JEE-Main-28-06-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: If n arithmetic means are inserted between a and 100 then ratio of first AM and n^{th} AM is $1:7$ and $a+n=33$. Find n.

Options:

(a) 21

(b) 22

(c) 23

(d) 24

Answer: (c)

Solution:

$$d=\frac{100-a}{n+1}$$

$$A_1=a+\frac{100-a}{n+1}$$

$$A_n=a+n\left(\frac{100-a}{n+1}\right)$$

$$\frac{A_1}{A_n}=\frac{an+100}{a+100n}=\frac{1}{7}$$

$$\Rightarrow 7an+700=a+100n$$

$$a+n=33$$

$$\Rightarrow a=33-n$$

$$\Rightarrow 7(33-n)n+700=33-n+100n$$

$$\Rightarrow 231n-7n^2+700=33+99n$$

$$\Rightarrow 7n^2-132n-667=0$$

$$\Rightarrow n=23,\ \frac{-29}{7}$$

$$23\times\alpha=\frac{-667}{7}$$

$$\Rightarrow n=23$$

Question: Find the area enclosed by x-axis & $y=3-|x+1|-\left|x-\frac{1}{2}\right|$.

Options:

(a) $\frac{27}{8}$

(b) $\frac{23}{8}$

(c) $\frac{25}{8}$

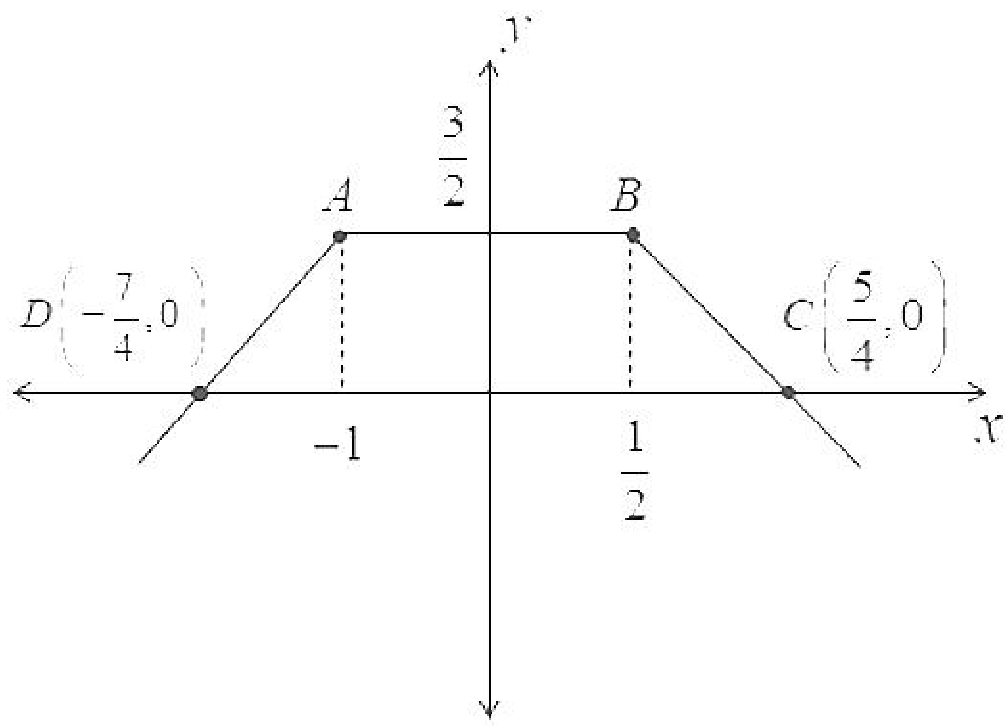
(d) $\frac{27}{4}$

Answer: (d)

Solution:

$$\begin{cases} 3+x+1+x-\frac{1}{2} & ; \quad x<-1 \\ 3-x-1+x-\frac{1}{2} & ; \quad -1 \le x < \frac{1}{2} \\ 3-x-1-x+\frac{1}{2} & ; \quad x \ge \frac{1}{2} \end{cases}$$

$$\begin{cases} 2x+\frac{7}{2} & ; \quad x<-1 \\ \frac{3}{2} & ; \quad -1 \le x < \frac{1}{2} \\ -2x+\frac{5}{2} & ; \quad x \ge \frac{1}{2} \end{cases}$$



Required area $(ABCD)=\frac{1}{2}\times\left(3+\frac{3}{2}\right)\frac{3}{2}$

$=\frac{27}{8}$

Question: $f(x)+f(x+k)=n,\ I_1=\int_0^{4nk} f(x)dx,\ I_2=\int_{-k}^{3k} f(x+k)dx$

Options:

(a) $I_1 = 2I_2$

(b) $I_1 = nI_2$

(c)

(d)

Answer: (b)

Solution:

$f(x)+f(x+k)=n$

$x \to x+k$

$f(x+k)=f(x+2k)=n$

$\Rightarrow f(x)=f(x+2k)$

Period $=2k$

$I_1 = \int_0^{4nk} f(x)dx = 2n\int_0^{2k} f(x)dx$

$I_2 = \int_{-k}^{3k} f(x+k)dx = \int_0^{4k} f(t)dt$

$=2\int_0^{2k} f(x)dx$

$I_1 = nI_2$

Question: There are 30 candidates to be distributed among 4 children C_1, C_2, C_3, C_4 such that C_2 gets at least 4 and at most 7 candy and C_3 gets atleast 2 and almost 6 candy. The number of ways to distribute it.

Answer: ${}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3$

Solution:

$x_1 + x_2 + x_3 + x_4 = 30$

$4 \le x_2 \le 7$

$t_2 = x_2 - 4$

$2 \le x_3 \le 6$

$t_3 = x_3 - 2$

$\Rightarrow x_1 + t_2 + t_3 + x_4 = 24$

${}^{24+4-1}C_{4-1} = {}^{27}C_3$

$x_1 + x_2 + x_3 + x_4 = 30$

$8 \le 2,\ 2 \le x_3$

$x_1 + t_2 + t_3 + x_4 = 20$

$^{20+4-1}C_{4-1} = {}^{23}C_3$

$x_1 + x_2 + x_3 + x_4 = 30$

$4 \le x_2,\ 7 \le x_3$

$x_1 + t_2 + t_3 + x_4 = 19$

$^{19+4-1}C_{4-1} = {}^{22}C_3$

$x_1 + x_2 + x_3 + x_4 = 30$

$8 \le x_2,\ 7 \le x_3$

$x_1 + t_1 + t_2 + x_4 = 15$

$^{15+4-1}C_{4-1} = {}^{18}C_3$

Required answer $= {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3$

Question: $f(x)$ is a quadratic polynomial. If $f(-2) + f(3) = 0$ and one root of equation is -1. Find the sum of roots.

Answer: $\frac{11}{3}$

Solution:

$f(x) = ax^2 + bx + c$

$f(-1) = 0 \Rightarrow a - b + c \quad ....(1)$

$f(-2) + f(3) = 0$

$\Rightarrow (4a - 2b + c) + (9a + 3b + c) = 0$

$13a + b + 2c = 0 \quad ....(2)$

Eq. (2) – 2 (Eq. 1)

$\Rightarrow 11a + 3b = 0$

$\Rightarrow b = \frac{-11a}{3}$

Sum of roots $= \frac{-b}{a} = \frac{11}{3}$

Question: $\lim_{n\to\infty} 6\tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2 + 3r + 3}\right)\right)$

Answer: 3.00

Solution:

Given, $\lim_{n\to\infty} 6\tan\left(\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2 + 3r + 3}\right)\right)$

$\lim_{n\to\infty} 6\tan\left(\sum_{r=1}^{n} \tan^{-1}\left[\frac{(r+2)-(r+1)}{1+(r+2)(r+1)}\right]\right)$

$$= \lim_{n\to\infty} 6\tan\left(\sum_{r=1}^{n} \tan^{-1}(r+2) - \tan^{-1}(r+1)\right)$$

$$= \lim_{n\to\infty} 6\tan\left[\tan^{-1}(n+2) - \tan^{-1}(2)\right]$$

$$= \lim_{n\to\infty} 6\tan\left[\tan^{-1}\left(\frac{n}{1+2(n+2)}\right)\right]$$

$$= \lim_{n\to\infty} \frac{6n}{2n+5} = 3$$

Question: If $\lim_{x\to 1}\left(\frac{\sin(3x^2-4x+1)-4x+1}{2x^3-7x^2+ax+b}\right) = -2$ then $a-b=?$

Answer: 11.00

Solution:

$$\lim_{x\to 1}\left[\frac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b}\right] = -2$$

$$\Rightarrow a+b=5$$

$$\lim_{x\to 1}\frac{\cos(3x^2-4x+1)(6x-4)-2x}{6x^2-14x+a} = -2$$

$$\Rightarrow a=8,\ b=-3$$

$$\therefore a-b=11$$

Question: For a parabola directrix: $3x-4y=21$, vertex (2, -1). Find length of Latus Rectum.

Answer: $\frac{32}{5}$

Solution:

Directrix: $3x-4y=21$; vertex (2, -1)

$$a = \left|\frac{3(2)-4(-1)-21}{5}\right| = \left|\frac{6+4-21}{5}\right| = \left|\frac{11}{5}\right|$$

$$\therefore \text{ LLR } = 4a = \frac{44}{5}$$

Question: $\cot\alpha = 1$; $\alpha \in \left(\pi, \frac{3\pi}{2}\right)$, $\sec\beta = \frac{-5}{3}$, $\beta \in \left(\frac{\pi}{2}, \pi\right)$. Find $\tan(\alpha+\beta)$.

Answer: $\frac{-1}{7}$

Solution:

$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}=\frac{1-\frac{4}{3}}{1+\frac{4}{3}}=\frac{-1}{7}$$

Question: $A=\{a,b,c,d\}, B=\{1,2,3,4,5\}, f:A\to B$ is one-one. Find the probability that $f(a)+2f(b)-f(c)=f(d)$.

Answer: $\frac{1}{20}$

Solution:

$$f(b)=\frac{f(c)+f(d)-f(a)}{2}$$

a	b	c	d
1	3	2	5
5	1	3	4
4	2	3	5
5	1	4	3
1	3	5	2
4	2	5	3

Favourable cases = 6

Total cases $=5\times4\times3\times2$

$\therefore$ Required probability $=\frac{6}{5!}=\frac{1}{20}$

Question: Find the coefficient of term independent of x in $\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$.

Answer: $\frac{33}{200}$

Solution:

General term of $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ is:

$$T_{r+1}={}^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(-\frac{1}{5x^2}\right)^r$$

$$={}^{11}C_r(-1)^r\cdot\frac{5^{11-2r}}{22^{11-r}}\cdot x^{33-5r}$$

Coefficient of term independent of $x=$ Coefficient of x^0 in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

-Coefficient of x^{-2} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}+3\times$ Coefficient of x^{-3} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

$$=0-{}^{11}C_7(-1)^7\cdot\frac{5^{-3}}{2^4}+0=\frac{330}{5^3\cdot 2^4}=\frac{33}{200}$$

Question: If vertex of parabola is (2, -1) and equation of its directrix is $4x-3y=21$, then the length of latus rectum is

Answer: 8.00

Solution:

Length if Latus rectum = 4 (perpendicular distance of (2, -1) from $4x-3y-21=0$)

$$=4\frac{|8+3-21|}{5}=8$$

Question: Find the equation of plane passing through the points $(2,-1,0)$ and perpendicular to planes $2x-3y+z=0$ and $2x-y-3z=0$.

Answer: 6.00

Solution:

Let DRs be a,b,c

Now $2a-3b+c=0$

$2a-b-3c=0$

$\frac{a}{5}=\frac{b}{4}=\frac{c}{2}$

Equation of required plane is

$5(x-2)+4(y+1)+2(z-0)=0$

$\Rightarrow 5x+4y+2z=6$

Question: In an infinite GP, $a=n^2, r=\frac{1}{(n+1)^2}, \frac{1}{26}+\sum_{n=0}^{50}\left(S_n-\frac{2}{n+1}-n-1\right)=?$, where S_n is sum of given GP.

Answer: 41652.00

Solution:

$$a=n^2, r=\frac{1}{(n+1)^2};\ s_n=\frac{a}{1-r}=\frac{n(n+1)^2}{(n+2)}$$

$$\Rightarrow \frac{1}{26}+\sum_{n=0}^{50}\left[\frac{n(n+1)^2}{(n+2)}+\left(\frac{2}{n+1}\right)-(n+1)\right]$$

$$\Rightarrow \frac{1}{26}+\sum_{n=0}^{50}\left[\left(n^2-n-1\right)+\left(\frac{n}{n+2}\right)+\left(\frac{2}{n+1}\right)\right]$$

$$\Rightarrow \frac{1}{26}+\sum_{n=0}^{50}\left[n^2-n-2\left(\frac{1}{n+2}-\frac{1}{n+1}\right)\right]$$

$$\Rightarrow \frac{1}{26}+\left[\frac{50\cdot 51\cdot 101}{6}-\frac{50\cdot 51}{2}-2\sum_{n=0}^{50}\left(\frac{1}{n+2}-\frac{1}{n+1}\right)\right]$$

$$\Rightarrow \frac{1}{26}+\left[41650-2\left(\frac{1}{52}-1\right)\right]=41652$$