

# PEA

## 2019

1. Robinson Crusoe will live this period (period 1) and the next period (period 2) as the only inhabitant of his island completely isolated from the rest of the world. His only income is a crop of 100 coconuts that he harvests at the beginning of each period. Coconuts not consumed in the current period spoil at the rate of 20% per period. Crusoe's preference over consumption in period 1 ( $c_1$ ) and consumption in period 2 ( $c_2$ ) is given by the utility function  $u(c_1, c_2) = \min\{5c_1, 6c_2\}$ . Crusoe's utility maximizing consumption choice is given by

(a)  $c_1 = \frac{200 \times 6}{11}$ ,  $c_2 = \frac{200 \times 5}{11}$ .

(b)  $c_1 = 90$ ,  $c_2 = 108$ .

(c)  $c_1 = 100$ ,  $c_2 = 100$ .

(d) none of the above.

2. The domestic supply and demand equations for a commodity in a country are as follows: Supply:  $P = 50 + Q$ , Demand:  $P = 200 - 2Q$ , where  $P$  is the price in rupees per kilogram and  $Q$  is the quantity in thousands of kilograms. The country is a small producer in the world market where the price (which will not be affected by anything done by this country) is Rs. 60 per kilogram. The government of this country introduces a "Permit Policy" which works as follows. The government issues a fixed number of Permits – each Permit allows its owner to sell exactly 100 kilograms of the commodity in this country's market. An exporter from a foreign country cannot sell this commodity in this country unless she purchases such a Permit. Suppose the government issues 300 Permits. What is the *maximum* price an exporter is willing to pay for a Permit?

(a) Rs. 3000.

- (b) Rs. 2000.
- (c) Rs. 1500.
- (d) Rs. 1000.
3. SeaTel provides cellular phone service in Delhi and has some monopoly power in the sense that it has its captive customer base with each customer's weekly demand being given by:  $Q = 60 - P$ , where  $Q$  denotes hours of cell phone calls per week and  $P$  is the price per hour. SeaTel's total cost of providing cell phone service is given by  $C = 20Q$ , so that the marginal cost is  $MC = 20$ . Suppose SeaTel offers a "Call-As-Much-As-You-Wish" deal: it charges only a flat *weekly access fee*, and once a customer pays the flat access fee, he/she can call as much as he/she wishes without paying any extra usage fee per hour. The *weekly access fee* that SeaTel should charge to maximize its profit is given by
- (a) 1800.
- (b) 1200.
- (c) 800.
- (d) 40.
4. A bus stop has to be located on the interval  $[0, 1]$ . There are three individuals located at points 0.2, 0.3 and 0.9 on the interval. If the bus stop is located at point  $x$ , then the utility of an individual located at  $y$  is  $-|y - x|$ , that is, the negative of the distance between the bus stop and the individual's location. A relocation of the bus stop is said to be *Pareto improving* if at least one individual is better off and no individual is worse off from the relocation. A location of the bus stop is said to be *Pareto efficient* if there does not exist any Pareto improving relocation. Then

- (a) 0.5 is the only Pareto efficient location.
- (b)  $\frac{0.2+0.3+0.9}{3}$  is the only Pareto efficient location.
- (c) Median of 0.2, 0.3 and 0.9 is the only Pareto efficient location.
- (d) none of the above.
5. Consider three goods: (a) cable television, (b) a fish in international waters, and (c) a burger. Also consider four descriptions of the goods: (A) non-rival and non-excludable, (B) rival and excludable, (C) non-rival and excludable, and (D) rival and non-excludable. In what follows we match goods to possible descriptions. Choose the correct match.
- (a) (a)-(A), (b)-(C), (c)-(B).
- (b) (a)-(C), (b)-(D), (c)-(A).
- (c) (a)-(C), (b)-(B), (c)-(A).
- (d) (a)-(C), (b)-(D), (c)-(B).
6. Consider an economy consisting of three individuals – 1, 2 and 3, two goods – A and B, and a single monopoly firm that can produce both goods at zero cost. Each individual would like to buy exactly 1 unit of the goods A and B, if at all. An individual's *valuation* of a good is defined as the maximum amount she is willing to pay for one unit of the good. Individual 1's valuation of good A is Rs. 10 and that of good B is Rs. 1. Individual 2's valuation of good A is Rs. 1, and that of good B is Rs. 10. Individual 3's valuation is Rs. 7 for good A, and Rs. 7 good B. The firm can charge a single price  $p_A$  for good A, a single price  $p_B$  for good B, and a bundled price  $p_{AB}$  such that if an individual pays  $p_{AB}$  then she gets the bundle consisting of one unit each of

goods A and B. If the monopolist sets  $p_A$ ,  $p_B$  and  $p_{AB}$  to maximize its profit then

- (a)  $p_A = 11, p_B = 11, p_{AB} = 11$ .
- (b)  $p_A = 11, p_B = 11, p_{AB} = 14$ .
- (c)  $p_A = 10, p_B = 10, p_{AB} = 11$ .
- (d) none of the above.

7. Consider a Bertrand duopoly with two firms, 1 and 2. Both firms produce the same good that has a market demand function  $p = 10 - q$ . The market is equally shared in case the firms charge the same price, otherwise the lower priced firm gets the entire demand. A firm must satisfy all the demand coming to it. The cost function of firm 1 is  $3q_1$ , that of firm 2 is  $2q_2$ . Suppose prices vary along the following grid,  $\{0, 0.1, 0.2, \dots\}$ . The Bertrand equilibrium is given by

- (a)  $p_1 = 2, p_2 = 2$ .
- (b)  $p_1 = 3, p_2 = 2$ .
- (c)  $p_1 = 3, p_2 = 2.9$ .
- (d)  $p_1 = 3, p_2 = 3$ .

8. Consider a monopolist with a market demand function  $p = 20 - q$ . It is a multi-plant monopolist with two plants, plant 1 and plant 2, where the plant specific cost function of plant  $i$ ,  $i = 1, 2$ , is

$$c_i(q_i) = \begin{cases} 2 + 4q_i, & \text{if } q_i > 0, \\ 0, & \text{otherwise.} \end{cases}$$

The optimal monopoly profit is given by

- (a) 60.

- (b) 64.
- (c) 68.
- (d) 62.
9. Consider a closed economy in which an individual's labour supply ( $L$ ) to firms is determined by the amount which maximizes her utility function  $u(C, L) = C^\alpha(1 - L)^\beta$ , where  $\alpha, \beta > 0$ ,  $\alpha + \beta < 1$ , and  $C$  is consumption expenditure which is taken to be equal to wage income ( $wL$ ). Then
- (a) labour supply does not depend on the wage rate  $w$ .
- (b) labour supply is directly proportional to the wage rate  $w$ .
- (c) labour supply is inversely proportional to the wage rate  $w$ .
- (d) more information is needed to derive the labour supply.
10. In the scenario described in Question 9, assume that the economy is Keynesian, that is, investment expenditure ( $I$ ) is autonomous and output ( $Y$ ) is determined by aggregate demand,  $Y = C + I$ . The aggregate production function is given by  $Y = AL^\theta$ , where  $A > 0$  is a productivity parameter and  $0 < \theta < 1$ . [Note that the firm's employment of labour is obtained by equating the marginal product of labour to  $w$ .] Then the marginal propensity to consume is
- (a)  $\frac{\alpha + \beta}{\theta}$ .
- (b)  $\frac{\beta}{\theta}$ .
- (c)  $\alpha$ .
- (d)  $\theta$ .
11. Consider a Solow growth model (in continuous time) with a production function with labour augmenting technological change,

$Y_t = F(K_t, A_t L_t)$ , where  $Y_t$  denotes output,  $K_t$  denotes the capital stock,  $A_t$  denotes the level of total factor productivity (TFP), and  $L_t$  denotes the stock of the labour force. Assume that  $L_t$  grows at the rate  $n > 0$  and  $A_t$  grows at the rate  $g > 0$ , that is,  $\frac{\dot{L}}{L} = n$  and  $\frac{\dot{A}}{A} = g$ , and the capital accumulation equation is given by  $\dot{K} = sY_t - \delta K_t$ , where  $s \in [0, 1]$  is the exogenous savings rate, and  $\delta \in [0, 1]$  is the depreciation rate of capital. [Note that for any variable  $x$ ,  $\dot{x}$  denotes  $\frac{dx}{dt}$ .] Define capital in efficiency units to be  $Z \equiv \frac{K}{AL}$ . Then the expression for  $\frac{\dot{Z}}{Z}$  is given by

- (a)  $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - (n + g)$ .
- (b)  $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - (\delta + n + g)$ .
- (c)  $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z}$ .
- (d)  $\frac{\dot{Z}}{Z} = \frac{sf(Z)}{Z} - n$ .

12. In the Solow growth model described in Question 11, the growth rate of  $Y$  at the steady state is given by

- (a)  $n + g$ .
- (b)  $\delta + n + g$ .
- (c) zero.
- (d)  $n$ .

13. Consider an IS-LM model where the IS curve is represented by  $0.25Y = 500 + G - i$ , and money demand function is given by  $\frac{M}{P} = \frac{2Y}{e^i}$ . The notations are standard:  $Y$  denotes output,  $G$  denotes government expenditure,  $i$  denotes the interest rate,  $P$  is the price level and  $e$  is the exponential. Suppose the government wants to increase spending and therefore the central bank decides to change the money supply accordingly such that the interest

rate remains the same in the short run. Then the change in money supply satisfies the following condition:

(a)  $\frac{dM}{dG} = e$ .

(b)  $\frac{dM}{dG} = \frac{Ye}{M}$ .

(c)  $\frac{dM}{dG} = \frac{Y}{M}$ .

(d)  $\frac{dM}{dG} = \frac{4M}{Y}$ .

14. An agent lives for two periods. Her utility from consumption in period 1 ( $c_1$ ) and consumption in period 2 ( $c_2$ ) is given by  $u(c_1, c_2) = \log(c_1) + \beta \log(c_2)$ , where  $0 < \beta < 1$  is the discount factor reflecting her time preference. The agent earns incomes  $w_1$  in period 1 and  $w_2$  in period 2. The rate of interest is  $r > 0$ . The agent chooses  $c_1$  and  $c_2$  so as to maximize  $u(c_1, c_2)$  subject to her budget constraint. Consider a *temporary* increase in income where  $w_1$  increases but the agent does not change her expectations about  $w_2$ . Then the marginal propensity to consume of present consumption with respect to  $w_1$ ,  $\frac{dc_1}{dw_1}$ , is given by

(a)  $\frac{1}{1+\beta} \left(1 + \frac{1}{1+r}\right)$ .

(b)  $\left(\frac{1}{1+\beta}\right) \left(\frac{1}{1+r}\right)$ .

(c)  $\frac{1}{1+\beta}$ .

(d) 1.

15. In the scenario described in Question 14, consider a *permanent* increase in income where  $w_1$  increases and the agent expects that  $w_2$  will also increase by the same amount. Then  $\frac{dc_1}{dw_1}$  is given by

(a)  $\frac{1}{1+\beta} \left(1 + \frac{1}{1+r}\right)$ .

(b)  $\left(\frac{1}{1+\beta}\right) \left(\frac{1}{1+r}\right)$ .

- (c)  $\frac{1}{1+\beta}$ .
- (d) 1.
16. For what values of  $a$  are the vectors  $(0, 1, a), (a, 1, 0), (1, a, 1)$  in  $\mathbb{R}^3$  linearly dependent?
- (a) 0.
- (b) 1.
- (c) 2.
- (d)  $\sqrt{2}$ .
17. Which of the following set of vectors form a basis of  $\mathbb{R}^2$ ?
- (a)  $\{(2, 1)\}$ .
- (b)  $\{(1, 1), (2, 2)\}$ .
- (c)  $\{(1, 1), (1, 2), (2, 1)\}$ .
- (d)  $\{(1, 1), (2, 3)\}$ .
18. If a candidate is good he is selected in MSQE examination with probability 0.9. If a candidate is bad he is selected in MSQE examination with probability 0.2. Suppose every candidate is equally likely to be good or bad. If you meet a candidate who is selected in the MSQE examination, what is the probability that he will be good?
- (a)  $\frac{11}{20}$ .
- (b)  $\frac{9}{10}$ .
- (c)  $\frac{9}{11}$ .
- (d)  $\frac{11}{12}$ .



19. Let  $S_1 = \{2, 3, 4, \dots, 9\}$ . First, an integer  $s_1$  is drawn uniformly at random from  $S_1$ . Then  $s_1$  and all its factors are removed from  $S_1$ . Let the new set be  $S_2$ . Next an integer  $s_2$  is drawn uniformly at random from  $S_2$ . Then  $s_2$  and all its factors are removed from  $S_2$ . Let the new set be  $S_3$ . Finally, an integer  $s_3$  is drawn uniformly at random from  $S_3$ . What is the probability that  $s_1 = 2, s_2 = 3, s_3 = 5$ ?

(a)  $\frac{1}{8}$ .

(b)  $\frac{1}{64}$ .

(c)  $\frac{1}{16}$ .

(d)  $\frac{1}{72}$ .

20. Mr. A and B are independently tossing a coin. Their coins have a probability 0.25 of coming HEAD. After each of them tossed the coin twice, we see a total of 2 HEADS. What is the probability that Mr. A had exactly one HEAD?

(a)  $\frac{2}{3}$ .

(b)  $\frac{1}{2}$ .

(c)  $\frac{1}{4}$ .

(d)  $\frac{1}{3}$ .

21. Consider the following function  $f : \mathbb{R} \rightarrow \mathbb{R}$ .

$$f(x) = \begin{cases} x & \text{if } x \leq e \\ x(\log_e x) & \text{if } x > e. \end{cases}$$

Which of the following is true for  $f$ ?

(a)  $f$  is not continuous at  $e$ .

- (b)  $f$  is not differentiable at  $e$ .
- (c)  $f$  is neither continuous nor differentiable at  $e$ .
- (d)  $f$  is continuous and differentiable at  $e$ .
22. Let  $f : [-1, 1] \rightarrow \mathbb{R}$  be a continuous and weakly increasing function such that  $\int_{-1}^1 f(x)dx = 2 \int_{-1}^1 f(-x)dx$ . Suppose  $f(-1) = 0$ , then  $f(1)$  is
- (a) 0.
- (b) 1.
- (c)  $\frac{1}{2}$ .
- (d) none of the above.
23. Let  $A \subseteq \mathbb{R}$  and  $f : A \rightarrow \mathbb{R}$  be a twice continuously differentiable function. Let  $x^* \in A$  be such that  $\frac{\partial f}{\partial x}(x^*) = 0$ . Consider the following two statements: (i) if  $\frac{\partial^2 f}{\partial^2 x}(x^*) \leq 0$ , then  $x^*$  is a point of local maximum of  $f$ ; (ii) if  $x^*$  is a point of local maximum of  $f$ , then  $\frac{\partial^2 f}{\partial^2 x}(x^*) < 0$ . Which of the following is true?
- (a) both (i) and (ii) are correct.
- (b) both (i) and (ii) are incorrect.
- (c) (i) is correct but (ii) is incorrect.
- (d) (ii) is correct but (i) is incorrect.
24. Consider the function  $f(x) = e^x$  for all  $x \in \mathbb{R}$ . Which of the following is true?
- (a)  $f$  is quasi-convex.
- (b)  $f$  is quasi-concave.
- (c)  $f$  is neither quasi-convex nor quasi-concave.

(d)  $f$  is both quasi-convex and quasi-concave.

25. Consider the following matrix  $A$ .

$$A = \begin{bmatrix} x & 0 & k \\ 1 & x & k-3 \\ 0 & 1 & 1 \end{bmatrix}$$

Suppose determinant of  $A$  is zero for two distinct real values of  $x$ . What is the least positive integer value of  $k$ ?

(a) 1.

(b) 9.

(c) 10.

(d) 8.

26. Define the following function on the set of all positive integers.

$$f(n) = \begin{cases} 2 \times 4 \times \dots \times (n-3) \times (n-1) & \text{if } n \text{ is odd} \\ 1 \times 3 \times \dots \times (n-3) \times (n-1) & \text{if } n \text{ is even.} \end{cases}$$

What is the value of  $f(n+2)f(n+1)$ ?

(a)  $n!$ .

(b)  $(n+1)!$ .

(c)  $(n+2)!$ .

(d)  $(n+2)(n!)$ .

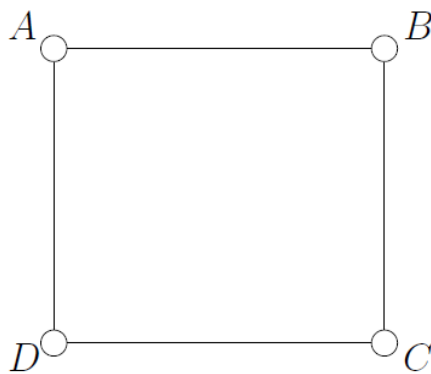
27. The sequence  $\{x_n\}_{n \geq 0}$  is defined as follows. We set  $x_0 = 1$  and  $x_n = \sum_{j=0}^{n-1} x_j$  for each integer  $n \geq 1$ . Then the value of the expression  $\sum_{j=0}^{\infty} \frac{1}{x_j}$  is equal to

- (a)  $\infty$ .
- (b) 2.
- (c) 3.
- (d)  $\frac{7}{4}$ .

28. For what values of  $p$  does the following quadratic equation have more than two solutions (variable in this equation is  $x$ )?

$$(p^2 - 16)x^2 - (p^2 - 4p)x + (p^2 - 5p + 4) = 0$$

- (a) No such value of  $p$  exists.
  - (b)  $-4$  and  $4$ .
  - (c)  $1$  and  $4$ .
  - (d)  $4$ .
29. Consider the square with vertices  $A, B, C, D$  as shown in the following figure. Call a pair of vertices in the square adjacent if they are connected by an edge in the figure. You have four colours: RED, BLUE, GREEN, YELLOW. How many ways can you colour the vertices  $A, B, C, D$  such that no adjacent vertices share the same colour?



- (a) 84.
  - (b) 24.
  - (c) 72.
  - (d) 108.
30. Two players  $P_1$  and  $P_2$  are playing a game which involves filling the entries of an  $n \times n$  matrix, where  $n \geq 2$  is an **even** integer. Starting with  $P_1$ , each player takes turn to fill an unfilled entry of the matrix with a real number. The game ends when all entries are filled. Player  $P_1$  wins if the determinant of the final matrix is non-zero. Else, player  $P_2$  wins. A player  $i \in \{1, 2\}$  has a **winning strategy** if irrespective of what the other player does,  $i$  wins by following this strategy. Which of the following is true?
- (a) Player 1 has a winning strategy.
  - (b) Player 2 has a winning strategy.
  - (c) No player has a winning strategy.
  - (d) None of the above.