqrt{1+t^2}\right)$$

Hence, the value of $g(2) = \log\left(2 + \sqrt{5}\right)$

15. Sol:

The given function is,

$$f(2-x) = f(2+x)$$

And,

$$f(4-x) = f(4+x)$$

Replace x with $(x+2)$,

$$f(-x) = f(4+x)$$

$$= f(4-x)$$

Replace x with $(-x)$,

$$f(x) = f(x+4)$$

Hence, the function $f(x)$ is periodic with period 4.

Then the required value,

$$\int_{10}^{50} f(x)dx = 10\int_{10}^{14} f(x)dx$$

$$= 10 \times 2\int_{0}^{2} f(x)dx$$

$$= 20 \times 5$$

$$= 100$$

16. Sol:

The given function is,

$$\int_{0}^{\sin x} f(t)dt = \frac{\sqrt{3}}{2}x$$

Differentiate the above function with respect to x,

$$f(\sin x)\cos x = \frac{\sqrt{3}}{2}$$

Hence, for the required value substitute $x = \frac{\pi}{3}$,

$$f\left(\sin\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right)=\frac{\sqrt{3}}{2}$$

$$f\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)=\frac{\sqrt{3}}{2}$$

$$f\left(\frac{\sqrt{3}}{2}\right)=\sqrt{3}$$

17. Sol:

The differential equation is given by,

$$ydx-\left(x+2y^2\right)dy=0$$

Rewrite the above equation.

$$\frac{ydx-xdy}{y^2}=2dy$$

$$d\left(\frac{x}{y}\right)=2dy$$

Integrate above equation,

$$\frac{x}{y}=2y+c$$

At $(-1,1)$,

$$-1=-2+c$$

$$c=1$$

The solution of the differential equation is,

$$\frac{x}{y} = 2y + 1$$

$$x = 2y^2 + y$$

$$f(y) = 2y^2 + y$$

Therefore, the value is,

$$f(1) = 2(1)^2 + 1$$

$$= 3$$

18. Sol:

Let the slope of the straight line L be m.

The equation of the given line L_1 is,

$$\sqrt{3x} + y = 1$$

$$y = \left(-\sqrt{3}\right)x + 1$$

Slope of the line L_1 is,

$$m_1 = -\sqrt{3}$$

The between L and L_1 is $60°$.

$$\tan 60^\circ = \left|\frac{m-\left(-\sqrt{3}\right)}{1+\left(-m\sqrt{3}\right)}\right|$$

$$\sqrt{3} = \left|\frac{m+\sqrt{3}}{1-m\sqrt{3}}\right|$$

$$\left|\sqrt{3}-3m\right| = \left|m+\sqrt{3}\right|$$

Then,

$$\sqrt{3}-3m = -m-\sqrt{3}$$

$$2m = 2\sqrt{3}$$

$$m = \sqrt{3}$$

Or

$$m = 0$$

Therefore, the equation of the line L is,

$$y+2 = \sqrt{3}\left(x-3\right)$$

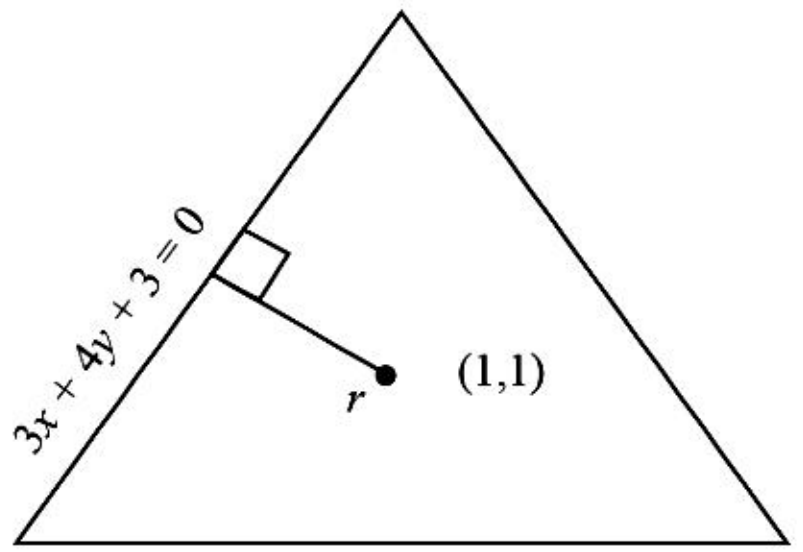
$$y-\sqrt{3}x+2+3\sqrt{3} = 0$$

19. Sol:

In an equilateral triangle, incentre and circumcenter are same and,

$$R = 2r$$

The figure below represents the equilateral triangle,



Now,

$$r = \frac{|3+4+3|}{\sqrt{9+16}}$$

$$= 2$$

Therefore,

$$R = 2r$$

$$= 4$$

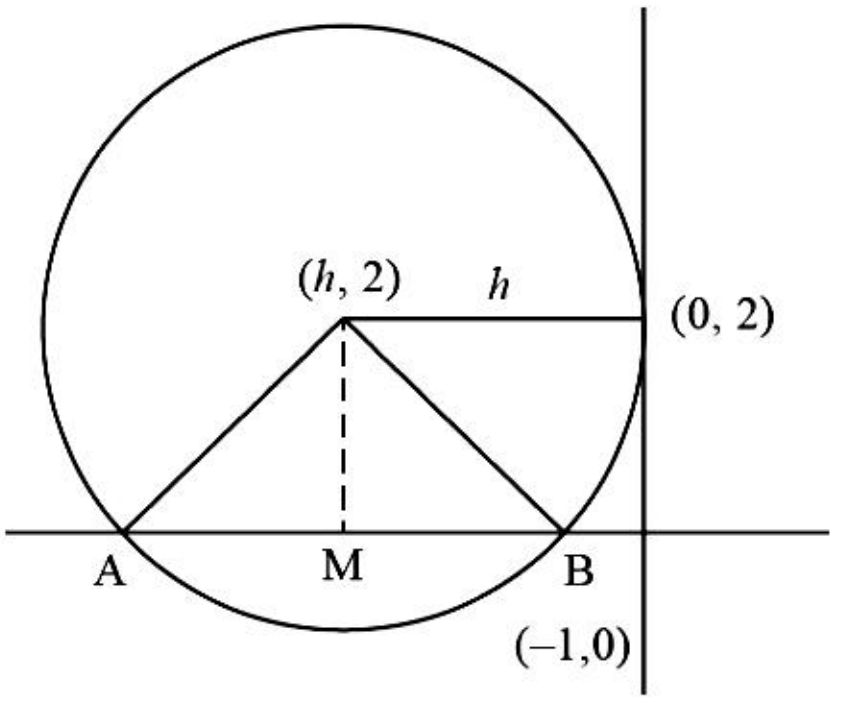
Thus, the equation of the circumcentre of this triangle is,

$$(x-1)^2 + (y-1)^2 = 4^2$$

$$x^2 + y^2 - 2x - 2y - 14 = 0$$

20. Sol:

The figure below represents the circle passing through (-1,0) touches with y axis at (0,2)



Apply the Pythagoras theorem,

$$(h+1)^2+2^2=h^2$$

$$h^2+1+2h+4=h^2$$

$$2h+5=0$$

$$h=\frac{-5}{2}$$

The length of the chord of the circle along the x-axis is,

$$\text{AB}=2(\text{AM})$$

$$=2\sqrt{(h^2-2^2)}$$

$$=2\left(\frac{3}{2}\right)$$

$$=3$$

21. Sol:

The equation of the ellipse is,

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

From the given statement,

$$ae=\frac{b^2}{2a}$$

$$e=\frac{b^2}{2a^2}$$

$$=\frac{a^2\left(1-e^2\right)}{2a^2}$$

$$e=\frac{1-e^2}{2}$$

Rewrite the above equation.

$$2e=1-e^2$$

$$e^2+2e-1=0$$

$$e=\frac{-2\pm\sqrt{4+4}}{2}$$

$$e=-1\pm\sqrt{2}$$

Thus, the eccentricity of the ellipse is,

$e>0$, then $e=\sqrt{2}-1$

22. Sol:

The equation of the parabola is given by,

$$y^2 = -4x$$

Let the points $P\left(-at_1^2, 2at_1\right)$, $Q\left(-at_2^2, 2at_2\right)$ and $R(h,k)$.

Here,

$$h = -at_1^2$$

And,

$$k = \frac{-2at_1}{3}$$

So,

$$9k^2 = -4h$$

Hence, the locus of R is $9y^2 = -4x$

23. Sol:

The equation of z-axis is,

$$\frac{x}{0} = \frac{y}{0} = \frac{z}{1}$$

The equation of the line is,

$$x + y + 2z - 3 = 0 = 2x + 3y + 4z - 4$$

The figure below represents the given line.

The shortest distance between the z-axis and the given line is,

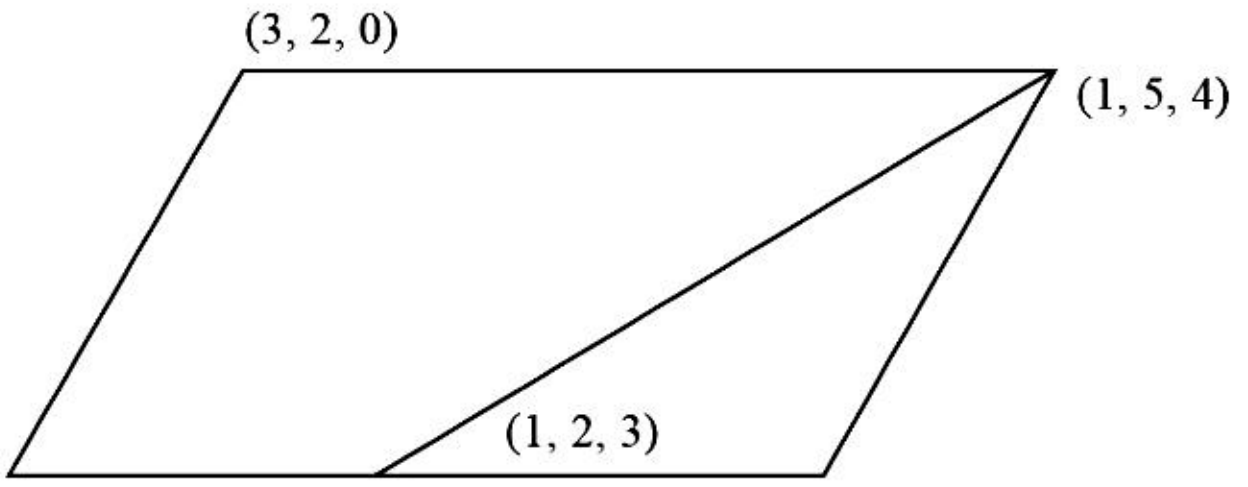
$$\text{Shortest distance} = \left|\frac{(5i-2i).2j}{2}\right|$$

$$= 2$$

24. Sol:

The plane which contains the point $(3,2,0)$.

The given equation of line is $\frac{x-1}{1} = \frac{y-2}{5} = \frac{z-3}{4}$.



The equation of the plane passing though the three points is,

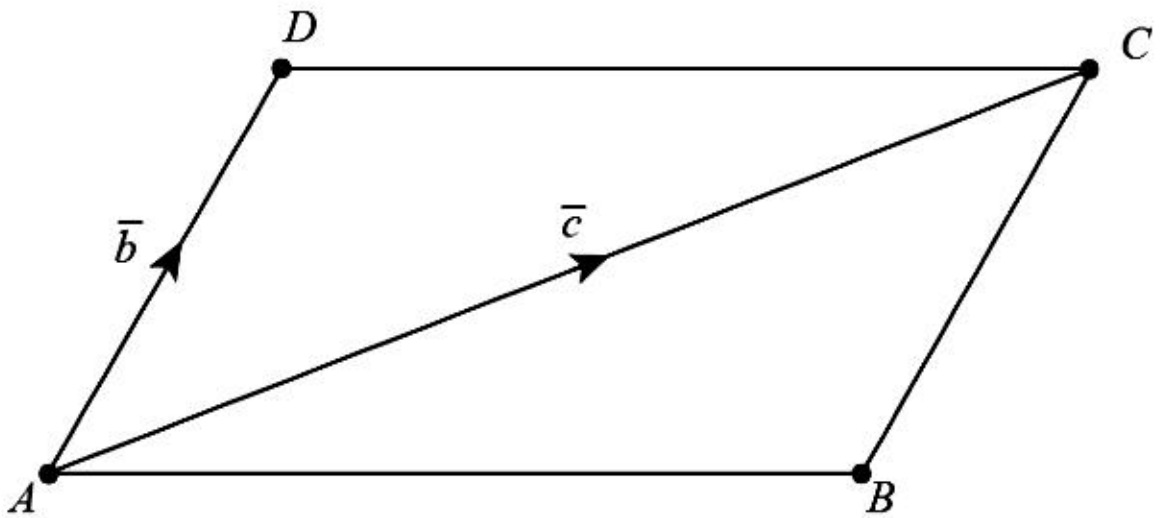
$15x-11y+10z=23$

Thus, the plane contains the point is,

$P(0,7,10)$

25. Sol:

The figure of the parallelogram with the direction of the vector,



Here,

$$\left|\overrightarrow{AB}\right| = a$$

$$\left|\overrightarrow{AD}\right| = b$$

And,

$$\left|\overrightarrow{AC}\right| = c$$

Since,

$$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$$

$$\left|\overrightarrow{AB}\right|^2 + \left|AD\right|^2 + 2\overrightarrow{AB} \cdot \overrightarrow{AD} = \left|AC\right|^2$$

$$a^2 + b^2 + 2\overrightarrow{AB} \cdot \left(\overrightarrow{AB} + \overrightarrow{BD}\right) = c^2$$

$$a^2 + b^2 + 2a^2 + 2\overrightarrow{AB} \cdot \overrightarrow{BD} = c^2$$

Further, simplify,

$$2\overrightarrow{AB}\cdot\overrightarrow{BD}=c^2-3a^2-b^2$$

$$2\overrightarrow{AB}\cdot\overrightarrow{DB}=3a^2+b^2-c^2$$

$$\overrightarrow{AB}\cdot\overrightarrow{DB}=\frac{1}{2}\left(3a^2+b^2-c^2\right)$$

$$\overrightarrow{DB}\cdot\overrightarrow{AB}=\frac{1}{2}\left(3a^2+b^2-c^2\right)$$

26. Sol:

Let S be the sample space then probability of occurrence of an event E is,

$$P(E)=\frac{n(E)}{n(S)}$$

$$P(E)\le 1$$

$$P(E)=\lim_{x\to\infty}\left(\frac{r}{n}\right)$$

Here, n repeated experiment and E occurs r times.

Consider 21 cases of isosceles triangle each case occurring thrice,

Total cases for $a+b>c$

$$\left\{\begin{matrix}(1,1,1),(2,2,1),(2,\ 2,\ 2),(2,\ 2,\ 3),(3,\ 3,\ 1),...,(3,\ 3,\ 5),\\(4,\ 4,\ 1),.....,(4,\ 4,\ 6),(5,\ 5,\ 1)\ ...(5,\ 5,\ 6),(6,\ 6,\ 1),.....,(6,\ 6,\ 6)\end{matrix}\right\}$$

Hence, required probability is,

$$P = \frac{1}{21}$$

If consider equilateral triangle, there are 63 occurrences of non-equilateral triangle and 6 occurrences of the equilateral triangle.

Hence, the required probability is,

$$P = \frac{1}{27}$$

27. Sol:

The given value of the mean and variance is,

Mean,

$$np = 2$$

Variance

$$npq = 1$$

Divide variance by mean,

$$p = \frac{1}{2}$$

Then,

$$q = \frac{1}{2}$$

And,

$n = 4$

Hence, the probability that X takes a value greater than or equal to 1 is,

$$P(x \geq 1) = {}^4C_1 p^1 q^3 + {}^4C_2 p^2 q^2 + {}^4C_3 p^3 q^1 + {}^4C_1 p^4 q^0$$

$$= 1 - {}^4C_1 p^0 q^4$$

$$= 1 - \left(\frac{1}{2}\right)^4$$

$$= \frac{15}{16}$$

28. Sol:

Rewrite the given equations.

$$\cos\alpha + \cos\beta = \frac{3}{2}$$

$$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{3}{2}$$

And,

$$\sin\alpha + \sin\beta = \frac{1}{2}$$

$$2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right) = \frac{1}{2}$$

Divide the given equations.

$$\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{3}$$

$$\tan\theta=\frac{1}{3}$$

Hence, the required value is,

$$\sin 2\theta+\cos 2\theta=\frac{2\tan\theta}{1+\tan^2\theta}+\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

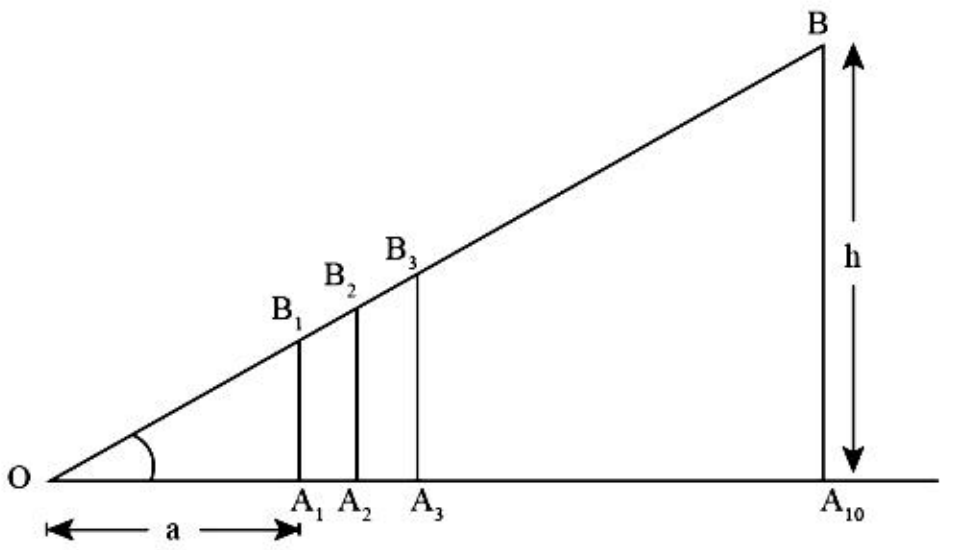
$$=\frac{2\tan\theta+1-\tan^2\theta}{1+\tan^2\theta}$$

$$=\frac{2\left(\frac{1}{3}\right)+1-\left(\frac{1}{9}\right)}{1+\left(\frac{1}{9}\right)}$$

$$=\frac{7}{5}$$

29. Sol:

The figure below represents the vertical poles at equal distance on a straight line.



From the similar triangles,

$$\frac{h_1}{a_1}=\frac{h_2}{a_2}=\frac{h_3}{a_3}=.....=\frac{h_{10}}{a_{10}}=\tan\alpha$$

So,

$$h_{10}=a_{10}\tan\alpha \qquad \ldots\ldots (1)$$

And,

$$h_1=a_1\tan\alpha \qquad \ldots\ldots (2)$$

Thus, apply the Arithmetic progression for the given ten poles,

$$h_{10}=(a_1+9d)\tan\alpha$$

$$d=\frac{h_{10}-a_1\tan\alpha}{9\tan\alpha}$$

$$=\frac{h-a\tan\alpha}{9\tan\alpha}$$

$$=\frac{h\cos\alpha-a\sin\alpha}{9\sin\alpha}$$

30. Sol:

Given statement is equivalent to,

$$(P\wedge\sim R)\leftrightarrow Q$$

This is same as,

$$Q\leftrightarrow(P\wedge\sim R)$$

Negation of the above statement is,

$$\sim Q \leftrightarrow P \wedge \sim R$$

JEE MAINS-11-APRIL-2015
PHYSICS

1. If electronic charge e, electron mass m, speed of light in vacuum c and plank's constant h are taken as fundamental quantities , the permeability of vacuum μ_0 can be expressed in units of:

(1) $\left(\frac{hc}{me^2}\right)$

(2) $\left(\frac{h}{me^2}\right)$

(3) $\left(\frac{h}{ce^2}\right)$

(4) $\left(\frac{mc}{he^2}\right)$

2. A vector $\vec{A}$ is rotated by a small angle $\Delta\theta$ radians ($\Delta\theta << 1$) to get a new vector $\vec{B}$. In that case $\left|\vec{B}-\vec{A}\right|$ is :

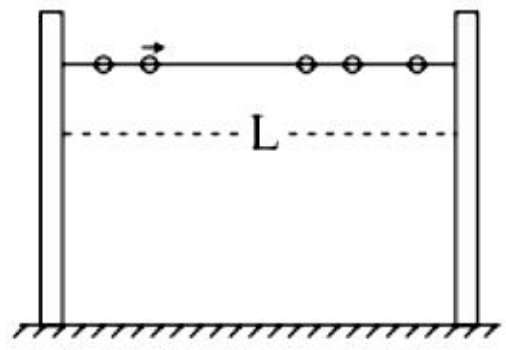
(1) 0

(2) $\left|\vec{A}\right|\left(1-\frac{\Delta\theta^2}{2}\right)$

(3) $\left|\vec{A}\right|\Delta\theta$

(4) $\left|\vec{B}\right|\Delta\theta - \left|\vec{A}\right|$

3. A large number (n) of identical beads, each of mass m and radius r are strung on a thin smooth rigid horizontal rod of length L (L >> r) and are at rest at random positions. The rod is mounted between two rigid supports (see figure). If one of the beads is now given a speed v, the average force experienced by each support after a long time is (assume all collisions are elastic):



(1) $\dfrac{mv^2}{L-nr}$

(2) $\dfrac{mv^2}{L-2nr}$

(3) $\dfrac{mv^2}{2(L-nr)}$

(4) zero

4. A particle is moving in a circle of radius r under the action of force $F = \alpha r^2$ which is directed towards centre of the circle. Total mechanical energy (kinetic energy + potential energy) of the particle is (take potential energy = 0 for r = 0) :

(1) αr^3

(2) $\frac{1}{2}\alpha r^3$

(3) $\frac{4}{3}\alpha r^3$

(4) $\frac{5}{6}\alpha r^3$

5. A uniform thin rod AB of length L has linear mass density $\mu(x) = a + \frac{bx}{L}$, where x is measured from A. If the CM of the rod lies at a distance of $\left(\frac{7}{12}L\right)$ from A, then a and b are related as:

(1) a=b

(2) a=2b

(3) 2a=b

(4) $3a=2b$

6. A particle of mass 2 kg is on a smooth horizontal table and moves in a circular path of radius 0.6 m. The height of the table from the ground is 0.8 m. If the angular speed of the particle is 12 rad s^{-1} , the magnitude of its angular momentum about a point on the ground right under the centre of the circle is:

(1) 8.64 kg m^2 s^{-1}

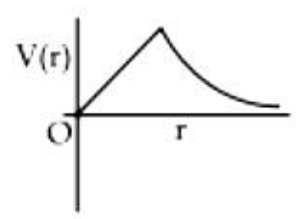
(2) 11.52 kg m^2 s^{-1}

(3) 14.4 kg m^2 s^{-1}

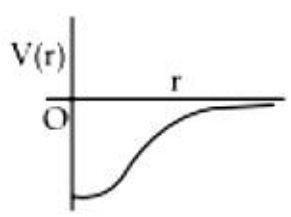
(4) 20.16 kg m^2 s^{-1}

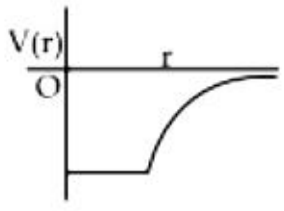
7. Which of the following most closely depicts the correct variation of the gravitation potential V(r) due to a large planet of radius R and uniform mass density? (*figures are not drawn to scale)*

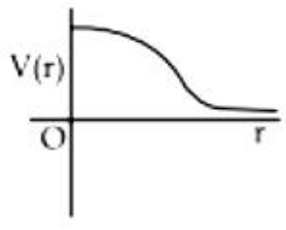
(1)



(2)



(3) 

(4) 

8. A cylindrical block of wood (density = 650 kg m^{-3}), of base area 30 cm^2 and height 54 cm, floats in a liquid of density 900 kg m^{-3}. The block is depressed slightly and then released. The time period of the resulting oscillations of the block would be equal to that of a simple pendulum of length (nearly) :

(1) 65 cm

(2) 52 cm

(3) 39 cm

(4) 26 cm

9. A beaker contains a fluid of density ρ kg/m^3, specific heat S J/kg °C and viscosity η. The beaker is filled up to height h. To estimate the rate of heat transfer per unit are $\left(\dot{Q}/A\right)$ by convection when beaker is put on a hot plate, a student

proposes that it should depend on η, $\left(\frac{S\Delta\theta}{h}\right)$ and $\left(\frac{1}{\rho g}\right)$ when $\Delta\theta$ (in°C) is the difference in the temperature between the bottom and top of the fluid. In that situation the correct option for $\left(\dot{Q}/A\right)$ is:

(1) $\eta\frac{S\Delta\theta}{h}$

(2) $\eta\left(\frac{S\Delta\theta}{h}\right)\left(\frac{1}{\rho g}\right)$

(3) $\frac{S\Delta\theta}{\eta h}$

(4) $\left(\frac{S\Delta\theta}{\eta h}\right)\left(\frac{1}{\rho g}\right)$

10. An experiment takes 10 minutes to raise the temperature of water in a container from 0 °C to 100 °C and another 55 minutes to convert it totally into steam by a heater supplying heat at a uniform rate. Neglecting the specific heat of the container and taking specific heat of water to be 1 cal/g °C, the heat o vaporization according to this experiment will come out to be:

(1) 530 cal/g

(2) 540 cal/g

(3) 550 cal/g

(4) 560 cal/g

11. Using equipartition of energy, the specific heat (in J kg^{-1} k^{-1}) of aluminium at room temperature can be estimated to be (atomic weight of aluminium =27)

(1) 25

(2) 410

(3) 925

(4) 1850

12. A pendulum with time period of 1 s is losing energy due to damping. At a certain time its energy is 45 J. If after completing 15 oscillations, its energy has become 15 J, its damping constant (in s^{-1}) is:

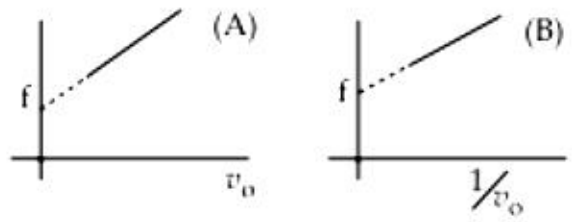
(1) $\frac{1}{30}\ln 3$

(2) $\frac{1}{15}\ln 3$

(3) 2

(4) $\frac{1}{2}$

13. A source of sound emits sound waves at frequency f_0 . It is moving towards an observer with fixed speed v_s ($v_s < v$, where v is the speed of sound in air). If the observer were to move towards the source with speed v_0 ,one of the following two graphs (A and B) will given the correct variation of the frequency f heard by the observer as v_0 is changed.



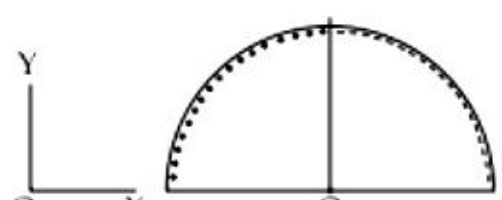
(1) graph A with slope = $\frac{f_0}{(v - v_s)}$

(2) graph A with slope = $\frac{f_0}{(v + v_s)}$

(3) graph B with slope = $\frac{f_0}{(v - v_s)}$

(4) graph B with slope = $\frac{f_0}{(v + v_s)}$

14. A wire, of length L(=20 cm), is bent into a semi-circular arc. If the two equal halves, of the arc, were each to be uniformly charged with charges $\pm Q$, $|Q| = 10^3\ \varepsilon_0$ Coulomb where ε_0 is the permittivity (in SI units) of free space the net electric field at the centre O of the semi-circular arc would be:



(1) $\left(50\times10^3\ \text{N/C}\right)\hat{j}$

(2) $\left(25\times10^3\ \text{N/C}\right)\hat{i}$

(3) $\left(25\times10^3\ \text{N/C}\right)\hat{j}$

(4) $\left(50\times10^3\ \text{N/C}\right)\hat{i}$

15. An electric field $\vec{E}=\left(25\hat{i}+30\hat{j}\right)\text{NC}^{-1}$ exists in a region of space.

If the potential at the origin is taken to be zero then the potential at x=2 m, y=2 m is:

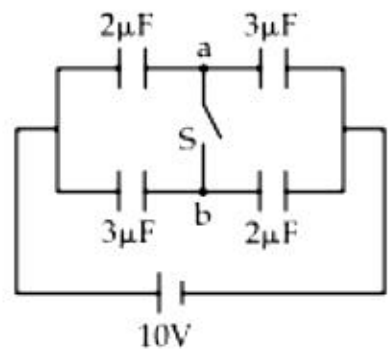
(1) -130 J

(2) -120 J

(3) -140 J

(4) -110 J

16. In figure is shown a system of four capacitors connected across a 10 V battery. Charge that will flow from switch S when it uis closed is:



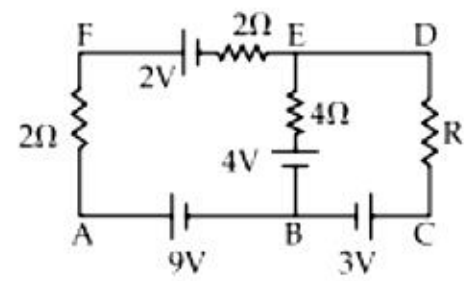
(1) 5 μC from b to a

(2) 20 μC from a to b

(3) 5 μC from a to b

(4) zero

17. In the electric network shown, when no current flows through the 4Ω resistor in the arm EB, the potential difference between the points A and D will be :



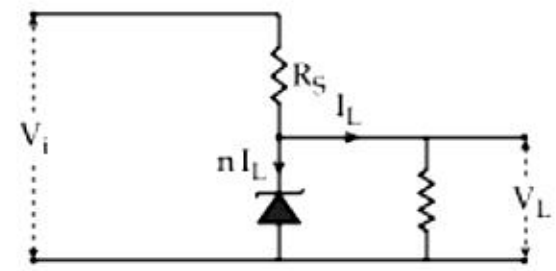
(1) 3 V

(2) 4 V

(3) 5 V

(4) 6 V

18. The value of resistor, R_S, needed in the dc voltage regulator circuit shown here, equals:



(1) $(V_i - V_L)/nI_L$

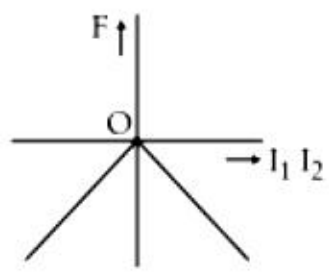
(2) $(V_i + V_L)/nI_L$

(3) $(V_i - V_L)/(n+1)I_L$

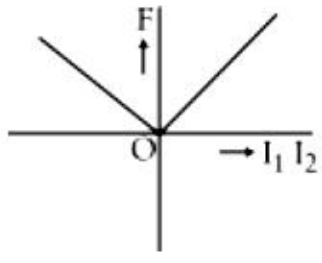
(4) $(V_i + V_L)/(n+1)I_L$

19. Two long straight parallel wires, carrying (adjustable) currents I_1 and I_2, are kept at a distance d apart. If the force 'F' between the two wires is taken as 'positive' when the wires repel each other and 'negative' when the wires attract each other, the graph showing the dependence of 'F', on the product I_1I_2, would be:

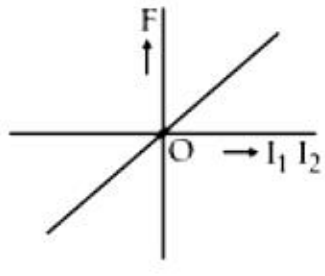
(1)



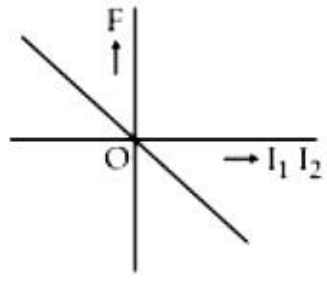
(2)



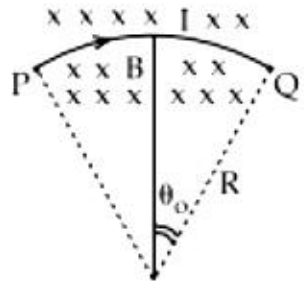
(3)



(4)



20. A wire carrying current I is tied points P and Q and is in the shape of a circular arch of radius R due to a uniform magnetic field B (perpendicular to the plane of the paper, shown by xxx) in the vicinity of the wire. If the wire subtends an angle $2\theta_0$ at the centre of the circle (of which it forms an arch) then the tension in the wire is :



(1) IBR

(2) $\frac{IBR}{\sin\theta_0}$

(3) $\frac{IBR}{2\sin\theta_0}$

(4) $\frac{IBR\theta_0}{\sin\theta_0}$

21. A short bar magnet is placed in the magnetic meridian of the earth with north pole pointing north. Neutral points are found at a distance of 30 cm from the magnet on the East - West line, drawn through the middle point of the magnet. The magnetic moment of the magnet in Am^2 is close to :

(Given $\frac{\mu_0}{4\pi} = 10^{-7}$ in SI units and B_H = Horizontal component of earth's magnetic field = 3.6×10^{-5} Tesla)

(1) 9.7

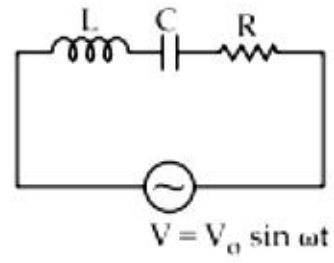
(2) 4.9

(3) 19.4

(4) 14.6

22. For the LCR circuit, shown here, the current is observed to lead the applied voltage. An additional capacitor C', when

joined with the capacitor C present in the circuit, makes the power factor of the circuit unity. The capacitor C', must have been connected in :



(1) series with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$

(2) series with C and has a magnitude $\frac{C}{\left(\omega^2 LC-1\right)}$

(3) parallel with C and has a magnitude $\frac{C}{\left(\omega^2 LC-1\right)}$

(4) parallel with C and has a magnitude $\frac{1-\omega^2 LC}{\omega^2 L}$

23. For plane electromagnetic waves propagating in the z direction, which one of the following combination gives the correct possible direction for $\vec{E}$ and $\vec{B}$ field respectively?

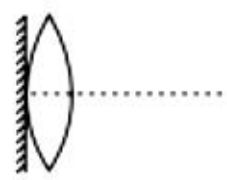
(1) $\left(\hat{i}+2\hat{j}\right)$ and $\left(2\hat{i}-\hat{j}\right)$

(2) $\left(-2\hat{i}-3\hat{j}\right)$ and $\left(3\hat{i}-2\hat{j}\right)$

(3) $\left(2\hat{i}+3\hat{j}\right)$ and $\left(\hat{i}+2\hat{j}\right)$

(4) $\left(3\hat{i}+4\hat{j}\right)$ and $\left(4\hat{i}-3\hat{j}\right)$

24. A thin convex lens of focal length 'f' is put on a plane mirror as shown in the figure. When an object is kept at a distance 'a' from the lens - mirror combination, its image is formed at a distance 3 a in front of the combination. The value of 'a' is :



(1) f

(2) 2f

(3) 3f

(4) $\frac{3}{2}$f

25. In a Young's double slit experiment with light of wavelength λ the separation of slits is d and distance of screen is D such that $D >> d >> \lambda$. If the fringe width is β, the distance from point of maximum intensity to the point where intensity falls to half of maximum intensity on either side is:

(1) $\frac{\beta}{2}$

(2) $\frac{\beta}{4}$

(3) $\frac{\beta}{3}$

(4) $\frac{\beta}{6}$

26. Unpolarized light of intensity I_0 is incident on surface of a block of glass at Brewster's angle. In that case, which one of the following statements is true?

(1) transmitted light is partially polarized with intensity

(2) transmitted light is completely polarized with intensity less than $\frac{I_0}{2}$

(3) reflected light is completely polarized with intensity less than $\frac{I_0}{2}$

(4) reflected light is partially polarized with intensity $\frac{I_0}{2}$

27. The de-Broglie wavelength associated with the electron in the n = 4 level is :

(1) two times the de-Broglie wavelength of the electron in the ground state

(2) four times the de-Broglie wavelength of the electron in the ground state

(3) half of the de-Broglie wavelength of the electron in the ground state

(4) $\frac{1}{4}$th of the de-Broglie wavelength of the electron in the ground state.

28. Let N_β be the number of β particles emitted by 1 gram of Na^{24} radioactive nuclei (half life = 15 hrs) in 7.5 hours, N_β is close to (Avogadro number = 6.023×10^{23}/g. mole) :

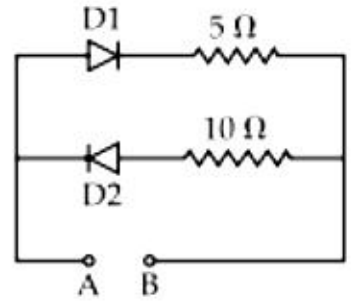
(1) 6.2×10^{21}

(2) 7.5×10^{21}

(3) 1.25×10^{22}

(4) 1.75×10^{22}

29. A 2V battery is connected across AB as shown in the figure. The value of the current supplied by the battery when in one case battery's positive terminal is connected to A and in other case when positive terminal of battery is connected to B will respectively be :



(1) 0.2 A and 0.1 A

(2) 0.4 A and 0.2 A

(3) 0.1 A and 0.2 A

(4) 0.2 A and 0.4 A

30. The AC voltage across a resistance can be measured using a :

(1) potentiometer

(2) moving coli galvanometer

(3) moving magnet galvanometer

(4) hot wire voltmeter

PART-2

1. Sol:

The dimension of the electron in [MLT] is,

$e = IT$

The dimension of the mass in [MLT] is,

$m = M$

The dimension of the speed of light in [MLT] is,

$c = LT^{-1}$

The dimension of the Plank's constant in [MLT] is,

$h = ML^2T^{-1}$

The expression for the permeability of the vacuum is,

$$\mu_0 = e^a m^b c^c h^d$$

$$\left(MLT^{-2}T^{-3}\right)=\left(IT\right)^a\left(M\right)^b\left(LT^{-1}\right)^c\left(ML^2T^{-1}\right)^d$$

From equating the powers of [MLT],

$a = -2$

$b = 0$

$c = -1$

$d = 1$

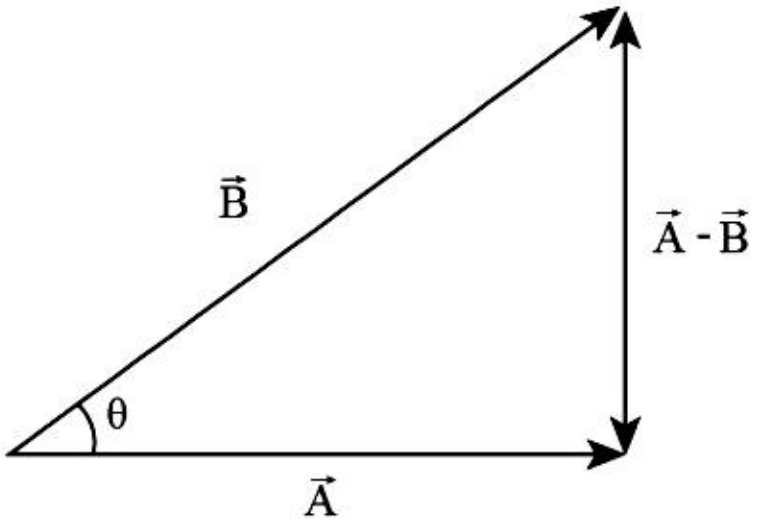
Thus,

$$\mu_0 = e^{-2}m^0c^{-1}h^1$$

$$= \left(\frac{h}{ce^2}\right)$$

2. Sol:

Consider the figure for the given statement,



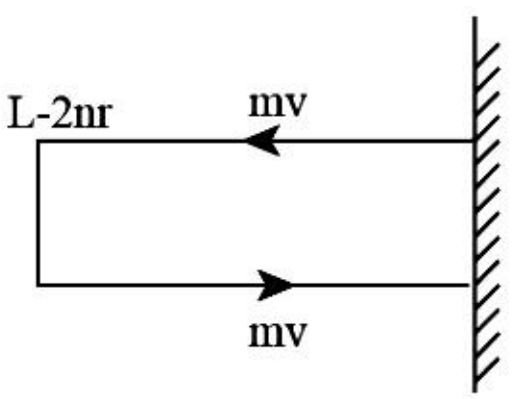
From the triangle in the figure, the arc length can be calculated as,

$$\text{arc length} = \text{radius} \times \text{angle}$$

$$\left|\vec{B} - \vec{A}\right| = \left|\vec{A}\right| \Delta\theta$$

3. Sol:

The required figure for the given statement,



From the above figure, the distance between the supports for motion of beads is,

$$d = L - 2nr$$

The expression for the average force experience by each support is,

$$F = \frac{2mv}{\left(\frac{2(L-2nr)}{v}\right)}$$

$$= \frac{mv^2}{(L-2nr)}$$

4. Sol:

The potential energy can be calculated as,

$$dU = F \cdot dr$$

$$U = \int_0^r F \cdot dr$$

$$= \int_0^r \alpha r^2 dr$$

$$= \frac{\alpha r^3}{3}$$

The centripetal force can be calculated as,

$$F = \frac{mv^2}{r}$$

$$\alpha r^2 = \frac{mv^2}{r}$$

$$mv^2 = \alpha r^3$$

$$m^2v^2 = m\alpha r^3$$

The kinetic energy can be calculated as,

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2}\alpha r^3$$

The total energy can be calculated as,

$$E = U + KE$$

$$= \frac{\alpha r^3}{3} + \frac{1}{2}\alpha r^3$$

$$= \frac{5}{6}\alpha r^3$$

5. Sol:

The centre of mass of a body can be calculated as is,

$$x_{CM} = \frac{\int_0^L \left(ax + \frac{bx^2}{L} \right) dx}{\int_0^L \left(a + \frac{bx}{L} \right) dx}$$

$$= \frac{\frac{aL^2}{2} + \frac{bL^2}{3}}{aL + \frac{bL}{2}}$$

$$= \frac{L\left(\frac{a}{2} + \frac{b}{3} \right)}{\left(a + \frac{b}{2} \right)}$$

Rewrite the above equation.

$$x_{CM} = \frac{L\left(\frac{a}{2} + \frac{b}{3} \right)}{\left(a + \frac{b}{2} \right)}$$

$$\frac{7}{12}L = \frac{L\left(\frac{a}{2} + \frac{b}{3} \right)}{\left(a + \frac{b}{2} \right)}$$

$$b = 2a$$

6. Sol:

The angular momentum of a body can be calculated as is,

$$L = mvl\sin 90°$$
$$= m(r\omega)l$$
$$= (2\times 0.6\times 0.8\times 12)$$
$$= 11.52\ \text{kg}\cdot\text{m}^2/\text{s}$$

7. Sol:

The expression for the gravitational potential because of a large radius outside the uniform sphere is,

$$V_0 = -\frac{Gm}{r} \qquad (r > R)$$

The expression for the gravitational potential because of a large radius on the surface of uniform sphere is,

$$V_s = -\frac{Gm}{R} \quad (r = R)$$

The expression for the gravitational potential because of a large radius inside the uniform sphere is,

$$V_i = -\frac{3}{2}\frac{Gm}{R}\left(1-\frac{r^2}{3R^2}\right) \quad (r < R)$$

From the options, graph (2) most closely represents the correct variation of the gravitational potential.

8. Sol:

Let the block's length immersed in the water be h.

Apply the equilibrium of forces in the vertical direction; the buoyant force is balanced by the gravitation force of the block.

Thus,

$$mg = F_B$$

$$\rho_{block} Vg = \rho_{liquid} Vg$$

$$\rho_{block} (Al) = \rho_{liquid} (Ah)$$

Rewrite the above equation.

$$h = \frac{\rho_{block} l}{\rho_{liquid}}$$

$$= \frac{650 \times 54}{900}$$

$$= 39 \text{ cm}$$

9. Sol:

From the given statement, the rate of heat transfer per unit area is,

$$\left(\frac{\dot{Q}}{A}\right) = \eta^a \left(\frac{S\Delta\theta}{h}\right)^b \left(\frac{1}{\rho g}\right)^c$$

Write the dimension of the quantity in [MLT] form,

$$\left(MT^{-3}\right) = \left(ML^{-1}T^{-1}\right)^a \left(LT^{-2}\right)^b \left(M^{-1}L^2T^{-2}\right)^c$$

By equating the powers of [MLT],

$a = 1$

$b = 1$

$c = 0$

Thus,

$$\left(\frac{\dot{Q}}{A}\right) = \eta\left(\frac{S\Delta\theta}{h}\right)$$

10. Sol:

The heat required to raise the temperature of water in the container can be calculated as,

$$Pt = mC\Delta T$$

$$P \times 10 \times 60 = mC \times 100 \quad \ldots\ldots (1)$$

The heat that converts the water into steam can be calculated as,

$$Pt = mL$$

$$P \times 55 \times 60 = mL \quad \ldots\ldots (2)$$

From the above equation (1) and (2),

$$\frac{10}{55}=\frac{C\times100}{L}$$

$$L=\frac{C\times100\times55}{10}$$

$$=\frac{1\times100\times55}{10}$$

$$=550\ \text{cal/g}$$

11. Sol:

Due to collision of atoms of the metal, the potential energy of the atom is equal to the kinetic energy of the atom.

Therefore,

$$PE=KE$$

$$=\frac{3}{2}RT$$

The total mechanical energy of the metal is,

$$E=KE+PE$$

$$mCT=\frac{3}{2}RT+\frac{3}{2}RT$$

$$\frac{6}{2}RT=mCT$$

Rewrite the above expression.

$$C = \frac{3R}{m}$$
$$= \frac{3 \times 8.314}{27 \times 10^{-3}}$$
$$\cong 925 \text{ J/kg} \cdot \text{K}$$

12. Sol:

The loss in energy due to damping is,

$$E = E_0 e^{\frac{-bt}{m}}$$
$$15 = 45e^{\frac{-b \times 15}{m}}$$
$$\frac{1}{3} = e^{\frac{-b \times 15}{m}}$$

Apply logarithmic in the above equation on both sides,

$$\ln(3) = \left(\frac{b \times 15}{m}\right)$$
$$\frac{b}{m} = \frac{1}{15}\ln(3)$$

13. Sol:

The expression for the apparent frequency is,

$$f = \left(\frac{v + v_0}{v - v_s}\right) f_0$$

Rewrite the above equation.

$$f = \left(\frac{f_0}{v - v_S}\right) v_0 + \left(\frac{v f_0}{v - v_S}\right)$$

The expression for the linear equation is,

$$y = mx + c$$

On comparing above two equations, the slope of graph A is,

$$m = \left(\frac{f_0}{v - v_S}\right)$$

14. Sol:

The linear charge density can be calculated as is,

$$\lambda = \frac{q}{A}$$

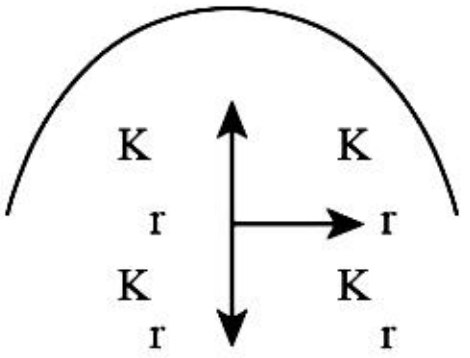
$$= \frac{2Q}{\pi r}$$

The length of the wire is,

$$L = \pi r$$

$$r = \frac{L}{\pi}$$

The required diagram for the statement is,



Due to linear infinite charged conductor, the electric field at the centre of semicircular arc is,

$$E = \left(\frac{2K\lambda}{r}\right)$$

$$= \frac{2K\left(\frac{2Q}{\pi r}\right)}{r}$$

$$= \frac{4KQ}{\pi r^2}$$

$$= \frac{4\pi^2 KQ}{\pi L^2}$$

Further, solve.

$$E = \frac{4\pi kQ}{L^2}$$

$$= \frac{4\pi \times 9\times 10^9 \times 10^3 \varepsilon_0}{\left(20\times 10^{-2}\right)^2}$$

$$= \frac{4\pi \times 9\times 10^9 \times 10^3 \times 8.85\times 10^{-12}}{\left(20\times 10^{-2}\right)^2}$$

$$= 25022.8$$

Rewrite the above equation.

$$E \cong \left(25\times10^{3}\ \text{N/C}\right)\hat{i}$$

15. Sol:

The potential difference can be calculated as,

$$E = -\frac{dV}{dx}$$

$$V = -\int_{0}^{(2,2)} 25x\ dx + 30y\ dy$$

Solve the above integration,

$$V = -\left(\int_{0}^{(2)} 25x\ dx + \int_{0}^{(2)} 30y\ dy\right)$$

$$= -\left(25\left(\frac{x^2}{2}\right)_0^2 + 30\left(\frac{y^2}{2}\right)_0^2\right)$$

$$= -110\ \text{J}$$

16. Sol:

When the switch S is closed:

When the switch S is closed,

The potential difference across the each capacitor in series is 5 V.

The charge on $2\ \mu\text{F}$ capacitor can be calculated as,

$$Q = CV$$
$$= 2 \times 5$$
$$= 10\ \mu C$$

The charge on $3\ \mu F$ capacitor can be calculated as,

$$Q = CV$$
$$= 3 \times 5$$
$$= 15\ \mu C$$

The total charge is,

$$Q' = 15 - 10$$
$$= 5\ \mu C$$

When the switch S is open:

The potential difference across $3\ \mu F$ capacitor can be calculated as,

$$V_1 = 10 \times \left(\frac{2}{2+3} \right)$$
$$= 4\ V$$

The potential difference across $2\ \mu F$ capacitor can be calculated as,

$$V_2 = 10 \times \left(\frac{3}{2+3} \right)$$
$$= 6\ V$$

The potential difference across each capacitor of $2\,\mu F$ is $6\,V$ and across each capacitor of $3\,\mu F$ is $4\,V$.

The charge on $3\,\mu F$ capacitor can be calculated as,

$$Q_1 = 3 \times 4$$
$$= 12\,\mu C$$

The charge on $2\mu F$ capacitor can be calculated as,

$$Q_2 = 2 \times 6$$
$$= 12\,\mu C$$

The total charge is,

$$Q'' = Q_2 - Q_1$$
$$= 12 - 12$$
$$= 0$$

Thus, the charge of $5\,\mu C$ will flow from b to a.

17. Sol:

The potential at point E and D will be zero because there is no current flow in $4\,\Omega$ resistor; that is in branch ED.

$$V_E = IR$$
$$= 0 \times 4$$
$$= 0$$

Similarly,

$$V_D = IR$$
$$= 0 \times R$$
$$= 0$$

The potential at point B can be calculated as,

$$V_B = -4 \text{ V}$$

The potential at point A can be calculated as,

$$V_A = 9 + V_B$$
$$= 9 - 4$$
$$= 5 \text{ V}$$

The potential difference between point A and D will be,

$$V = V_A - V_D$$
$$= 5 - 0$$
$$= 5 \text{ V}$$

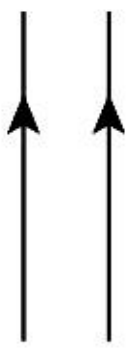
18. Sol:

The voltage across the R_S can be calculated as,

$$V_{R_S} = V_i - V_L$$
$$(I_L + nI_L)R_S = V_i - V_L$$
$$R_S = \frac{V_i - V_L}{(I_L + nI_L)}$$
$$= \frac{V_i - V_L}{I_L(n+1)}$$

19. Sol:

Let the current is flowing in same direction as shown in figure below.



The force of attraction is taken as negative. Therefore the multiplication of I_1I_2 will be positive.

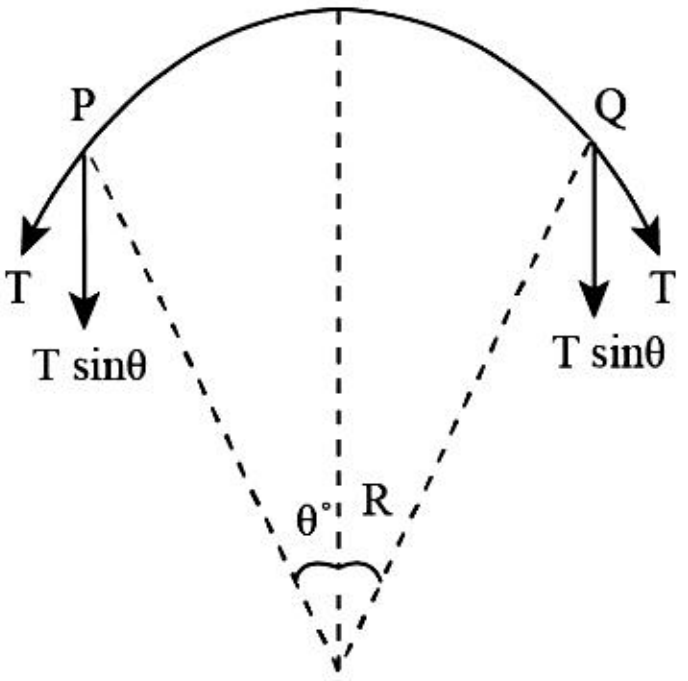
Let the current is flowing in opposite direction as shown in figure below.

The force of repulsion is taken as negative. Therefore the multiplication of I_1I_2 will be negative.

Thus, graph 4 represents the exact dependency of the force on the product of I_1I_2.

20. Sol:

The figure below represents the circular arc wire.



The expression for the force due to magnetic field is,

$F = BIL$

The expression for the arc length is,

$L = R\theta_0$

For a very small arc length,

$BIR2\theta_0 = 2T\sin\theta_0$

$T\sin\theta_0 = BIR$

$T = IBR$

21. Sol:

The magnetic field due to magnetic moment can be calculated as,

$$B_H = \frac{\mu_0 M}{4\pi r^3}$$

$$3.6\times10^{-5} = \frac{10^{-7}\times M}{\left(30\times10^{-2}\right)^3}$$

$$M = \left(\frac{3.6\times10^{-5}\times\left(30\times10^{-2}\right)^3}{10^{-7}}\right)$$

$$= 9.7$$

22. Sol:

The single equivalent capacitance when a capacitor C'' connected parallel to the circuit is,

$$C_1 = C + C''$$

The impedance of the given circuit can be calculated as,

$$Z = \sqrt{R^2 + \left(X_L - X_{C_1}\right)}$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega\left(C + C'\right)}\right)^2}$$

The power factor can be calculated as,

$$\cos\phi = \frac{R}{Z}$$

$$1 = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega(C + C')}\right)^2}}$$

$$R^2 + \left(\omega L - \frac{1}{\omega(C + C')}\right)^2 = R^2$$

$$C' = \frac{1 - \omega^2 LC}{\omega^2 L}$$

23. Sol:

The electric field and magnetic field is perpendicular to each other. Therefore, the dot product of electric field and magnetic field is equal to zero,

$\vec{E} \cdot \vec{B} = 0$

Thus, the cross product of the vector $\vec{E}$ and $\vec{B}$ must have unit vector $\hat{k}$ to propagate in *z* direction. Then, from options check for the cross product that will result in $\hat{k}$.

Therefore,

$$\vec{E}\times\vec{B}=\left(-2\hat{i}-3\hat{j}\right)\times\left(3\hat{i}-2\hat{j}\right)$$

$$=\left(-2\hat{i}\times3\hat{i}\right)+\left(-2\hat{i}\times-2\hat{j}\right)+\left(-3\hat{j}\times3\hat{i}\right)+\left(-3\hat{j}\times-2\hat{j}\right)$$

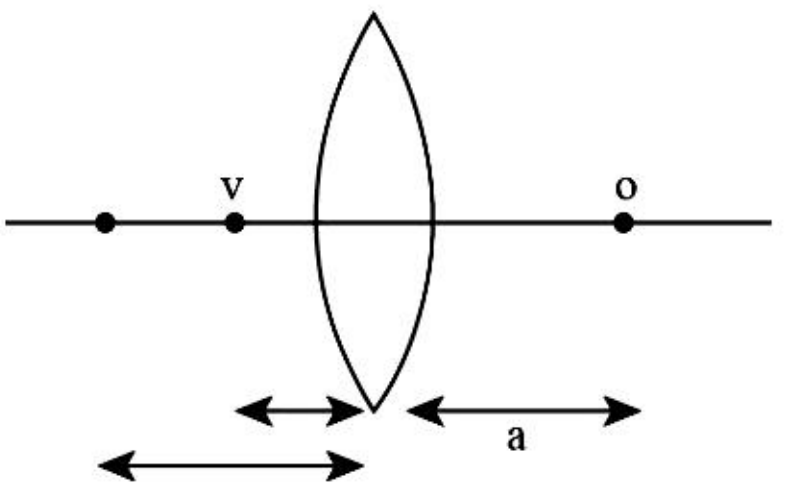
$$=0-4\hat{k}+9\hat{k}$$

$$=5\hat{k}$$

Thus, option (2) results in unit vector $\hat{k}$ to propagate in z direction.

24. Sol:

The figure below represents an object, which is placed at distance "a" from the lens.



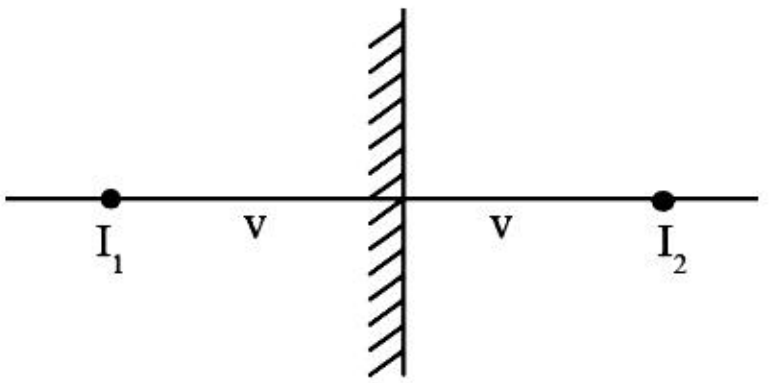
From the formula of lens,

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{(-a)}=\frac{1}{f}$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{a}$$

The mirror forms image at equal distance from the mirror.



Similarly,

$$\frac{1}{u}-\frac{1}{v}=\frac{1}{f}$$

$$\frac{3}{a}-\left(\frac{1}{f}-\frac{1}{a}\right)=\frac{1}{f}$$

$$\frac{3}{a}-\frac{1}{f}+\frac{1}{a}=\frac{1}{f}$$

$$a=2f$$

25. Sol:

The expression for the intensity is,

$$2I_0=4I_0\cos^2\left(\frac{\Delta\phi}{2}\right)$$

Here,

$$\Delta\phi=\frac{\pi}{2}$$

The phase difference can be calculated as,

$$\Delta\phi = \frac{2\pi}{\lambda(\Delta x)}$$

$$\frac{\pi}{2} = \frac{2\pi}{\lambda(\Delta x)}$$

$$\Delta x = \frac{\lambda}{4}$$

The path difference can be calculated as,

$$\Delta x = \frac{dy}{D}$$

$$\frac{\lambda}{4} = \frac{dy}{D}$$

$$\lambda = \frac{4dy}{D}$$

The fringe width can be calculated as,

$$\beta = \frac{\lambda D}{d}$$

$$= \frac{\left(\frac{4dy}{D}\right)D}{d}$$

$$= 4y$$

$$y = \frac{\beta}{4}$$

26. Sol:

If an unpolarized light incident on a surface at the Brewster's angle, then light reflected by it is completely polarized. Therefore the intensity of the reflected light becomes less than half of the light that is incident.

27. Sol:

The expression for the de-Broglie wavelength of the electron is,

$$\lambda = \frac{h}{mv}$$

The speed of the wave is inversely proportional to the number of level. Then,

$$v \propto \frac{1}{n}$$

Therefore,

$$\lambda = \frac{h}{m\left(\frac{1}{n}\right)}$$

$$\lambda \propto n$$

Now,

$$4\lambda = 4\lambda_1$$

Here, λ_1 is the de-Broglie wavelength of the electron in the ground level.

28. Sol:

The expression for the number of particle for time t is,

$$N = N_0 e^{-\lambda t}$$

The number of molecules left after the 7.5 hrs,

$$N = N_0 e^{-\lambda 7.5}$$

It is given that,

$$N = \frac{N}{2} \text{ at } t = t_{\frac{1}{2}}$$

Now,

$$N = \frac{N_0}{\sqrt{2}}$$

Therefore, the number of particle emitted is,

$$N' = N_0 - \frac{N_0}{\sqrt{2}}$$

$$= N_0 \frac{\left(\sqrt{2}-1\right)}{\sqrt{2}}$$

$$= \frac{\left(\sqrt{2}-1\right)}{\sqrt{2}} \times \frac{6.023 \times 10^{23}}{24}$$

$$\approx 7.5 \times 10^{21}$$

29. Sol:

When the positive terminal connected to A, then the diode D1 becomes forward biased. Therefore, the current through the circuit can be calculated as,

$$I = \frac{V}{R}$$
$$= \frac{2}{5}$$
$$= 0.4 \text{ A}$$

When the positive terminal connected to B, then the diode D2 becomes forward biased. Therefore, the current through the circuit can be calculated as,

$$I = \frac{2}{10}$$
$$= 0.2 \text{ A}$$

30. Sol:

When DC voltmeters are connected across the AC circuits, then the reading measured by the DC voltmeters is zero.

The average value of the alternative voltage in a DC circuit is zero because of a complete full cycle.

The AC circuits comprise of RMS values of the current and voltage, so the hot wire voltmeter measures the AC voltage across the resistance. The hot wire voltmeter measures only RMS values of the alternating voltages.