JEE-Main-28-07-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: If $P\left(\frac{B}{A}\right)=\frac{5}{7}$; $P\left(\frac{A}{B}\right)=\frac{7}{9}$ and $P(A\cap B)=\frac{1}{9}$. Given $S_1 \equiv P(A'\cup B)=\frac{5}{6}, S_2 \equiv P(A'\cap B')=\frac{1}{18}$, then:

Options:

(a) Both S_1 and S_2 are correct

(b) S_1 is true and S_2 is false

(c) S_1 is false and S_2 is true

(d) Both S_1 and S_2 are false

Answer: (d)

Solution:

Given: $\frac{P(A\cap B)}{P(A)}=\frac{5}{7}$ & $\frac{P(A\cap B)}{P(B)}=\frac{7}{9}$

As $P(A\cap B)=\frac{1}{9}$, we get

$P(A)=\frac{7}{45}$ & $P(B)=\frac{1}{7}$

$P(A'\cap B')=1-P(A\cap B)$

$=1-\left(\frac{7}{45}+\frac{1}{7}-\frac{1}{9}\right)$

$=\frac{256}{315}$

Given, $S_1 \equiv P(A'\cup B)=\frac{5}{6}$

$S_2 \equiv P(A'\cap B')=\frac{1}{18}$, then:

As $P(A\cap B)=\frac{1}{9}$, we get $P(A)=\frac{7}{45}$ & $P(B)=\frac{1}{7}$

$P(A'\cup B)=1-(P(A)-P(A\cap B))$

$=1-\left(\frac{7}{45}-\frac{1}{9}\right)=\frac{43}{45}$

$\therefore S_1$ is wrong.

Question: Absolute maximum value of $f(x)=\tan^{-1}(\sin x-\cos x)$ is:

Options:
(a) 0
(b) $\tan^{-1}\frac{1}{\sqrt{2}}-\frac{\pi}{4}$
(c) $\frac{\pi}{4}$
(d) $\tan^{-1}\sqrt{2}$
Answer: (d)
Solution:
$\because \sin x-\cos x\in\left[-\sqrt{2},\sqrt{2}\right]$
So maximum value of $\tan^{-1}\left(\sin x-\cos x\right)$ is $\tan^{-1}\sqrt{2}$

Question: The values of λ for which the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{\lambda^2}$ and $\frac{x-3}{1}=\frac{y-2}{\lambda^2}=\frac{z-1}{2}$ are coplanar, are:
Options:
(a) $\pm\sqrt{3}$
(b) $\pm\sqrt{5}$
(c) ± 2
(d) $\pm\sqrt{2}$
Answer: (d)
Solution:
For coplanarity

$$\begin{vmatrix} 1 & 2 & \lambda^2 \\ 1 & \lambda^2 & 2 \\ 2 & 0 & 4 \end{vmatrix}=0$$

$\Rightarrow 1\left(4\lambda^2\right)-2\left(0\right)+\lambda^2\left(-2\lambda^2\right)=0$
$\Rightarrow -2\lambda^4+4\lambda^2=0$
$\Rightarrow \lambda^2\left(\lambda^2-2\right)=0$
$\Rightarrow \lambda=\pm\sqrt{2}$

Question: From point $\left(2,0\right)$ tangents are drawn on $2y^2=-x$. These tangents also touches the circle $\left(x-5\right)^2+y^2=r^2$. The value of $17r^2$ is:
Options:
(a) 1
(b) 12
(c) 9
(d) 4
Answer: (c)

Solution:

$P \equiv y^2 = \frac{-x}{2}$

Equation of tangent $\equiv y = mx + \left(-\frac{1}{8m}\right)$

$16m^2 = 1 \Rightarrow m = \pm\frac{1}{4}$

$\therefore$ Tangent are $y = \frac{1}{4}x - \frac{1}{2}$ & $y = -\frac{x}{4} + \frac{1}{2}$

Equation of tangent to $(x-5)^2 + y^2 = r^2$ are:

$x - 4y = 2$ & $4y + x = 2$

Using $d_c = r,$ we get

$\left|\frac{5-0-2}{\sqrt{17}}\right| = r$ or $\left|\frac{5+0-2}{\sqrt{17}}\right| = r$

$\therefore r^2 = \frac{9}{17}$

$\Rightarrow 17r^2 = 9$

Question: Let $f(x) = \lim_{n\to\infty} \frac{\cos 2\pi x - x^{2n}\sin(x-1)}{1 + x^{2n+1} - x^{2n}}$, is continuous at:

Options:

(a) $R - \{1\}$

(b) $R - \{-1, 1\}$

(c) $R - \{0, 1\}$

(d) $R - \{0\}$

Answer: (b)

Solution:

For $|x| < 1, f(x) = \cos 2\pi x$

For $|x| > 1, f(x) = -\frac{\sin(x-1)}{x-1}$

For $|x| = 1,\ f(x) = \begin{cases} 1 & \text{if} \quad x = 1 \\ \frac{(1+\sin 2)}{-1} & \text{if} \quad x = -1 \end{cases}$

$\lim_{x\to 1^+} f(x) = -1,\ \lim_{x\to 1^-} f(x) = 1$

So f is discontinuous at $x = 1$

$\lim_{x\to -1^+} f(x) = 1,\ \lim_{x\to -1^-} f(x) = -\frac{\sin 2}{2}$

So $f(x)$ is discontinuous at $x = -1$

Question: A class have B boys and G girls, 3 boys and 2 girls selected at random and number of ways of selecting 3 boys and 2 girls are 168. Then $B+3G$ is equal to ______.

Answer: 17.00

Solution:

Given that

$^{B}C_3 \cdot {^{G}C_3} = 168$

$\Rightarrow \frac{B(B-1)(B-2)}{6} \cdot \frac{G(G-1)}{2} = 168$

$\Rightarrow B(B-1)(B-2)G(G-1) = 7 \cdot 6 \cdot 4 \cdot 3 \cdot 2 \cdot 2$

$\Rightarrow B(B-1)(B-2)G(G-1) = 8 \cdot 7 \cdot 6 \cdot 3 \cdot 2$

$\therefore B = 8 \ \& \ G = 3$

$\Rightarrow B+3G = 8+9 = 17$

Question: Let $f(x) = ax^2 + bx + c$ and $f(1) = 3,\ f(-2) = \lambda, f(3) = 4,$ then the value of λ for which $f(0)+f(1)+f(-2)+f(3) = 14$ is ____.

Answer: 4.00

Solution:

Given, $f(x) = ax^2 + bx + c$

$f(0) = c$

$f(1) = a+b+c = 3 \quad ....(1)$

$f(-2) = 4a-2b+c = \lambda \quad ....(2)$

$f(3) = 9a+3b+c = 4 \quad ....(3)$

By solving (2) & (3)

$a+b = \frac{4-\lambda}{5}$

Also, $c = 3+\lambda+4 = 14$

$c = 7-\lambda$

$\therefore$ Putting in (1)

$\frac{4-\lambda}{5} + 7 - \lambda = 3$

$6\lambda = 24$

$\lambda = 4$