PERMUTATION AND COMBINATION

In our day-to-day life, we have various things to do. We can find all the possible ways to do those works by using permutation and combination.

**Permutation**

**Factorial**

It can be defined as the product of all-natural numbers up to that number i.e., \( n! \)

\[
= n \times (n - 1) \times (n - 2) \times (n - 3) \times \ldots \times 1.
\]

Example:

\[
4! = 4 \times 3 \times 2 \times 1 = 24 \\
5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \\
5! = 5 \times 4! = 5 \times 4 \times 3! = 5 \times 4 \times 3 \times 2! = 5 \times 4 \times 3 \times 2 \times 1 = 120
\]

**Permutation**

It is defined as the possible number of different arrangements which can be made by taking some or all given things at a time. In permutation order of things matters i.e., if two numbers A and B are arranged then AB and BA are counted as two different permutations.

**Formula of Permutation:**

\[
\text{\(#P_r = \frac{n!}{(n - r)!}\)}
\]

**Points to Remember**

- \( ^nP_n = n! \)
- \( ^nP_0 = 1 \)

**Combination**

It is defined as the possible number in which given things can be selected. In combination order of things does not matter i.e., if two numbers A and B are to be selected then AB and BA are counted as one combination.

**Formula of Combination:**

\[
\text{\(#C_r = \frac{n!}{r!(n - r)!}\)}
\]
Points to Remember

- \( nC_n = 1 \)
- \( ^nC_0 = 1 \)
- \( nC_r = nC_n - r \)
- \( nC_0 + nC_1 + nC_2 + \ldots + nC_n = 2^n \)

Permutation vs Combination

Points to Remember

- Whenever we want to arrange \( n \) things at \( n \) places, we can arrange it in \( n! \) ways.
- Whenever we want to select \( r \) things out of \( n \) things, we can select it in \( ^nC_r \) ways.
- Whenever we want to arrange \( r \) things out of \( n \) things, we can arrange it in \( ^nP_r \) ways.
- Number of ways in which \( r \) objects can be arranged out of \( n \) objects if \( q \) things are similar = \( ^nP_r/q! = n!/q!(n-r)! \)
- If, \( ^nC_x = ^nC_y \), then either \( x = y \) or \( (x + y) = n \)
- Number of circular permutations of \( n \) different objects = \( (n - 1)! \)
- Number of circular permutations of \( n \) different objects if clockwise or anti-clockwise arrangement are not considered = \( (n - 1)!/2 \)

Example 1: 5 letters A, B, C, D and E are given, then find
A. In how many ways 5 letters can be arranged?
B. In how many ways 3 of 5 letters can be selected.
C. In how many ways 3 of 5 letters can be arranged

Solution:

We can find the number of possible arrangements by using factorial = 5! = 120
We can find a number of ways of selection by using combination, \(^5C_3 = 5!/(2! \times 3!) = (5 \times 4 \times 3!)/(2 \times 3!) = 10 \)
We can find the required arrangement by using permutation, \( ^5P_3 = 5! / 3! = (5 \times 4 \times 3!)/3! = 20 \)

**Example 2:** In how many different ways can the letters of the word ‘HAPPY’ be arranged?

**Solution:**

HAPPY = 5 Letter word with letter "P" repeating two times

Hence, No. of ways 5 letters word can be arranged with a letter repeating itself 2 times = \( 5!/2! = 120/2 = 60 \)

**Example 3:** Find the number of different ways of forming a committee consisting of 3 men and 3 women from 6 men and 5 women.

**Solution:**

No. of ways of selecting 3 men out of 6 men = \( ^6C_3 \)

And, No. of ways of selecting 3 women out of 5 women = \( ^5C_3 \)

So, the no. of ways of forming a committee = \( ^6C_3 \times ^5C_3 = 200 \)

**Example 4:** Find the number of ways in which 5 beads can be strung into a necklace.

**Solution:**

Total number of beads, \( n = 5 \)

Required number of ways = \( (5 - 1)!/2 = 12 \)

**Example 5:** In a meeting 5 persons are present, find the number of handshakes if each person shakes his hand with every other.

**Solution:**

Number of ways in which 5 people can shake hand with each other once = \( ^5C_2 = 10 \)

**Example 6:** Find the number of straight lines that can be formed by 10 non-collinear points.

**Solution:** Number of straight lines formed by 10 non-collinear points = \( ^{10}C_2 = 45 \)

**Example 7:** Find the number of triangles that can be made by using 10 points in a plane, out of which 3 are collinear.

**Solution:** Number of triangles formed by 10 points out of which 3 are collinear points = \( ^{10}C_3 - ^3C_3 = 120 - 1 = 119 \)