Page No.
Date: / /

MATRICES (1 + 3 + 5 = 9m)

1 marker : Definitions or construction of matrix
3 marker : Find values (p, q, r, x, y, z)
: A = P + Q (sum of symmetric & skew symmetric matrices)
: A^{-1} by elementary transformation
5 marker : Proofs on properties
$(AB)' = B'A'$
$A(BC) = (AB)C$
$A(B+C) = AB + AC$

Definition: Matrix is a rectangular arrangement of elements in rows and columns.
Horizontal elements form rows and vertical elements form columns.
Matrices are represented by capital alphabets A, B, C ... and the elements are enclosed in brackets like [ ], ( ), ⟨ ⟩, ‖ ‖.
General form of a matrix:
A_{mn} : matrix A with m rows and n columns

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & & \\ a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots a_{in} \\ \vdots & & & & \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

a_{ij} - element present in i^{th} row and j^{th} column.

1] Row matrix : A matrix which has a single row and is of order "$1 \times n$".
$A = \begin{bmatrix} 0 & 1 & -1 & 3 & 5 \end{bmatrix}_{1 \times 5}$

2] Column matrix : A matrix which has a single column and is of order "m x 1"

$$A = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3\times1}$$

3] Zero/ Null / Void matrix ('O') : A matrix of any order in which all the elements are equal to zero.

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times3}$$

4] Rectangular matrix : A matrix in which no of rows is not equal to no. of columns i.e, $m \neq n$.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & -1 & 2 \end{bmatrix}_{2\times4}$$

5] Square matrix : A matrix in which no. of rows is equal to no. of columns i.e, $m = n$

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2\times2} \rightarrow 2^{nd} \text{ order matrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}_{3\times3} \rightarrow 3^{rd} \text{ order matrix}$$

NOTE: Principal diagnol

$$A = \begin{bmatrix} 1 & 4 & 5 \\ -2 & 2 & 0 \\ -5 & 7 & 3 \end{bmatrix} \rightarrow pd$$

The diagnol in a matrix from left top to right bottom is called its principal diagnol. (The other diagnol is not considered for any property)

6] Diagnol matrix: It is a square matrix in which all the elements are equal to zero except the principal diagnol elements.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

7] Scalar matrix: It is a diagnol matrix in which all the principal diagnol elements are equal

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

8] Identity / Unit matrix (I): It is a scalar matrix in which all the principal diagnol elements are equal to 1 and is represented by "I".

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

NOTE: Transpose of a matrix A is obtained by interchanging rows and columns of matrix A and is denoted by A'.

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad A' = \begin{bmatrix} 1 & 2 & 7 \\ 3 & 4 & 8 \\ 5 & 6 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \end{bmatrix} \quad B' = \begin{bmatrix} 1 & 0 \\ 3 & 1 \\ 5 & 2 \\ 7 & 3 \end{bmatrix}$$

If order $(A) = m \times n$ then order $(A') = n \times m$

** $(AB)' = B'A'$ – Demorgan's Law

If element of A is a_{ij} then element of A' becomes a_{ji}

9] Symmetric matrix : A matrix in which $a_{ij} = a_{ji}$ i.e, $A' = A$

$$A = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 4 & -1 \\ 7 & -1 & 5 \end{bmatrix} \quad A' = \begin{bmatrix} 3 & 2 & 7 \\ 2 & 4 & -1 \\ 7 & -1 & 5 \end{bmatrix}$$

10] Skew Symmetric matrix : A matrix in which $a_{ij} = -a_{ji}$ i.e, $A' = -A$ and all the principal diagnol elements are equal to zero.

$$A = \begin{bmatrix} 0 & 1 & -7 \\ -1 & 0 & 5 \\ 7 & -5 & 0 \end{bmatrix} \quad A' = \begin{bmatrix} 0 & -1 & 7 \\ 1 & 0 & -5 \\ -7 & 5 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -7 \\ -1 & 0 & 5 \\ 7 & -5 & 0 \end{bmatrix}$$

$$= -A$$

11] Triangular matrix :

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 2 & 0 \\ 5 & 6 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

lower triangular matrix upper triangular matrix

Equality of matrices :

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(i) They are of same order

(ii) Each element of A is equal to the corresponding element of B i.e, $a_{ij} = b_{ij}$ for all i and j.

$$\begin{bmatrix} x & 3 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 1 & a \\ b & 2 \end{bmatrix}$$

$\Rightarrow x = 1, \ a = 3$

$b = 0, \ y = 2$

EXERCISE 3.1

1] In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$ write :

(i) order of matrix = 3×4

(ii) no. of elements = 12

iii] $a_{13} = 19$ $\quad a_{24} = 12$

$a_{21} = 35$ $\quad a_{23} = 5/2$

$a_{33} = -5$ $\quad a_{34} = 17$

**2] If a matrix has 24 elements, what are the possible orders it can have ? What, if it has 13 elements.

Sol] Given : 24 elements

Possible orders : 1×24, 24×1

2×12, 12×2

3×8, 8×3

4×6, 6×4

Given : 13 elements

Possible orders : 1×13, 13×1

**3] If a matrix has 18 elements, what are the possible orders it can have ? What, if it has 5 elements.

Sol] Given : 18 elements

Possible orders : 1×18, 18×1

2×9, 9×2

3×6, 6×3

Given : 5 elements

Possible orders : 1×5, 5×1

4] Construct 2×2 matrix $A = [a_{ij}]$ whose elements are given by;

(i) $a_{ij} = \frac{(i+j)^2}{2}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 2 & 9/2 \\ 9/2 & 8 \end{bmatrix}$$

(ii) $a_{ij} = \frac{i}{j}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$$

(iii) $a_{ij} = \frac{(i+2j)^2}{2}$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$$

5] Construct a 3×4 matrix, whose elements are given by:

(i) $a_{ij} = \frac{1}{2}|-3i+j|$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ \cancel{a_{41}} & \cancel{a_{42}} & \cancel{a_{43}} & \cancel{a_{44}} \end{bmatrix} = \begin{bmatrix} 1 & 1/2 & 0 & 1/2 \\ 5/2 & 2 & 3/2 & 1 \\ 4 & 7/2 & 3 & 5/2 \\ \cancel{11/2} & \cancel{5} & \cancel{11/2} & \cancel{4} \end{bmatrix}$$

(ii) $a_{ij} = 2i-j$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

6] Find the values of x, y, z from the following equations.

(i) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

$\Rightarrow x = 1,\ y = 4,\ z = 3$

(ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

$x + y = 6 \qquad 5 + z = 5 \Rightarrow \boxed{z = 0} \qquad xy = 8$

$y = 6 - x \qquad x(6-x) = 8$

$y = 6 - 2$ (or) $6 - 4 \qquad 6x - x^2 - 8 = 0$

$\boxed{y = 4, 2} \qquad x^2 - 6x - 8 = 0$

$(x-4)(x-2) = 0$

$\boxed{x = 2, 4}$

$\therefore\ x = 2,\ y = 4,\ z = 0$

$x = 4,\ y = 2,\ z = 0$

(iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

$\Rightarrow x + y + z = 9$

$x + z = 5 \qquad y + z = 7 \qquad x + z = 5$

$\Rightarrow y + 5 = 9 \qquad z = 7 - 4 \qquad x = 5 - 3$

$\boxed{y = 4} \qquad \boxed{z = 3} \qquad \boxed{x = 2}$

7] Find the value of a, b, c, d:

$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

$a - b = -1 \qquad \cancel{b = 1} \qquad 2a + c = 5 \qquad 3c + d = 13$

$\underline{-2a \mp b = -0} \qquad a - b = -1 \qquad c = 5 - 2 \qquad d = 13 - 9$

$-a = -1 \qquad -b = -1 - 1 \qquad \boxed{c = 3} \qquad \boxed{d = 4}$

$\boxed{a = 1} \qquad \boxed{b = 2}$

8] $A = [a_{ij}]_{m\times n}$ is a square matrix if
(a) $m < n$ (b) $m > n$ ~~(c)~~ $m = n$ (d) none

9] Which of the given values of x, y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

(a) $x = \frac{-1}{3}, y = 7$ (c) $y = 7, x = \frac{-2}{3}$
~~(b)~~ not possible (d) $x = \frac{-1}{3}; y = \frac{-2}{3}$

$3x + 7 = 0$ $2 - 3x = 4$
$\Rightarrow x = \frac{-7}{3}$ $x = \frac{-2}{3}$

x has two different values which is not valid.

**10] The no. of all possible matrices of order 3×3 with each entry 0 or 1 is
(a) 27 (c) 81
(b) 18 ~~(d)~~ 512

Ans] Every entry has two possibilities (0 or 1)
No. of elements = 9
$\therefore$ no. of all possible matrices $= 2^9 = 512$

*] Consider a 2×2 matrix whose elements
(i) $a_{ij} = \frac{(i-2j)^2}{2}$ (ii) $a_{ij} = \frac{|2i-3j|}{2}$

*] Construct a 2×3 matrix such that (a) $a_{ij} =$ $e^{ix}\sin jx$
Given: $a_{ij} = e^{ix}\sin jx$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} e^{x}\sin x & e^{x}\sin 2x & e^{x}\sin 3x \\ e^{2x}\sin x & e^{2x}\sin 2x & e^{2x}\sin 3x \end{bmatrix}$$

(b) $a_{ij} = |i-j|$ $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

*] Construct a 4×3 matrix whose elements are given by

(a) $a_{ij} = \begin{cases} i^2 & \text{if } i<j \\ \frac{i}{j} & \text{if } i=j \\ j^2 & \text{if } i>j \end{cases}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 4 \\ 1 & 4 & 1 \\ 1 & 4 & 9 \end{bmatrix}$$

(b) $a_{ij} = \begin{cases} ij-j & \text{if } i<j \\ \frac{i}{j} & \text{if } i=j \\ ij-i & \text{if } i>j \end{cases}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{41} & a_{42} & a_{43} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \\ 0 & 4 & 8 \end{bmatrix}$$

Operation of matrices:

Addition of matrices:

$A = \begin{bmatrix} \sqrt{3} & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 2 & \sqrt{5} & 1 \\ -2 & 3 & 1/2 \end{bmatrix}$

find (i) $A + 2B$ (ii) $2A - B$

(ii) $\begin{bmatrix} 2\sqrt{3} & 2 & -2 \\ 4 & 6 & 0 \end{bmatrix} - \begin{bmatrix} 2 & \sqrt{5} & 1 \\ -2 & 3 & 1/2 \end{bmatrix} = \begin{bmatrix} 2(\sqrt{3}-1) & 2-\sqrt{5} & -3 \\ 6 & 3 & -1/2 \end{bmatrix}$

(i) $\begin{bmatrix} \sqrt{3} & 1 & -1 \\ 2 & 3 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 2\sqrt{5} & 2 \\ -4 & 6 & 1 \end{bmatrix} = \begin{bmatrix} 4+\sqrt{3} & 1+2\sqrt{5} & 1 \\ -2 & 9 & 1 \end{bmatrix}$

To add or subtract two matrices, the matrices have to be of same order.
We add or subtract the corresponding elements of the two matrices.

Multiplication of a matrix by a scalar k:
To multiply a matrix A by a scalar k, every element of A is multiplied by k i.e, $kA = k[a_{ij}]_{m\times n}$
$= [k(a_{ij})]_{m\times n}$

Negative of a matrix.
Negative of a matrix A is −A.
$-A = (-1)A$, in which sign of every element of A is changed.

Properties of Matrix Addition:
i] Commutative Law: $A+B = B+A$
ii] Associative Law: $A+(B+C) = (A+B)+C$
iii] Existence of Identity: Null matrix 'O' is identity element under matrix addition
$\because A+O = O+A = A$
iv] Existence of Inverse: '−A' is additive inverse of '+A'
$\because -A+A = +A-A = O$

*] If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}$ find $2A-B$

$$\begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 5 & 3 \\ 5 & 6 & 0 \end{bmatrix}$$

*] If $A = \begin{bmatrix} 8 & 0 \\ 4 & -2 \\ 3 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 2 & -2 \\ 4 & 2 \\ -5 & 1 \end{bmatrix}$ find matrix X

such that $2A + 3X = 5B$

$$3X = 5B - 2A$$

$$= \begin{bmatrix} 10 & -10 \\ 20 & 10 \\ -25 & 5 \end{bmatrix} + \begin{bmatrix} -16 & 0 \\ -8 & 4 \\ -6 & -12 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & -10 \\ 12 & 14 \\ -31 & -7 \end{bmatrix}$$

$$X = \begin{bmatrix} -2 & -10/3 \\ 4 & 14/3 \\ -31/3 & -7/3 \end{bmatrix}$$

*] Find X and Y if $\underbrace{X + Y = \begin{bmatrix} 5 & 2 \\ 0 & 9 \end{bmatrix}}_{①}$, $\underbrace{X - Y = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix}}_{②}$

$$① + ② \Rightarrow 2X = \begin{bmatrix} 8 & 8 \\ 0 & 8 \end{bmatrix}$$

$$X = \begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix}$$

$$② \Rightarrow X - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix} = Y$$

$$\begin{bmatrix} 4 & 4 \\ 0 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix} = Y$$

*] Find the values of x & y :

$$2\begin{bmatrix} x & 5 \\ 7 & y-3 \end{bmatrix} + \begin{bmatrix} 3 & -4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ 15 & 14 \end{bmatrix}$$

$2x + 3 = 7$ $\qquad$ $2y - 6 + 2 = 14$

$2x = 4$ $\qquad$ $2y = 18$

$\boxed{x = 2}$ $\qquad$ $\boxed{y = 9}$

Pix Multiplication of matrices :

2 matrices A & B are compatible for multiplication iff no. of columns in A = no. of rows in B.

order (A) = $m \times n$

order (B) = $n \times p$

order (AB) = $m \times p$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 8 & 9 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} -3 & -4 \\ 11 & 16 \end{bmatrix}$$

$AB \neq BA$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 3 & 5 \\ 4 & 2 & 5 & 7 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 13 & 19 \\ 4 & 2 & 5 & 7 \end{bmatrix}$$

$$CD = \begin{bmatrix} 3 & 2 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 11 & 17 \\ 22 & 15 \end{bmatrix}$$

$$DC = \begin{bmatrix} 1 & 5 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 13 & 27 \\ 14 & 13 \end{bmatrix}$$

$CD \neq DC$

*] $A = \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1\times 4}$ $\quad B = \begin{bmatrix} 4 \\ 0 \\ -1 \\ 2 \end{bmatrix}_{4\times 1}$

$$AB = [4 + 0 - 3 + 8] = [9]$$

$$BA = \begin{bmatrix} 4 \\ 0 \\ -1 \\ 2 \end{bmatrix}_{4\times 1} \begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}_{1\times 4} = \begin{bmatrix} 4 & 8 & 12 & 16 \\ 0 & 0 & 0 & 0 \\ -1 & -2 & -3 & -4 \\ 2 & 4 & 6 & 8 \end{bmatrix}_{4\times 4}$$

$(AB)' = [9]' = [9]$

$B'A' = \begin{bmatrix} 4 & 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} = [4+0-3+8] = [9]$

$\Rightarrow (AB)' = B'A'$

*] $A = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}_{2\times 3}$ $B = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}_{3\times 2}$ Show that $AB \neq BA$

$AB = \begin{bmatrix} 2-8+6 & 3-10+3 \\ -8+8+10 & -12+10+5 \end{bmatrix} = \begin{bmatrix} 0 & -4 \\ 10 & 3 \end{bmatrix}$

$BA = \begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix} = \begin{bmatrix} 2-12 & -4+6 & 6+15 \\ 4-20 & -8+10 & 12+25 \\ 2-4 & -4+2 & 6+5 \end{bmatrix} = \begin{bmatrix} -10 & 2 & 21 \\ -16 & 2 & 37 \\ -2 & -2 & 11 \end{bmatrix}$

***] $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}$

Show that $(AB)C = A(BC)$

Sol] $AB = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}_{3\times 3} \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 1+0+1 & 3+2-4 \\ 2+0-3 & 6+0+12 \\ 3+0-2 & 9-2+8 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 18 \\ 1 & 15 \end{bmatrix}$

$(AB)C = \begin{bmatrix} 2 & 1 \\ -1 & 18 \\ 1 & 15 \end{bmatrix}_{3\times 2} \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}_{2\times 4} = \begin{bmatrix} 2+2 & 4+0 & 6-2 & -8+1 \\ -1+36 & -2+0 & -3-36 & 4+18 \\ 1+30 & 2+0 & 3-30 & -4+15 \end{bmatrix}$

$= \begin{bmatrix} 4 & 4 & 4 & -7 \\ 35 & -2 & -39 & 22 \\ 31 & 2 & -27 & 11 \end{bmatrix} \rightarrow (1)$

$$BC = \begin{bmatrix} 1 & 3 \\ 0 & 2 \\ -1 & 4 \end{bmatrix}_{3\times 2} \begin{bmatrix} 1 & 2 & 3 & -4 \\ 2 & 0 & -2 & 1 \end{bmatrix}_{2\times 4} = \begin{bmatrix} 1+6 & 2+0 & 3-6 & -4+3 \\ 0+4 & 0+0 & 0-4 & 0+2 \\ -1+8 & -2+0 & -3-8 & 4+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 2 & -3 & -1 \\ 4 & 0 & -4 & 2 \\ 7 & -2 & -11 & 8 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ 3 & -1 & 2 \end{bmatrix}_{3\times 3} \begin{bmatrix} 7 & 2 & -3 & -1 \\ 4 & 0 & -4 & 2 \\ 7 & -2 & -11 & 8 \end{bmatrix}_{3\times 4}$$

$$= \begin{bmatrix} 7+4-7 & 2+0+2 & -3-4+11 & -1+2-8 \\ 14+0+21 & 4+0-6 & -6-0-33 & -2+0+24 \\ 21-4+14 & 6+0-4 & -9+4-22 & -3-2+16 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 4 & 4 & -7 \\ 35 & -2 & -39 & 22 \\ 31 & 2 & -27 & 11 \end{bmatrix} \longrightarrow ②$$

$\therefore$ ① = ② $\Rightarrow (AB)C = A(BC)$

*] $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix}$ Show that $A^3 - 23A - 40I = 0$

Sol] $$A^2 = \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+6+12 & 2-4+6 & 3+2+3 \\ 3-6+4 & 6+4+2 & 9-2+1 \\ 4+6+4 & 8-4+2 & 12+2+1 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 4 & 8 \\ 1 & 12 & 8 \\ 14 & 6 & 15 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 3 & -2 & 1 \\ 4 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 19+12+32 & 38-8+16 & 57+4+8 \\ 1+36+32 & 2-24+16 & 3+12+8 \\ 14+18+60 & 28-12+30 & 42+6+15 \end{bmatrix}$$

$$= \begin{bmatrix} 63 & 46 & 69 \\ 69 & -6 & 23 \\ 92 & 46 & 63 \end{bmatrix} = A^3 \longrightarrow (B)$$

$$23A = 23\begin{bmatrix}1 & 2 & 3\\ 3 & -2 & 1\\ 4 & 2 & 1\end{bmatrix} = \begin{bmatrix}23 & 46 & 69\\ 69 & -46 & 23\\ 92 & 46 & 23\end{bmatrix} \longrightarrow ②$$

$$40I = 40\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}40 & 0 & 0\\ 0 & 40 & 0\\ 0 & 0 & 40\end{bmatrix} \longrightarrow ③$$

From ① & ②, ① − ② =

$$\begin{bmatrix}63 & 46 & 69\\ 69 & -6 & 23\\ 92 & 46 & 63\end{bmatrix} - \begin{bmatrix}23 & 46 & 69\\ 69 & -46 & 23\\ 92 & 46 & 23\end{bmatrix} = \begin{bmatrix}40 & 0 & 0\\ 0 & 40 & 0\\ 0 & 0 & 40\end{bmatrix} \longrightarrow ④$$

④ − ③ =

$$\begin{bmatrix}40 & 0 & 0\\ 0 & 40 & 0\\ 0 & 0 & 40\end{bmatrix} - \begin{bmatrix}40 & 0 & 0\\ 0 & 40 & 0\\ 0 & 0 & 40\end{bmatrix} = \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} = 0$$

*] Construct a 2×2 matrix whose elements are given by (i) $a_{ij} = \frac{(i-2j)^2}{2}$

$$A = \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} = \begin{bmatrix}1/2 & 9/2\\ 0 & 2\end{bmatrix}$$

(ii) $a_{ij} = \frac{|2i-3j|}{2}$

$$A = \begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix} = \begin{bmatrix}1/2 & 2\\ 1/2 & 1\end{bmatrix}$$

Q18 EXERCISE 3.2

1] $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}, C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

Find : (i) $A+B$ (ii) $A-B$ (iii) $3A-C$ (iv) AB (v) BA

(i) $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$

(iii) $\begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$

(iv) $AB = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$

(v) $BA = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$

2] (i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix} = 2\begin{bmatrix} a & b \\ 0 & a \end{bmatrix}$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix} = \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 5 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$

(iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

3] (i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}\begin{bmatrix} a & -b \\ b & a \end{bmatrix} = \begin{bmatrix} a^2+b^2 & 0 \\ 0 & a^2+b^2 \end{bmatrix} = a^2+b^2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= (a^2+b^2)I$

(ii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\begin{bmatrix} 2 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$

(v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}_{3\times 2}\begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}_{2\times 3} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$

(vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix}_{2\times 3}\begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}_{3\times 2} = \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$

**4] $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$

Verify: $A+(B-C) = (A+B)-C$

Sol] $A+(B-C) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & +2 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix} \rightarrow ①$

$(A+B)-C = \begin{bmatrix} 4 & 1 & -1 \\ 9 & +2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix} \rightarrow ②$

$\because ① = ②$

$\therefore A+(B-C) = (A+B)-C$

14, 5 H.W

6] Simplyfy $\cos\theta\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix} + \sin\theta\begin{bmatrix}\sin\theta & -\cos\theta\\ \cos\theta & \sin\theta\end{bmatrix}$

Sol] $\begin{bmatrix}\cos^2\theta & \cos\theta\sin\theta\\ -\cos\theta\sin\theta & \cos^2\theta\end{bmatrix} + \begin{bmatrix}\sin^2\theta & -\cos\theta\sin\theta\\ \cos\theta\sin\theta & \sin^2\theta\end{bmatrix}$

$= \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

7] (i) $X+Y = \begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix} \rightarrow ②$ $\quad X-Y = \begin{bmatrix}3 & 0\\ 0 & 3\end{bmatrix} \rightarrow ①$

$② + ① = 2X = \begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix} \Rightarrow X = \begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$

$① \Rightarrow Y = \begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix} - X = \begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix} - \begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix} = \begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$

(ii) $2X + 3Y = \begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix} \rightarrow ①$ $\quad 3X + 2Y = \begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix} \rightarrow ②$

$2X + 3Y = \begin{bmatrix}2 & 3\\ 4 & 0\end{bmatrix} \Big\} \times 3$

$3X + 2Y = \begin{bmatrix}2 & -2\\ -1 & 5\end{bmatrix} \Big\} \times 2$

$\Rightarrow \cancel{6X} + 9Y = \begin{bmatrix}6 & 9\\ 12 & 0\end{bmatrix}$

$\cancel{6X} + 4Y = \begin{bmatrix}4 & -4\\ -2 & 10\end{bmatrix}$

$\ominus$

$5Y = \begin{bmatrix}2 & 13\\ 14 & -10\end{bmatrix}$

$Y = \begin{bmatrix}2/5 & 13/5\\ 14/5 & -2\end{bmatrix}$

① ⇒ $2X = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - 3Y$

$2X = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 6/5 & 39/5 \\ 42/5 & -6 \end{bmatrix}$

$2X = \begin{bmatrix} 4/5 & -24/5 \\ -22/5 & 6 \end{bmatrix}$

$X = \begin{bmatrix} 2/5 & -12/5 \\ -11/5 & 3 \end{bmatrix}$

8] Find X, $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$, $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Sol] $2X = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

$= \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$

$X = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$

9] Find x, y if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Sol] $2x + 2 = 8$ $\qquad$ $2 + y = 5$

$x = \frac{6}{2} = 3$ $\qquad$ $y = 5 - 2 = 3$

$\boxed{x = 3}$ $\qquad$ $\boxed{y = 3}$

10] Solve: $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Sol] $2x + 3 = 9$ $\quad 2z - 3 = 15$ $\quad 2y + 0 = 12$

$2x = 6$ $\quad 2z = 18$ $\quad 2y = 12$

$\boxed{x = 3}$ $\quad \boxed{z = 9}$ $\quad \boxed{y = 6}$

$2t + 6 = 18$

$2t = 18 - 6$

$\boxed{t = 6}$

11] $x\begin{bmatrix} 2 \\ 3 \end{bmatrix} + y\begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$. find x, y.

Sol] $2x - \cancel{y} = 10$

$3x + \cancel{y} = 5$

$5x = 15$

$\boxed{x = 3}$

$6 - 10 = y$

$\boxed{y = -4}$

12] $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$, find x, y, z, w.

Sol] $3x = x + 4$ $\quad 3y = 6 + x + y$ $\quad 3z = -1 + z + w$

$2x = 4$ $\quad 2y = 8$ $\quad 2z = -1 + w$

$\boxed{x = 2}$ $\quad \boxed{y = 4}$ $\quad 2z = 2 \Rightarrow \boxed{z = 1}$

$3w = 2w + 3$

$\boxed{w = 3}$

13] $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$ ST $F(x) \cdot F(y) = F(x+y)$

$F(x) \cdot F(y) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y & +\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix} = F(x+y) = RHS$$

15] $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ +1 & -1 & 0 \end{bmatrix}$ Find $A^2 - 5A + 6I$.

Sol] $A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$

$5A = \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix}$

$6I = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$

$A^2 - 5A = \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix}$

$\begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = \begin{bmatrix} +1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$

16] $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$ ST : $A^3 - 6A^2 + 7A + 2I = O$

Sol] $A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$

$A^3 = A^2(A) = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$

$6A^2 = \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix}$ $7A = \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix}$

$2I = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

LHS $A^3 - 6A^2 + 7A + 2I = 0$

$\begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \theta = \text{RHS}$

17] $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ find k so that $A^2 = kA - 2I$

Sol] LHS : $A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$

RHS : $kA - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = kA - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$kA = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$

$k = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Eg 21] $A = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix}_{3\times 1}$ $B = \begin{bmatrix} 1 & 3 & -6 \end{bmatrix}_{1\times 3}$, verify $(AB)' = B'A'$

Sol] $AB = \begin{bmatrix} -2 \\ 4 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 3 & -6 \end{bmatrix} = \begin{bmatrix} -2 & -6 & 12 \\ 4 & 12 & -24 \\ 5 & 15 & -30 \end{bmatrix}$

$(AB)' = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix} \rightarrow (1)$

$B' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix}$ $A' = \begin{bmatrix} -2 & 4 & 5 \end{bmatrix}$

$\therefore B'A' = \begin{bmatrix} 1 \\ 3 \\ -6 \end{bmatrix} \begin{bmatrix} -2 & 4 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 4 & 5 \\ -6 & 12 & 15 \\ 12 & -24 & -30 \end{bmatrix} \rightarrow (2)$

$\because (1) = (2)$

$\therefore (AB)' = B'A'$

5] $A = \begin{bmatrix} 2/3 & 1 & 5/3 \\ 1/3 & 2/3 & 4/3 \\ 7/3 & 2 & 2/3 \end{bmatrix}$ $B = \begin{bmatrix} 2/5 & 3/5 & 1 \\ 1/5 & 2/5 & 4/5 \\ 7/5 & 6/5 & 2/5 \end{bmatrix}$. Find : $3A - 5B$

Sol] $3A = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$

$5B = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$

$\therefore 3A - 5B = \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

14] (i) Show that $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

Sol] $\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix} \neq \begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

Sol] $\begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \neq \begin{bmatrix} -1 & -1 & -3 \\ 1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$

EXERCISE 3.3

1] Find transpose:

(i) $A = \begin{bmatrix} 5 \\ 1/2 \\ -1 \end{bmatrix}_{3\times1}$ $A' = \begin{bmatrix} 5 & 1/2 & -1 \end{bmatrix}_{1\times3}$

NOTE: (i) If order $(A) = m\times n$ then order $(A') = n\times m$

(ii) $(A')' = A$

(iii) $(kA)' = kA'$

(iv) $(A+B)' = A'+B'$

(v) $(AB)' = B'A'$

**(vi) Every square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

(ii) $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ $A' = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$ $A' = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

2] $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ Verify that

(i) $(A+B)' = A' + B'$ (ii) $(A-B)' = A' - B'$

Sol] i] $A+B = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

$(A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$

$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ $B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$A' + B' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix}$ $\therefore (A+B)' = A' + B'$

ii] $A-B = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$ $(A-B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ $B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$A' - B' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix}$ $\therefore (A-B)' = A' - B'$

3] $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ then verify

(i) $(A+B)' = A' + B'$ (ii) $(A-B)' = A' - B'$

Sol] i] $A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$A + B = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$ $(A+B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$

$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$ $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$

$A' + B' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix}$ $\therefore (A+B)' = A' + B'$

ii] $A - B = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$ $(A-B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

$A' - B' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ $\therefore (A-B)' = A' - B'$

5] Prove: $(AB)' = B'A'$

i] $A = \begin{bmatrix} 1 \\ -4 \\ 5 \end{bmatrix}_{3\times1}$ $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}_{1\times3}$

$AB = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -5 & 10 & 5 \end{bmatrix}$ $(AB)' = \begin{bmatrix} -1 & 4 & -5 \\ 2 & -8 & 10 \\ 1 & -4 & 5 \end{bmatrix}$

$B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$ $A' = \begin{bmatrix} 1 & -4 & 5 \end{bmatrix}$

$B'A' = \begin{bmatrix} -1 & 4 & -5 \\ 2 & -8 & 10 \\ 1 & -4 & 5 \end{bmatrix}$ $\therefore (AB)' = A'B'$

ii] $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$ $(AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$

$B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$ $A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$

$B'A' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix}$ $\therefore (AB)' = A'B'$

4] $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$ Find $(A+2B)'$

Sol] $A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$ $2B = \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$

$A + 2B = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

$(A+2B)' = \begin{bmatrix} -4 & 5 \\ 1 & 6 \end{bmatrix}$

6] Verify that $A'A = I$

i] $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ $A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

NOTE: For any square matrix, if $A'A = I \Rightarrow A$ is orthogonal matrix

ii] $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$A'A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

7] (i) Show that $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is symmetric.

Symmetric $\Rightarrow A = A'$

$A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$

$\because A = A' \Rightarrow A$ is a symmetric matrix.

(ii) Show that $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is skew symmetric matrix.

$A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$

8] $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ show that

(i) $(A+A')$ is symmetric

(ii) $(A-A')$ is skew symmetric

Sol] i] $A = \begin{bmatrix} 1 & 5 \\ -6 & 7 \end{bmatrix}$ $A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$A + A' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

Consider $(A+A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} = A + A'$

$\because (A+A')' = A+A'$, $A+A'$ is symmetric.

ii] $A - A' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

Consider $(A-A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} = -(A-A')$

$\therefore (A-A')' = -(A-A')$

$\therefore$ $A-A'$ is skew symmetric.

(3m) ** 10] Express the following as the sum of symmetric and skew symmetric matrix

Given: $B = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ $B' = \begin{bmatrix} 2 & -1 & 1 \\ -2 & 3 & -2 \\ -4 & 4 & -3 \end{bmatrix}$

$P = \frac{1}{2}(B+B') = \frac{1}{2}\begin{bmatrix} 4 & -3 & -3 \\ -3 & 6 & 2 \\ -3 & 2 & -6 \end{bmatrix} = \begin{bmatrix} 2 & -3/2 & -3/2 \\ -3/2 & 3 & 1 \\ -3/2 & 1 & -3 \end{bmatrix}$

$P' = \begin{bmatrix} 2 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 3 & 1 \\ -\frac{3}{2} & 1 & -3 \end{bmatrix} = P$

$\Rightarrow P$ is symmetric

$Q = \frac{1}{2}(B-B') = \frac{1}{2}\begin{bmatrix} 0 & -1 & -5 \\ 1 & 0 & 6 \\ 5 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix}$

$$Q' = \begin{bmatrix} 0 & \frac{1}{2} & \frac{5}{2} \\ -\frac{1}{2} & 0 & -3 \\ -\frac{5}{2} & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix} = -Q$$

$Q' = -Q \Rightarrow Q$ is skew symmetric

Consider:

$$P+Q = \begin{bmatrix} 2 & -\frac{3}{2} & -\frac{3}{2} \\ -\frac{3}{2} & 3 & 1 \\ -\frac{3}{2} & 1 & -3 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & 0 & 3 \\ \frac{5}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = B$$

$\therefore$ Given matrix has been expressed as sum of a symmetric and a skew symmetric matrix.

i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A \qquad A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

$$P = \frac{1}{2}(A+A') = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$P' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} = P \Rightarrow P$ is symmetric

$$Q = \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = -Q$$

$Q' = -Q \Rightarrow Q$ is skew symmetric

Consider:

$$P+Q = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$$

$\therefore$ Given matrix has been expressed as sum of a symmetric and a skew symmetric matrix.

iv] $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = A \qquad A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

$P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$

$P' = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} = P \quad \Rightarrow P$ is symmetric

$Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = -Q$

$Q' = -Q \quad \Rightarrow Q$ is skew symmetric

Consider

$P + Q = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

iii] $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} \qquad A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

$P = \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$

$P' = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} = P \quad \Rightarrow P$ is symmetric

$Q = \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix} = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix}$

$Q' = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} = -Q$

$Q' = -Q \Rightarrow Q$ is skew symmetric

Consider

$$P+Q = \begin{bmatrix} 3 & \frac{1}{2} & -\frac{5}{2} \\ \frac{1}{2} & -2 & -2 \\ -\frac{5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ -\frac{5}{2} & 0 & 3 \\ -\frac{3}{2} & -3 & 0 \end{bmatrix} = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

ii] $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$ $\quad A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$$P = \frac{1}{2}(A+A') = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$P' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = P \Rightarrow P$ is symmetric

$$Q = \frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Q' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = -Q$$

$Q' = -Q \Rightarrow Q$ is skew symmetric

Consider,

$$P+Q = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$$

classmate
Date
Page

9] Find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, when

$$A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

Sol] $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ $A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

$$\frac{1}{2}(A+A') = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\frac{1}{2}(A-A') = \frac{1}{2}\begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix} = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

(3m) ** Invertible Matrices

If A is a square matrix of order m and if there exists another square matrix B of same order m, such that $AB = BA = I$, then B is called the inverse matrix of A and is denoted by A^{-1}.

Properties under matrix multiplication:

Associative property: matrix X^n is always ass've
$A(BC) = (AB)C$

Distributive property: $A(B+C) = AB + AC$

** Commutative property: In general, matrix X^n is not commutative i.e., $AB \neq BA$

Existence of Identity:
Unit/Identity matrix 'I' is multiplicative identity in matrices
$\because AI = IA = A$

Existence of Inverse:

(a) If $AB = I$ then $A^{-1} = B$ and $B^{-1} = A$

(b) If A is singular matrix then $A^{-1} = \frac{1}{|A|}(\text{adj } A)$

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then

(a) $|A| = ad - bc$

(b) $\text{adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(c) $A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Find the inverse of the following:

(1) $A = \begin{bmatrix} 2 & 3 \\ -5 & 1 \end{bmatrix}$

$|A| = 2-(-15) = 17$

$\text{adj } A = \begin{bmatrix} 1 & -3 \\ 5 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{17}\begin{bmatrix} 1 & -3 \\ 5 & 2 \end{bmatrix}$

(2) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $|A| = 4-6 = -2$

$\text{adj } A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

** (3) $A = \begin{bmatrix} 1 & 6 \\ 2 & 12 \end{bmatrix}$ $|A| = 12-12 = 0$

$\Rightarrow$ A is singular matrix

$\therefore A^{-1}$ does not exist

(4) $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $|A| = 4-4 = 0$

A^{-1} does not exist

(5) $A = \begin{bmatrix} 5 & -6 \\ 4 & 3 \end{bmatrix}$ $|A| = 15-(-24) = 39$

$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{39}\begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix}$

(6) $A = \begin{bmatrix} -1 & 3 \\ 5 & -7 \end{bmatrix}$ $|A| = 7-15 = -8$

$A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-8}\begin{bmatrix} -7 & -3 \\ -5 & -1 \end{bmatrix}$

$= \frac{1}{8}\begin{bmatrix} 7 & 3 \\ 5 & 1 \end{bmatrix}$

(7) $A = \begin{bmatrix} 1 & 2 \\ -1 & 0 \end{bmatrix}$ $|A| = 0-(-2) = 2$

$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{2}\begin{bmatrix} 0 & -2 \\ 1 & 1 \end{bmatrix}$

(8) $A = \begin{bmatrix} 4 & 3 \\ 2 & 7 \end{bmatrix}$ $|A| = 28-6 = 22$

$A^{-1} = \frac{1}{22}\begin{bmatrix} 7 & -3 \\ -2 & 4 \end{bmatrix}$

(9) $A = \begin{bmatrix} 10 & 12 \\ -14 & -15 \end{bmatrix}$ $|A| = -150-(-168) = 18$

$A^{-1} = \frac{1}{18}\begin{bmatrix} -15 & -12 \\ 14 & 10 \end{bmatrix}$

(10) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $|A| = 1-0 = 1$

$A^{-1} = 1\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

NOTE: Properties of 'I'

(a) $I^n = I$

(b) $I^{-1} = I$

(11) $A = \begin{bmatrix} 1 & -1 \\ 2 & -4 \end{bmatrix}$ $|A| = -4-(-2) = -2$

$A^{-1} = \frac{1}{-2}\begin{bmatrix} -4 & 1 \\ -2 & 1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 4 & -1 \\ 2 & -1 \end{bmatrix}$

Find the inverses of the following by elementary operation

NOTE: (a) Interchange of 2 rows or 2 columns is allowed

$R_i \leftrightarrow R_j \,/\, C_i \leftrightarrow C_j$

(b) Scalar X" or Scalar division of any row or column is allowed i.e, kR_i or $\frac{R_i}{k}$ / kC_i or $\frac{C_i}{k}$

(c) A multiple of one row can be added or subtracted to another row and vice versa.

ie, $R_i \to R_i + kR_j \,/\, C_i \to C_i - kC_j$

(d) To get inverse, we either follow only row transformations or only column transformations.

(e) If row transformation is followed, consider

(i) $A = IA$

(ii) Use row transformations to convert A in LHS to I, and in the process we get $I = BA$

Hence, $A^{-1} = B$.

$A = IA$

$$\begin{bmatrix} 1 & b \\ c & d \end{bmatrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & b \\ 0 & d \end{bmatrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = BA$$

(f) If column transformation, consider

$A = AI$

$$\begin{bmatrix} 1 & \\ & 1 \end{bmatrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 \\ & \end{bmatrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 \\ & 1 \end{bmatrix}$$

$\downarrow$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I = AB$$

$A^{-1} = B$

EXERCISE 3.4

Find the inverses of each matrices :

1] $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

$A = IA$

$\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$R_2 \to R_2 - 2R_1$

$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

$\div R_2$ by 5

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$

$R_1 \to R_1 + R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$

$I = BA$

$\therefore A^{-1} = B = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

8] $A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

$A = IA$

$\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$R_1 \to R_1 - R_2$

$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

$R_2 \to R_2 - 3R_1$

$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A$$

$I = BA$

$\Rightarrow A^{-1} = B = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$

6] $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

$A = IA$

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$$

$R_1 \to R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$$

$I = BA$

$A^{-1} = B = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$

2] $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

$A = IA$

$$\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix} = \begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix} A$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix} A$$

$$I = BA$$

$$A^{-1} = B = \begin{bmatrix}1 & -1\\ -1 & 2\end{bmatrix}$$

3] $A = \begin{bmatrix}1 & 3\\ 2 & 7\end{bmatrix}$ $\qquad A^{-1} = \dfrac{1}{|A|}\,\text{adj}\,A = \dfrac{1}{1}\begin{bmatrix}7 & -3\\ -2 & 1\end{bmatrix}$

$$A = IA$$

$$\begin{bmatrix}1 & 3\\ 2 & 7\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix}1 & 3\\ 0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\ -2 & 1\end{bmatrix} A$$

$R_1 \to R_1 - 3R_2$

$$\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} = \begin{bmatrix}7 & -3\\ -2 & 1\end{bmatrix} A$$

$$I = BA$$

$$A^{-1} = B = \begin{bmatrix}7 & -3\\ -2 & 1\end{bmatrix}$$

** 4] $A = \begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix}$ $\qquad A^{-1} = \dfrac{1}{|A|}\,\text{adj}\,A = -\dfrac{1}{1}\begin{bmatrix}7 & -3\\ -5 & 2\end{bmatrix} = \begin{bmatrix}-7 & 3\\ 5 & -2\end{bmatrix}$

$$A = AI$$

$$\begin{bmatrix}2 & 3\\ 5 & 7\end{bmatrix} = A\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$\cancel{R_2 \leftrightarrow R_2}$ $C_2 \to C_2 - C_1$

$$\begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$C_1 \leftrightarrow C_2$

$$\begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix} = A\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

$C_2 \to C_2 - 2C_1$

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = A\begin{bmatrix} -1 & 3 \\ 1 & -2 \end{bmatrix}$$

$C_1 \to C_1 - 2C_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

$I = AB$

$$A^{-1} = B = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$$

5] $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$ $\qquad A^{-1} = \frac{1}{|A|}\, adj A = \frac{1}{1}\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$

$A = AI$

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$C_1 \to C_1 - C_2$

$$\begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$C_2 \to C_2 - C_1$

$$\begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$C_1 \to C_1 - 3C_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

$I = AB$

$$A^{-1} = B = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

7] $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

$A^{-1} = \frac{1}{|A|} \text{adj} A = \frac{1}{1}\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

$A = AI$

$\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$C_1 \leftrightarrow C_2$

$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} = A\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$C_2 \to C_2 - 3C_1$

$\begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix} = A\begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$

$C_2 \to C_2/-1$

$\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = A\begin{bmatrix} 0 & -1 \\ 1 & 3 \end{bmatrix}$

$C_1 \to C_1 - 2C_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

$I = AB$

$A^{-1} = B = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$

9] $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

$A = IA$

$\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}A$

$R_1 \to R_1 - R_2$

$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}A$

$R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A$$

$R_1 \to R_1 - 3R_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$$

$I = BA$

$$A^{-1} = B = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

10] $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$ $\qquad A^{-1} = \frac{1}{2}\begin{bmatrix} 2 & 1 \\ 4 & 3 \end{bmatrix}$

$A = AI$

$$\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$C_1 \leftrightarrow C_2$

$$\begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix} = A\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$-4 = 3(-2)$

$-4+6 = 2$

$C_1 \to (-1)C_1$

$$\begin{bmatrix} 1 & 3 \\ -2 & -4 \end{bmatrix} = A\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$C_2 \to C_2 - 3C_1$

$$\begin{bmatrix} 1 & 0 \\ -2 & +2 \end{bmatrix} = A\begin{bmatrix} 0 & 1 \\ -1 & 3 \end{bmatrix}$$

$C_2 \to C_2/2$

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = A\begin{bmatrix} 0 & \frac{1}{2} \\ -1 & \frac{3}{2} \end{bmatrix}$$

$C_1 \to C_1 + 2C_2$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

$\therefore I = AB$

$$A^{-1} = B = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$

11] $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$ $A^{-1} = \frac{1}{2}\begin{bmatrix} -2 & 6 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$

$A = IA$

$\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$R_1 \to R_1 - R_2$

$\begin{bmatrix} 1 & -4 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

$R_2 \to R_2 - R_1$

$\begin{bmatrix} 1 & -4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$

$R_2 \to R_2/2$

$\begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -\frac{1}{2} & 1 \end{bmatrix} A$

$R_1 \to R_1 + 4R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} A$

$I = BA$

$\therefore A^{-1} = B = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$

12] $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

$|A| = 6 - 6 = 0$

$\Rightarrow$ A is a sigular matrix

$\therefore$ Inverse of A does not exist

13] $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ $A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

$A = IA$

~~$R_1 \to R_1 + R_2$~~

$\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$R_1 \to R_1 + R_2$

$\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$

$R_2 \to R_2 + R_1$

$\begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$

$R_1 \to R_1 + R_2$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$

$I = BA$

$\therefore A^{-1} = B = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

14] $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

$|A| = 4 - 4 = 0$

$\Rightarrow$ A is a singular matrix

$\therefore$ Inverse of matrix A does not exist

15] $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = A$

$A = IA$

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 3 \\ 2 & -3 & 3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} A$$

$R_2 \to R_2 - 2R_1$; $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 5 \\ 0 & -5 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 0 & -2 \end{bmatrix} A$$

$R_3 \to R_3 / -5$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 5 \\ 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -2 \\ -\frac{3}{5} & 0 & \frac{2}{5} \end{bmatrix} A$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -\frac{3}{5} & 0 & \frac{2}{5} \\ 2 & 1 & -2 \end{bmatrix} A$$

$R_1 \to R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{3}{5} & 0 & \frac{2}{5} \\ 2 & 1 & -2 \end{bmatrix} A$$

$R_3 \to R_3/5$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{3}{5} & 0 & \frac{2}{5} \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A$$

$R_2 \to R_2 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix} A$$

$I = BA$

$$A^{-1} = B = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ \frac{2}{5} & \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

17] $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$A = AI$

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_1 \to C_1 + C_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 5 & 1 & 0 \\ +3 & 1 & 3 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$C_3 \to C_3 + C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 5 \\ 3 & 1 & 6 \end{bmatrix} = A \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ +1 & 0 & 2 \end{bmatrix}$$

$C_1 \to C_1 - C_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 5 \\ -3 & 1 & 6 \end{bmatrix} = A \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 2 \end{bmatrix}$$

$C_3 \to C_3 - 5C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 1 & 1 \end{bmatrix} = A \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -5 \\ -1 & 0 & 2 \end{bmatrix}$$

$C_2 \to C_2 - C_3$; $C_1 \to C_1 + 3C_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$I = AB$

$$A^{-1} = B = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

16] $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

$A = IA$

$$\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2 + 3R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

$R_2 \to R_2/9$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & -\frac{11}{9} \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

$R_1 \to R_1 - 3R_2$

$$\begin{bmatrix} 1 & 0 & \frac{15}{9} \\ 0 & 1 & -\frac{11}{9} \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & \frac{15}{9} \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & \frac{25}{9} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{5}{3} & \frac{1}{9} & 1 \end{bmatrix} A$$

$R_3 \to R_3\left(\frac{9}{25}\right)$

$$\begin{bmatrix} 1 & 0 & \frac{15}{9} \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{3} & 0 \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$R_1 \to R_1 - \left(\frac{15}{9}\right)R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{11}{9} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ \frac{1}{3} & \frac{1}{9} & 0 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$R_2 \to R_2 + \left(\frac{11}{9}\right)R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$I = BA$

$$\therefore A^{-1} = B = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

Mis Eg28) $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$

Find matrix D such that $CD - AB = O$

Sol) D is of order $2 \times 2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\therefore \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2a+5c & 2b+5d \\ 3a+8c & 3b+8d \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2a+5c-3 & 2b+5d-0 \\ 3a+8c-43 & 3b+8d-22 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\therefore$ $2a + 5c - 3 = 0 \longrightarrow (1)$

$2b + 5d = 0 \longrightarrow (2)$

$3a + 8c - 43 = 0 \longrightarrow (3)$

$3b + 8d - 22 = 0 \longrightarrow (4)$

Consider (3) − (1)

$3a + 8c = 43$

$\ominus 2a \pm 5c \ominus 3$

$a + 3c = 40$

$\Rightarrow a = 40 - 3c \longrightarrow (5)$

Substitute (5) in (1)

$2a + 5c = 3$

$2(40 - 3c) + 5c = 3$

$80 - 6c + 5c = 3$

$80 - 3 = c$

$\boxed{c = 77} \longrightarrow (6)$

Substitute (6) in (5)

$a = 40 - 3c$

$a = 40 - 3(77)$

$a = 40 - 231$

$\boxed{a = -191}$

Consider ④ – ②

$3b + 8d = 22$

$\ominus 2b \pm 5d \mathrel{\ominus}= 0$

$b + 3d = 22$

$\Rightarrow b = 22 - 3d \rightarrow ⑦$

Substitute ⑦ in ②

$2b + 5d = 0$

$2(22-3d) + 5d = 0$

$44 - 6d + 5d = 0$

$44 - d = 0$

$\boxed{d = 44} \rightarrow ⑧$

Substitute ⑧ in ⑦

$b = 22 - 3d$

$b = 22 - 3(44)$

$b = 22 - 132$

$\boxed{b = -110}$

$\therefore \quad D = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$

MISCELLANEOUS EXERCISE

7] For what values of x : $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$?

LHS =

Sol] $\begin{bmatrix} 6 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 4 + 4x$

$4 + 4x = 0$

$4x = -4$

$\boxed{x = -1}$

8] $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = O$

Sol] $A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & \cancel{6}5 \\ -5 & 3 \end{bmatrix}$

$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$7I = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$\therefore A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O = RHS$

9] Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = O$?

Sol] LHS $= \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix}\begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} x^2 - 2x + (-40) + 2x - 8 \end{bmatrix}$

$\cancel{x^2 - 2x - 40}$ $x^2 - \cancel{2x} - 40 + \cancel{2x} - 8 = 0$

$x^2 - 48 = 0$

$x = \sqrt{48}$

$x = \pm 4\sqrt{3}$

11] Find matrix X so that $X \underbrace{\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}}_{Z}$

Sol] order $(X) = m \times n$ $\therefore (m \times n)(2 \times 3) = (2 \times 3)$

order $(Y) = 2 \times 3$ According to multiplication of matrix

order $(Z) = 2 \times 3$ $m = 2$ and

$n = 2$

$\therefore X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

$\Rightarrow a+4b = -7 \rightarrow (1)$

$c+4d = 2 \rightarrow (2)$

$2a+5b = -8 \rightarrow (3)$

$2c+5d = 4 \rightarrow (4)$

Consider (3) − (1)

$2a + 5b = -8$

$a + 4b = -7$

$a + b = -1$

$a = -1 - b \rightarrow (5)$

Substitute (5) in (1)

$-1 - b + 4b = -7$

$-1 + 3b = -7$

$3b = -6$

$\boxed{b = -2} \rightarrow (6)$

Substitute (6) in (5)

$a = -1 - (-2)$

$\boxed{a = 1}$

Consider (4) − (2)

$2c + 5d = 4$

$c + 4d = 2$

$c + d = 2$

$c = 2 - d \rightarrow (7)$

Substitute (7) in (2)

$2 - d + 4d = 2$

$2 + 3d = 2$

$3d = 0$

$\boxed{d = 0} \rightarrow (8)$

Substitute (8) in (7)

$c = 2 - 0$

$\boxed{c = 2}$

$\therefore\ X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$

Eg 25] Find P^{-1} if it exists, given $P = \begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$

Sol] $|P| = 10 - 10 = 0$

$\Rightarrow P$ is singular matrix

$\therefore P^{-1}$ does not exist.

Eg 24]. Obtain the inverse of $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$

Sol] $A = IA$

$$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$R_3 \rightarrow R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

$R_1 \rightarrow R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} A$$

$R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} A$$

$R_3 \to R_3 \cancel{R_1} \div 2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A$$

$R_2 \to R_2 - 2R_3$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A$$

$R_1 \to R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix} A$$

$I = BA$

$$A^{-1} = B = \begin{bmatrix} \frac{1}{2} & \frac{-1}{2} & \frac{1}{2} \\ -4 & 3 & -1 \\ \frac{5}{2} & \frac{-3}{2} & \frac{1}{2} \end{bmatrix}$$

Eg 11] 2 farmers R & S cultivate 3 varieties of rice namely A, B, C. The sale of these variaties by bot[h] in month of Sept & Oct are given by the following matrices.

Sept sales in ₹ Oct sales in ₹

$$X = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 10000 & 20000 & 30000 \\ 50000 & 30000 & 10000 \end{bmatrix} \end{array} \begin{array}{c} R \\ S \end{array} \quad Y = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix} \end{array} \begin{array}{c} R \\ S \end{array}$$

(i) Find the combined sales in Sept & Oct for each farmer in each variety.

Ans: Combined sales $= X + Y = \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 15000 & 30000 & 36000 \\ 70000 & 40000 & 20000 \end{bmatrix} \end{array} \begin{array}{c} R \\ S \end{array}$

(ii) Find the decrease in sales from Sept to Oct.

Ans: Decrease in sales $= X - Y = \begin{array}{c|ccc} & A & B & C \\ R & 5000 & 10000 & 24000 \\ S & 30000 & 20000 & 0 \end{array}$

(iii) If both farmers receive 2% profit, compute the profit for each farmer, for each variety sold in Oct.

$2\% = \frac{2}{100} = 0.02$

Profit obtained from Oct sales $= 0.02(Y)$

$= 0.02 \begin{bmatrix} 5000 & 10000 & 6000 \\ 20000 & 10000 & 10000 \end{bmatrix}$

$= \begin{bmatrix} 100 & 200 & 1200 \\ 400 & 200 & 200 \end{bmatrix}$

Eg 19] In a LA election, A political grp hired a PRF to promote its candidates in 3 ways – telephone, house calls, letters. The cost per contact in paise is given in matrix A as

$A = \begin{array}{c|c} \text{cost per contact} & \\ \hline 40 & \text{telephone} \\ 100 & \text{house call} \\ 50 & \text{letter} \end{array}$

The no. of contacts of each type made in two cities X & Y is given by

$B = \begin{array}{c|ccc} & \text{telephone} & \text{house call} & \text{letter} \\ X & 1000 & 500 & 5000 \\ Y & 3000 & 1000 & 10000 \end{array}$

Find the total amount spent by the group in 2 cities X & Y.

Total money spent $= BA$

$= \begin{bmatrix} 1000 & 500 & 5000 \\ 3000 & 1000 & 10000 \end{bmatrix} \begin{bmatrix} 40 \\ 100 \\ 50 \end{bmatrix}$

$= \begin{bmatrix} 340000 \\ 720000 \end{bmatrix}$

Amount spent on X = 340000 paise

Y = 720000 paise.

Theorem 1: For any square matrix A with real no. entries, $A+A'$ is a symmetric matrix and $A-A'$ is a skew symmetric matrix

(a) TPT : $P = A + A'$ is symmetric $\Rightarrow P' = P$

$P' = (A+A')'$ $\quad [(A+B)' = A'+B']$

$= A' + (A')'$ $\quad [(A')' = A]$

$P' = A' + A$

$P' = A + A'$ $\quad (A+B = B+A)$

$P' = P$

$\therefore P$ is symmetric

i.e, $A+A'$ is symmetric.

(b) $Q = A - A'$ is skew symmetric

i.e, $Q' = -Q$

$Q' = (A-A')'$

$Q' = A' - (A')'$

$= A' - A$

$= -(A - A')$

$Q' = -Q$

Theorem 2: Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

** Theorem 3: (Uniqueness of Inverse) : Inverse of a square matrix if it exists is unique.

Proof: Let $A = [a_{ij}]$ be a square matrix of order m

If possible let B & C be two inverses of A.

TPT : $B = C$

Since B is inverse of A : $AB = BA = I \rightarrow (1)$

Since C is inverse of A = $AC = CA = I \rightarrow (2)$

$B = BI$

$= B(AC)$ [from (2)]

$= (BA)C$ (Associative axiom)

$B = IC$

$B = C$

$\Rightarrow A$ has unique inverse.

**Theorem 4: If A and B are invertible matrices of same order, then $(AB)^{-1} = B^{-1}A^{-1}$

TPT : $(AB)^{-1} = B^{-1}A^{-1}$

Proof : $(AB)(AB)^{-1} = I$

Premultiply by A^{-1} on both sides

$A^{-1}(AB)\cdot(AB)^{-1} = A^{-1}I$

$(A^{-1}A)B\cdot(AB)^{-1} = A^{-1}$

$(IB)\cdot(AB)^{-1} = A^{-1}$

$B(AB)^{-1} = A^{-1}$

Premultiply by B^{-1} on both sides

$(B^{-1}B)(AB)^{-1} = B^{-1}A^{-1}$

$I(AB)^{-1} = B^{-1}A^{-1}$

$(AB)^{-1} = B^{-1}A^{-1}$