MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Coefficient of x^{301} in $(1 + x)^{500} + x(1 + x)^{499} + x^2(1 + x)^{498} + ... + x^{500}$ is equal to

(1) $^{506}C_{306}$

(2) $^{501}C_{300}$

(3) $^{501}C_{301}$

(4) $^{500}C_{300}$

Answer (3)

Sol. Coeff of $x^{301} = {}^{500}C_{301} + {}^{499}C_{300} + {}^{498}C_{299} + ... + {}^{199}C_0$

$= {}^{500}C_{199} + {}^{499}C_{199} + {}^{498}C_{199} + ... + {}^{199}C_{199}$

$= {}^{501}C_{200}$

$= {}^{501}C_{301}$

2. $\tan 15° + \frac{1}{\tan 165°} + \frac{1}{\tan 105°} + \tan 195° = 2a$, then value of $\left(a + \frac{1}{a}\right)$ is

(1) $4 - 2\sqrt{3}$

(2) $\frac{-4}{\sqrt{3}}$

(3) 2

(4) $5 - \frac{3}{2}\sqrt{3}$

Answer (2)

Sol. tan15° + cot165° + cot105° + tan195°

= tan15° – cot15° – tan15° + tan15°

= tan15° – cot15°

$= -2\sqrt{3}$

$\Rightarrow \quad a = -\sqrt{3}$

$a + \frac{1}{a} = -\sqrt{3} - \frac{1}{\sqrt{3}} = \frac{-4}{\sqrt{3}}$

3. If set $A = \{a, b, c\}$

$R : A \to A$

$R = \{(a,b),(b,c)\}$

How many elements should be added for making it symmetric and transitive.

(1) 2

(2) 3

(3) 4

(4) 7

Answer (4)

Sol. For symmetric

$(a, b), (b, c) \in R$

$\Rightarrow (b, a), (c, b) \in R$

For transitive.

$(a, b), (b, c) \in R$

$\Rightarrow (a, c) \in R$

Now,

$(a, c) \in R$

$\Rightarrow (c, a) \in R$ {For symmetric}

$(a, b), (b, a) \in R$

$\Rightarrow (a, a) \in R$

$(b, c), (c, b) \in R$

$\Rightarrow (b, b) \in R$

$(c, b), (b, c) \in R$

$\Rightarrow (c, c) \in R$

$\therefore$ elements to be added

$\{(b, a)\ (c, b)\ (b, b)\ (a, a)\ (a, c)\ (c, a)\ (c, c)\}$

Total 7 elements

4. Let $P(h, k)$ be two points on $x^2 = 4y$ which is at shortest distance from $Q(0, 33)$ then difference of distances of $P(h, k)$ from directrix of $y^2 = 4(x + y)$ is

(1) 2

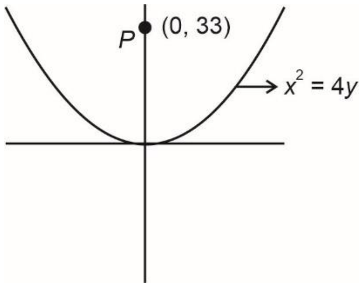
(2) 4

(3) 6

(4) 8

Answer (2)

Sol. For normal through (0, 33)



Normal at point $(2t, t^2)$

$x = -ty + 2at + at^3$

$0 = -t \cdot 33 + 2t + t^3$

$\Rightarrow \quad t = 0 \quad \text{OR} \quad \pm\sqrt{31}$

Points at which normal are drawn are

$A(0, 0), B\left(2\sqrt{31}, 31\right), C(-2\sqrt{31}, 31)$

Shortest distance

$$= PB = PC = \sqrt{124 + 4} = 8\sqrt{2} \text{ units}$$

Given parabola $(y-2)^2 = 4(x+1)$

Directrix is $x = -2$, that is line L

$B_L - C_L = \left|\left(-2 + 2\sqrt{31}\right) - \left(2 + 2\sqrt{31}\right)\right|$

$= 4$

5. Area bounded by larger part in I quadrant by $x = 4y^2$, $x = 2$ and $y = x$ is A then $3A$ equals

(1) $6 + \frac{1}{32} - 2\sqrt{2}$

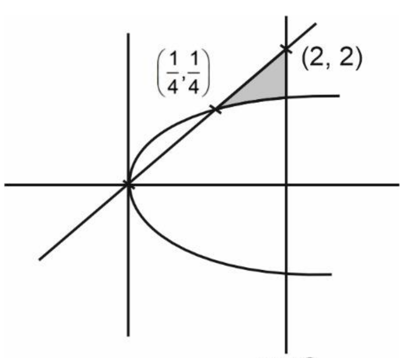
(2) $2 + \frac{1}{96} - \frac{2\sqrt{2}}{3}$

(3) $\frac{2\sqrt{2}}{3}$

(4) 96

Answer (1)

Sol.



Shaded area is the required area

$$A = \int_{1/4}^{2} \left(x - \frac{\sqrt{x}}{2}\right) dx$$

$$= \left.\frac{x^2}{2} - \frac{x^{3/2}}{3}\right|_{1/4}^{2}$$

$$= \left(2 - \frac{2\sqrt{2}}{3}\right) - \left(\frac{1}{32} - \frac{1}{24}\right)$$

$$= 2 + \frac{1}{96} - \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \quad 3A = 6 + \frac{1}{32} - 2\sqrt{2} \text{ sq. units.}$$

6. A die with points (2, 1, 0, –1, –2, 3) is thrown 5 times. The probability that the product of outcomes on all throws is positive is

(1) $\frac{521}{2592}$

(2) $\frac{16}{81}$

(3) $\frac{41}{288}$

(4) $\frac{28}{81}$

Answer (1)

Sol. Either all outcomes are positive or any two are negative.

The required probability $= {}^5C_5\left(\frac{1}{2}\right)^5 + {}^5C_2\left(\frac{1}{3}\right)^2\left(\frac{1}{2}\right)^3$

$+ {}^5C_4\left(\frac{1}{3}\right)^4\left(\frac{1}{2}\right)^1 = \frac{5}{162} + \frac{1}{32} + \frac{5}{36} = \frac{521}{2592}$

7. Let $S = \{1, 2, 3, 4, 5\}$

if $f : S \rightarrow P(S)$, where $P(S)$ is power set of S. Then number of one-one functions f can be made is

(1) $(32)^5$

(2) $\frac{32!}{27!}$

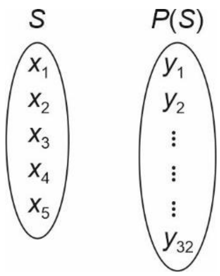
(3) ${}^{32}C_{27}$

(4) ${}^{32}P_{27}$

Answer (2)

Sol. $n(S) = 5$

$n(P(S)) = 2^5 = 32$



$\therefore$ No. of one-one function $= 32 \times 31 \times 30 \times 24 \times 28$

$$= \frac{32!}{27!}$$

8. A line is cutting x axis and y axis at two points A and B, respectively, where $OA = a$, $OB = b$. A perpendicular is drawn from O (origin) to AB at an angle of $\frac{\pi}{6}$ from positive x-axis. If area of triangle $OAB = \frac{98\sqrt{3}}{3}$ sq. units, then $\sqrt{3}a + b$ is equal to

(1) 28 (2) 14

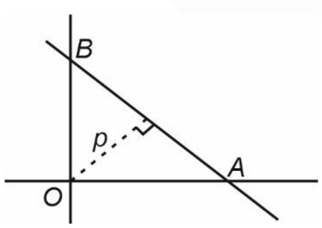
(3) 12 (4) 7

Answer (1)

Sol. Let the perpendicular distance of line from origin is p.

$\Rightarrow$ Equation of $AB: \frac{x\sqrt{3}}{2} + \frac{y}{2} = p$

$$\Rightarrow \frac{x}{\frac{2p}{\sqrt{3}}} + \frac{y}{2p} = 1$$



$$OA = \frac{2p}{\sqrt{3}}, OB = 2p$$

$$\frac{1}{2} \cdot \frac{2p}{\sqrt{3}} \cdot 2p = \frac{98}{\sqrt{3}}$$

$\Rightarrow p = 7$

$$OA = a = \frac{14}{\sqrt{3}}$$

$OB = b = 14$

$\sqrt{3}a + b$

$\Rightarrow 14 + 14 = 28$

9. For solution of differential equation

$$\frac{dy}{dx} - \frac{3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} y = -\frac{x^3 \tan^{-1} x^3}{\sqrt{1+x^6}}$$

given that $y(0) = 0$ then $y(1)$ is

(1) $1 - e^{\frac{\pi}{4\sqrt{2}}}$

(2) $1 - e^{\left(\frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}\right)}$

(3) $e^{\frac{1}{\sqrt{2}}} - e^{\frac{\pi}{4\sqrt{2}}}$

(4) $e^{\frac{\pi}{4\sqrt{2}}}$

Answer (2)

Sol. IF $= \int \frac{-3x^5 \tan^{-1}(x^3)}{(1+x^6)^{\frac{3}{2}}} dx$

Let $\tan^{-1}(x^3) = t$

IF $= e^{-\int t \sin t} = e^{(t\cos t - \sin t)}$

Solution of Differential equation

$$y \cdot e^{(t\cos t - \sin t)} = \int e^{(t\cos t - \sin t)} (-t \sin t) dt$$

$$y \cdot e^{(t\cos t - \sin t)} = e^{(t\cos t - \sin t)} + c$$

$t = 0 \rightarrow y = 0$

$\therefore$ $c = -1$

When $x = 1, t = \frac{\pi}{4}$

$$y \cdot e^{\left(\frac{\pi}{4\sqrt{2}} - \frac{1}{\sqrt{2}}\right)} = e^{\left(\frac{\pi}{4\sqrt{2}} - \frac{1}{\sqrt{2}}\right)} - 1$$

$$y = 1 - e^{\left(\frac{1}{\sqrt{2}} - \frac{\pi}{4\sqrt{2}}\right)}$$

10. $\frac{3(e-1)}{e} \int_1^2 x^2 e^{[x]+[x^3]} dx$ equals

(1) $e^9 - e$ (2) $e^8 - 1$

(3) $e^8 - e$ (4) $e^9 - 1$

Answer (3)

Sol. $I = \int_1^2 x^2 e^{[x]+[x^3]}dx = e\int_1^2 x^2 \cdot e^{[x^3]}dx$

Let $x^3 = t$

$I = e\int_1^8 \frac{dt}{3}e^{[t]} = \frac{e}{3}\left(e + e^2 + ... + e^7\right)$

$= \frac{e^2}{3}\left(\frac{e^7 - 1}{e - 1}\right)$

So, $\frac{3(e-1)}{e}\cdot\frac{e^2}{3}\cdot\frac{e^7-1}{e-1} = e^8 - e$

11. $\hat{n}$ is a vector, $\vec{a} \neq 0, \vec{b} \neq 0$. If $\vec{n} \perp \vec{c}, \vec{a} = \alpha\vec{b} - \hat{n}$ and $\vec{b}\cdot\vec{c} = 12$ then the value of $|\vec{c}\times(\vec{a}\times\vec{b})|$ equals (where $\hat{n}$ represents unit vector in the direction of $\vec{n}$)

(1) 144

(2) $\sqrt{12}$

(3) 12

(4) 24

Answer (3)

Sol. $\vec{a} = \alpha\vec{b} - \hat{n}$

$\Rightarrow \vec{a}\times\vec{b} = -\hat{n}\times\vec{b}$

Now,

$|\vec{c}\times(\vec{a}\times\vec{b})|$

$= |\vec{c}\times\left(-\hat{n}\times\vec{b}\right)|$

$= |\hat{n}(12) - \vec{b}(0)|$

= 12

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. $\lim_{x\to 0}\frac{\int_0^x \frac{t^3}{1+t^6}dt}{x^4}$ equals

Answer (12)

Sol. $\lim_{x\to 0}\frac{48\int_0^x \frac{t^3}{t^6+1}dt}{x^4}$

As $\frac{0}{0}$ form, applying L' hospital rule we get

$\lim_{x\to 0} 48\frac{x^3}{\left(x^6+1\right)\cdot 4x^3} = 48\cdot\frac{1}{4} = 12$

22. If $a_n = \frac{-2}{4n^2 - 16n + 15}$ and $a_1 + a_2 + ... + a_{25} = \frac{m}{n}$ where m and n are coprime, then the value of $m + n$ is

Answer (191)

Sol. $a_n = \frac{-2}{4n^2 - 16n + 15} = \frac{-2}{(2n-3)(2n-5)}$

$= \frac{1}{2n-3} - \frac{1}{2n-5}$

$a_1 + a_2 + ... + a_{25} = \left(\frac{1}{-1} - \frac{1}{-3}\right) + ... \left(\frac{1}{47} - \frac{1}{45}\right)$

$= \frac{1}{47} + \frac{1}{3} = \frac{50}{141}$

$\therefore \; m + n = 191$

23. If $z = 1 + i$ and $z_1 = \frac{i+\bar{z}(1-i)}{\bar{z}(1-z)} = z_1$, then find the value of $\frac{12}{\pi}\arg(z_1)$.

Answer (3)

Sol. $z_1 = \frac{i+\bar{z}(1-i)}{\bar{z}(1-z)} = \frac{i+(1-i)(1-i)}{(1-i)(-i)} = \frac{1}{1-i}$

$\arg z_1 = \arg\left(\frac{1}{1-i}\right) = -\arg(1-i) = \frac{\pi}{4}$

$\frac{12}{\pi}\arg(z_1) = \frac{12}{\pi}\times\frac{\pi}{4} = 3$

24. Mean & Variance of 7 observations are 8 & 16 respectively, if number 14 is omitted then a & b are new mean & variance. The value of $a + b$ is

Answer (19)

Sol. Let $x_1, \ldots x_7$ are observation

New mean $= \frac{8\times 7-14}{6} = 7$

$\therefore \frac{\sum_{i-1}^{n} x_i^2}{7} - 64 = 16 \Rightarrow \sum x_i^2 = 560$

$\sum x_{i(\text{new})}^2 = 560 - 14^2$

$\therefore b = \frac{364}{6} - 7^2 = \frac{70}{6} = \frac{35}{3}$

$\therefore a + b = 7 + \frac{35}{3} = \frac{56}{3} = 18.67$

Rounding off gives 19

25. If coefficient of x^{15} in expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to coefficient of x^{-15} in expansion of $\left(ax^{1/3} + \frac{1}{bx^3}\right)^{15}$ then $|ab - 5|$ is equal to

Answer (04.00)

Sol. $a_n\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15} \Rightarrow T_{r+1} = {}^{15}C_r a^{15-r}\left(x^3\right)^{15-r} b^{-r} x^{\frac{-r}{3}}$

$45 - 3r - \frac{r}{3} = 15 \Rightarrow \frac{10r}{3} = 30$

$\boxed{r = 9}$

$a_n\left(ax^{\frac{1}{3}} + \frac{1}{bx^3}\right)^{15} \Rightarrow T_{r+1} = {}^{15}C_r a^{15-r} x^{\frac{15-r}{3}} b^{-r} x^{-3r}$

$\frac{15-r}{3} - 3r = -15$

$15 - r - 9r = -45$

$\Rightarrow r = 6$

So, ${}^{15}C_9 a^6 b^{-9} = {}^{15}C_6 a^9 b^{-6}$

$\Rightarrow a^{-3}b^{-3} = 1$

or $\boxed{ab = 1}$

$|ab - 5| = 4$

26. Using 1, 2, 3, 5, 4-digit numbers are formed, where repetition is allowed. How many of them is divisible by 15?

Answer (21)

Sol. Units digit will be 5

$\underline{a}\ \underline{b}\ \underline{c}\ \underline{5}$

$a + b + c = (3\lambda + 1)$ type

For (a, b, c) possibilities are

(2, 2, 3) (1, 1, 5) (1, 1, 2)

(3, 3, 1) (5, 5, 3) (2, 3, 5)

For (2, 2, 3) $\Rightarrow \frac{3!}{2!} = 3$

For (1, 1, 5) $\Rightarrow \frac{3!}{2!} = 3$

For (1, 1, 2) $\Rightarrow \frac{3!}{2!} = 3$

For (3, 3, 1) $\Rightarrow \frac{3!}{2!} = 3$

For (5, 5, 3) $\Rightarrow \frac{3!}{2!} = 3$

For (2, 3, 5) $\Rightarrow 3! = 6$

Total = 21

27. If $5f(x+y)=f(x)\cdot f(y)$ and $f(3)=320$, then the value of $f(1)$ is

Answer (20)

Sol. $5f(x+y)=f(x)\cdot f(y)$ ...(i) $f(3) = 320$

Put $x = 1$, $y = 2$ in (i)

$5f(3)=f(1)\cdot f(2)$

$\Rightarrow f(1)\cdot f(2)=5\times 320=1600$ ...(ii)

Put $x = y = 1$ in (i)

$5f(2)=(f(1))^2$

$\Rightarrow f(2)=\dfrac{(f(1))^2}{5}$ ...(iii)

Using (iii) in (ii),

$f(1)\cdot\dfrac{(f(1))^2}{5}=1600$

$(f(1))^3=8000$

$f(1)=20$

28. If for $\log_{\cos x}(\cot x)-4\log_{(\sin x)}\cot x=1$,

$x=\sin^{-1}\left(\dfrac{\alpha+\sqrt{\beta}}{2}\right)$. Find $(\alpha + \beta)$, given $x\in\left(0,\ \dfrac{\pi}{2}\right)$

Answer (04.00)

Sol. $\log_{\cos x}\cot x-4\log_{\sin x}\cot x=1$

$1-\log_{\cos x}\sin x-4(\log_{\sin x}\cos x-1)=1$

Let $\log_{\cos x}\sin x=t$

$-t-4\left(\dfrac{1}{t}-1\right)=0$

$\Rightarrow t+\dfrac{4}{t}=4$

$\Rightarrow t = 2$

$\log_{\cos x}\sin x=2$

$\Rightarrow \cos^2 x = \sin x$

$\Rightarrow 1 - \sin^2 x - \sin x = 0$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

So, $\sin x=\dfrac{-1\pm\sqrt{5}}{2}$

$\alpha = -1$, $\beta = 5$

$\alpha + \beta = 4$

29.

30.

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