Sample Paper

3

...(i)

	ANSWERKEY																		
1	(a)	2	(c)	3	(a)	4	(b)	5	(d)	6	(c)	7	(d)	8	(b)	9	(a)	10	(d)
11	(b)	12	(d)	13	(a)	14	(b)	15	(a)	16	(c)	17	(c)	18	(d)	19	(c)	20	(b)
21	(b)	22	(c)	23	(d)	24	(a)	25	(b)	26	(b)	27	(a)	28	(d)	29	(d)	30	(a)
31	(b)	32	(d)	33	(b)	34	(d)	35	(d)	36	(d)	37	(a)	38	(b)	39	(d)	40	(a)
41	(c)	42	(a)	43	(a)	44	(a)	45	(b)	46	(d)	47	(a)	48	(b)	49	(c)	50	(d)

SOLUTIONS

 (a) Let speed of boat in still water = x km/hr and speed of stream = y km/hr According to question,

time
$$(t_1) = \frac{9}{x + y} = 2$$
 (for down = rate) ... (i)

and time $(t_2) = \frac{9}{x - y} = 6$ (for up = rate) ... (ii)

Solving equations (i) & (ii), we get x = 3 km/hr and y = 1.5 km/hr Speed of the boat = 3 km/hr Speed of the current = 1.5 km/hr

- 2. (c) P(raining on both day) = $0.2 \times 0.3 = 0.06$ (Because both independent event)
- 3. (a) Statement given in option (a) is false.
- **4. (b)** $2\pi r_1 = 503$ and $2\pi r_2 = 437$

$$\therefore$$
 $r_1 = \frac{503}{2\pi}$ and $r_2 = \frac{437}{2\pi}$

Area of ring = $\pi (r_1 + r_2) (r_1 - r_2)$

$$\,=\,\pi\Bigg(\frac{503+437}{2\pi}\Bigg)\!\Bigg(\frac{503-437}{2\pi}\Bigg)$$

$$=\frac{940}{2}\left(\frac{66}{2\pi}\right)=235\times\frac{66}{22}\times7=235\times21=4935$$
 sq. cm.

5. (d)

6. (c) ::
$$\tan^2 \theta = 1 - e^2$$

$$\Rightarrow \sec\theta = \sqrt{1 + \tan^2 \theta} = \sqrt{1 + 1 - e^2}$$
$$\Rightarrow \sec\theta = \sqrt{2 - e^2}$$

$$\therefore \sec\theta + \tan^3\theta \csc\theta = \frac{1}{\cos\theta} + \tan^2\theta \cdot \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta}$$

$$= \frac{1}{\cos \theta} (1 + \tan^2 \theta) = \frac{\sec^2 \theta}{\cos \theta} = \sec^3 \theta = (2 - e^2)^{3/2} \text{ [from (i)]}$$

- 7. **(d)** L.C.M × H.C.F = First number × second number
 Hence, required number = $\frac{36 \times 2}{18}$ = 4.
- 8. (b)
- 9. (a)
- 10. (d) Sum is $888 \Rightarrow$ unit's digit should add up to 8. This is possible only for option (d) as "3" + "5" = "8".
- 11. **(b)** In $\triangle KPN$ and $\triangle KLM$, we have

$$\angle KNP = \angle KML = 46^{\circ}$$

$$\angle K = \angle K$$
 (Common)

 \therefore $\triangle KNP \sim \triangle KML$ (By A A criterion of similarity)

$$\Rightarrow \frac{KN}{KM} = \frac{NP}{MI} \Rightarrow \frac{c}{b+c} = \frac{x}{a}$$

12. (d) Let the fraction be $\frac{x}{y}$

According to given conditions,

$$\frac{x+1}{y+1} = 4$$
 ... (i)

and
$$\frac{x-1}{y-1} = 7$$
 ... (ii)

Solving (i) and (ii), we have x = 15, y = 3 i.e. numbers = 15

13. (a) Let the radii of the two circles be r_1 and r_2 , then $r_1 + r_2 = 15$ (given) (i)

and $\pi r_1^2 + \pi r_2^2 = 153\pi$ (given)

$$\Rightarrow r_1^2 + r_2^2 = 153$$
 (ii)

On solving, we get

$$r_1 = 12, \ r_2 = 3$$

Required ratio = 12:3=4:1

- 14. **(b)** $x^2 (m+3)x + mx m(m+3) = 0$ $\Rightarrow x[x - (m+3)] + m[x - (m+3)] = 0$ $\Rightarrow (x+m)[x - (m+3)] = 0$
 - x + m = 0 x = -m x (m+3) = 0 x = m+3
- **15.** (a) $\csc x \sin x = a \& \sec x \cos x = b$

$$\csc x - \frac{1}{\csc x} = a \& \sec x - \frac{1}{\sec x} = b$$

$$\Rightarrow \frac{\csc^2 x - 1}{\csc x} = a \& \frac{\sec^2 x - 1}{\sec x} = b$$

$$\Rightarrow \frac{\cot^2 x}{\csc x} = a \& \frac{\tan^2 x}{\sec x} = b$$

$$\frac{\cos^2 x}{\sin x} = a \, \& \, \frac{\sin^2 x}{\cos x} = b$$

Now,
$$a^2b = \frac{\cos^4 x}{\sin^2 x} \cdot \frac{\sin^2 x}{\cos x} = \cos^3 x$$

$$\Rightarrow \cos x = (a^2b)^{1/3} \Rightarrow \cos^2 x = (a^2b)^{2/3}$$

Similarly, $\sin^2 x = (ab^2)^{2/3}$

We know that, $\sin^2 x + \cos^2 x = 1$

$$\Rightarrow (ab^2)^{2/3} + (a^2b)^{2/3} = 1$$

16. (c) We have, sum of zeroes

$$= a + b = -\frac{(-4)}{2} = 2$$

Product of zeroes = $ab = \frac{3}{2}$

$$a^2b + ab^2 = ab(a+b) = \frac{3}{2} \times 2 = 3$$

17. (c) Since, $DE \parallel BC \implies \triangle ADE \sim \triangle ABC$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{1.5}{3} = \frac{1}{EC} \Rightarrow EC = 2 \text{ cm}$$

- **18.** (d) $3^{13} 3^{10} = 3^{10} (3^3 1) = 3^{10} (26) = 2 \times 13 \times 3^{10}$ Hence, $3^{13} - 3^{10}$ is divisible by 2, 3 and 13.
- 19. (c) Let the ages of father and son be 7x, 3x After 10 years,

(7x + 10) : (3x + 10) = 2 : 1 or x = 10

 \therefore Age of the father is 7x i.e. 70 years.

20. (b) 24 out of the 90 two digit numbers are divisible by '3' and not by '5'.

The required probability is therefore, $\frac{24}{90} = \frac{4}{15}$.

- **21. (b)** $\pi d_1 + \pi d_2 = \pi d \Rightarrow d_1 + d_2 = d$
- 22. (c) We have, $p(x) = x^2 10x 75 = x^2 15x + 5x 75$ = x(x-15) + 5(x-15) = (x-15)(x+5)

p(x) = (x-15)(x+5)

So, p(x) = 0 when x = 15 or x = -5. Therefore required zeroes are 15 and -5.

23. (d) Let $\csc x - \cot x = \frac{1}{3}$

$$\Rightarrow \frac{1}{\sin x} - \frac{\cos x}{\sin x} = \frac{1}{3}$$

$$\Rightarrow \frac{1-\cos x}{\sin x} = \frac{1}{3} \Rightarrow \frac{2\sin^2 \frac{x}{2}}{2\sin \frac{x}{2}\cos \frac{x}{2}} = \frac{1}{3}$$

 \Rightarrow $\tan \frac{x}{2} = \frac{1}{3}$

Consider

$$\tan x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}} = \frac{\frac{2}{3}}{1 - \frac{1}{9}} = \frac{3}{4}$$

Thus
$$\sin x = \frac{3}{5}$$
, $\cos x = \frac{4}{5}$

$$\therefore \quad \cos^2 x - \sin^2 x = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

- **24.** (a) $x^3 3x^2 10x + 24$
 - : Last term = (product of roots)
 - \therefore Factorising $24 = 2 \times 4 \times 3$

Also sum of roots must be "3"

- \therefore Possible factors are (2, 4, -3)
- $\therefore \text{ Factorization of } x^3 3x^2 10x + 24$

$$=(x-2)(x+3)(x-4)$$

- **25. (b)** The point satisfy the line 4y = x + 1
- **26. (b)** Let salary of *Y* be = *A* and of *X* is = $\frac{A}{2}$

 $\therefore \text{ Total salary of } X \text{ and } Y = \frac{3A}{2} \qquad \dots (i)$

Let X' and Y' be the new salary after increment, then we get

$$X' = \frac{3A}{4}$$
 and $Y' = \frac{5A}{4} \Rightarrow X' + Y' = 2A$... (ii)

 $\therefore \text{ Required percentage increase} = \frac{\left(\frac{2A - 3A}{2}\right) \times 100}{\frac{3A}{2}}$

[from (i) & (ii) eqns.]

$$= \frac{1}{3} \times 100 \Rightarrow 33\frac{1}{3}\%$$

27. (a) Perimeter of sector = 25 cm

$$\Rightarrow 2r + \frac{\theta}{360^{\circ}} \times 2\pi r = 25$$

$$\Rightarrow 2r + \frac{90^{\circ}}{360^{\circ}} \times 2 \times \frac{22}{7} \times r = 25$$

$$\Rightarrow$$
 2r + $\frac{11}{7}$ r = 25 \Rightarrow $\frac{25}{7}$ r = 25 \Rightarrow r = 7

Area of minor segment = $\left(\frac{\pi\theta}{360^{\circ}} - \frac{\sin\theta}{2}\right)r^2$

$$= \left(\frac{22}{7} \times \frac{90^{\circ}}{360^{\circ}} - \frac{\sin 90^{\circ}}{2}\right) (7)^{2}$$
$$= \left(\frac{11}{14} - \frac{1}{2}\right) \times 49 = \frac{4}{14} \times 49 = 14 \text{ cm}^{2}.$$

28. (d)
$$\therefore \frac{\text{Perimeter of } \triangle ABC}{\text{Perimeter of } \triangle POR} = \frac{AB}{PO} = \frac{BC}{OR} = \frac{AC}{PR}$$

29. (d) We have, $\tan \theta = \frac{a \sin \phi}{1 - a \cos \phi}$

$$\Rightarrow \cot \theta = \frac{1}{a \sin \phi} - \cot \phi \Rightarrow \cot \theta + \cot \phi = \frac{1}{a \sin \phi} ...(i)$$

and
$$\tan \phi = \frac{b \sin \theta}{1 - b \cos \theta}$$

$$\Rightarrow \cot \phi = \frac{1}{b \sin \theta} - \cot \theta$$

$$\Rightarrow \cot \phi + \cot \theta = \frac{1}{h \sin \theta}$$
 ...(ii)

From (i) and (ii), we have

$$\frac{1}{a\sin\phi} = \frac{1}{b\sin\theta} \implies \frac{a}{b} = \frac{\sin\theta}{\sin\phi}$$

30. (a) The number divisible by 15, 25 and 35 = L.C.M. (15, 25, 35) = 525

Since, the number is short by 10 for complete division by 15, 25 and 35.

Hence, the required least number = 525 - 10 = 515.

- **31. (b)** [**Hint.** One digit prime numbers are 2, 3, 5, 7. Out of these numbers, only the number 2 is even.]
- **32. (d)** Work ratio of A: B = 100: 160 or 5: 8

 \therefore time ratio = 8:5 or 24:15

If *A* takes 24 days, *B* takes 15 days. Hence, *B* takes 30 days to do double the work.

- **33. (b)** Hypotenuse = 270m
 - \Rightarrow Hypotenuse² = Side² + Side² = 2 Side²
 - \Rightarrow Side² = $(270)^2/2 = 72900/2 = 36450$

or Side = 190.91m

 \Rightarrow Required area = $1/2 \times 190.91 \times 190.91$

$$= 36446.6/2 = 18225 \text{ m}^2 \text{ (approx)}.$$

- **34. (d)** Out of n and n + 2, one is divisible by 2 and the other by 4, hence n (n + 2) is divisible by 8. Also n, n + 1, n + 2 are three consecutive numbers, hence one of them is divisible by 3. Hence, n (n + 1) (n + 2) must be divisible by 24. This will be true for any even number n.
- 35. (d) $(\cos^4 A \sin^4 A) = (\cos^2 A)^2 (\sin^2 A)^2$ = $(\cos^2 A - \sin^2 A)(\cos^2 A + \sin^2 A)$

$$= (\cos^2 A - \sin^2 A)(1) = \cos^2 A - (1 - \cos^2 A)$$
$$= 2 \cos^2 A - 1$$

- **36. (d)** The L.C.M. of 16, 20 and 24 is 240. The least multiple of 240 that is a perfect square is 3600 and also we can easily eliminate choices (a) and (c) since they are not perfect number. Hence, the required least number which is also a perfect square is 3600 which is divisible by each of 16, 20 and 24.
- 37. (a) Since, $\triangle ABC \sim \triangle PQR$

$$\therefore \frac{\operatorname{ar}(\Delta PQR)}{\operatorname{ar}(\Delta ABC)} = \frac{PR^2}{AC^2} = \frac{QR^2}{BC^2} = \frac{9}{1} \left[\because \frac{QR}{BC} = \frac{3}{1} \right] = 9$$

38. (b) Area of rectangle = $28 \times 23 = 644 \text{ cm}^2$

Radius of semi-circle = $28 \div 2 = 14$ cm

Radius of quadrant = 23 - 16 = 7 cm

Area of unshaded region

$$= \left(\frac{1}{2} \times \frac{22}{7} \times 14 \times 14\right) + \left(2 \times \frac{1}{4} \times \frac{22}{7} \times 7 \times 7\right) = 385 \text{ cm}^2$$

:. Shaded area = $(644 - 385) = 259 \text{ cm}^2$

39. (d)
$$\frac{1}{5} = \frac{2}{k} \neq \frac{-3}{7}$$

$$\Rightarrow k = 10$$

$$\therefore \text{ For inconsistent}$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

40. (a) Required probability $=\frac{4}{6} = \frac{2}{3}$.

41. (c) (0, 0)

42. (a) (4, 6)

43. (a) (6, 5)

44. (a) (16, 0)

45. (b) (-12, 6)

46. (d) parabola

47. (a) 2

48. (b) -1, 3

49. (c) $x^2 - 2x - 3$

50. (d) 0