# COURSE STRUCTURE
# &
# SYLLABI

for two-year postgraduate programme

**M.A./M.Sc.(Mathematics)**

Under

**Choice Based Credit System (CBCS)**

DEPARTMENT OF MATHEMATICS
ALIGARH MUSLIM UNIVERSITY
ALIGARH-202002

# STRUCTURE OF MODEL CURRICULUM

**Credits for each course: 4. Total Credits: 96. Periods/Week for each course: 5**
**Maximum Marks assigned for each course: 100 (Sessional: 30 & Exams: 70)**

## FIRST SEMESTER

| S. No. | Course No. | Course Title |
|---|---|---|
| 1. | MMM-1006 | Ordinary Differential Equations |
| 2. | MMM-1008 | Advanced Real Analysis |
| 3. | MMM-1009 | General Topology |
| 4. | MMM-1010 | Advanced Complex Analysis |
| 5. | MMM-1011 | Functional Analysis |
| 6. | MMM-1013 | Advanced Linear Algebra |

## SECOND SEMESTER

| S. No. | Course No. | Course Title |
|---|---|---|
| 1. | MMM-2002 | Measure Theory |
| 2. | MMM-2005 | Partial Differential Equations |
| 3. | MMM-2008 | Algebraic Topology |
| 4. | MMM-2009 | Advanced Functional Analysis |
| 5. | MMM-2010 | Differentiable Manifolds |
| 6. | MMM-2011 | Advanced Theory of Groups and Fields |

## THIRD SEMESTER

| S. No. | Course No. | Course Title |
|---|---|---|
| 1. | MMM-3003 | Mechanics |
| 2. | MMM-3005 | Nonlinear Functional Analysis |
| 3. | MMM-3006 | Advanced Ring Theory |
| 4. | MMM-3007 | Riemannian Geometry and Submanifolds |
| 5. & 6. | **Elective (Opt any TWO)** | |
| | MMM-3016 | Wavelet Analysis |
| | MMM-3018 | Theory of Semigroups |
| | MMM-3019 | Topological Vector Spaces |
| | MMM-3021 | Homological Algebra and Module Theory |

## FOURTH SEMESTER

| S. No. | Course No. | Course Title |
|---|---|---|
| 1,2&3. | **Elective (Opt any THREE)** | |
| | MMM-4016 | Structures on Manifolds |
| | MMM-4017 | Special Functions and Lie Theory |
| | MMM-4018 | Non-commutative Rings |
| | MMM-4020 | Variational Analysis and Optimization |
| | MMM-4021 | Advanced Discrete Mathematics |
| 4. | --- | Open Elective |
| 5. | MMM-4071 | Project |
| 6. | MMM-4072 | Viva-Voce |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY

## Syllabus of M.A./M.Sc. I Semester approved in BOS: 01-08-2019

| | |
|---|---|
| **Course Title** | **Ordinary Differential Equations** |
| **Course Number** | **MMM-1006** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **A course of Ordinary Differential Equations (UG Level)** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The goal is to understand the concepts relating to ODE and then applying those concepts to solve the equations and find the solutions. |
| **Course Outcomes** | After successful completion of the course students will be able to :<br>• Acquire understanding of ODE's and solve them.<br>• Find Laplace transforms and orthogonal trajectories for a family of curves.<br>• Will be able to apply the named theorems to find the solutions to the problems.<br>• Solve system of first order differential equations and non homogeneous linear system of equations. |

| **Contents of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Theory of Homogeneous and Nonhomogeneous L.D.E.**<br>Initial value problem, Boundary value problem, Linear differential equations with constant as well as variable coefficient, Linear dependence and independence of solutions, Wronskian, Variation of parameter, Method of undetermined coefficients, Reduction of the order. | **12** |
| **UNIT II: First Order Initial Value Problems**<br>Method of successive approximation, Lipschitz's condition, Gronwall's inequality, Picard's theorem, Dependence of solution on initial conditions and on function, Existence and uniqueness of solution for a system of linear equations. | **12** |
| **UNIT III: Second Order Boundary Value Problems**<br>Orthogonal set of functions, Strum-Liouville problem, Legendre and Bessel functions and their orthogonal properties, Green's function and its applications to boundary value problems, Some oscillation theorems such as: Strum theorem, Strum comparison theorem and related results. | **12** |
| **UNIT IV: System of Linear Differential Equations**<br>System of first order differential equations, Fundamental matrix, Non-homogeneous linear system, Linear system with constant as well as periodic coefficients. | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ Reference Books** | 1. *E. A. Coddington: An introduction to Ordinary Differential Equations, Prentice Hall of India, New Delhi, 1991.<br>2. *S. C. Deo, Y. Lakshminathan and V. Raghavendra: Text Book of Ordinary Differential Equation, 2nd Ed, Tata McGraw Hill, New Delhi (Chapters IV, VII and VIII).<br>3. P. Haitman: Ordinary Differential Equations, Wiley, New York, 1964.<br>4. E. A. Coddington and H. Davinson: Theory of Ordinary Differential Equations, McGraw Hill, NY, 1955.<br>5. S. L. Ross: Differential Equations, Blaisdell Publishing Company, Londan, 1964. |

**DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY**

**Syllabus of M.A./M.Sc. I Semester implemented w.e.f. Session 2020-21**

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|---|---|
| **Course Title** | **Advanced Real Analysis** |
| **Course Number** | **MMM-1008** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **Courses of Real Analysis (UG Level)** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | To understand the uniform convergence, sequence and series of real valued functions, the properties of certain real-valued functions, the generalization of Riemann integration of bounded functions on a closed and bounded interval and its extension to the cases where either the interval of integration is infinite, or the integrand has infinite limits at a finite number of points on the interval of integration. |
| **Course Outcomes** | The course will enable the students to learn about:<br>• The valid situations for the inter-changeability of differentiability and integrability with infinite sum, and approximation of transcendental functions in terms of power series.<br>• Some special real functions and their properties.<br>• Some of the families and properties of Riemann-Stieltjes integrable functions, and the applications of the fundamental theorems of integration. |

| **Contents of syllabus** | **No. of Lectures** |
|---|---|
| **Unit I: Properties of Real Functions**<br>Characterizations of continuous functions via open sets and closed sets, Dini's derivative, Applications of Taylor's theorem, Bounded functions, Monotone functions, Functions of bounded variation and their properties, Variation function, Jordon theorem, Absolute continuous functions and their properties, Exponential, Logarithmic, Generalized power, Trigonometric, Inverse trigonometric functions and their properties. | **12** |
| **Unit II: Sequence of Functions and Applications to Approximation Theorems**<br>Pointwise and uniform convergence of sequence of functions, Uniform norm on a set of bounded functions, Cauchy's Criterion for uniform convergence, Interchange of the limit and continuity, interchange of the limit and derivative and interchange of the limit and integral of a sequence of functions, Bounded convergence theorem, Dini's Theorem, Tietze's Extension Theorem, Weierstrass Approximation Theorem, Stone-Weierstrass Theorem. | **12** |
| **Unit III: Series of Functions**<br>Pointwise and uniform convergence of series of functions, Cauchy's Criterion for uniform convergence, Weierstrass M-test, Abel's test, Dirichlet's test for uniform convergence, Continuity, Derivability and integrability of the sum function of a series of functions, Uniform convergence of Fourier series, power series, Taylor series and binomial series. | **12** |
| **Unit IV: Riemann-Stieltjes Integrals**<br>Concept and properties of Riemann-Stieltjes integral, Integration by parts, Change of variables, Concept of Riemann integrals, Reduction of Riemann-Stieltjes integration into Riemann integration, Riemann condition of integrability, Integral as a limit of sums, Existence of Reimann-Stieltjes integrals, Mean value theorems, Second fundamental theorem of integral calculus, Interchanging the order of integration. | **12** |
| **Total no of lectures** | **48** |

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|---|---|
| **Text Books*/ Reference Books** | 1. *T. M. Apostol: Mathematical Analysis, Addison-Wesley Series in Mathematics, 1974.<br>2. *Rudin: Principles of Mathematical Analysis, Third Edition, McGraw Hill, New York, 3rd Ed, 1976.<br>3. *R. G. Bartle and D. R. Sherbert: Introduction to Real Analysis, John Wiley and Sons, Singapore, 3rd Ed, 2003.<br>4. S. C. Malik and Savita Arora: Mathematical Analysis, New Age International, 2017.<br>5. D. Somasundaram: A Second Course in Mathematical Analysis, Narosa Publishing House, 2010.<br>6. H. L. Royden: Real Analysis, Macmillan, 1993. |

# DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. I Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **General Topology** |
| **Course Number** | **MMM-1009** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **UG Level Courses of Real Analysis & Metric Space** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | To introduce basic concepts of point set topology, basis and subbasis for a topology and order topology. Further, to study continuity, homeomorphisms, open and closed maps, product and box topologies and introduce notions of connectedness, path connectedness, local connectedness, local path connectedness, countability axioms and compactness of spaces. |
| **Course Outcomes** | After studying this course the student will be able to<br>• determine interior, closure, boundary, limit points of subsets and basis and subbasis of topological spaces.<br>• check whether a collection of subsets is a basis for a given topological spaces or not, and determine the topology generated by a given basis.<br>• identify the continuous maps between two spaces and maps from a space into product space and determine common topological property of given two spaces.<br>• determine the connectedness and path connectedness of the product of an arbitrary family of spaces.<br>• find Hausdorff spaces using the concept of net in topological spaces and learn about first and second countable spaces, separable and Lindelöf spaces.<br>• learn Bolzano-Weierstrass property of a space and prove Tychonoff theorem |

| Contents of Syllabus | No. of Lectures |
|---|---|
| **UNIT I: Basic Concepts and Point Set Topology**<br>Definitions of topology and topological spaces, Examples of topology including discrete topology, indiscrete topology, standard topology on IR, lower limit and upper limit topology, co-finite topology and co-countable topology, Topology induced by a metric, Basis for topology, Subspace topology, K-topology, Order Topology, Product Topology on X×Y, Topology generated by the sub-basis, Closed sets and limit points, Neighbourhoods, Interior, exterior and boundary points, Derived sets, Hausdorff spaces. | 12 |
| **UNIT II: Continuity, Connectedness and Compactness**<br>Continuous functions, Pasting lemma, Homeomorphisms, Convergence in topological spaces, Connected spaces, Connected sets in the real line, Intermediate value theorem, Components and Local connectedness, Path connected, Path components, Locally path connected spaces, Properties of Continuous functions on Connected sets, Compact spaces and their basic properties, Finite intersection property, Compact subspaces of the real line, Extreme value theorem, Lebesgue number, Uniform continuity, Limit point compactness, Sequential compactness, Local compactness, Properties of continuous functions on compact sets. | 12 |
| **UNIT III: Countability and Separation Axioms**<br>First and second countable spaces, Lindelof spaces, T-1, T-2 (Hausdorff), T-3 (Regular), T-4 (Normal), T-3.5 (Completely regular) spaces and their characterizations and basic properties, Urysohn's lemma, Tietze extension theorem. | 12 |
| **UNIT IV: Product Spaces and Quotient Spaces**<br>Product topology (finite and infinite number of spaces), Tychonoff product, Projection maps, Stone Cech Compactification, Comparison of the Box and Product topologies, Quotient topology, Quotient (Identification) spaces with some examples. | 12 |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Book*/ References Books** | **1.** *James R. Munkres: Topology, A first course, Prentice Hall of India Pvt. Ltd., New Delhi, 2000.<br>**2.** Martin D. Crossley.: Essential Topology, Springer Undergraduate Mathematics Series.<br>**3.** M. A. Armstrong: Basic Topology, Undergraduate Text in Mathematics, 1983.<br>**4.** Mohammed Hichem Mortad: Introductory Topology, Second Edition, World Scientific. |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY

## Syllabus of M.A./M.Sc. I Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Advanced Complex Analysis** |
| **Course Number** | **MMM-1010** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **A course of Complex Analysis (UG Level)** |
| **Contact Hours** | **4 Lectures+1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The primary aim of this course is to understand the theory of complex functions, analytic functions, Meromorphic functions, entire functions, Conformal mappings and Möbius transformations. Particular emphasis has been laid on Cauchy's theorems, Cauchy's integral formula, series expansions, calculation of residues, singularities, evaluation of contour integration, maximum principle, Schwarz' lemma, Liouville's theorem, Argument principle, Rouche's theorem and Mittag-Leffler expansion theorem. |
| **Course Outcomes** | **After successful completion of the course students will be able to:**<br>• Understand the importance of complex variables in analysis.<br>• Apply the appropriate techniques of complex integration for establishing theoretical results and for solving related problems.<br>• Understand the concepts and results related to singularities and residues and their use in integration.<br>• Understand the general theory of conformal mappings, Möbius transformations and their applications. |

| **Contents of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Complex Integration**<br>Curves in the complex plane, Properties of complex line integrals, Fundamental theorem of line integrals (or contour integration), Simplest version of Cauchy's theorem, Cauchy-Goursat theorem, Symmetric, starlike, convex and simply connected domains, Cauchy's theorem for a disk, Cauchy's integral theorem, Index of a closed curve, Advanced versions of Cauchy integral formula and applications, Cauchy's estimate, Morera's theorem (Revisited), Riemann's removability theorem, Examples. | **12** |
| **UNIT II: Series Expansions and Singularities**<br>Convergence of sequences and series of functions, Weierstrass' M-test, Power series as an analytic function, Root test, Ratio test, Uniqueness theorem for power series, Zeros of analytic functions; Identity theorem and related results, Maximum/Minimum modulus principles and theorems, Schwarz' lemma and its consequences, Advanced versions of Liouville's theorem, Fundamental theorem of algebra, Isolated and non-isolated singularities, Removable singularities, Poles, Characterization of singularities through Laurent's series, Examples. | **12** |
| **UNIT III: Calculus of Residues**<br>Residue at a finite point, Results for computing residues, Residue at the point at infinity, Cauchy's residue theorem, Residue formula, Meromorphic functions, Number of zeros and poles, Argument principle, Evaluation of integrals, Rouche's theorem, Mittag-Leffer expansion theorem, Examples. | **12** |
| **UNIT IV: Conformal Mappings and Transformation**<br>Introduction and preliminaries, Conformal mappings, Special types of transformations, Basic properties of Möbius maps, Images of circles and lines under Mobius maps, Fixed points, Characterizations of Möbius maps in terms of their fixed points, Triples to triples under Möbius maps, Cross-ratio and its invariance property, Mappings of half-planes onto disks, Inverse function theorem and related results, Examples. | **12** |
| **Total No. of Lectures** | **48** |

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| **Text Books*/ Reference Books** | 1. *Lars V. Ahlfors: Complex Analysis, McGraw-Hill Book Company Inc, New York, 1986.<br>2. John B. Conway: Functions of One Complex Variable, 2nd Ed, Springer International Student, Narosa Publishing House, 1980.<br>3. S. Ponnusamy: Foundations of Complex Analysis, 2nd Ed, Narosa Publishing House, 2005 |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY

## Syllabus of M.A./M.Sc. I Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Functional Analysis** |
| **Course Number** | **MMM-1011** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **UG Level Courses of Real Analysis & Metric Space** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | To familiarize with the basic tools of Functional Analysis involving normed spaces, Banach spaces and Hilbert spaces, their properties dependent on the dimension and the bounded linear operators from one space to another**.** |
| **Course Outcomes** | After studying this course the student will be able to<br>• verify the requirements of a norm, completeness with respect to a norm, relation between compactness and dimension of a space, check boundedness of a linear operator and relate to continuity, convergence of operators by using a suitable norm, compute the dual spaces.<br>• distinguish between Banach spaces and Hilbert spaces, decompose a Hilbert space in terms of orthogonal complements, check totality of orthonormal sets and sequences, represent a bounded linear functional in terms of inner product.<br>• extend a linear functional under suitable conditions, check reflexivity of a space, ability to apply uniform boundedness theorem, open mapping theorem and closed graph theorem, check the convergence of operators and functional and weak and strong convergence of sequences. |

| **Contact of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Normed Spaces**<br>Normed spaces, Banach spaces and their examples, Examples of incomplete normed spaces, Subspace of normed spaces, Isometry on normed spaces, Completion of normed linear spaces, Quotient spaces, Product spaces, Schauder basis, Infinite series in normed space: convergence and absolute convergence, Finite dimensional normed spaces, Equivalent norms, Compactness, Riesz Lemma, Denseness and separability properties. | **12** |
| **UNIT II: Operators on Banach Spaces**<br>Bounded linear operators and bounded linear functionals with their norms and properties, Unbounded linear operators, Space of bounded linear operators, Dual basis, Algebraic and topological duals and relevant results, Duals of some standard normed spaces, Second duals and canonical embedding, Reflexive normed spaces and their properties, Separability of dual space. | **12** |
| **UNIT III**: **Hilbert Spaces**<br>Inner product spaces and examples, Parallelogram law, Polarization identity and related results, Schwartz and triangle inequalities, Separability and reflexivity of Hilbert spaces, Orthonormal sets and sequences, Bessel inequality, Total orthonormal sets and sequences, Parseval relation, Bounded linear functionals on Hilbert spaces: Riesz representation theorem. | **12** |
| **UNIT IV: Fundamental Theorems**<br>Hahn-Banach theorem and its extended forms, Pointwise and uniform boundedness, Uniform boundedness principle and its applications, Weak convergence of sequences and weak topology in normed space, Open and closed maps, Graph of linear operators and closedness property, Open mapping and closed graph theorems, their consequences and applications. | **12** |
| **Total** | **48** |

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| **Text Books*/ Reference Books** | **1.** *E. Kreyszig: Introductory Functional Analysis with Applications, John Willey, 1978.<br>**2.** H. Siddiqi: Applied functional Analysis: Numerical Methods, Wavelet Methods, and Image Processing, CRC Press, 2003.<br>**3.** P. K. Jain and O. P. Ahuja: Functional Analysis, New Age International Publishers, 2nd Ed, 2010.<br>**4.** W. Rudin: Functional Analysis, Mc Graw Hill Education, 2nd Ed, 1991.<br>**5.** B.V. Limaye: Functional Analysis, New Age International Publishers, 3rd Ed, 2014 |

**DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY**

**Syllabus of M.A./M.Sc. I Semester implemented w.e.f. Session 2020-21**

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|---|---|
| **Course Title** | **Advanced Linear Algebra** |
| **Course Number** | **MMM-1013** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **A course of Linear Algebra (UG Level)** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | **The objective of this course is to introduce the following concepts and cognitive skills to the students:**<br>• to provide students with a good understanding of the concepts and methods of Linear Algebra described in details in the syllabus.<br>• to help the students to develop abilities to solve problems using algebraic tools.<br>• to develop critical reasoning by studying the logical proofs and axiomatic methods as applied to prove various theorems.<br>• to understand how abstract definitions are motivated by concrete examples, how result follows from the axiomatic definitions and are specialized back to the concrete examples, and how applications are woven in throughout.<br>• to understand basic proof and disproof techniques using proof by contradiction and disproof by counterexamples. |
| **Course Outcomes** | **Upon successful completion of this course the students will be able to**<br>• apply the concepts and methods described in syllabus, will be able to solve problems using methods in Linear algebra, and will know the application of Linear Algebra to follow complex logical arguments and develop modest logical argument.<br>• understand and compute transition matrices, dual basis, dual vector spaces and dual linear transformations.<br>• deal with the inner product spaces, orthonormal basis, Bessel's inequality and Riesz Representation theorem with applications.<br>• understand implications of the existence of various operators on inner product spaces viz. self adjoint operator, normal operator and their properties.<br>• apply diagonalization of matrices in various problems together with canonical and quadratic forms. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I**: **Vector Space, Linear Transformation and Dual Spaces**<br>Recall of vector space, basis, dimension and related properties, Algebra of Linear transformations, Vector space of Linear transformations L(U,V), Dimension of space of linear transformations, Change of basis and transition matrices, Linear functional, Dual basis, Computing of a dual basis, Dual vector spaces, Annihilator, Second dual space, Dual transformations. | **12** |
| **UNIT II: Inner Product Spaces**<br>Inner-product spaces, Normed space, Cauchy-Schwartz inequality, Pythagorean Theorem, Projections, Orthogonal Projections, Orthogonal complements, Orthonormality, Matrix Representation of Inner-products, Gram-Schmidt Orthonormalization Process, Bessel's Inequality, Riesz Representation theorem and orthogonal Transformation, Inner product space isomorphism. | **12** |
| **UNIT III: Operators on Vector Spaces**<br>Operators on Inner-product spaces, Isometry on Inner-product spaces and related theorems, Adjoint operator, Self-adjoint operators, Normal operator and their properties, Matrix of adjoint operator**,** Algebra of Hom(V,V), Minimal Polynomial, Invertible Linear transformation, Characteristic Roots**,** Characteristic Polynomial and related results. | **12** |

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| **UNIT IV: Canonical Forms and Quadratic Forms**<br>Diagonalization of Matrices, Invariant Subspaces, Cayley-Hamilton Theorem, Canonical form, Jordan Form. Forms on vector spaces, Bilinear Functionals, Symmetric Bilinear Forms, Skew Symmetric Bilinear Forms, Rank of Bilinear Forms, Quadratic Forms, Classification of Real Quadratic forms. | **12** |
| **Total No. of Lectures** | **48** |

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| **Text Books*/ References Books** | **1.** *Kenneth Hoffman and Ray Kunze: Linear Algebra, 2nd Ed.<br>**2.** Sheldon Alexer: Linear Algebra Done Right, Springer, 3rd Ed.<br>**3.** I. N. Herstein: Topics in Algebra |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. II Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Measure Theory** |
| **Course Number** | **MMM-2002** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **Real Analysis** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | **After studying this course the student will be able to**<br>• know and understand the basic concepts of the theory of measure and integration.<br>• understand the main proof techniques in the field, and apply the theory abstractly and concretely.<br>• to write elementary proofs himself, as well as more advanced proofs under guidance.<br>• to use measure theory in Riemann integration and calculus.<br>• work with Lebesgue measure and to exploit its special properties. |
| **Course Outcomes** | The theory leads to a new perspective on integration of functions, which is not only more general than the Riemann setting when working on the real line, but also allows one to integrate in an abstract setting. This is of crucial importance for the development of functional analysis and probability theory. Thus, the students will learn a lot about the advancement of basic integration theory and will also learn some application of this theory. |

| **Contact of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Lebesgue Measure**<br>Lebesgue outer measure, Lebesgue measurable sets, Lebesgue measure, Non-measurable sets, Lebesgue measurable functions, Borel Lebesgue measurability. | **12** |
| **UNIT II: Lebesgue Integral**<br>Revisit of Riemann integral, Lebesgue integral of simple function, bounded function (over a set of finite measure) and nonnegative function, General Lebesgue integral, Differentiation and integration: Differentiation of an integral. | **12** |
| **UNIT III: Abstract Measure**<br>Ring, algebra, σ-ring and σ-algebra. Set functions, Measure, Measure space and Measurable spaces, Measurable functions, General integration, General Convergence Theorem, Outer measure and measurability, Extension of a measure, Uniqueness of measure. | **12** |
| **UNIT IV: Spaces of Lebesgue Integrable Functions**<br>$L^p$-spaces, Jensen's inequality, Minkowski inequality, Hölder inequality, Convergence in $L^p$, Completeness of $L^p$, $L^p(\mu)$spaces and their properties. | **12** |
| **Total** | **48** |

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|---|---|
| **Text Books*/ Reference Books** | **1.** *H. L. Royden: Real Analysis, Macmillan, 1993.<br>**2.** *P. R. Halmos: Measure Theory, Van Nostrand, Princeton, 1950.<br>**3.** G. de Barra: Measure Theory and Integration, New Age International (P) Ltd., NewDelhi, 2014.<br>**4.** I. K. Rana: An Introduction to Measure and Integration, Narosa, 1997.<br>**5.** S. Shirali: A Concise Introduction to Measure Theory, Springer, 2018.<br>**6.** P.K. Jain and V.P. Gupta: Lebesgue Measure and Integration, New Age International, 1986.<br>**7.** P. K. Jain and P. Jain: General Measure and Integration, New Age International, 2014. |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. II Semester implemented w.e.f. Session 2020-21

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|---|---|
| **Course Title** | **Partial Differential Equations** |
| **Course Number** | **MMM-2005** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **A course of Partial Differential Equations (UG level)** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The objective of this course is to form partial differential equations occurring in the various fields of science and engineering and to provide their analytic solutions. |
| **Course Outcomes** | **After studying this course student will be able to:**<br>• classify the second order linear partial differential equations.<br>• transform linear partial differential equations of hyperbolic type into canonical form and solve it by Riemann's method.<br>• formulate and solve heat, Laplace and wave equations into Cartesian, polar, cylindrical and spherical coordinates. |

| Content of Syllabus | No. of Lectures |
|---|---|
| **UNIT I: Mathematical Models, Linear Hyperbolic Equations and Cauchy Problems**<br>Classification of second order linear PDE, Canonical form of hyperbolic type linear PDE, Riemann's method and Goursat problem for linear hyperbolic equations, Derivations of heat equation and wave equation in one/two/three dimensions, Occurrence of Laplace equation in Physics, Mathematical modelling of gravitational potential, Poisson equation, conservation laws and Burger equation, Cauchy's problems for second order linear PDE, D'Alembert's solution of an infinite vibrating string problem, Initial value problem for heat flow in an infinite rod, Cauchy problem for Laplace equation. | **12** |
| **UNIT II: One-dimensional Initial BVP**<br>Semi-infinite string with a fixed end as well as a free end, Nonhomogeneous boundary conditions, Finite string with fixed ends, Cauchy problem for nonhomogeneous wave equations; Revisit to method of separation of variables and Fourier method, Fourier series solutions of finite vibrating string problems and finite heat-conducting rod problems, Nonhomogeneous heat and wave equations: Fourier method and Duhamel's principle. | **12** |
| **UNIT III: Higher-dimensional Initial BVP**<br>Double and triple Fourier series, Fourier series solutions of Initial BVP in vibrating membrane, vibrating cuboid, heat-conducting plate and heat-conducting cuboid; Dirichlet, Neumann and Robin problems and their Fourier series solutions, Steady state temperature distribution in a cuboid; Spherical mean, Mean value theorem and Maximum-Minimum principle for harmonic functions, Green's function for two dimensional Laplace equation. | **12** |
| **UNIT IV: BVP in Polar Coordinate Systems**<br>Transformation of two dimensional Laplace equation, heat equation and wave equation from Cartesian coordinates to polar coordinates, Transformation of three dimensional Laplace equation, heat equation and wave equation from Cartesian coordinates to cylindrical and spherical coordinates, Fourier series solutions of Laplace equation, heat equation and wave equation in polar, cylindrical and spherical coordinates. | **12** |
| **Total No. of Lectures** | **48** |

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| **Text Books*/ Reference Books** | 1. *N. Sneddon: Elements of Partial Differential Equations, McGraw Hill Book Company, 1957.<br>2. *Tyn Myint U and Lokenath Debnath: Linear Partial Differential Equations for Scientists and Engineers, Birkhäuser Boston, 4th Ed, 2007.<br>3. S. J. Farlow: Partial Differential Equations for Scientists and Engineers, Dover Publications Inc, 1993.<br>4. K. S. Rao: Introduction to Partial Differential Equations, PHI Learning Pvt Ltd, New Delhi, 3rd Ed, 2011.<br>5. T. Amaranath: An Elementary Course in Partial Differential Equations, Narosa Publishing House, New Delhi, 2nd Ed, 2003. |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY

## Syllabus of M.A./M.Sc. II Semester approved in BOS: 01-08-2019

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|---|---|---|
| **Course Title** | **Algebraic Topology** | |
| **Course Number** | **MMM-2008** | |
| **Credits** | **4** | |
| **Course Category** | **Compulsory** | |
| **Prerequisite Courses** | **General Topology, Group Theory** | |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** | |
| **Type of Course** | **Theory** | |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** | |
| **Course Objectives** | The student should get well versed with the importance of various topological tools and to feel the generalized notions of Nets and Filters with the notion of Sequences in metric spaces. | |
| **Course Outcomes** | After the completion of the course, students will feel the power of the various topological notions introduced in the course. | |
| **Content of Syllabus** | | **No. of Lectures** |
| **UNIT I: Metrization and Paracompactness**<br>Urysohn Metrization Theorem, Partitions of unity, Local finiteness, Nagota Metrization Theorem, Para-compactnes, Smirnov Metrization Theorem. | | **12** |
| **UNIT II: Nets and Filters**<br>Topology and convergence of nets, Hausdorffness and nets, compactness and nets, filters and their convergence, Canonical way connecting nets to filters and vice-versa, Ultra filters and compactness. | | **12** |
| **UNIT III: Fundamental Groups**<br>Homotopy, Relative homotopy, Path homotopy, Homotopy classes, Construction of fundamental groups for topological spaces and its properties. | | **12** |
| **UNIT IV: Covering Spaces**<br>Covering maps, Local homeomorphism, Covering spaces, Lifting lemma, The fundamental group of Circle, Torus and Punctured Plane, The fundamental Theorem of Algebra. | | **12** |
| | **Total No. of Lectures** | **48** |
| **Text Books*/ References Books** | 1. *J. M. Munkres: Topology-A first course, 1987. (for Unit I, III & IV)<br>2. *M. C. Gemignani: Elementary Topology. (for Unit II)<br>3. *Jheral O. Moore: Elementary General Topology. (for Unit II)<br>4. *J. Dugundji: Topology. (for Unit II)<br>5. *Sheldon W. Daves : Topology. (for Unit II)<br>6. *H. Schubert: Topology. (for Unit II) | |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. II Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Advanced Functional Analysis** |
| **Course Number** | **MMM-2009** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **Functional Analysis, Linear Algebra, Real Analysis** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | To discuss some advanced topics of Hilbert space, spectrum theory, geometric properties of Banach space and variant of differentiability on normed space which play central role in research and advancement of various topics in mathematics. |
| **Course Outcomes** | After undertaking this course, the students will be able to appreciate:<br>• understand the variational analysis and optimization.<br>• understand the concepts of compactness, self-adjointness and positivity of bounded linear operators.<br>• provide the basic tools for nonlinear functional analysis and operator theory.<br>• provide the motivation of the concept of differentiable manifolds. |

| Content of Syllabus | No. of Lectures |
|---|---|
| **UNIT-I: Orthogonal Projections, Bilinear Forms and Variational inequalities**<br>Orthogonal complements, Orthogonal projections, Projection theorem, Projection on convex sets, Sesquilinear forms, Bilinear forms and their basic properties, Lax-Miligram lemma, Variational inequalities and Lions-Stampacchia theorem, Variational inequalities for monotone operators, Minty lemma. | **12** |
| **UNIT II: Spectral Theory of Continuous Linear Operators**<br>Eigenvalues and eigenvectors, Resolvent operators, Spectrum, Spectral properties of bounded linear operators, Compact linear operators on normed spaces, Finite dimensional domain or range, Sequence of compact linear operators, Weak convergence, Spectral theory of compact linear operators. | **12** |
| **UNIT III: Geometry of Banach Spaces**<br>Strict convexity, Modulus of convexity, Uniform convexity, Duality mapping and its properties, Smooth Banach Space, Modulus of smoothness. | **12** |
| **UNIT IV: Differential Calculus on Normed Spaces**<br>Gateaux derivative, Gradient of a function, Fréchet derivative, Chain rule, Mean value theorem, Properties of Gateaux and Fréchet derivatives, Implicit function theorem, Tylor's formula, Inverse function theorem. | **12** |
| **Total No. of Lectures** | **48** |

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| **Text Books*/ Reference Books** | 1. *Q. H. Ansari: Topics in Nonlinear Analysis and Optimization, World Education, Delhi, 2012 (Sections 2.4, 2.5, 2.6, 2.7).<br>2. *C. Chidume: Geometric properties of Banach Spaces and Nonlinear Iterations, Springer, London, 2009 (Sections 1.2, 1.3, 1.4, 1.5, 2.2, 2.3).<br>3. *E. Kreyazig: Introductory Functional Analysis with Applications, John Wiley and Sons, New York, 1989 (Sections 7.1, 7.2, 7.3, 8.1, 8.3).<br>4. *A. H. Siddiqi: Applied functional Analysis, CRC Press, 2003 (Sections 3.3, 3.4, 3.5, 5.2, 9.3.1, 9.3.2).<br>5. M. C. Joshi and R. K. Bose: Some topics in Nonlinear Functional Analysis, Wiley Eastern Limited, New Delhi, 1985 (Sections 2.1, 2.2, 2.3). |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY

## Syllabus of M.A./M.Sc. II Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Differentiable Manifolds** |
| **Course Number** | **MMM-2010** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **Topology, Geometry of Curves and Surfaces** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The primary objective of this course is to provide basic knowledge of manifolds, submanifolds and geometry of manifolds. |
| **Course Outcomes** | This course will enable the students to understand about differentiation of functions of several variables, tangent vector, vector field, differential forms and Connections. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Calculus of $\mathbb{R}^n$**<br>Differentiable functions from $\mathbb{R}^n \to \mathbb{R}^m$, Chain rule, Directional derivatives, Differential of a map, Chain rule for differentials, Inverse mapping theorem, Implicit function theorem. | **12** |
| **UNIT II: Manifold and its differentiable structure**<br>Topological manifolds, Differentiable atlas, Smooth maps, Diffeomorphism, Equivalent atlases, Differentiable structure on a manifold, Space of smooth maps, Tangent vectors and tangent space, Differential of a smooth map. | **12** |
| **UNIT III: Submanifolds, Vector fields and Covectors**<br>Immersion, Embedding and Submanifolds, Vector fields, Lie algebra of vector fields, Integral curve of a vector field, Covectors and Cotangent spaces, Pull back of a linear differential form, One parameter group of transformation, Exponential map, Covariant and Contravariant tensors, Laws of transformation for the components of tensors. | **12** |
| **UNIT IV: Differential forms and Connection**<br>Differential forms, Exterior product, Grassman algebra of forms, Exterior derivative, Affine Connection, Parallelism, Geodesic Covariant differentiation of tensors, Torsion and Curvature of a Connection, Structure equation of Cartan, Bianchi's identities. | **12** |
| **Total No. of Lectures** | **48** |

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| **Text Books*/ Reference Books** | 1. *W. M. Boothby: An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, Revised Ed, 2003.<br>2. *S. I. Husain: Lecture Notes on Differentiable Manifolds.<br>3. K. Matsushima: Differentiable Manifolds.<br>4. S. Kumaresan: A Course in Differential Geometry and Lie groups |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. II Semester implemented w.e.f. Session 2020-21

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|---|---|
| **Course Title** | **Advanced Theory of Groups and Fields** |
| **Course Number** | **MMM-2011** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **UG level courses of Group Theory & Ring Theory** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | **This course aims to introduce students to the following concepts and cognitive skills:**<br>This course is divided into two major Parts, namely Units I and II; and Units III and IV. In the first part of the course aims to introduce the concepts of: Relation of conjugacy, Conjugate classes of a group, Number of elements in a conjugate class of an element of a finite group, Class equation in a finite group and related results, Partition of a positive integer, Conjugate classes in Symmetric groups, Sylow's theorems, External and Internal direct products and related results, Structure theory of finite abelian groups. Subgroup generated by a subset of a group, Commutator subgroup of a group, Subnormal series of a group, Refinement of a subnormal series, Length of a subnormal series, Solvable groups and related results. n-th derived subgroup, Upper central and lower central series of a group, Nilpotent groups, Relation between solvable and nilpotent groups, Composition series of a group, Zassenhaus theorem, Schreier refinement theorem, Jordan-Holder theorem for finite groups.<br>In the second part of the course the introduction: Field extensions, Finite extensions, Degree of extensions, Multiplicative property of degree of extensions, Finitely generated extensions, Simple extension and its properties, Relationship between two simple extensions, Quadratic extensions over field of characteristic different from 2, Algebraic and transcendental elements, Characterization of algebraic elements, Algebraic extensions, Composite field of any collection of subfields, Simple applications of algebraic extensions: Classical straightedge and compass constructions, Splitting field and its uniqueness, Normal extensions, Cyclotomic fields of nth roots of unity, Algebraic closures, Algebraically closed fields and their uniqueness, Separable and inseparable extensions, Perfect fields, Cyclotomic polynomials and extensions, The group of automorphisms of a field and fixed fields, Galois extension and its different characterizations, the Galois group of an extension, the Galois group of a polynomial.<br>The course discusses some important applications of these notions. |
| **Course Outcomes** | **On successful completion of this course, students should be able to:**<br>Understand these notions and apply them to get the fruitful decisions about finite groups of various orders and construct smallest field extensions having roots of polynomials and infer some important information about field extensions having roots of polynomials. |

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| **Content of Syllabus** | **No. of Lectures** |
| **Unit I: Conjugate Classes and Sylow's Theorems**<br>Relation of conjugacy, conjugate classes of a group, Number of elements in a conjugate class of an element of a finite group, Class equation in a finite group and related results, Partition of a positive integer, Conjugate classes in Symmetric groups, Sylow's theorems, External and Internal direct products and related results. | **12** |
| **Unit II: Series of Groups**<br>Structure theory of finite abelian groups, Subgroup generated by a subset of a group, Commutator subgroup of a group, Subnormal series of a group, Refinement of a subnormal series, Length of a subnormal series, Solvable groups and related results, n-th derived subgroup, Upper central and lower central series of a group, Nilpotent groups, Relation between solvable and nilpotent groups, Composition series of a group, Zassenhaus theorem, Schreier refinement theorem, Jordan-Holder theorem for finite groups. | **12** |
| **UNIT III: Algebraic Extensions of Fields**<br>Finite extensions, Degree of extensions, Multiplicative property of degree of extensions, Finitely generated extensions, Simple extension and its properties, Relationship between two simple extensions, Quadratic extensions over field of characteristic different from 2, Algebraic and transcendental elements, Characterization of algebraic elements, Algebraic extensions, Composite field of any collection of subfields, Simple applications of algebraic extensions: Classical straightedge and compass constructions. | **12** |
| **UNIT IV: Separable Extension and Galois Theory**<br>Splitting field and its uniqueness, Normal extensions, Cyclotomic fields of $n^{th}$ roots of unity, Algebraic closures, Algebraically closed fields and their uniqueness, Separable and inseparable extensions, Perfect fields, Cyclotomic polynomials and extensions, The group of automorphisms of a field and fixed fields, Galois extension and its different characterizations, the Galois group of an extension, the Galois group of a polynomial. | **12** |
| **Total No. of Lectures** | **48** |
| **Text Books*/ Reference Books** | 1. *N. Herstein: Topics in Algebra.<br>2. *Surjeet Singh and Qazi Zameeruddin: Modern Algebra.<br>3. *T. Adamson: Introduction to Field Theory.<br>4. D. S. Dummit and R. M. Foote: Abstract Algebra.<br>5. P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul: Basic Abstract Algebra.<br>6. J. S. Milne: Fields and Galois Theory. |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. III Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Mechanics** |
| **Course Number** | **MMM-3003** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **UG level course of Mechanics** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The course aims at understanding the various concepts of physical quantities and the related effects on different bodies using mathematical techniques. It emphasizes knowledge building for applying mathematics in physical world. |
| **Course Outcomes** | The course will enable the students to understand:<br>• The significance of mathematics involved in physical quantities and their uses;<br>• To study and to learn the cause-effect related to these; and<br>• The applications in observing and relating real situations/structures |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Mechanics of System of Particle and Rigid Bodies**<br>General force system, equipollent force system, equilibrium conditions, Reduction of force systems, couples, moments and wrenches, Necessary and sufficient conditions of rigid bodies, General motion of rigid body, Moments and products of inertia and their properties, Momentalellipsa, Kinetic energy and angular motion of rigid bodies. | **12** |
| **UNIT II: Elements of Classical Mechanics**<br>Moving frames of references and frames in general motion, Euler's dynamical equations, Motion of a rigid body with a fixed point under no force, Method of pointset, Constraints, Generalized coordinates, D'Alembert's principle and Lagrange's equations, Lagrangian formulation and its applications. | **12** |
| **UNIT III: Hamilton's Principle and Formulation**<br>Hamilton's principle, Techniques of calculus of variations, Lagrange's equations through Hamilton's principle, Cyclic coordinates and conservation theorems, Canonical equations of Hamilton, Hamilton's equations from variational principle, Principle of least action. | **12** |
| **UNIT IV: Special Theory of Relativity**<br>Galilean transformation, Postulates of special relativity, Lorentz transformation and its consequences, Length contraction, Time dilation, Addition of velocities, variation of mass with velocity, Equivalence of mass and energy, Four-dimensional formalism, Relativistic classification of particles, Maxwell's equations and their Lorentz invariance. | **12** |
| **Total No. of Lectures** | **48** |

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| **Text Books*/ Reference Books** | 1. *J. L. Synge and B. A. Griffith: Principle of Mechanics, McGraw-Hill Book Company, 1970.<br>2. *H. Goldstein: Classical Mechanics, 2$^{nd}$ Ed, Narosa Publishing House, 1980.<br>3. Zafar Ahsan: Lecture Notes on Mechanics, Seminar Library (Chapters III-VI). |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. III Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Nonlinear Functional Analysis** |
| **Course Number** | **MMM-3005** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **Functional Analysis, Metric Spaces** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | To introduce the basic concepts of fixed point theory and variational inequality. This is a research level course which begins with core results on fixed points and undertakes some recent results on metric fixed point theory including selected applications as well. However, in this course we introduce the concepts of set-valued maps and variational principle with their applications to fixed point theory |
| **Course Outcomes** | **After undertaking this course, the students will be able to appreciate:**<br>• provide the applicability in differential equations, integral equations and variational inequality problems.<br>• provide the basic tools for variational analysis and optimization.<br>• understand the topological properties of set-valued maps.<br>• understand the strong and weak convergence theorems in Banach space. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Contraction Principle, Variational Principle and Their Applications**<br>Banach contraction principle and its applications to system of linear equations, integral equations and differential equations; Contractive mappings and Eldestien Theorem, Boyd-Wong's fixed point theorem, Matkowski's fixed point theorem, Caristi's fixed point theorem, Ekeland's variational principle and its applications to fixed point theorems and optimization, Takahashi's minimization theorem. | **12** |
| **UNIT II: Set-Valued Maps and Related Fixed Point Theorems**<br>Definitions and examples of set-valued maps, Lower and upper semi-continuity of set-valued maps, Hausdorff metric, H-continuity of set-valued maps, Set-valued contraction maps, Nadler's fixed point theorem, DHM Theorem and some other fixed point theorems for set-valued maps. | **12** |
| **UNIT III: Classical Existence Theorems for Nonexpansive Mappings**<br>Browder fixed point theorem in Hilbert space, Approximate fixed point property, Asymptotic centre and radius, Browder-Göhde fixed point theorem, Normal structure property, Kirk fixed point theorem, Metrical variants of Kirk fixed point theorem. | **12** |
| **UNIT IV: Some Iterative Methods for Fixed Points**<br>Kransnoselskij iterative method, Mann iterative method, Ishikawa iterative method, Helpern iterative method and Browder iterative method with their convergence results. | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ References Books** | 1. *Q. H. Ansari: Metric Spaces-Including Fixed Point Theory and Set-valued Maps, Narosa Publishing House, New Delhi, 2010 (Chapters 7 & 9 for Unit I & Chapter 8 for Unit II).<br>2. *S. Almezel, Q. H. Ansari and M. A. Khamsi: Topics in Fixed Point Theory, Springer, New York, 2014 (Chapter 1 for Unit III & Chapter 8 for Unit IV).<br>3. *S. A. R. Al-Mezel, F. R. M. Al-Solamy and Q. H. Ansari: Fixed Point Theory, Variational Analysis, and Optimization, CRC Press, 2014 (Chapter 1 for Unit III).<br>4. K. Goebel: Concise Course on Fixed Point Theorems, Yokohama Publishers Inc, Yokohama, Japan, 2002.<br>5. V. Berinde: Iterative Approximation of Fixed Points, Springer, Berlin, 2007. |

**DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY**
**Syllabus of M.A./M.Sc. III Semester approved in BOS: 01-08-2019**

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|---|---|
| **Course Title** | **Advanced Ring Theory** |
| **Course Number** | **MMM-3006** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **Ring Theory (UG level)** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The objectives of this course are to give some basic definitions, state several fundamental properties and a few examples of rings. We also discuss some important concepts that play a central role in the theory of rings. We define the direct sum of a finite number of rings , the complete direct sum and also discrete direct sum of denumerably finite set of rings. We generalize these concepts by defining in a natural way the direct sum of an entirely arbitrary rings. We define Prime and semiprime ideals. |
| **Course Outcomes** | Some of the results that we prove in this course have direct applications to other branches of Mathematics. The knowledge obtained from study of advanced ring theory motivates to do further research work in the theory of rings, near rings and modules in future. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Theory of Ideals**<br>Examples and fundamental properties of rings (Review), Direct and discrete direct sum of rings, Ideals generated by subsets and their characterizations in terms of elements of the ring under different conditions, Sums and direct sums of ideals, Ideal products and nilpotent ideals, Minimal and maximal ideals. | **12** |
| **UNIT II: Complete Matrix Ring and Subdirect Sum**<br>Complete matrix ring, Ideals in complete matrix ring, Residue class rings, Homomorphisms, Subdirect sum of rings and its characterizations, Zorn's Lemma, Subdirectly irreducible rings, Boolean rings. | **12** |
| **UNIT III: Prime Ideals and Prime Radical**<br>Prime ideals and m-systems, Different equivalent formulation of prime ideals, Semi-prime ideals and n-systems, Equivalent formulation of semi prime ideals, Necessary and sufficient conditions for an ideal to be a prime ideal, Prime radical of a ring. | **12** |
| **UNIT IV: Prime Rings and Jacobson Radical**<br>Prime rings and its characterization in terms of prime ideals, Primeness of complete matrix rings, D.C.C. for ideals and the prime radical, Jacobson radical: Definition and simple properties, Relationship between Jacobson radical and prime radical of a ring, Primitive rings, Jacobson radical of primitive rings. | **12** |
| **Total No. of Lectures** | **48** |

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| **Text Books*/ References Books** | 1. *N. H. McCoy: The Theory of Rings.<br>2. Anderson and Fuller: Rings and Categories of Modules.<br>3. I. S. Luthar and I. B. S. Passi: Algebra Volume 2: Rings. |

**DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY**
**Syllabus of M.A./M.Sc. III Semester approved in BOS: 01-08-2019**

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|---|---|
| **Course Title** | **Riemannian Geometry and Submanifolds** |
| **Course Number** | **MMM-3007** |
| **Credits** | **4** |
| **Course Category** | **Compulsory** |
| **Prerequisite Courses** | **Differentiable Manifolds** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The study of differentiable manifolds is a basis of the study of differential geometry and differential topology and recent developments in various branches of mathematics have been one of the cornerstones of the edifice of modern mathematics. Further, differential geometric aspect of submanifolds with certain structures are vast and very fruitful fields of Riemannian geometry. The study of differentiable manifolds has been an important tool because of its application in the area of Physics, Astronomy and Relativity.<br>The purpose of the study of this course is to provide to students an introduction to the Riemannian structure on a manifold and the theory of submanifolds of manifolds having such a structure. |
| **Course Outcomes** | After the completion of the course the students shall be well equipped with the notion of Riemannian manifolds and the submanifolds of Riemannian manifolds. Also, they will be aware of the complex structure and the submanifolds of complex manifolds. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Riemannian Manifolds**<br>Partition of unity, Paracompactness, Riemannian metric on a paracompact manifold, First fundamental form on a Riemannian manifold, Riemannian connexion, Riemannian curvature, Ricci and scalar curvature. | **12** |
| **UNIT II: Submanifolds of Riemannian Manifolds**<br>Distribution on a manifold, Submanifold of a Riemannian manifold, Hypersurfaces, Gauss and Weingarten formulae, Equation of Gauss, Coddazi and Ricci.. | **12** |
| **UNIT III: Complex and Contact Manifolds**<br>Complex and almost manifolds, Nejenhuis tensor and integrability of a structure, Almost Hermitian, Kaehler and nearly Kaehler manifolds, Almost contact and Sasakian manifolds. | **12** |
| **UNIT IV: Submanifolds of Complex Manifolds**<br>Submanifolds of almost Hermitian manifolds, Invariant and Anti- Invariant distributions of a Hermitian manifold, CR submanifolds of Kaehler and nearly Kaehler, Generic and slant submanifolds of Kaehler manifold. | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ References Books** | 1. *B. Y. Chen: Geometry of Submanifolds, Marcel Dekker Inc, New York, 1973.<br>2. S. Kobayashi and K. Nomizu: Foundation of Geometry, Vol I, Interscience Publishers (John Wiley & Sons), Revised Ed, 1996.<br>3. *W. M. Boothby: An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, Revised Ed, 2003.<br>4. S. I. Husain: Lecture Notes on Differentiable Manifolds, Seminar Library, Deptt of Maths, AMU, Aligarh.<br>5. Kentaro Yano and Masahiro Kon: Structures on Manifolds, World Scientific Press. |

**DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY**

**Syllabus of M.A./M.Sc. III Semester approved in BOS: 01-08-2019**

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|---|---|
| **Course Title** | **Theory of Semigroups** |
| **Course Number** | **MMM-3018** |
| **Credits** | **4** |
| **Course Category** | **Optional** |
| **Prerequisite Courses** | **Some background about binary and associative operation plus basic knowledge of group and ring theoretic results.** |
| **Contact Hours** | **(4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | This course aims to expose the students to more liberal and powerful tools of Algebra that are applicable in the present-day life. |
| **Course Outcomes** | On successful completion of this course, students should be able to learn and feel that learnig further advance tools of this discipline will equip them to apply these tools to the huge world of Automata, Languages and Machines. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Introductory Ideas**<br>Basic definitions, Group with zero, Rectangular Band, Monogenic semigroups, Periodic semigroups, Partially ordered sets, Semilattices and lattices. | **12** |
| **UNIT II: Equivalences and Congruences**<br>Binary relations, Equivalences and related results, Congruences and related results, Free semigroups, Ideals and Rees congruences, Lattices of equivalences and congruences. | **12** |
| **UNIT III: Green's Equivalences and Regular Semigroups**<br>Green's Equivalences and related results, Structure of D-classes, Green's lemma and its corollaries, Green's theorem, Regular D-classes, Regular semigroups and related results. | **12** |
| **UNIT IV: 0-Simple Semigroups**<br>Sandwich set, Simple and 0-simple semigroups and related results, Completely simple and Completely 0-simple semigroups and related results. | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ References Books** | 1. *John M. Howie: Fundamentals of semigroup theory, Clarendon press, Oxford, 1995.<br>2. A. H. Clifford and G. B. Preston: The Algebraic theory of semi groups, Vol. 1, and 2, Mathematical surveys of the AMS, 1961 and 1967.<br>3. P. M. Higgins: Techniques of Semi Group Theory, Oxford University Press, 1992. |

# DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. III Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Topological Vector Spaces** |
| **Course Number** | **MMM-3019** |
| **Credits** | **4** |
| **Course Category** | **Optional** |
| **Prerequisite Courses** | **A Course of Functional Analysis and A Course of Topology** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The objective of this course is to teach how one can extend the results and concepts from Normed Spaces to Topological Spaces. The course gives the idea of topological vector spaces and locally convex topological vector spaces and their properties. It also covers several fixed point theorems for set-valued maps defined on a topological vector space. |
| **Course Outcomes** | A student can learn the concepts of Hahn Banach theorem in the setting of vector spaces, topological vector spaces, locally convex topological vector spaces and their properties. A student can also learn several important fixed point results in the setting of topological vector spaces, namely, KKM theorem, Browder fixed point theorem, Kakutani fixed point theorem, etc. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Some Concepts from Vector Space**<br>Subspaces, affine sets, convex sets, cones (pointed cone, convex cones etc), balanced sets, absorbent sets, hulls (linear hull, affine hull, convex hull, balanced hull) and their properties and characterizations: Hahn Banach theorem in vector spaces: Convex functions, Minkowski function and seminorm with their properties. | **12** |
| **UNIT II: Topological Vector Spaces**<br>Definition and general properties, product spaces and quotient spaces, bounded and totally bounded sets; Topological properties of convex sets, convex cones, compact sets, convex hull; Hyperplanes, closed half spaces and separation of convex hulls; Hahn Banach theorem on separation. Complete topological vector spaces: Metrizable topological vector spaces: Definition and properties; Normable topological vector spaces and finite spaces. | **12** |
| **UNIT III: Locally Convex Spaces**<br>Definition and general properties, subspaces, product spaces and quotient spaces; Convex and compact sets in locally convex spaces; Separation theorems in locally convex spaces; Continuous linear operators: General consideration on continuous linear operators, open operators and closed operators; Space of operators: Topologies of uniform convergence, properties of the space of continuous linear operators. | **12** |
| **UNIT IV: Dual Vector Spaces**<br>Definition and properties; Mackey topology; Strong topology: Definition and properties, semi-reflexive spaces and space and reflexive spaces, Some fixed points theorems in topological vector spaces: KKM theorem, Browder fixed point theorem, Kakutani fixed point theorem and related results. | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ References Books** | 1. *R. Cristescu: Topological Vector Spaces, Noordhoff International Publishing, Leyden, The Netherlands, 1977.<br>2. *L. Narici and E. Beckenstein: Topological Vector Spaces, Nacel Dekker, Inc., New York and Basel, 1985.<br>3. *Y. C. Wong: Introductory Theory of Topological Vector Spaces, Marcel Dekker, Inc., New York Basel and Hong Kong, 1992.<br>4. V. I. Bogachev and O.G. Smolyanov: Topological Vector Spaces and Their Applications, Springer International Publishing AG, 2017.<br>**5.** S. P. Singh, B. Watson and P. Srivastava: Fixed Point Theory and Best Approximation: The KKM-map Principle, Kluwer Academic Publishers, Dordrecht, Boston, London, 1977. |

**DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY, ALIGARH**

**Syllabus of M.A./M.Sc. III Semester implemented w.e.f. Session 2020-21**

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|---|---|
| **Course Title** | **Homological Algebra and Module Theory** |
| **Course Number** | **MMM-3021** |
| **Credits** | **4** |
| **Course Category** | **Optional** |
| **Prerequisite Courses** | **Linear Algebra, Ring Theory** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | To develop the concept of a module as a generalization of a vector space and an abelian group. Constructions such as direct sum and direct products. To provide a solid background by developing fundamental notions helpful in other areas such as number theory, algebraic geometry and homological algebra. |
| **Course Outcomes** | Module theory as linear algebra over general rings. Ability to handle modern algebraic notions like quotients and generators. Ability to discuss special classes of modules: Free modules, Divisible modules, Torsion modules and Bimodules. Ability to deal with module theory which is indispensable in wide range of mathematical disciplines such as algebra, topology, number theory, operator theory. |

| **Contents of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Basic Concepts**<br>Modules, Submodules, Factor modules, Module Homomorphisms, Correspondence theorem, Isomorphism theorems, Bimodules, Linear combinations and spanning set, Socle and radical of modules, Linearly independent sets, Bases and rank of module, Generators and Relations for modules, Annihilation. | **12** |
| **UNIT II: Elements of Homological Algebra**<br>External and internal direct sums, Direct summands, Direct products of modules, Idempotent endomorphisms, Natural maps, Splitting maps, Exact sequences, Short exact sequences, Splitting sequences, Four lemma, Five Lemma, Semi-Exactness, Products, coproducts and their universal property, Projections and injections. | **10** |
| **UNIT III: Some Special Types of Modules**<br>Simple modules, Cyclic modules, Unitary Modules, Chain conditions, Noetherian modules, Artinian modules, Hilbert Basis Theorem, Essential and superfluous submodules, Semi-simple modules, Torsion and torsion-free modules, Free modules, Homomorphism extension property, Basis and Rank of free modules, Divisible modules, Projective modules, Connection between projective and free modules, Direct sum of projective modules, Injective modules, Baer's characterization, Character modules of free modules, Connections to injectivity. | **14** |
| **UNIT IV: Structure Theory and Applications**<br>Free modules over PID's, Invariant factor theorem for sub modules, Finitely generated modules over principal ideal domains, Chain of invariant ideals, Fundamental structure theorem for finitely generated module over a PID, Applications to finitely generated abelian groups and linear transformations, Elementary divisors, Rational canonical forms. | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ Reference Books** | 1. *S. T. Hu: Introduction to Homological Algebra.<br>2. *M. E. Keating: A first course in Module Theory.<br>3. *P. B. Bhattacharya, S. K. Jain and S. R. Nagpaul: Basic Abstract Algebra.<br>4. F. W. Anderson and K. R. Fuller: Rings and Categories of Modules.<br>5. J. Lamback: Lectures on Rings and Modules.<br>6. N. Jacobson: Basic Algebra I and II. |

**DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY**
**Syllabus of M.A./M.Sc. IV Semester approved in BOS: 01-08-2019**

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|---|---|
| **Course Title** | **Structures on Manifolds** |
| **Course Number** | **MMM-4016** |
| **Credits** | **4** |
| **Course Category** | **Optional** |
| **Prerequisite Courses** | **Courses of Differentiable Manifolds, Riemannian Geometry and Submanifolds** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The study of structures on a manifold is an important and interesting topic of differential geometry and the study of submanifolds with some of the important structures is a very useful field to the study of Riemannian geometry.<br>The purpose of the study of this course is to provide to students an introduction to the theory of various differential geometric structures on manifolds and to collect and arrange the results on submanifolds of Riemannian manifolds with certain structures. |
| **Course Outcomes** | After the completion of the course the students will be well equipped with the Kaehler, Contact and Sasakian structures on a manifold and different type of curvatures of such manifolds. Also, they will know how a submanifold of such manifolds remain invariant or semi-invariant. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I**: **Kaehler Structure on Manifolds**<br>Infinitesimal auto-morphism of an almost complex structure, Holomorphic vector fields, Characterizations for a vector field to be an infinitesimal auto-morphism, Kaehler structure on a manifold, Holomorphic sectional curvature and the space of constant holomorphic sectional curvature (complex space form), Kaehler analogue of Schur's Theorem, An example of a Kaehler manifold, Kaehlerian submanifolds (Invariant submanifolds of Kaehler manifold). | **12** |
| **UNIT II: Contact Structure on Manifolds**<br>Almost contact structure on a smooth manifold, Contact manifolds, Torsion tensor of an almost contact manifold, Killing vector field, K-contact manifold, Sasakian manifolds, φ-sectional curvature, Sasakian space form, $\eta$-Einstein manifold. | **12** |
| **UNIT III: Invariant Submanifolds**<br>Invariant submanifolds of an almost contact manifold, $\eta$-parallel second fundamental form, Invariant submanifolds of Sasakian manifolds and Sasakiai space forms, $\eta$-parallel Ricci tensor of a Sasakian manifold, Anti-invariant submanifolds tangent to the structure vector field of Sasakian manifolds, Anti-invariant submanifolds normal to the structure vector field of Sasakian manifolds. | **12** |
| **UNIT IV: Semi-invariant Submanifolds**<br>Semi-invariant submanifolds of an almost contact manifold, Semi-invariant submanifolds of a Sasakian manifold, Integrability conditions for the distributions, Geodesic conditions for the leaves, Semi-invariant products of Sasakian manifolds, Generic semi-invariant product, Totally contact-geodesic submanifolds, Totally contact umbilical semi-invariant submanifolds in Sasakian manifolds. | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ References Books** | 1. *Kentaro Yano and Masahiro Kon: Structures on Manifolds, World Scientific Press.<br>2. Aurel Bejancu: Geometry of CR submanifolds, D. Reidel Publishing Company.<br>3. *S. Kobayashi and K. Nomizu: Foundations of Differential Geometry, Vol. II, Interscience Publishers (John Wiley & Sons).<br>4. *Y. Matsushima: Differentiable Manifolds, Marcel Dekker Inc.<br>5. D. E. Blair: Contact Manifolds in Riemannian Geometry, Lecture Notes in Maths. 509, Springer Verlag.<br>6. M. P. do Carmo: Riemannian Geometry, Birkhäuser Basel. |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. IV Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Special Functions and Lie Theory** |
| **Course Number** | **MMM-4017** |
| **Credits** | **4** |
| **Course Category** | **Optional** |
| **Prerequisite Courses** | **Ordinary Differential Equations** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | **To enable the students to learn the following:**<br>• The interplay between mathematical analysis and physical understanding.<br>• To investigate and derive the properties of special functions, inter-relations between such functions and their representations in various forms.<br>• Certain specific systems of orthogonal polynomials and their properties.<br>• The general concepts related to theory of Lie groups and Lie algebras and connection between Lie theory and special functions. |
| **Course Outcomes** | **After successful completion of the course students will be able to:**<br>• Solve, expand and interpret solutions of many types of important differential equations by making use of special functions and orthogonal polynomials.<br>• Derive the formulas and results of certain classical special functions and orthogonal polynomials by different methods.<br>• Derive the generating relations involving special functions by applying the Lie algebraic techniques.<br>• Achieve the knowledge to analyse the problems using the methods of special functions and orthogonal polynomials, which helps in exploring the role of special functions and orthogonal polynomials in other areas of mathematics. |

| Content of Syllabus | No. of Lectures |
|---|---|
| **UNIT I: Gamma, Hypergeometric, Bessel and Neumann Functions**<br>Introduction; Gamma Function; Hypergeometric Functions: Definition and special cases, convergence, analyticity, integral representation, differentiation, transformations and summation theorems; Bessel Functions: Definition, connection with hypergeometric function, differential and pure recurrence relations, generating function, integral representation; Neumann polynomials, Neumann series and related results; Examples on above topics. | **12** |
| **UNIT II: Legendre, Hermite and Laguerre Polynomials**<br>Legendre polynomials: (i) Generating function (ii) Special values (iii) Pure and differential recurrence relations (iv) Differential equation (v) Series definition (vi) Rodrigues' formula (vii) Integral representation; Hermite polynomials: Results (i) to (vii) and expansion of $x^n$ in terms of Hermite polynomials; Laguerre polynomials: Results (i) to (vii); Examples on above topics. | **12** |
| **UNIT III: Orthogonal Polynomials**<br>Simple sets of polynomials; Orthogonal polynomials: Equivalent condition for orthogonality; Zeros of orthogonal polynomials; Expansion of polynomials; Three-term recurrence relation; Christoffel-Darboux formula; Normalization and Bessel's inequality; Orthogonality of Legendre, Hermite and Laguerre polynomials; Ordinary and singular points of differential equations, Regular and irregular singular points of hypergeometric, Bessel, Legendre, Hermite and Laguerre differential equations; Examples on above topics. | **12** |
| **UNIT IV: Lie Theory**<br>Lie groups; Tangent vector; Lie bracket; Lie algebra; General linear and special linear groups and their Lie algebras; Exponential of matrix and its properties; Construction of partial differential equation; Linear differential operators; Group of operators; Extended forms of the group generated by the operators; Derivation of generating functions; Examples on above topics. | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ References Books** | 1. *E. D. Rainville: Special Functions, Chelsea Publishing Co., Bronx, New York, Reprint, 1971.<br>2. W. Jr. Miller: Lie Theory and Special Functions, Academic Press, New York and London, 1968.<br>3. E. B. McBride: Obtaining Generating Functions, Springer Verlag, Berlin Heidelberg, 1971. |

## DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. IV Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Non Commutative Rings** |
| **Course Number** | **MMM-4018** |
| **Credits** | **4** |
| **Course Category** | **Optional** |
| **Prerequisite Courses** | **Ring Theory** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The study of commutative rings constitutes the subject of commutative Algebra. In this course we focus on the noncommutative aspects of ring theory. We do not exclude commutative rings from our discussion. In the most of the cases the theorems Proved remain meaningful for commutative category. The main point, therefore is to find good notions and good tools to work with in the possible absence of commutativity, in order to develop a general theory of possibly noncommutative rings. The theorems that we prove are the attempts to extend results from the commutative setting to the general setting. |
| **Course Outcomes** | In a ring (possibly noncommutative ring) we add, subtract and multiply the elements but we cannot be able to divide one element by another. In a very natural sense most perfect objects in noncommutative ring theory are the division rings i.e. non zero rings in which each element has an inverse. From division rings we can build up matrix rings and form direct product of such matrix rings. Accordingly, to the Wedderburn-Artin Theorem, the rings obtained in this way comprise exactly the all important class of semisimple rings. This is one of the earliest and nicest complete classification of theorems in abstract Algebra and has served for decades as a model for many similar results in the structure theory of rings. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Basic Terminology and Examples**<br>Free k-rings, Rings with generators and relations, Twisted polynomial rings, Differential polynomial rings, Group rings, skew group rings. | **12** |
| **UNIT II: Semi-simple Rings**<br>Prime radical of a ring, Jacobson radical of a ring, Neotherian rings, Artinian rings, Simple rings, Semi-simple rings, Simple Artinian rings, Semi-simple Artinian rings. | **12** |
| **UNIT III: More on Semi-simple Rings**<br>Prime rings, Semiprime rings, Subdirectly irreducible rings, Primitive rings, Density Theorem, Wedderburn Artin's Theorem. | **12** |
| **UNIT IV: Commutativity Theorems**<br>Regular rings, Some commutativity Theorems, Wedderburn Theorem, Generalizations of Wedderburn Theorem. | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ References Books** | 1. *I. N. Herstein: Non-commutative Rings, John Wiley and Sons, INC.<br>2. T. Y. Lam: A First Course in Non-commutative Rings, Springer-Verlag. |

# DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY
## Syllabus of M.A./M.Sc. IV Semester approved in BOS: 01-08-2019

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|---|---|
| **Course Title** | **Variational Analysis and Optimization** |
| **Course Number** | **MMM-4020** |
| **Credits** | **4** |
| **Course Category** | **Optional** |
| **Prerequisite Courses** | **None** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | To give the concept from convex analysis, variational inequalities and optimization.<br>It gives the description of the application of convex analysis and variational inequalities to optimization. |
| **Course Outcomes** | A student will learn how to use the convex analysis and variational inequality technique to solve optimization problems. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Prerequisites of Convex Analysis**<br>Convex Set, Hyperplanes, Convex function and its characterizations, Generalized convex functions and their characterizations, Optimality criteria, Kuhn-Tucker optimality criteria. | **12** |
| **UNIT II: Subdifferentiability and Monotonicity**<br>Subgradients and subdifferentials, Monotone and generalized monotone maps, their generalizations and their relations with convexity. | **12** |
| **UNIT III: Classical Variational Inequalities**<br>Variational inequalities and related problems, Existence and uniqueness results, Solution methods. | **12** |
| **UNIT IV: Generalized Variational Inequalities**<br>Generalized variational inequalities and related topics, Basic existence and uniqueness results. | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ References Books** | *Q. H. Ansari, C. S. Lalitha and M. Mehta: Generalized Convexity, Nonsmooth Variational and Nonsmooth Optimization, CRC Press, Taylor and Francis Group, Boca Raton, London, New York, 2014. |

**DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY**

**Syllabus of M.A./M.Sc. IV Semester implemented w.e.f. Session 2020-21**

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|---|---|
| **Course Title** | **Advanced Discrete Mathematics** |
| **Course Number** | **MMM-4021** |
| **Credits** | **4** |
| **Course Category** | **Optional** |
| **Prerequisite Courses** | **Discrete Mathematics & Set Theory (UG level)** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | **This course aims to introduce students to the following concepts and cognitive skills:**<br>This course is divided into two major sections. In the first part of the course aims to introduce the concepts of partially ordered sets (posets), lattices and complete lattices, lattices as algebraic structures, modular and distributive lattices, Boolean algebras, Zessenhau's Lemma, Schreier's Refinement Theorem. The Isomorphism Theorem of Modular Lattices. A consensus of Fundamental products, Algorithm, Logic, Gates and Circuits, Boolean functions, and its truth table.<br>In the second part of the course the introduction to algebraic coding theory for channel capacity, source coding (data compression), error-detection and error-correction codes, linear block codes, cyclic codes and convolution codes, bounds for coding parameters; properties, coding and decoding of Hamming codes; Reed-Muller codes; BCH codes and Reed-Solomon codes for decoding.<br>The course discusses some important applications of lattices and coding theory in real-life situations. |
| **Course Outcomes** | **On successful completion of this course, students should be able to:**<br>• understand the partially ordered sets; their diagrams; maps between ordered sets; the duality principle; down-sets and up-sets; maximal and minimal elements; top and bottom; and building new ordered sets.<br>• lattices as ordered sets; complete lattices; chain conditions and completeness; and how to construct completions.<br>• How to form directed joins; how to construct algebraic closure operators; and how to complete partially ordered sets.<br>• Prove the existence of projective Intervals, Zessenhau's Lemma, Schreier's Refinement Theorem<br>• deal with lattices as algebraic structures; to form sublattices; products; homomorphisms and congruences.<br>• state and prove fundamental theorems about error-correcting codes given in the course,<br>• calculate the parameters of given codes and their dual codes using standard matrix and polynomial operations,<br>• encode and decode information by applying algorithms associated with well-known codes,<br>• compare the error-detecting/correcting facilities of given codes for a given binary symmetric channel,<br>• design simple linear or cyclic codes with required properties,<br>• solve mathematical problems involving error-correcting codes by linking them to concepts from elementary number theory, combinatorics, linear algebra, and elementary calculus. |
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| Content of Syllabus | | No. of Lectures |
|---|---|---|
| **UNIT I: Lattice Theory**<br>Partially Ordered Sets, Diagrams, Lower and Upper Bounds, Lattices, The Lattices Theoretical Duality Principle, Semi Lattices, Lattices as Partially Ordered Sets, Diagrams of Lattices, Sub Lattices, Lattice homomorphism, Axiom Systems of Lattices, Complete Lattices, Distributive Lattices, Modular Lattices, Characterization of Modular and Distributive Lattices, Similar Intervals, Projective Intervals, Zessenhau's Lemma, Schreier's Refinement Theorem, Independent Sets with properties, The Isomorphism Theorem of Modular Lattices. | | **12** |
| **UNIT II: Boolean Algebras**<br>Boolean Algebras, De Morgan Formulae, Complete Boolean Algebras, Boolean Algebras and Boolean Rings, The Algebra of Relations, Boolean Homomorphism, Representation Theorem, Boolean expression, Algorithm for finding sum-of-products form, Minimal sum-of-products, Consensus of Fundamental products, Algorithm, Logic, Gates and Circuits, Boolean functions and its truth table. | | **12** |
| **UNIT III: Linear Codes**<br>Brief introduction to coding theory, Linear codes, Hamming weight, Bases for linear codes, Generator matrix and Parity-check matrix, Equivalence of linear codes, Encoding with a linear code, Decoding of linear codes, Cosets, Nearest neighbor decoding for linear codes, Syndrome decoding. | | **12** |
| **UNIT IV: Cyclic Codes**<br>Definition of cyclic codes, Generator polynomials, Generator and parity-check matrices, Decoding of cyclic codes, Burst-error-correcting codes, BCH codes, Parameters of BCH codes, Decoding of BCH codes, Quaternary linear codes and their generator matrices. | | **12** |
| **Total No. of Lectures** | | **48** |
| **Text Books*/ References Books** | 1. *Nathan Jacobson: Lectures in Abstract Algebra, Vol. I- Basic Concepts.<br>2. Gabor Szasz: Introduction to Lattice Theory, Academic Press.<br>3. *Ling S. and C.P. Xing: Coding Theory: A First Course Cambridge University Press, Cambridge.<br>4. Zhe-Xian Wan: Quaternary codes, World Scientific, Publishing Company, Pvt. Ltd. | |

**DEPARTMENT OF MATHEMATICS, ALIGARH MUSLIM UNIVERSITY**

**Syllabus of M.A./M.Sc. IV Semester approved in BOS: 01-08-2019**

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|---|---|
| **Course Title** | **Elements of Elementary Calculus** |
| **Course Number** | **MMM-4091** |
| **Credits** | **4** |
| **Course Category** | **Open Elective** |
| **Prerequisite Courses** | **None** |
| **Contact Hours** | **4 Lectures & 1 Tutorial/week** |
| **Type of Course** | **Theory** |
| **Course Assessment** | **Sessional Tests 30%**<br>**Semester Examination 70%** |
| **Course Objectives** | The primary objective of this course is to introduce the basic tools of theory of differential and integral calculus with applications for those students which have not mathematical background at UG level. |
| **Course Outcomes** | **This course will enable the students:**<br>• to determine domain and range of real functions and to plot their graph.<br>• to calculate limit, continuity and differentiability.<br>• to provide the applications of calculus in real life. |

| **Content of Syllabus** | **No. of Lectures** |
|---|---|
| **UNIT I: Sets, Function and Limit**<br>Sets, and their properties, Functions and their properties, Some known functions, Domain, Range, Graph of Functions, Limit and its basic properties. | **12** |
| **UNIT II: Continuity and its Basic Properties**<br>Derivative (as rate of change and slope of a tangent), Properties of derivatives, Derivatives of some known functions, namely: polynomial, logarithmic functions, exponential functions, trigonometric functions. | **12** |
| **UNIT III: Applications of Derivatives**<br>Rate of change, Increasing and decreasing functions, maxima and minima of polynomials and trigonometric functions (first and second derivative test motivated geometrically), Simple problems (that illustrates basic problems and understanding of the subject as well as real life situations), Mean Theorem Functions. | **12** |
| **UNIT IV: Integration and its applications**<br>Indefinite integral, standard formulae of indefinite integral, Definite integral as a limit of sum, Basic properties and formulae of definite integral of simple functions (without proof), Applications finding the area of under simple curves, especially lines, area of circle, parabolas, ellipse (in standard form only). | **12** |
| **Total No. of Lectures** | **48** |

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|---|---|
| **Text Books*/ Reference Books** | 1. *Finney and Thomas: Calculus, Addison-Wesley Pub. Company.<br>2. *R. D. Sharma: Mathematics Vol. 1 and 2, Class 12, Dhanpat Rai and Sons.<br>3. I. A. Marun: Problems in Calculus of one variable, Arihant Publication. |