JEE MAINS-12-APRIL-2014
PHYSICS

1. From the following combinations of physical constants (expressed through their usual symbols) the only combination, that would have the same value in different systems of units, is :

(1) $\frac{ch}{2\pi \in_0^2}$

(2) $\frac{e^2}{2\pi \in_0 Gm_e^2}$ $(m_e = \text{mass of electron})$

(3) $\frac{\mu_0 \in_0}{c^2}\frac{G}{he^2}$

(4) $\frac{2\pi\sqrt{\mu_0 \in_0}}{ce^2}\frac{h}{G}$

2. A person climbs up a stalled escalator in 60 s. If standing on the same but escalator running with constant velocity he takes 40 s. How much time is taken by the person to walk up the moving escalator ?

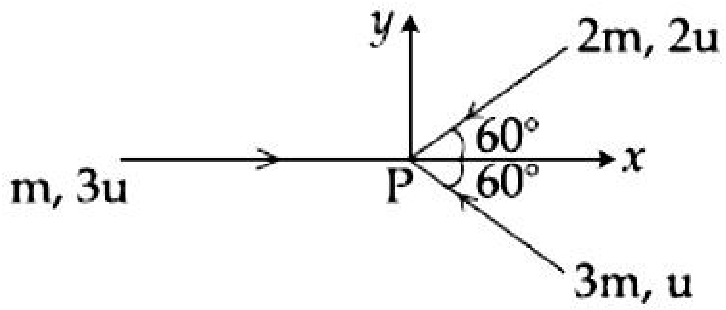
(1) 37 s

(2) 27 s

(3) 24 s

(4) 45 s

3. Three masses m, 2m and 3m are moving in x-y plane with speed 3u, 2u, and u respectively as shown in figure. The three masses collide at the same point P and stick together. The velocity of resulting mass will be :



(1) $\frac{u}{12}\left(\hat{i}+\sqrt{3}\hat{j}\right)$

(2) $\frac{u}{12}\left(\hat{i}-\sqrt{3}\hat{j}\right)$

(3) $\frac{u}{12}\left(-\hat{i}+\sqrt{3}\hat{j}\right)$

(4) $\frac{u}{12}\left(-\hat{i}-\sqrt{3}\hat{j}\right)$

4. A 4 g bullet is fired horizontally with a speed of 300 m/s into 0.8 kg block of wood at rest on a table. If the coefficient of friction between the block and the table is 0.3, how far will the block slide approximately ?

(1) 0.19 m

(2) 0.379 m

(3) 0.569 m

(4) 0.758 m

5. A spring of unstitched length l has a mass m with one end fixed to a rigid support. Assuming spring to be made of a uniform wire, the kinetic energy possessed by it if its free end is pulled with uniform velocity v is :

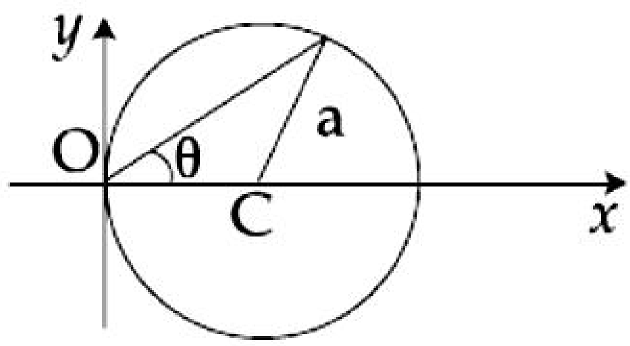
(1) $\frac{1}{2}mv^2$

(2) mv^2

(3) $\frac{1}{3}mv^2$

(4) $\frac{1}{6}mv^2$

6. A particle is moving in a circular path of radius a, with a constant velocity v as shown in the figure. The center of circle is marked by 'C. The angular momentum from the origin O can be written as :



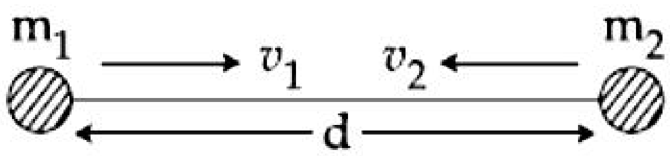
(1) $va(1+\cos 2\theta)$

(2) $va(1+\cos\theta)$

(3) $va\cos 2\theta$

(4) va

7.



Two hypothetical planets of masses m_1 and m_2 are at rest when they are infinite distance apart. Because of the gravitational force they move towards each other along the line joining their centres. What is their speed when their separation is 'd' ? (Speed of m_1 is v_1 and that of m_2 is v_2):

(1) $v_1 = v_2$

(2) $v_1 = m_2\sqrt{\dfrac{2G}{d(m_1+m_2)}}$

$v_2 = m_1\sqrt{\dfrac{2G}{d(m_1+m_2)}}$

(3) $v_1 = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$

$v_2 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$

(4) $v_1 = m_2\sqrt{\frac{2G}{m_1}}$

$v_2 = m_1\sqrt{\frac{2G}{m_2}}$

8. Steel ruptures when a shear of 3.5×10^8 Nm^{-2} is applied. The force needed to punch a 1 cm diameter hole in a steel sheet 0.3 cm thick is nearly :

(1) 1.4×10^4 N

(2) 2.7×10^4 N

(3) 3.3×10^4 N

(4) 1.1×10^4 N

9. A cylindrical vessel of cross-section A contains water to a height h. There is a hole in the bottom of radius 'a'. The time in which it will be emptied is :

(1) $\frac{2A}{\pi a^2}\sqrt{\frac{h}{g}}$

(2) $\frac{\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

(3) $\frac{2\sqrt{2}A}{\pi a^2}\sqrt{\frac{h}{g}}$

(4) $\frac{A}{\sqrt{2}\pi a^2}\sqrt{\frac{h}{g}}$

10. Two soap bubbles coalesce to form a single bubble. If V is the subsequent change in volume of contained air and S the change in total surface area, T is the surface tension and P atmospheric pressure, which of the following relation is **correct** ?

(1) $4PV + 3ST = 0$

(2) $3PV + 4ST = 0$

(3) $2PV + 3ST = 0$

(4) $3PV + 2ST = 0$

11. Hot water cools from 60°C to 50°C in the first 10 minutes and to 42°C in the next 10 minutes. The temperature of the surroundings is :

(1) 25°C

(2) 10°C

(3) 15°C

(4) 20°C

12. A Carnot engine absorbs 1000 J of heat energy from a reservoir at 127°C and rejects 600 J of heat energy during each cycle. The efficiency of engine and temperature of sink will be :

(1) 20% and –43ºC

(2) 40% and –33ºC

(3) 50% and –20ºC

(4) 70% and –10ºC

13. At room temperature a diatomic gas is found to have an r.m.s. speed of 1930 ms^{-1}. The gas is :

(1) H_2

(2) Cl_2

(3) F_2

(4) O_2

14. Which of the following expressions corresponds to simple harmonic motion along a straight line, where x is the displacement and a, b, c are positive constants ?

(1) $a + bx - cx^2$

(2) bx^2

(3) $a - bx + cx^2$

(4) $-bx$

15. A source of sound A emitting waves of frequency 1800 Hz is falling towards ground with a terminal speed v. The observer B on the ground directly beneath the source receives waves of frequency 2150 Hz. The source A receives waves, reflected from ground, of frequency nearly : (Speed of sound =343 m/s)

(1) 2150 Hz

(2) 2500 Hz

(3) 1800 Hz

(4) 2400 Hz

16. A spherically symmetric charge distribution is characterised by a charge density having the following variation :

$\rho(r) = \rho_0\left(1 - \frac{r}{R}\right)$ for $r < R$

$\rho(r) = 0$ for $r \geq R$

Where r is the distance from the centre of the charge distribution and ρ_0 is a constant. The electric field at an internal point $(r < R)$ is :

(1) $\frac{\rho_0}{4\epsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

(2) $\frac{\rho_0}{\epsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

(3) $\frac{\rho_0}{3\epsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

(4) $\frac{\rho_0}{12\epsilon_0}\left(\frac{r}{3} - \frac{r^2}{4R}\right)$

17. The space between the plates of a parallel plate capacitor is filled with a 'dielectric' whose 'dielectric constant' varies with distance as per the relation :

$K(x) = K_o + \lambda x$ (λ= a constant)

The capacitance C, of this capacitor, would be related to its 'vacuum' capacitance C_o as per the relation :

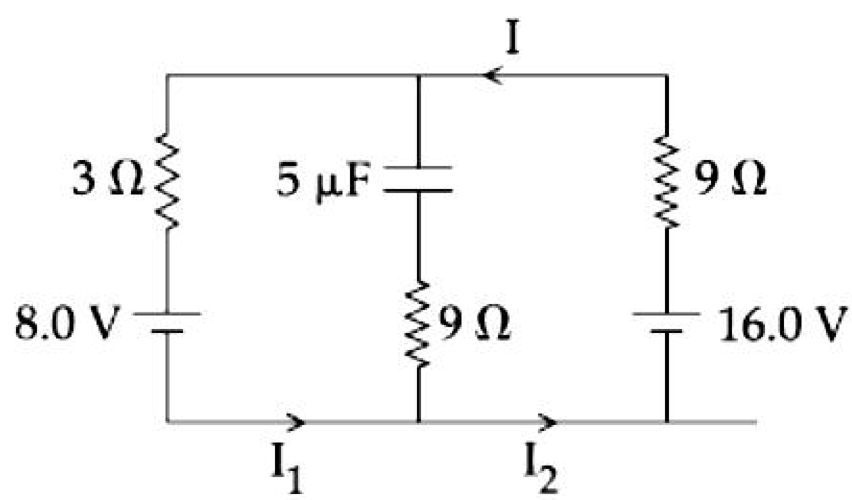
(1) $C = \dfrac{\lambda d}{\ln(1 + K_o \lambda d)} C_o$

(2) $C = \dfrac{\lambda}{d \cdot \ln(1 + K_o \lambda d)} C_o$

(3) $C = \dfrac{\lambda d}{\ln(1 + \lambda d / K_o)} C_o$

(4) $C = \dfrac{\lambda}{d \cdot \ln(1 + K_o / \lambda d)} C_o$

18. The circuit shown here has two batteries of 8.0 V and 16.0 V and three resistors 3 Ω, 9 Ω and 9 Ω and a capacitor 5.0 μF.



How much is the current I in the circuit in steady state ?

(1) 1.6 A

(2) 0.67 A

(3) 2.5 A

(4) 0.25 A

19. A positive charge 'q' of mass 'm' is moving along the $+x$ axis. We wish to apply a uniform magnetic field B for time Δt so that the charge reverses its direction crossing the y axis at a distance d. Then :

(1) $B = \frac{mv}{qd}$ and $\Delta t = \frac{\pi d}{v}$

(2) $B = \frac{mv}{2qd}$ and $\Delta t = \frac{\pi d}{2v}$

(3) $B = \frac{2mv}{qd}$ and $\Delta t = \frac{\pi d}{2v}$

(4) $B = \frac{2mv}{qd}$ and $\Delta t = \frac{\pi d}{v}$

20. Consider two thin identical conducting wires covered with very thin insulating material. One of the wires is bent into a loop and produces magnetic field B_1, at its centre when a current (I) passes through it. The second wire is bent into a coil with three identical loops adjacent to each other and produces magnetic field B_2 at the centre of the loops when current 1/3 passes through it. The ratio $B_1 : B_2$ is :

(1) 1 : 1

(2) 1 : 3

(3) 1 : 9

(4) 9 : 1

21. A sinusoidal voltage V(t) = 100 sin (500t) is applied across a pure inductance of L = 0.02 H. The current through the coil is :

(1) 10 cos (500t)

(2) – 10 cos (500t)

(3) 10 sin (500t)

(4) – 10 sin (500t)

22. A lamp emits monochromatic green light uniformly in all directions. The lamp is 3% efficient in converting electrical power to electromagnetic waves and consumes 100 W of power. The amplitude of the electric field associated with the electromagnetic radiation at a distance of 5 m from the lamp will be nearly :

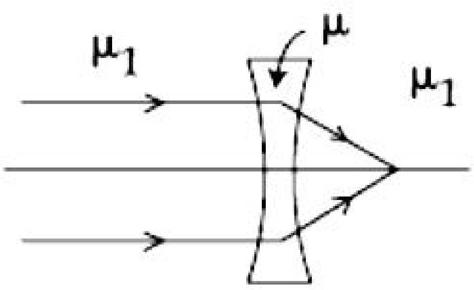
(1) 1.34 V/m

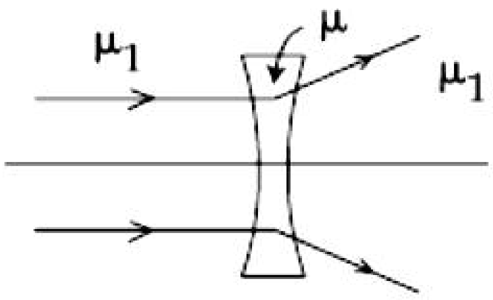
(2) 2.68 V/m

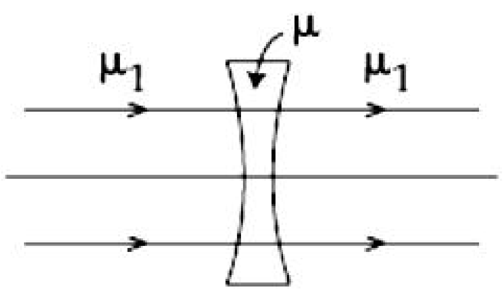
(3) 4.02 V/m

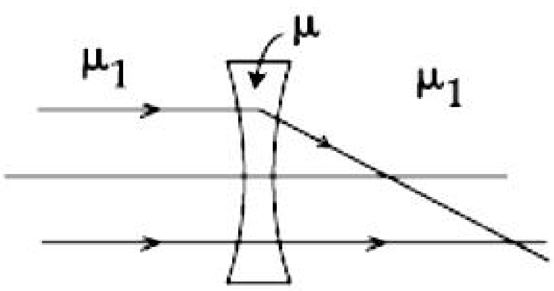
(4) 5.36 V/m

23. The refractive index of the material of a concave lens is μ. It is immersed in a medium of refractive index μ_1. A parallel beam of light is incident on the lens. The path of the emergent rays when $\mu_1 > \mu$ is :

(1) 

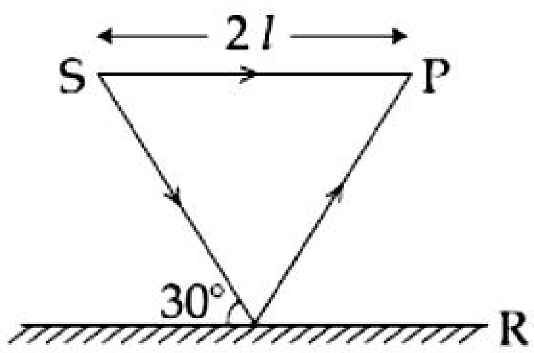
(2) 

(3) 

(4) 

24. Interference pattern is observed at 'P' due to superimposition of two rays coming out from a source 'S' as shown in the figure.

The value 'l' for which maxima is obtained at 'P' is (R is perfect reflecting surface) :



(1) $l = \frac{2n\lambda}{\sqrt{3}-1}$

(2) $l = \frac{(2n-1)\lambda}{2(\sqrt{3}-1)}$

(3) $l = \frac{(2n-1)\lambda\sqrt{3}}{4(2-\sqrt{3})}$

(4) $l = \frac{(2n-1)\lambda}{\sqrt{3}-1}$

25. In an experiment of single slit diffraction pattern, first minimum for red light coincides with first maximum of some other wavelength. If wavelength of red light is 6600 Å , then wavelength of first maximum will be :

(1) 3300 Å

(2) 4400 Å

(3) 5500 Å

(4) 6600 Å

26. A beam of light has two wavelengths 4972 Å and 6216 Å with a total intensity of 3.6×10^{-3} Wm^{-2} equally distributed among the two wavelengths. The beam falls normally on an area of 1 cm^2 of a clean metallic surface of work function 2.3 eV. Assume that there is no loss of light by reflection and that each capable photon ejects one electron. The number of photo electrons liberated in 2s is approximately :

(1) 6×10^{11}

(2) 9×10^{11}

(3) 11×10^{11}

(4) 15×10^{11}

27. A piece of bone of an animal from a ruin is found to have ^{14}C activity of 12 disintegrations per minute per gm of its carbon content. The ^{14}C activity of a living animal is 16 disintegrations per minute per gm. How long ago nearly did the animal die ? (Given half life of ^{14}C is $t_{1/2}$ = 5760 years):

(1) 1672 years

(2) 2391 years

(3) 3291 years

(4) 4453 years

28. For LED's to emit light in visible region of electromagnetic light, it should have energy band gap in the range of :

(1) 0.1 eV to 0.4 eV

(2) 0.5 eV to 0.8 eV

(3) 0.9 eV to 1.6 eV

(4) 1.7 eV to 3.0 eV

29. For sky wave propagation, the radio waves must have a frequency range in between :

(1) 1 MHz to 2 MHz

(2) 5 MHz to 25 MHz

(3) 35 MHz to 40 MHz

(4) 45 MHz to 50 MHz

30. In the experiment of calibration of voltmeter, a standard cell of e.m.f. 1.1 volt is balanced against 440 cm of potentiometer wire. The potential difference across the ends of resistance is found to

balance against 220 cm of the wire. The corresponding reading of voltmeter is 0.5 volt. The error in the reading of voltmeter will be :

(1) – 0.15 volt

(2) 0.15 volt

(3) 0. 5 volt

(4) – 0.15 volt

Part-2

1. Sol:

$\left[M^2L^6T^{-5}I^{-2}\right]$ is the dimensional formula of $\frac{ch}{2\pi\varepsilon_0^2}$.

$\left[M^0L^0T^0\right]$ is the dimensional formula of $\frac{e^2}{2\pi\varepsilon_0 G m_e^2}$.

$\left[L^4T^{-7}I^{-3}\right]$ is the dimensional formula of $\frac{\mu_0\varepsilon_0}{c^2}\frac{G}{he^2}$.

$\left[M^{-3}L^{-1}T^{-3}A^{-4}\right]$ is the dimensional formula of $\frac{2\pi\sqrt{\mu_0\varepsilon_0}}{ce^2}\frac{h}{G}$.

A dimensionless and unitless term will have same value in all system of units. Option (2) is a dimensionless term. Thus, this term would have same value in different systems of units.

2. Sol:

The expression of time taken by man is,

$$t = \frac{d}{v}$$

$$60 = \frac{d}{v_m}$$

The expression for time taken by escalator is,

$$40 = \frac{d}{v_{es}}$$

So the total time is,

$$t_{total} = \frac{d}{v_{total}}$$

$$t_{total} = \frac{d}{(v_m + v_{es})}$$

$$\frac{1}{t_{total}} = \frac{v_m}{d} + \frac{v_{es}}{d}$$

Substitute the values.

$$\frac{1}{t_{total}} = \frac{1}{60} + \frac{1}{40}$$

$$= \frac{1}{24}$$

Thus, $t_{total} = 24\ s$.

3. Sol:

According to the law of conservation of momentum, Left momentum = Right momentum

$$3mu\hat{i} = \begin{bmatrix} 3m\left[-u\cos 60^\circ \hat{i} + u\sin 60^\circ \hat{j}\right] + \\ 2m\left[-2u\cos 60^\circ \hat{i} - 2u\sin 60^\circ \hat{j}\right] \end{bmatrix}$$

$$= \begin{bmatrix} 3mu\hat{i} + 3m\left[-\frac{u}{2}\hat{i} + \frac{0\times\sqrt{3}}{2}\hat{j}\right] + \\ 2m\left[-\frac{2u}{2}\hat{i} - \frac{2u\sqrt{3}}{2}\hat{j}\right] \end{bmatrix}$$

$$= \frac{u}{12}\left(-\left(\hat{i}\right) - \sqrt{3}\hat{j}\right)$$

4. Sol:

According to the law of conservation of momentum,

$$mv = Mv'$$

$$0.004 \times 300 = 0.8v'$$

$$v' = 1.5 \text{ m/s}$$

From kinetics, write the expression for the velocity,

$$v^2 = v'^2 + 2as$$

$$0 = 1.5^2 + 2 \times 3 \times s$$

$$s \approx 0.379 \text{ m}$$

5. Sol:

The kinetic energy in elemental form is,

$$d(KE) = \frac{1}{2} dm (dv)^2$$

The velocity in elemental form is,

$$dv = \frac{v}{l} x$$

The mass in elemental form is,

$$dm = \frac{m}{l} dx$$

The kinetic energy in integral form is,

$$KE = \int_0^1 \frac{1}{2} dm \left(\frac{v}{l} x \right)^2$$

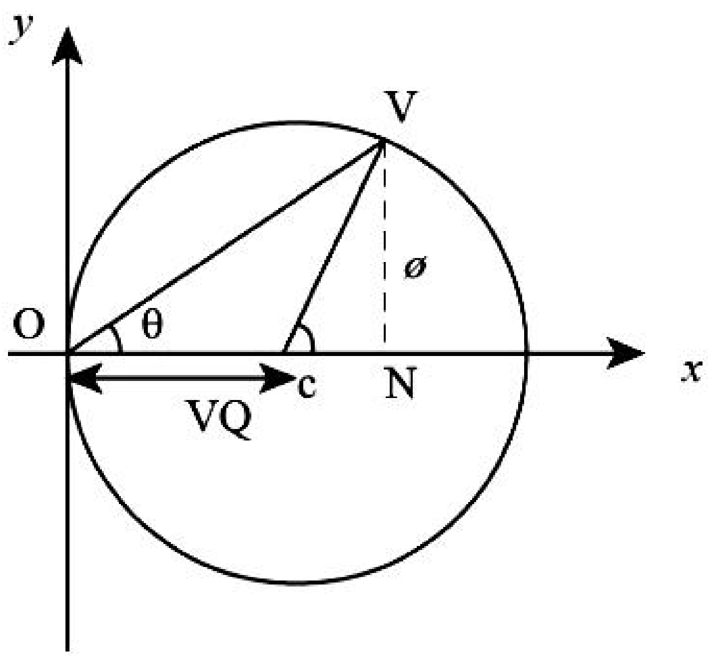
$$= \frac{1}{2} \int_0^1 \frac{m}{l} dx \left(\frac{v^2}{l^2} x^2 \right)$$

$$= \frac{mv^2}{2l^3} \left(\frac{x^3}{3} \right)_0^1$$

$$= \frac{mv^2}{6}$$

6. Sol:

Consider the diagram shown below,



The expression of the component of angular velocity through OC is,

$$OC = v \times a \qquad \ldots\ldots(1)$$

By using the property of triangle,

$$\theta = \frac{\phi}{2}$$

$$2\theta = \phi$$

The expression for the angular velocity for an arbitrary point N from the figure is,

$$ON = OC + CN \qquad \ldots\ldots(2)$$

The expression of the component of angular velocity through CN is,

$$CN = va\cos\phi$$
$$= va\cos 2\theta$$

Substitute the values into equation (2)

$$\text{Angular momentum} = va + va\cos 2\theta$$
$$= va(1+\cos 2\theta)$$

7. Sol:

Let reference point is at infinity. The total energy at infinity will be 0.

So, initial energy of the system is 0.

The expression for the final energy of the system will be,

$$KE - \frac{GM_1M_2}{d}$$

From law of conservation of energy,

$$0 = -\frac{GM_1M_2}{d} + KE$$
$$KE = \frac{GM_1M_2}{d} \qquad \ldots\ldots (1)$$

As the momentum is conserved for the system, the kinetic energy will be inversely proportional to the mass.

The expression of the kinetic energy for mass m_1 is,

$$KE_1 = \frac{1}{2}m_1v_1^2 \times \frac{1}{\frac{m_2}{m_1+m_2}}$$

Substitute above expression in equation (1),

$$\frac{Gm_1m_2}{d} = \frac{1}{2}m_1v_1^2 \times \frac{1}{\frac{m_2}{m_1+m_2}}$$

$$v_1 = m_2\sqrt{\frac{2G}{d(m_1+m_2)}}$$

Similarly for mass m_2 KE is,

$$KE_2 = \frac{1}{2}m_2v_2^2 \times \frac{1}{\left(\frac{m_1}{m_1+m_2}\right)}$$

Solve the above expression.

$$\left(\frac{Gm_1m_2}{d}\right) = \frac{1}{2}m_2v_2^2 \times \frac{1}{\left(\frac{m_1}{m_1+m_2}\right)}$$

$$v = m_1\sqrt{\frac{2G}{d(m_1+m_2)}}$$

8. Sol:

The expression for stress is,

$$stress = \frac{\text{Force}}{\text{Area}}$$
$$= 3.5\times10^8 \text{ N/m}^2$$

The expression for area is,

$$\text{Area} = 2\pi rt$$
$$= 2\pi \times 0.5 \times 0.3$$
$$= 0.3\pi \times 10^{-4}\ \text{m}^2$$

The expression for the force is,

$$\text{Force} = \text{A} \times stress$$
$$= \left(0.3\pi \times 10^{-4}\right) \times \left(3.5 \times 10^{8}\right)$$
$$= 3.3 \times 10^{4}\ \text{N}$$

9. Sol:

If the liquid is at x height from orifice at time t then, the volume coming from the orifice is given by the expression,

$$V = vA \qquad \ldots\ldots (1)$$

The expression of the velocity of water is,

$$v = \sqrt{2gx}$$

From equation (1)

$$V = \sqrt{2gx} \times \pi r^2 \qquad \ldots\ldots (2)$$

Write expression for rate of volume.

$$\text{A} \times \left(\frac{-dx}{dt}\right)$$

Compare equation (2) with the above expression.

$$\sqrt{2gx}\left(\pi a^2\right) = A\left(\frac{-dx}{dt}\right)$$

$$2A\int_h^0 \frac{dx}{\sqrt{x}} = \left(\pi a^2\right)\sqrt{2g}\int_0^t dt$$

$$2A \times \left(\sqrt{x}\right)_h^0 = \left(\pi a^2\right)\sqrt{2g} \times t$$

$$2A \times \sqrt{h} = \left(\pi a^2\right)\sqrt{2g} \times t$$

Solve the above expression.

$$t = \frac{A}{\left(\pi a^2\right)} \times \frac{\sqrt{2h}}{\sqrt{g}}$$

10. Sol:

Let the radius of the soap bubble is r and the radius of the larger bubble is R. So, the surface tension is given by,

$$T = P\frac{\left(R^3 - 2r^3\right)}{4\left(2r^2 - R^2\right)}$$

By solving the above expression,

$$T = P\frac{\pi\left(R^3 - 2r^3\right)}{4\pi\left(2r^2 - R^2\right)}$$

$$= \frac{3PV}{-4S}$$

Rearrange the above expression to form,

$3PV + 4TS = 0$

11. Sol:

The rate of change temperature is given as,

$$\frac{\Delta T}{t} = K\left[\frac{T_1 + T_2}{2} - T\right]$$

Substitute the values.

$$\frac{10}{10} = K\left(\frac{60+55}{2} - T\right)$$

$$\frac{10}{10} = K(55 - T) \qquad \ldots\ldots (1)$$

Similarly,

$$\frac{8}{10} = K(46 - T) \qquad \ldots\ldots (2)$$

Thus,

$T = 10\ °C$

12. Sol:

The values for heat in Carnot engine is,

$Q_{abs} = 1000\ J$

$Q_{rej} = 600\ J$

The expression for efficiency is,

$$\eta = 1 - \frac{Q_{rej}}{Q_{abs}}$$

$$= 1 - \frac{600}{1000}$$

$$= 40\%$$

So,

$$\frac{T_1}{T_2} = \frac{Q_{abs}}{Q_{rej}}$$

$$\frac{400}{T_2} = \frac{1000}{600}$$

$$T_2 = 240\text{ K} - 273$$

$$= -33\ ^{o}C$$

13. Sol:

The equation of rms velocity is,

$$v_{rms} = \sqrt{\frac{3p}{\rho}}$$

From ideal gas equation,

$$pv = \frac{MRT}{M_o}$$

$$\frac{p}{(M/V)} = \frac{RT}{M_o}$$

$$\frac{p}{\rho} = \frac{RT}{M_o}$$

Thus,

$$1930 = \sqrt{\frac{3RT}{M_o}}$$

$$M_o = \frac{3 \times 8.314 \times 300}{1930 \times 1930}$$

$$= 2 \times 10^{-3} \text{ kg or } 2\text{g}$$

The molar mass of hydrogen is 2 g.

14. Sol:

Only linear equations represent SHM along a straight line; and the degree of x as 1 is present only in option (4) so it is a linear equation. Thus option (4) is correct.

15. Sol:

By using Doppler effect,

$$f = \left| \frac{f_0(c)}{(c+v)} \right|$$

The velocity from the above expression is,

$$\left| \frac{343}{343+v} \right| \times 1800 = 2150$$

$$v = 55.8 \text{ m/s}$$

The reflected frequency is,

$$\left| \frac{343+55.8}{343} \right| \times 2150$$

$$= 2500 \text{ Hz}$$

16. Sol:

The expression of charge for a symmetrically charged spherical body,

$$dq = \rho \times 4\pi r^2 dr$$

The equation of Charge density is,

$$\rho = \rho_0 \left(1 - \frac{r}{R} \right)$$

So, the total charge is given by,

$$\oint \varepsilon \times ds = \text{total charge}$$

$$\varepsilon\left(4\pi r^2\right) = \rho\int_0^r \left(1-\frac{r}{R}\right)4\pi r^2 \times dr$$

$$= 4\pi\rho\left[\frac{r^3}{3}-\frac{r^2}{4R}\right]_0^q$$

$$E = \frac{\rho_o}{\varepsilon_o}\left(\frac{r}{3}-\frac{r^2}{4R}\right)$$

17. Sol:

The equation of potential is,

$$V = \int_0^d \frac{\sigma}{K}dx$$

$$= \sigma\int_0^d \frac{1}{\left(K_o+\lambda x\right)}dx$$

$$= \frac{\sigma}{\lambda}\left[\ln\left(K_o+\lambda d\right)-\ln K_o\right]$$

$$= \frac{\sigma}{\lambda}\ln\left(1+\frac{\lambda d}{K_o}\right)$$

Let the charge density is σ and the surface area is s.

The capacitance is given by,

$$C = \frac{\sigma \times s}{V}$$

$$= \frac{\sigma \times s}{\frac{\sigma}{\lambda} \ln\left(1 + \frac{\lambda d}{K_o}\right)}$$

Solve above expression by multiply the denominator and numerator by d in the right hand side.

$$C = s\lambda \times \frac{d}{d}\left(\frac{1}{\ln\left(1 + \frac{\lambda d}{K_o}\right)}\right)$$

$$C = \frac{\lambda d}{\ln\left(1 + \frac{\lambda d}{K_o}\right)} \times C_o$$

18. Sol:

The current would not flow through the branch with capacitor in the steady state. Therefore, net potential is 8 V and equivalent resistance is $12\,\Omega$.

So,

$$I = \frac{8}{12}$$
$$= 0.67\ A$$

19. Sol:

Write expression for the energy balance for the charge,

$$\frac{mv^2}{r} = qvB$$

$$r = \frac{mv}{qB}$$

$$\frac{d}{2} = \frac{mv}{qB} \quad \ldots\ldots (1)$$

$$B = \frac{2mv}{qd}$$

The equation of the total time for charge particles is,

$$T = \frac{2\pi m}{Bq} \quad \ldots\ldots (2)$$

The equation of the time for reversal of direction is,

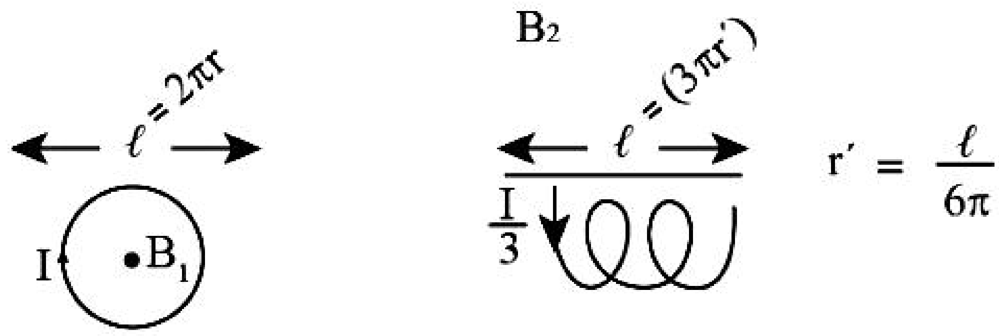
$$\Delta t = \frac{T}{2}$$

Solve equation (1) and (2),

$$\Delta t = \frac{\pi d}{2v}$$

20. Sol:

Consider the following diagram,



The equation of magnetic field for loop wire is,

$$B_1 = \frac{\mu_o I \times 2\pi}{2\pi l}$$

$$= \frac{\mu_o I}{l}$$

The equation of magnetic field for coiled wire is,

$$B_2 = 3 \times \frac{\mu_o (I/3)}{2\pi l} \times 6\pi$$

$$= \frac{3\mu_o I}{l}$$

Taking ratio,

$$\frac{B_1}{B_2} = \frac{\frac{\mu_o I}{l}}{\frac{3\mu_o I}{l}}$$

$$= \frac{1}{3}$$

21. Sol:

The inductive reactance is,

$$X_L = \omega L$$

Substitute the values.

$$X_L = 500 \times 0.2$$
$$= 10\,\Omega$$

The current lags behind voltage by $\frac{\pi}{2}$ because the circuit is pure inductive. So,

$$I = 10\sin\left(\omega t - \frac{\pi}{2}\right)$$
$$= -10\cos(\omega t)$$
$$I = -10\cos(500t)$$

22. Sol:

The intensity of electric field is,

$$I = \frac{P}{4\pi r^2} \qquad \ldots\ldots (1)$$

Also,

$$I = \frac{1}{2}\varepsilon E^2 C$$

From equation (1),

$$\frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon E^2 C$$

$$E^2 = \frac{P}{2\pi r^2 \varepsilon C}$$

$$E = \sqrt{\frac{P}{2\pi r^2 \varepsilon C}}$$

$$= \sqrt{\frac{3}{2\pi(5)^2 \times 8.85\times10^{-12} \times 3\times10^{8}}}$$

$$= 2.68\ V/m$$

The electric field intensity will be half of the total intensity and thus,

$$E' = \frac{2.68}{2}$$

$$= 1.34\ V/m$$

23. Sol:

The light bends towards the normal as it passes through rarer to denser medium and as it goes through lens to medium.

Thus, option (1) is correct.

24. Sol:

From the geometry of the given figure,

$l = x\cos 30^{o}$

$$x = \frac{2l}{\sqrt{3}}$$

The expression of total path difference by light is,

$$2x + \frac{\lambda}{2} - 2l = n\lambda$$

For maxima, the path difference should be $n\lambda$,

$$2x + \frac{\lambda}{2} - 2l = n\lambda$$

$$2\left(\frac{2l}{\sqrt{3}}\right) + \frac{\lambda}{2} - 2l = n\lambda$$

$$l = \frac{(2n-1)\lambda\sqrt{3}}{4\left(2-\sqrt{3}\right)}$$

25. Sol:

Write the expression for the first minima of red.

$$\sin\theta = \frac{\lambda_r}{x} \qquad \ldots\ldots (1)$$

Write the expression for maxima for a wavelength λ_2.

$$\sin\theta_2 = \frac{3\lambda_2}{2x} \qquad \ldots\ldots (2)$$

As the light coincides, So compare (1) and (2).

$$\frac{3\lambda_2}{2x} = \frac{\lambda_r}{x}$$

$$\lambda_2 = \frac{2}{3} \times 6600$$

$$= 4400 \times 10^{-10} \text{ m}$$

$$= 4400 \overset{0}{\text{A}}$$

26. Sol:

Write the expression for the number of photons per second by a monochromatic light.

$$n = \frac{I \Delta t \lambda}{hc}$$

Substitute the values,

$$n = \frac{\left(1.8 \times 10^{-3}\right) \times 1 \times 4972 \times 10^{-10} \times 10^{-4}}{6.64 \times 10^{-34} \times 3 \times 10^{8}}$$

$$= 4.52 \times 10^{11} \text{ photons per second}$$

The number of photons ejected in 2 s is,

$$= 2 \times 4.52 \times 10^{11}$$

$$= 9 \times 10^{11} \text{ photons}$$

27. Sol:

The radioactivity in t^{th} time,

$$R = \frac{R_o}{2^{\frac{t}{T_H}}}$$

$$2^{\frac{t}{T_H}} = \frac{R_o}{R}$$

Substitute the values.

$$2^{\frac{t}{T_H}} = \frac{16}{12}$$

$$= \frac{4}{3}$$

Take log both sides, and use property of log.

$$\frac{t}{T_H} = \log\left(\frac{4}{3}\right)$$

$$t = \frac{T_H}{\log 2}\left[2\log 2 - \log 3\right]$$

$$= \frac{5760}{\log 2}\left[2\log 2 - \log 3\right]$$

$$= 2391 \text{ years}$$

28. Sol:

λ should lie between 4000×10^{-10} to 7600×10^{-10} m for emitting light. The equation of minimum energy is,

$$E_{min} = \frac{hc}{\lambda \times 1 \text{ Coulomb}}$$

Substitute the values in above expression,

$$E_{min} = \frac{hc}{\lambda \times 1.6 \times 10^{-19}}$$

$$= \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{7600 \times 10^{-10} \times 1.6 \times 10^{-19}}$$

$$= 1.7 \text{ eV}$$

Similarly,

$$E_{max} = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{4000 \times 10^{-10} \times 1.6 \times 10^{-19}}$$

$$= 3 \text{ eV}$$

29. Sol:

The range of frequency for radio wave is $5\,\text{MHz}$ to $25\,\text{MHz}$ for sky wave propagation. Thus, option (2) is correct.

30. Sol:

The potential gradient is,

$$\frac{V}{l} = \frac{1.1}{440} \text{ V/cm}$$

The voltage across 220 cm is,

$$V = \frac{V}{l} \times 220$$

$$= \frac{1.1}{440} \times 220$$

$$= 0.55 \text{ V}$$

The error in reading is,

$$0.5 - 0.55 = -0.05 \text{ V}$$

31. If m and e are the mass and charge of the revolving electron in the orbit of radius r for hydrogen atom, the total energy of the revolving electron will be :

(1) $\frac{1}{2}\frac{e^2}{r}$

(2) $-\frac{e^2}{r}$

(3) $\frac{me^2}{r}$

(4) $-\frac{1}{2}\frac{e^2}{r}$

32. The de-Broglie wavelength of a particle of mass 6.63 g moving with a velocity 100 ms^{-1} is :

(1) $10^{-33\ m}$

(2) $10^{-35\ m}$

(3) $10^{-31\ m}$

(4) $10^{-25\ m}$

33. What happens when an inert gas is added to an equilibrium keeping volume unchanged ?

(1) More product will form

(2) Less product will form

(3) More reactant will form

(4) Equilibrium will remain unchanged

34. The amount of $BaS0_4$ formed upon mixing 100 mL of 20.8% $BaCl_2$ solution with 50 mL of 9.8% H_2SO_4 solution will be :

(Ba = 137, Cl = 35.5, S=32, H = 1 and O = 16)

(1) 23.3 g

(2) 11.65 g

(3) 30.6 g

(4) 33.2 g

35. The rate coefficient (k) for a particular reaction is $1.3\times10^{-4}\ M^{-1}s^{-1}$ at 100 C, and $1.3\times10^{-3}\ M^{-1}s^{-1}$ at 150 C. What is the energy of activation (E_A) (in kJ) for this reaction? (R = molar gas constant = 8.314 $JK_{-1}mol_{-1}$)

(1) 16

(2) 60

(3) 99

(4) 132

36. How many electrons would be required to deposit 6.35 g of copper at the cathode during the electrolysis of an aqueous solution of copper sulphate ? (Atomic mass of copper = 63.5 u, N_A = Avogadro's constant) :

(1) $\frac{N_A}{20}$

(2) $\frac{N_A}{10}$

(3) $\frac{N_A}{5}$

(4) $\frac{N_A}{2}$

37. The entropy (S°) of the following substances are :

CH_4 (g) 186.2 J K^{-1} mol^{-1}

O_2 (g) 205.0 J K^{-1} mol^{-1}

CO_2 (g) 213.6 J K^{-1} mol^{-1}

H_2O (*l*) 69.9 J K^{-1} mol^{-1}

The entropy change (ΔS°) for the reaction

CH_4 (g) + $2O_2 \rightarrow CO_2$ (g) + 2H2O (l) is :

(1) $-312.5\ JK^{-1}\ mol^{-1}$

(2) $-242.8\ JK^{-1}\ mol^{-1}$

(3) $-108.1\ JK^{-1}\ mol^{-1}$

(4) $-37.6\ JK^{-1}\ mol^{-1}$

38. The conjugate base of hydrazoic acid is :

(1) N^{-3}

(2) N_3^-

(3) N_2^-

(4) HN_3^-

39. In a monoclinic unit cell, the relation of sides and angles are respectively :

(1) $a = b \neq c$ and $\alpha = \beta = \gamma = 90^o$

(2) $a \neq b \neq c$ and $\alpha = \gamma = \beta = 90^o$

(3) $a \neq b \neq c$ and $\beta = \gamma = 90^o \neq \alpha$

(4) $a \neq b \neq c$ and $\alpha \neq \beta \neq \gamma \neq 90^o$

40. The standard enthalpy of formation ($\Delta_f H^o_{298}$) for methane, CH_4 is– 74.9 kJ mol^{-1}. In order to calculate the average energy given out in the formation of a C–H bond from this it is necessary to know which one of the following?

(1) the dissociation energy of the hydrogen molecule, H_2.

(2) the first four ionisation energies of carbon.

(3) the dissociation energy of H_2 and enthalpy of sublimation of carbon (graphite).

(4) the first four ionisation energies of carbon and electron affinity of hydrogen.

41. Which of the following xenon-OXO compounds may not be obtained by hydrolysis of xenon fluorides ?

(1) XeO_2F_2

(2) $XeOF_4$

(3) XeO_3

(4) XeO_4

42. Excited hydrogen atom emits light in the ultraviolet region at 2.47×10^{15} Hz. With this frequency, the energy of a single photon is :

($h = 6.63 \times 10^{-34}$ Js)

(1) 8.041×10^{-40} J

(2) 2.680×10^{-40} J

(3) 1.640×10^{-40} J

(4) 6.111×10^{-40} J

43. Which one of the following exhibits the largest number of oxidation states ?

(1) Ti (22)

(2) V (23)

(3) Cr (24)

(4) Mn (25)

44. Copper becomes green when exposed to moist air for a long period. This is due to :

(1) the formation of a layer of cupric oxide on the surface of copper.

(2) the formation of a layer of basic carbonate of copper on the surface of copper.

(3) the formation of a layer of cupric hydroxide on the surface of copper.

(4) the formation of basic copper sulphate layer on the surface of the metal.

45. Among the following species the one which causes the highest CFSE, Δo as a ligand is :

(1) CN^{-1}

(2) NH_3

(3) F^-

(4) CO

46. Similarity in chemical properties of the atoms of elements in a group of the Periodic table is most closely related to :

(1) atomic numbers

(2) atomic masses

(3) number of principal energy levels

(4) number of valence electrons

47. Which of the following arrangements represents the increasing order (smallest to largest) of ionic radii of the given species O^{2-}, S^{2-}, N^{3-}, P^{3-} ? :

(1) $O^{2-} < N^{3-} < S^{2-} < P^{3-}$

(2) $O^{2-} < P^{3-} < N^{3-} < S^{2-}$

(3) $N^{3-} < O^{2-} < P^{3-} < S^{2-}$

(4) $N^{3-} < S^{2-} < O^{2-} < P^{3-}$

48. Global warming is due to increase of :

(1) methane and nitrous oxide in atmosphere

(2) methane and CO_2 in atmosphere

(3) methane and O_3 in atmosphere

(4) methane and CO in atmosphere

49. Hydrogen peroxide acts both as an oxidising and as a reducing agent depending upon the nature of the reacting species. In which of the following cases H_2O_2 acts as a reducing agent in acid medium ?

(1) MnO_4^-

(2) $Cr_2O_7^{2-}$

(3) SO_3^{2-}

(4) KI

50. Which one of the following complexes will most likely absorb visible light ?

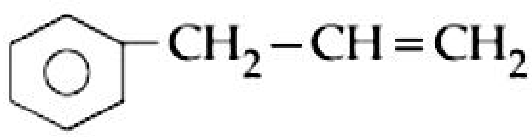
(At nos. Sc = 21, Ti = 22, V = 23, Zn = 30)

(1) $[Sc(H_2O)_6]^{3+}$

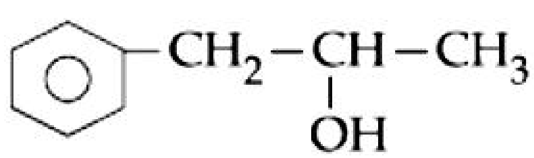
(2) $[Ti(NH_3)_6]^{4+}$

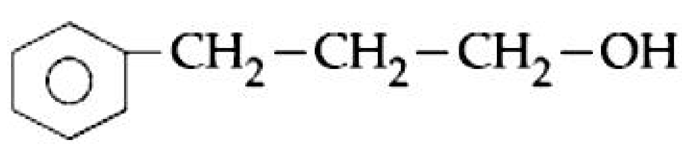
(3) $[V(NH_3)_6]^{3+}$

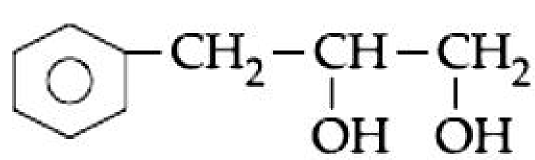
(4) $Zn(NH_3)_6]^{2+}$

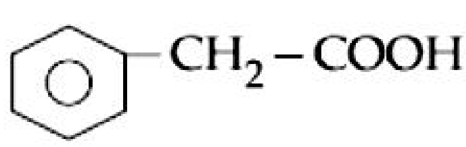
51. 

on mercuration-demercuration produces the major product :

(1) 

(2) 

(3) 

(4) 

52. In the Victor-Meyer's test, the colour givem by 1°, 2° and 3 alcohols are respectively :

(1) Red, colourless, blue

(2) Red, blue, colourless

(3) Colourless, red, blue

(4) Red, blue, violet

53. Conversion of benzene diazonium chloride to chloro benzene is an example of which of the following reactions ?

(1) Claisen

(2) Friedel-craft

(3) Sandmeyer

(4) Wurtz

54. In the presence of peroxide, HCl and HI do not give anti-Markownikoff s addition to alkenes because :

(1) One of the steps is endothermic in HCl and HI

(2) Both HCl and HI are strong acids

(3) HCl is oxidizing and the HI is reducing

(4) All the steps are exothermic in HCl and HI

55. The major product obtained in the photo catalysed bromination of 2-methylbutane is :

(1) 1-bromo-2-methylbutane

(2) 1-bromo-3-methylbutane

(3) 2-bromo-3-methylbutane

(4) 2-bromo-2-methylbutane

56. Which of the following molecules has two sigma (σ) and two pi(π)bonds :

(1) C_2H_4

(2) N_2F_2

(3) $C_2H_2Cl_2$

(4) HCN

57. Which one of the following acids does not exhibit optical isomerism ?

(1) Lactic acid

(2) Tartaric acid

(3) Maleic acid

(4) α-amino acids

58. Aminoglycosides are usually used as :

(1) antibiotic

(2) analgesic

(3) hypnotic

(4) antifertility

59. Which of the following will not show mutarotation?

(1) Maltose

(2) Lactose

(3) Glucose

(4) Sucrose

60. Phthalic acid reacts with resorcinol in the presence of concentrated H_2SO_4 to give :

(1) Phenolphthalein

(2) Alizarin

(3) Coumarin

(4) Fluorescein

Part-2

31.Sol:

The total energy of a revolving electron in the hydrogen atom is calculated by adding the potential energy and kinetic energy of the electron.

$$TE = PE + KE$$

$$= -\frac{Ze^2}{r} + \frac{Ze^2}{2r}$$

$$= -\frac{Ze^2}{2r}$$

The value of Z for hydrogen atom is 1.

$$E = -\frac{(1)e^2}{2r}$$

$$= -\frac{e^2}{2r}$$

32. Sol:

The momentum of the particle is,

$$p = m \cdot v$$
$$= 6.63 \times 10^{-3}\ \text{kg} \times 100\ \text{m/s}$$
$$= 0.663\ \text{kg} \cdot \text{m/s}$$

The de Broglie wavelength of the given particle,

$$p = \frac{h}{\lambda}$$
$$\lambda = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{0.663\ \text{kg} \cdot \text{m/s}}$$
$$= 10.00 \times 10^{-34}\ \text{m} \left(= 10^{-33}\ \text{m}\right)$$

33. Sol:

The molar concentration of reactants and products does not change by the addition of inert gas at constant volume, and due to which the state of equilibrium remains unchanged. Thus, the addition of an inert gas at constant volume does not affect the equilibrium.

34. Sol:

The number of moles of $BaCl_2$ is,

$$\text{Moles of } BaCl_2 = \frac{2.08\ \text{g}}{137\ \text{g/mol} + 2 \times 35.5\ \text{g/mol}}$$
$$= 0.01\ \text{mol}$$

The molarity of the given $BaCl_2$ solution is,

$$\text{Molarity of } BaCl_2 = \frac{0.01\text{ mol}}{0.1\text{ L}}$$
$$= 0.1\text{ M}$$

The number of moles of H_2SO_4 is,

$$\text{Moles of } H_2SO_4 = \frac{0.98\text{ g}}{2\times 1\text{ g/mol} + 32\text{ g/mol} + 4\times 16\text{ g/mol}}$$
$$= 0.01\text{ mol}$$

The molarity of the given H_2SO_4 solution is,

$$\text{Molarity of } H_2SO_4 = \frac{0.01\text{ mol}}{0.05\text{ L}}$$
$$= 0.2\text{ M}$$

The number of moles of $BaSO_4$ formed from $BaCl_2$ and H_2SO_4 solution is 0.1 mol because the number of moles of barium is limited. The mass of $BaSO_4$ formed in the given system is calculated by multiplying its moles with the molar mass.

$$\text{Mass of } BaSO_4 = 0.1\text{ mol}\times\left(137\text{ g/mol} + 32\text{ g/mol} + 4\times 16\text{ g/mol}\right)$$
$$= 23.3\text{ g}$$

35. Sol:

The activation energy of the given reaction is,

$$\ln\left(\frac{k_1}{k_2}\right)=\left(\frac{E_a}{R}\right)\left(\frac{1}{T_2}-\frac{1}{T_1}\right)$$

$$\ln\left(\frac{1.3\times10^{-4}\ M^{-1}s^{-1}}{1.3\times10^{-3}\ M^{-1}s^{-1}}\right)=\left(\frac{E_a}{8.314\ J/mol\cdot K}\right)\left(\frac{1}{423\ K}-\frac{1}{373\ K}\right)$$

$$E_a=-26235.5\times\ln\left(\frac{1.3\times10^{-4}\ M^{-1}s^{-1}}{1.3\times10^{-3}\ M^{-1}s^{-1}}\right)$$

$$E_a=60409\ J/mol\,(=60.4\ kJ/mol)$$

36. Sol:

The number of molecules of copper formed in the given electrolysis process is,

$$\text{Molecules of Cu}=\frac{6.35\ g\times N_A}{63.5\ g/mol}$$

$$=0.1N_A$$

$$=\frac{N_A}{10}$$

Since one copper molecule is formed by 2 electrons. So, the number of electrons is double the number of molecules of copper. Hence, the number of electrons required in the given system is calculated as,

$$\text{Number of electrons} = 2 \times \frac{N_A}{10}$$

$$= \frac{N_A}{5}$$

37. Sol:

The standard entropy change of the given reaction is,

$$\Delta S^o = \sum nS^o(\text{products}) - \sum mS^o(\text{reactants})$$

$$= \left(S^o_{CO_2(g)} + 2S^o_{H_2O(l)}\right) - \left(S^o_{CH_4(g)} + 2S^o_{O_2(g)}\right)$$

$$= (1\ \text{mol} \times 213.6\ \text{J/mol}\cdot\text{K} + 2\ \text{mol} \times 69.9\ \text{J/mol}\cdot\text{K})$$

$$-(1\ \text{mol} \times 186.2\ \text{J/mol}\cdot\text{K} + 2\ \text{mol} \times 205.0\ \text{J/mol}\cdot\text{K})$$

$$= -242.8\ \text{J/mol}\cdot\text{K}$$

38. Sol:

The expression of the dissociation of hydrazoic acid is,

$$HN_3 \rightarrow H^+ + N_3^-$$

Thus, the conjugate base of hydrazoic acid is N_3^-.

39. Sol:

In a monoclinic unit cell, the relation between sides is $a \neq b \neq c$, and the relationship between angles is $\gamma = \beta = 90°$ and $\alpha \neq 90°$.

40. Sol:

Write the expression of the balanced chemical equation for the formation of methane.

$$C(s) + H_2(g) \rightarrow CH_4(g) \qquad \Delta_f H^o{}_{298} = -74.9 \text{ kJ/mol}$$

If the values of enthalpy of sublimation of carbon and bond dissociation energy of H_2 are known, the average energy released in the given reaction can be calculated.

41. Sol:

The chemical reactions of xenon fluoride with water are,

$$XeF_6 + H_2O \rightarrow XeOF_4 + 2HF$$
$$XeF_6 + 2H_2O \rightarrow XeO_2F_2 + 4HF$$
$$XeF_6 + 3H_2O \rightarrow XeO_3 + 6HF$$

The oxidation state of xenon in XeO_4 is $+8$.

42. Sol:

The energy of a photon is,

$$E = h\nu$$
$$= 6.63 \times 10^{-34}\ \text{J s} \times 2.47 \times 10^{15}\ \text{Hz}$$
$$= 1.640 \times 10^{-18}\ \text{J}$$

43. Sol:

The number of oxidation states exhibited by an element depends upon the number of unpaired electrons present in the d orbital. Since, these are five unpaired electrons in the d orbital of manganese that is larger than that of titanium, vanadium, and chromium. Thus, manganese exhibits the largest number of oxidation states among the given compounds.

44. Sol:

The copper metal in presence of air for a longer time reacts with carbon dioxide, water, and oxygen to form copper carbonate and copper hydroxide. The color of copper changes to green due to the formation of copper carbonate (green color).

The chemical equation involved in the reaction of copper with moist air is given as,

$$2Cu + H_2O + CO_2 + O_2 \rightarrow CuCO_3 + Cu(OH)_2$$

45. Sol:

The crystal field splitting energy of ligand depends upon the strength of ligands. The splitting of molecular orbitals occurs due to the higher strength of ligands, cause results in the high crystal field splitting energy. Since the strength of CO ligand is higher than that of the CN^-, NH_3 and F^-. Therefore, CO causes highest crystal field splitting energy out of the given ligands.

46. Sol:

According to the modern periodic law, the similarity in the chemical properties is shown by the elements having an equal number of valence electrons in their valence shell. Thus, the similarity in the chemical properties of elements in a group is related to the number of valence electrons as all elements present in a group have an equal number of valence electrons in their outermost shell.

47. Sol:

The atomic radii of O^{2-}, N^{3-} and S^{2-} is smaller than that of P^{3-} due to the atomic radius of elements decreases along the period. The atomic radius of O^{2-} is smaller than that of N^{3-} because both the ions have an equal number of electrons but the number of protons is greater in O^{2-} as compared to N^{3-}. Thus, the increasing order of atomic radii of the given elements is $O^{2-} < N^{3-} < S^{2-} < P^{3-}$.

48. Sol:

The increase in the amount of methane and carbon dioxide gas in the atmosphere causes global warming. The increase in the amount of these gases warms the earth's atmosphere which results in the global warming.

49. Sol:

The reducing agent in the reaction with permanganate ion in the acidic medium is hydrogen peroxide as shown in the chemical equation,

$$5H_2O_2 + 2MnO_4^- + 6H^+ \rightarrow 2Mn^{2+} + 8H_2O + 5O_2$$

50. Sol:

The electronic configuration of central metal ions in the given complexes is,

$Sc^{3+} = 1s^2 2s^2 2p^6 3s^2 3p^6$

$Ti^{4+} = 1s^2 2s^2 2p^6 3s^2 3p^6$

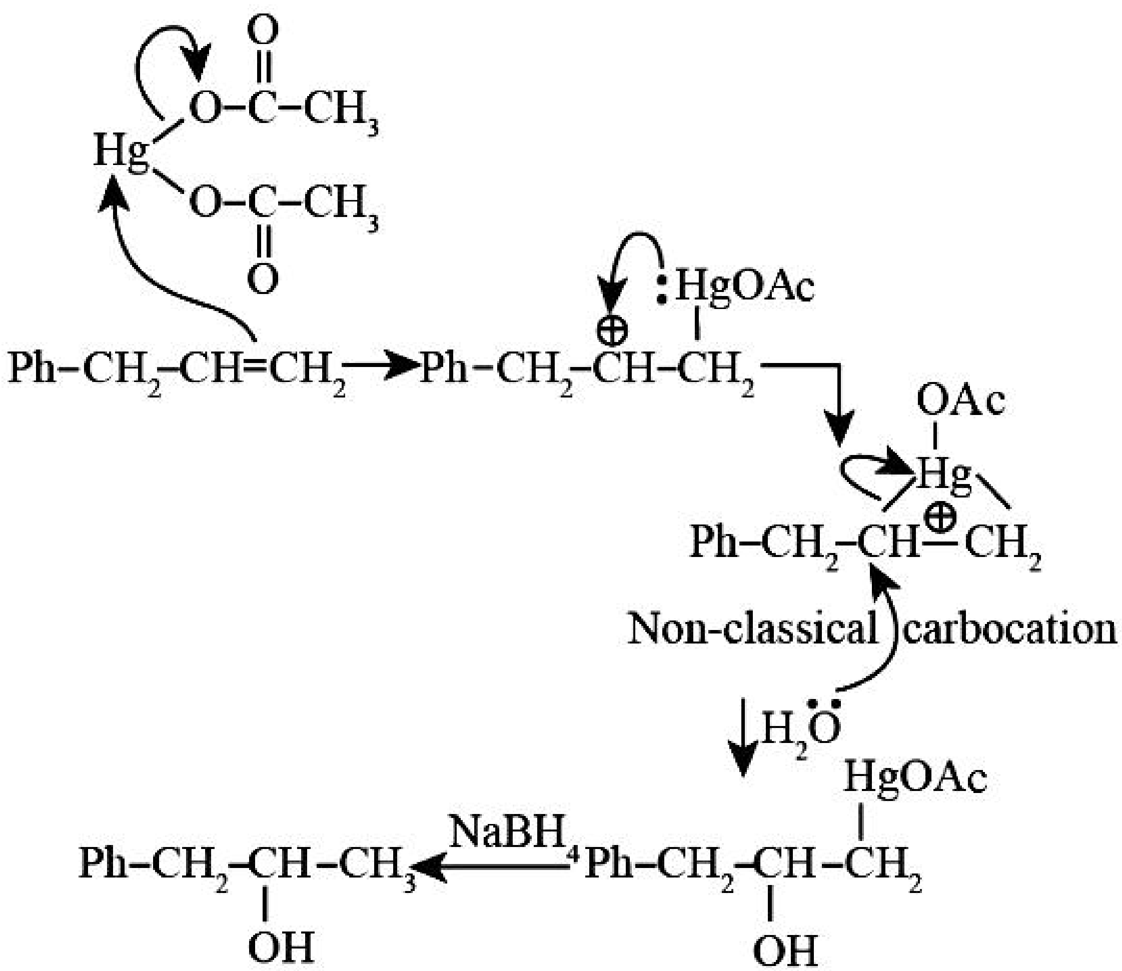
$V^{3+} = 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$

$Sc^{3+} = 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10}$

The central metal ion present in the complex $\left[V\left(NH_3\right)_6\right]^{3+}$ has its valence electrons in the $4s$ orbital that can easily excite to the $3d$ orbital and can absorb visible light, but in case of other complexes, the central metal ion has fully filled electronic configuration. Thus, the complex $\left[V\left(NH_3\right)_6\right]^{3+}$ is most likely to absorb visible light among the given complexes.

51. Sol:

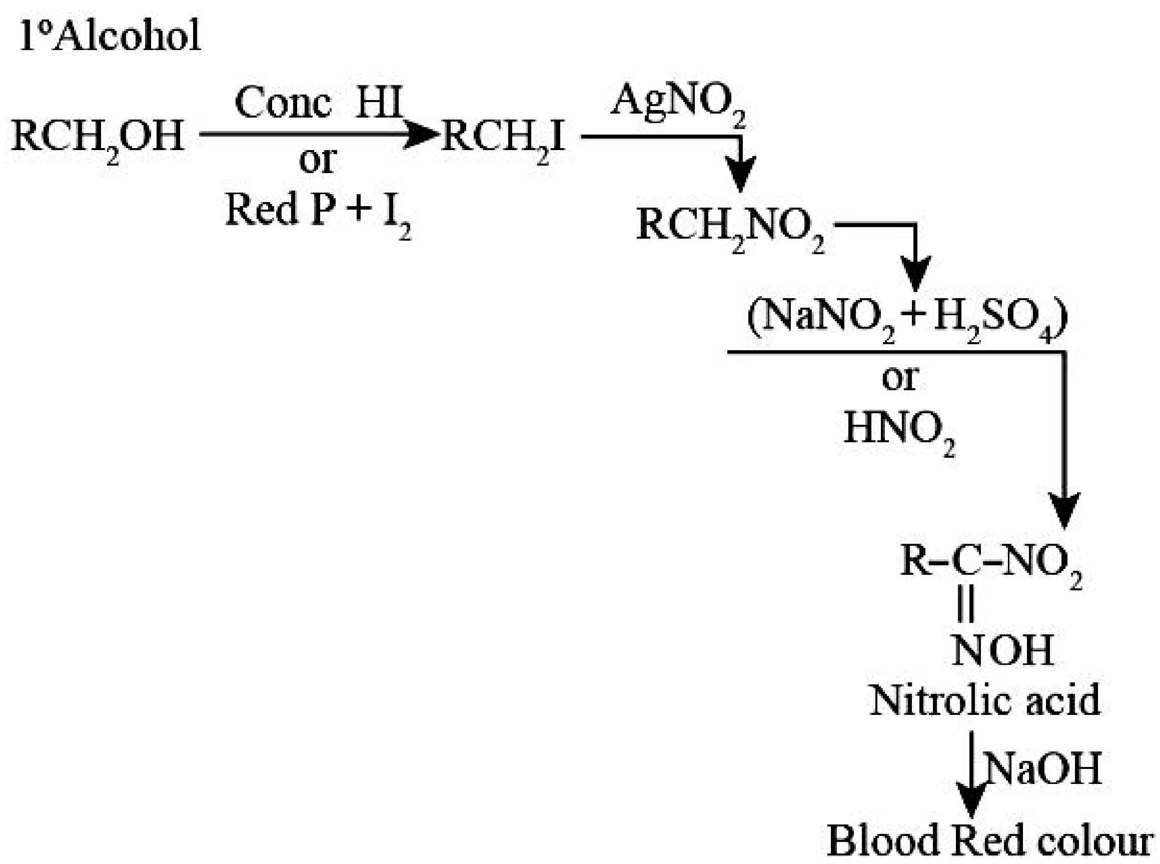
The mechanism involved in the mercuration-demercuration of the given compound is,



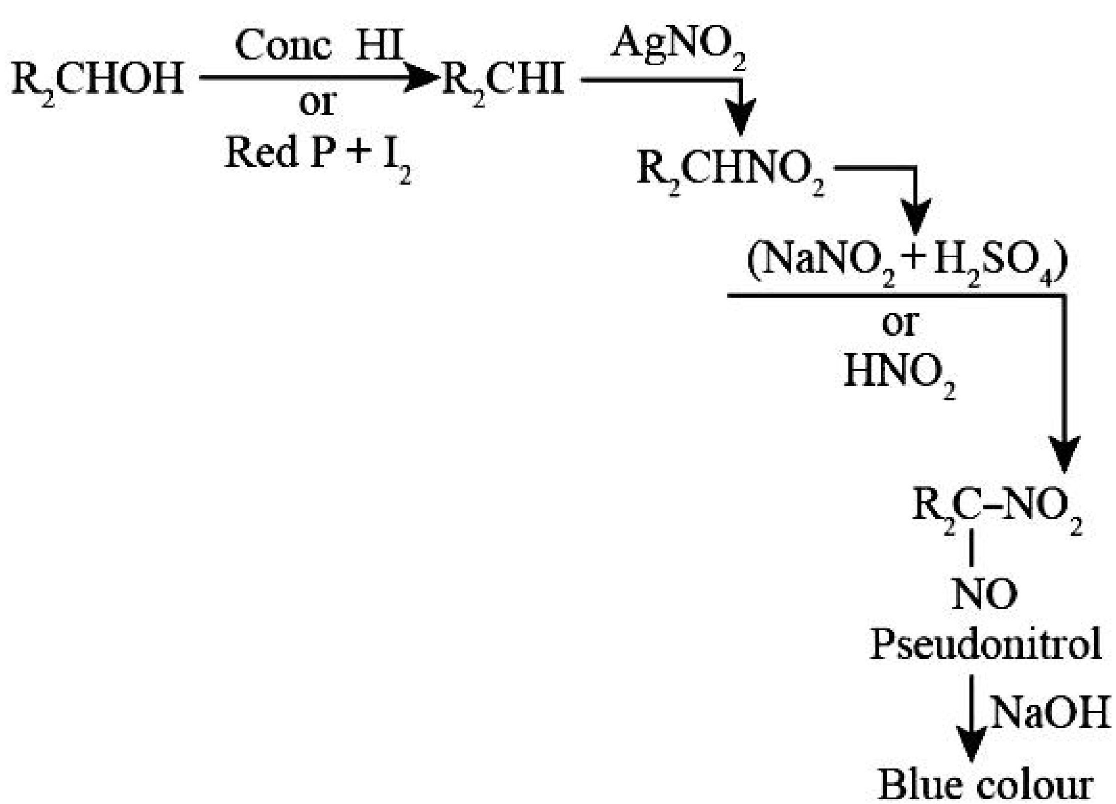
The mercuration-demercuration reaction of the given compound involves the formation of cyclic carbocation intermediate. This cyclic carbocation intermediate cannot undergo rearrangement because it is a type of non-classical carbocation.

52. Sol:

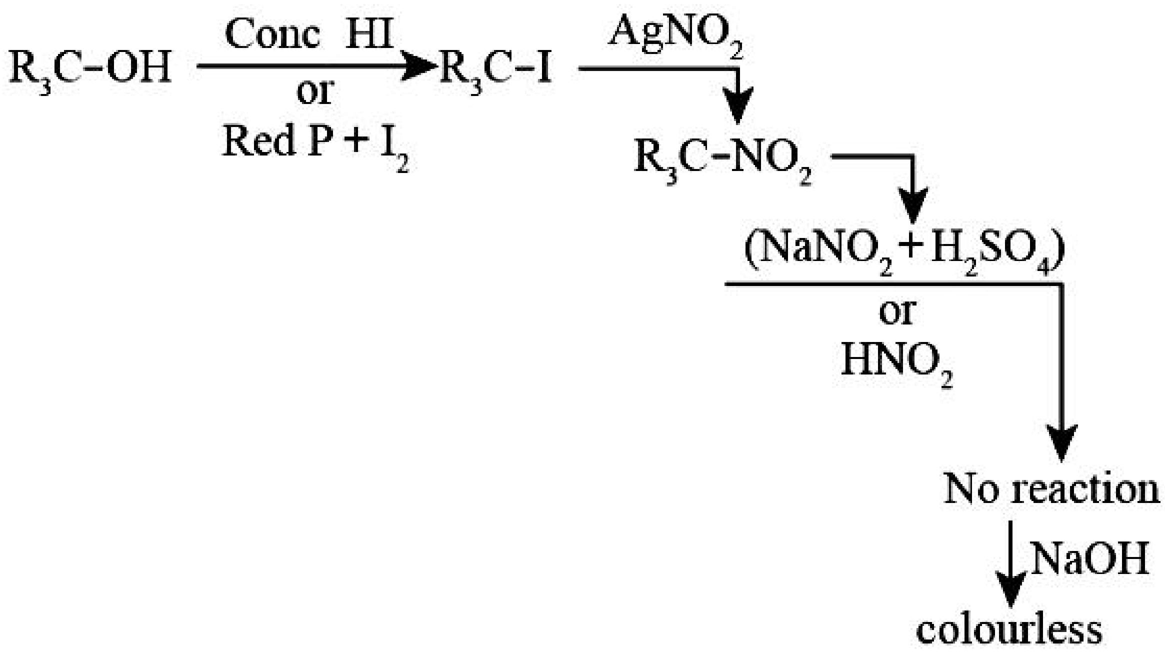
In the Victor Meyer's test, primary alcohols give blood red color solution of nitrolic acid in sodium hydroxide as shown below.



In the Victor Meyer's test, secondary alcohols give a blue color solution of pseudonitrol in sodium hydroxide as shown below.



At the end of Victor Meyer’s test, tertiary alcohols form a colorless solution as shown below.



53. Sol:

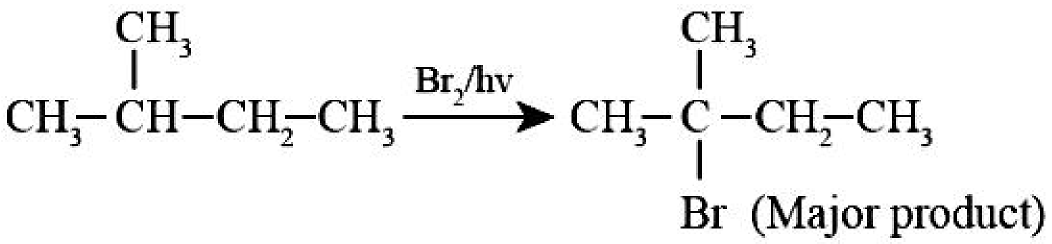
Sandmeyer reaction involves the reaction of benzene diazonium salts with the solution of copper halide, and hydrochloric acid leads to the formation of halogen substituted benzene. The conversion of benzene diazonium salt to chlorobenzene is shown below.

54. Sol:

The steps involved in the antimarkovnikov addition of hydrogen chloride and hydrogen iodide on alkenes in the presence of peroxide are endothermic in nature due to which the antimarkovnikov addition of hydrogen chloride and hydrogen iodide is not favorable.

55. Sol:

The photocatalyzed bromination of 2 – methylbutane proceeds via free radical mechanism. Since the stability of tertiary free radical is greater than that of primary and secondary radical, the major product obtained in the given reaction corresponds to that of tertiary free radical.



2-Bromo-2-methyl butane

Thus, the major product formed in the given reaction is 2 – bromo – 2 – methylbutane.

56. Sol:

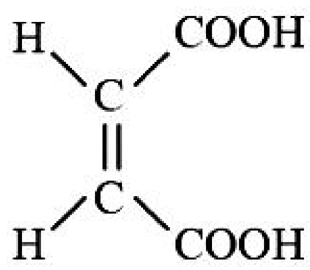
The structure of HCN is shown below.

$$H-C\equiv N$$

In the structure of HCN, there are two sigma bonds and two pi bonds.

57. Sol:

The structure of maleic acid is shown below.



Maleic acid exhibits only geometrical isomerism because only cis and trans arrangement is possible in its structure due to the presence of two groups at each carbon atom. So, the compound which does not exhibit optical isomerism is maleic acid.

58. Sol:

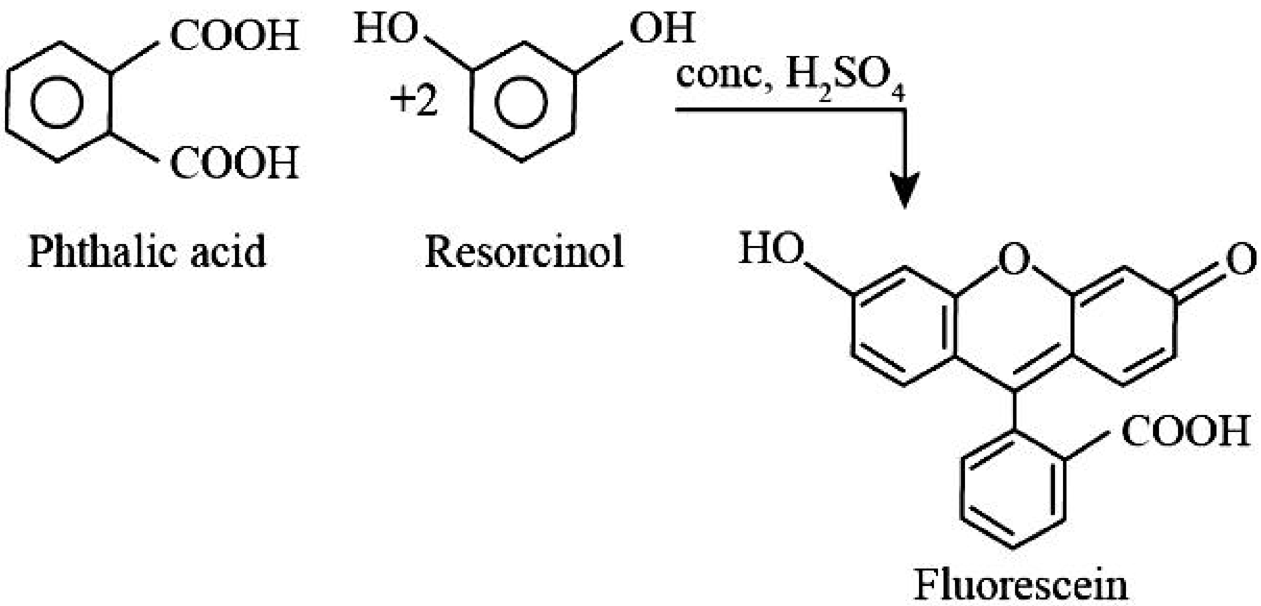
Aminoglycosides are a class of antibiotics due to which they are very useful as antibiotics. Thus, the aminoglycosides are usually used as antibiotics.

59. Sol:

Sucrose does not show mutarotation because the bond present between glucose and fructose is between the alpha hydroxyl group of both glucose and fructose due to which it cannot exists in alpha and beta form. Thus, sucrose does not undergo mutarotation.

60. Sol:

The formation of fluorescein takes place due to the reaction of phthalic acid with resorcinol as shown below.



Answer keys

1	2	19	3	37	2	55	4	73	1
2	3	20	2	38	2	56	4	74	2
3	4	21	2	39	3	57	3	75	4
4	2	22	1	40	3	58	1	76	1
5	4	23	1	41	4	59	4	77	4
6	1	24	3	42	3	60	4	78	3
7	2	25	2	43	4	61	2	79	3
8	3	26	2	44	2	62	3	80	4
9	2	27	2	45	4	63	3	81	4
10	2	28	4	46	4	64	3	82	4
11	2	29	2	47	1	65	1	83	2
12	2	30	4	48	2	66	2	84	3
13	1	31	4	49	1	67	2	85	2
14	4	32	1	50	3	68	1	86	2
15	2	33	4	51	1	69	3	87	1
16	2	34	1	52	2	70	2	88	2
17	3	35	2	53	3	71	3	89	3
18	2	36	3	54	1	72	2	90	2

JEE MAINS-12-APRIL-2014
MATHEMATICS

61. A relation on the set $A = \{x : |x| < 3, x \in Z\}$, where Z is the set of integers is defined by $R = \{(x, y) : y = |x|, x \neq 1\}$. Then the number of elements in the power set of R is :

(1) 32

(2) 16

(3) 8

(4) 64

62. Let $z \neq -i$ be any complex number such that $\frac{z-i}{z+i}$ is a purely imaginary number. Then $z + \frac{1}{z}$ is :

(1) 0

(2) any non-zero real number other than 1.

(3) any non- zero real number

(4) a purely imaginary number.

63. The sum of the roots of the equation, $x^2 + |2x - 3| - 4 = 0$, is :

(1) 2

(2) –2

(3) $\sqrt{2}$

(4) $-\sqrt{2}$

64. If $\begin{vmatrix} a^2 & b^2 & c^2 \\ (a+\lambda)^2 & (b+\lambda)^2 & (c+\lambda)^2 \\ (a+\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}, \lambda \neq 0$, then k is equal to :

(1) $4\lambda abc$

(2) $-4\lambda abc$

(3) λ^2

(4) $-4\lambda^2$

65. If $A = \begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$ be such that $AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$, then :

(1) $y = 2x$

(2) $y = -2x$

(3) $y = x$

(4) $y = -x$

66. 8-digit numbers are formed using the digits 1, 1, 2, 2, 2, 3, 4, 4. The number of such number in which the odd digits do not occupy odd places, is :

(1) 160

(2) 120

(3) 60

(4) 48

67. If $\left(2+\frac{x}{3}\right)^{55}$ is expanded in the ascending powers of x and the coefficients of powers of x in two consecutive terms of the expansion are equal, then these terms are :

(1) 7^{th} and 8^{th}

(2) 8^{th} and 9^{th}

(3) 28^{th} and 29^{th}

(4) 27^{th} and 28^{th}

68. Let G be the geometric mean of two positive numbers a and b, and M be the arithmetic mean of $\frac{1}{a}$ and $\frac{1}{b}$. If $\frac{1}{M}:G$ is 4 : 5, then a : b can be :

(1) 1 : 4

(2) 1 : 2

(3) 2 : 3

(4) 3 : 4

69. The least positive integer n such that $1-\frac{2}{3}-\frac{2}{3^2}-......-\frac{2}{3^{n-1}}<\frac{1}{100}$, is :

(1) 4

(2) 5

(3) 6

(4) 7

70. Let $f, g : R \to R$ be two functions defined by $f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$, and $g(x)=xf(x)$:

Statement I : f is a continuous function at $x = 0$.

Statement II : g is a differentiable function at $x = 0$.

(1) Both statements I and II are false.

(2) Both statement I and II are true.

(3) Statement I is true and statement II is false.

(4) Statement I is false and statement II is true.

71. If $f(x) = x^2 - x + 5, x > \frac{1}{2}$, and $g(x)$ is its inverse function, then $g'(7)$ is equals :

(1) $-\frac{1}{3}$

(2) $\frac{1}{13}$

(3) $\frac{1}{3}$

(4) $-\frac{1}{13}$

72. Let f and g be two differentiable functions on $\mathbf{R}$ such that $f'(x) > 0$ and $g'(x) < 0$, for all $x \in \mathbf{R}$. Then for all x :

(1) $f(g(x)) > f(g(x-1))$

(2) $f(g(x)) > f(g(x+1))$

(3) $g(f(x)) > g(f(x-1))$

(4) $g(f(x)) < g(f(x+1))$

73. If $1+x^4+x^5=\sum_{i=0}^{5} a_i (1+x)^i$, for all x in **R,** then a_2 is :

(1) –4

(2) 6

(3) –8

(4) 10

74. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^3 x+\cos^3 x)^2} dx$ is equal to :

(1) $\frac{1}{(1+\cot^3 x)}+c$

(2) $-\frac{1}{3(1+\tan^3 x)}+c$

(3) $\frac{\sin^3 x}{(1+\cos^3 x)}+c$

(4) $-\frac{\cos^3 x}{3(1+\sin^3 x)}+c$

75. If [ ] denote the greatest integer function, then the integral $\int_0^{\pi} [\cos x] dx$ is equal to :

(1) $\frac{\pi}{2}$

(2) 0

(3) -1

(4) $-\frac{\pi}{2}$

76. If for a continuous function $f(x)$, $\int_{-\pi}^{t}(f(x)+x)dx=\pi^2-t^2$, for all $t\geq-\pi$, then $f\left(-\frac{\pi}{3}\right)$ is equal to :

(1) π

(2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$

(4) $\frac{\pi}{6}$

77. The general solution of the differential equation, $\sin 2x\left(\frac{dy}{dx}-\sqrt{\tan x}\right)-y=0$, is :

(1) $y\sqrt{\tan x} = x + c$

(2) $y\sqrt{\cot x} = \tan x + c$

(3) $y\sqrt{\tan x} = \cot x + c$

(4) $y\sqrt{\cot x} = x + c$

78. If a line intercepted between the coordinate axes is trisected at a point A (4, 3), which is nearer to x-axis, then its equation is :

(1) $4x - 3y = 7$

(2) $3x + 2y = 18$

(3) $3x + 8y = 36$

(4) $x + 3y = 13$

79. If the three distinct lines $x + 2ay + a = 0$, $x + 3by + b = 0$ and $x + 4ay + a = 0$ are concurrent, then the point (a, b) lies on a :

(1) circle

(2) hyperbola

(3) straight line

(4) parabola

80. For the two circles $x^2 + y^2 = 16$ and $x^2 + y - 2y = 0$, there is/are:

(1) one pair of common tangents

(2) two pairs of common tangents

(3) three common tangents

(4) no common tangents

81. Two tangents are drawn from a point (–2,–1) to curve $y^2 = 4x$.

If α is the angle between them, then $|\tan\alpha|$ is equal to :

(1) $\frac{1}{3}$

(2) $\frac{1}{\sqrt{3}}$

(3) $\sqrt{3}$

(4) 3

82. The minimum area of a triangle formed by any tangent to the ellipse $\frac{x^2}{16}+\frac{y^2}{81}=1$ and the coordinate axes is :

(1) 12

(2) 18

(3) 26

(4) 36

83. A symmetrical form of the line of intersection of the planes $x = ay + b$ and $z = cy + d$ is :

(1) $\frac{x-b}{a} = \frac{y-1}{1} = \frac{z-d}{c}$

(2) $\frac{x-b-a}{a} = \frac{y-1}{1} = \frac{z-d-c}{c}$

(3) $\frac{x-a}{b} = \frac{y-0}{1} = \frac{z-c}{d}$

(4) $\frac{x-b-a}{b} = \frac{y-1}{0} = \frac{z-d-c}{d}$

84. If the distance between planes, $4x - 2y - 4z + 1 = 0$ and $4x - 2y - 4z + d = 0$ is 7, then d is :

(1) 41 or – 42

(2) 42 or – 43

(3) – 41 or 43

(4) – 42 or 44

85. If $\hat{x}, \hat{y}$ and $\hat{z}$ are three unit vectors in three-dimensional space, then the minimum value of $|\hat{x} + \hat{y}|^2 + |\hat{y} + \hat{z}|^2 + |\hat{z} + \hat{x}|^2$ is :

(1) $\frac{3}{2}$

(2) 3

(3) $3\sqrt{3}$

(4) 6

86. Let $\overline{X}$ and M.D. be the mean and the mean deviation about $\overline{X}$ of n observation x_i, $i = 1, 2, ..., n$. If each of the observation is increased by 5, then the new mean and the mean deviation about the new mean respectively , are :

(1) $\overline{X}$, MD.

(2) $\overline{X} + 5$, MD.

(3) $\overline{X}$, MD. + 5

(4) $\overline{X} + 5$, MD. + 5

87. A number x is chosen at random from the set $\{1, 2, 3, 4, ..., 100\}$. Define the event : A = the chosen number x satisfies $\frac{(x-10)(x-50)}{(x-30)} \geq 0$ Then P(A) is :

(1) 0.71

(2) 0.70

(3) 0.51

(4) 0.20

88. **Statement I** : The equation $\left(\sin^{-1}x\right)^3+\left(\cos^{-1}x\right)^3-a\pi^3=0$ has a solution for all $a\geq\frac{1}{32}$.

Statement II : For any $x\in\mathbf{R}$, $\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}$ and $0\leq\left(\sin^{-1}x-\frac{\pi}{4}\right)^2\leq\frac{9\pi^2}{16}$.

(1) Both statement I and II are true.

(2) Both statements I and II are false.

(3) Statement I is true and statement II is false

(4) Statement I is false and statement II is true

89. If $f(\theta)=\begin{vmatrix}1 & \cos\theta & 1\\ -\sin\theta & 1 & -\cos\theta\\ -1 & \sin\theta & 1\end{vmatrix}$ and A and B are respectively the maximum and the minimum values of $f(\theta)$, then (A, B) is equal to :

(1) $(3,-1)$

(2) $\left(4, 2-\sqrt{2}\right)$

(3) $\left(2+\sqrt{2}, 2-\sqrt{2}\right)$

(4) $\left(2+\sqrt{2}, -1\right)$

90. Let p, q, r denote arbitrary statements. Then the logically equivalent of the statement $p \Rightarrow (q \vee r)$ is :

(1) $(p \vee q) \Rightarrow r$

(2) $(p \Rightarrow q) \vee (p \Rightarrow r)$

(3) $(p \Rightarrow \sim q) \wedge (p \Rightarrow r)$

(4) $(p \Rightarrow q) \wedge (p \Rightarrow \sim r)$

PART-2

61. Sol:

For $|x|<3$, the set A is,

$A=\{-2,-1,0,1,2\}$

For $y=|x|$ and $x\neq-1$, the set R is,

$R=\{(-2,2)(0,0)(1,1)(1,2)\}$

The power set of R is,

$2^4=16$

62. Sol:

Let's consider $z=x+iy$

The expression is,

$$z+\frac{1}{z}=x+iy+\frac{1}{x+iy}$$

$$=x+iy+\frac{1\times(x-iy)}{x+iy(x-iy)}$$

$$=x+iy+\frac{(x-iy)}{x^2+y^2}$$

This term cannot be purely imaginary. Thus, $z+\frac{1}{z}$ is any non-zero real number.

63. Sol:

Write the equation when $x \geq \frac{3}{2}$.

$$x^2 + |2x - 3| - 4 = 0$$
$$x^2 + 2x - 7 = 0$$

Write the equation when $x < \frac{3}{2}$.

$$x^2 + |2x - 3| - 4 = 0$$
$$x^2 - 2x + 3 - 4 = 0$$
$$x^2 - 2x - 1 = 0$$

Compute the roots of the quadratic equations when $x \geq \frac{3}{2}$,

$$\begin{aligned} x_1 &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times (-7)}}{2 \times 1} \\ &= \frac{-2 \pm \sqrt{32}}{2} \\ &= -1 + 2\sqrt{2} \end{aligned}$$

Compute the roots of the quadratic equations when $x < \frac{3}{2}$,

$$x_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(2)^2 - 4 \times 1 \times (-1)}}{2 \times 1}$$

$$= \frac{2 \pm \sqrt{8}}{2}$$

$$= 1 - \sqrt{2}$$

Compute the sum of the roots as,

$$x_1 + x_2 = -1 + 2\sqrt{2} + 1 - \sqrt{2}$$

$$= \sqrt{2}$$

64. Sol:

By gauss elimination, apply the first row transformation $R_2 \to R_2 - R_1$ and $R_1 \to R_1 - R_3$.

$$\begin{vmatrix} \lambda(2a-\lambda) & \lambda(2b-\lambda) & \lambda(2c-\lambda) \\ 4a\lambda & 4b\lambda & 4c\lambda \\ (a-\lambda)^2 & (b-\lambda)^2 & (c-\lambda)^2 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

By gauss elimination, apply the second row transformation $R_3 \to R_3 + R_1$ and $R_1 \to R_1 - \frac{1}{2}R_2$.

$$\begin{vmatrix} -\lambda^2 & -\lambda^2 & -\lambda^2 \\ 4a\lambda & 4b\lambda & 4c\lambda \\ a^2 & b^2 & c^2 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$$-4\lambda^3 \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$$4\lambda^3 \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix} = k\lambda \begin{vmatrix} a^2 & b^2 & c^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$

$$k = 4\lambda^2$$

65. Sol:

The multiplication of AB is,

$$AB = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & x \\ 3 & -1 & 2 \end{bmatrix} \begin{bmatrix} y \\ x \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} y + 2x + x \\ 3y - x + 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

Solve the above matrix for first equation.

$$y + 2x + x = 6$$
$$3x + y = 6$$

Solve the above matrix for second equation.

$$3y - x + 2 = 8$$
$$3y - x = 6$$

Solve the above equations.

$$3x + y = 3y - x$$
$$4x = 2y$$
$$y = 2x$$

66. Sol:

The expression for the ways to select 3 odd places out of 4 odd places is,

$$n = {}^{4}C_{3} \times \frac{3!}{2!} \times \frac{5!}{3!2!}$$
$$= \frac{4!}{3!(4-3)!} \times \frac{3 \times 2!}{2!} \times \frac{5 \times 4 \times 3!}{3!2!}$$
$$= 4 \times 3 \times \frac{5 \times 4}{2}$$
$$= 120$$

67. Sol:

The expression for the general term from Binomial theorem is,

$$P(r) = {}^{n}C_{r} \cdot p^{r} \cdot q^{n-r}$$

$$\left(2+\frac{x}{3}\right)^{55} = {}^{55}C_{r} \cdot \left(\frac{x}{3}\right)^{r} \cdot 2^{55-r}$$

Let's consider the coefficients of $T_{r+1} = T_{r+2}$.

$${}^{55}C_{r} \cdot \left(\frac{1}{3}\right)^{r} \cdot 2^{55-r} = {}^{55}C_{r+1} \cdot \left(\frac{1}{3}\right)^{r+1} \cdot 2^{55-r-1}$$

$${}^{55}C_{r} \cdot \left(\frac{1}{3}\right)^{r} \cdot 2^{55-r} = {}^{55}C_{r+1} \cdot \left(\frac{1}{3}\right)^{r+1} \cdot 2^{54-r}$$

$$\frac{55!}{(55-r)!r!} \cdot 2 = \frac{55!}{(55-(r+1))!(r+1)!} \cdot \left(\frac{1}{3}\right)$$

$$r = 7$$

Thus, the first term is given as,

$$(r+1)^{\text{th}} = (7+1)^{\text{th}}$$
$$= 8^{\text{th}}$$

And, the second term is given as,

$$(r+2)^{\text{th}} = (7+2)^{\text{th}}$$
$$= 9^{\text{th}}$$

68. Sol:

Write expression for the geometric mean of numbers a and b.

$$G = \sqrt[2]{ab}$$

The value of b is ar.

Thus, the expression becomes,

$$G = \sqrt[2]{a \cdot ar}$$
$$= a\sqrt{r}$$

Calculate the arithmetic mean of numbers $\frac{1}{a}$ and $\frac{1}{b}$ as,

$$M = \frac{\frac{1}{a}+\frac{1}{b}}{2}$$
$$= \frac{\frac{1}{a}+\frac{1}{b}}{2}$$
$$= \frac{b+a}{2ab}$$
$$\frac{1}{M} = \frac{2ab}{b+a}$$

The ratio of $\frac{1}{M} : G$ is,

$$\frac{1}{M}:G=\frac{\frac{2ab}{b+a}}{ab}$$

$$\frac{\frac{2a\cdot ar}{ar+a}}{a\sqrt{r}}=\frac{4}{5}$$

$$\frac{a^2r}{a\sqrt{r}\cdot a(r+1)}=\frac{2}{5}$$

$$\frac{\sqrt{r}}{(r+1)}=\frac{2}{5}$$

Solve above equation,

$$5\sqrt{r}=2(r+1)$$

$$25r=4(r^2+1+2r)$$

$$4r^2+4+8r=25r$$

$$4r^2-17r+4=0$$

The roots of the above equation are,

$$4r(r-4)-1(r-4)=0$$

$$(4r-1)(r-4)=0$$

$$r=\frac{1}{4}\text{ and }4$$

Thus, the value of a:b is,

$$a = br$$

$$\frac{a}{b} = r$$

$$= \frac{1}{4}$$

69. Sol:

Write the general binomial expression.

$$1 - \frac{2}{3} - \frac{2}{3^2} - \ldots - \frac{2}{3^{n-1}} < \frac{1}{100}$$

$$1 - 2\left(\frac{1}{3} - \frac{1}{3^2} - \ldots - \frac{1}{3^{n-1}}\right) < \frac{1}{100}$$

Solve the above expression.

$$1 - 2\left(\frac{1}{3}\right)\frac{\left[1 - \frac{1}{3^{n-1}}\right]}{1 - \frac{1}{3}} < \frac{1}{100}$$

$$1 - 2\left(\frac{1}{3}\right)\frac{\left[1 - \frac{1}{3^{n-1}}\right]}{\frac{2}{3}} < \frac{1}{100}$$

$$1 - 1 + \frac{1}{3^{n-1}} < \frac{1}{100}$$

$$3^{n-1} > 100$$

Further solve the above expression,

$$3^{n-1} > 100$$

$$\frac{3^n}{3} > 100$$

$$n-1=5$$

$$n=6$$

70. Sol:

Write the given function.

$$f(x)=\begin{cases} x\sin\left(\frac{1}{x}\right) & x\neq 0 \\ 0 & x=0 \end{cases}$$

The value of the function $f(x)$ at $x\to 0$.

$$\lim_{x\to 0} x\sin\left(\frac{1}{x}\right)=0$$

$$=f(0)$$

So, the $f(x)$ is continuous function at $x=0$.

The value of the function $g(x)$ at $x\to 0$.

$$g(0)=0$$

$$=\lim_{x\to 0} g(x)$$

The expression for the left hand limit of the function $g(x)$ is,

$$\lim_{x\to 0^-} g(x) = \lim_{x\to 0^-} \frac{g(0-h)-g(0)}{-h}$$

$$= \lim_{x\to 0^-} \frac{h^2 \sin\left(-\frac{1}{h}\right)-0}{-h}$$

$$= 0$$

The expression for the right hand limit of the function $g(x)$ is,

$$\lim_{x\to 0^+} g(x) = \lim_{x\to 0^+} \frac{g(0+h)-g(0)}{h}$$

$$= \lim_{x\to 0^+} \frac{h^2 \sin\left(\frac{1}{h}\right)-0}{h}$$

$$= 0$$

Since, the left hand limit is equal to the right hand limit. Thus, the function $g(x)$ at $x=0$ is differentiable.

71. Sol:

Write the given function.

$$f(x) = x^2 - x + 5$$

Compute the value of x at which the value of $f(x)$ is 7 as,

$$7 = x^2 - x + 5$$

$$2 = x(x-1)$$

$$x = 2 \text{ and } 3$$

The value of the function $g(x)$ is,

$$g(f(x)) = x$$

Differentiate the above expression.

$$g'(f(x)) = \frac{1}{f'(x)}$$

$$= \frac{1}{f'(x^2 - x + 5)}$$

$$= \frac{1}{2x - 1}$$

The value of the function $g'(7)$ is,

$$g'(7) = \frac{1}{2 \times 2 - 1}$$

$$= \frac{1}{3}$$

72. Sol:

The term $f'(x) > 0$ shows that it is an increasing function.

The term $g'(x) < 0$ shows that it is an decreasing function.

In case (1),

$$x > x - 1$$

$$g(x) > g(x - 1)$$

$$f(g(x)) < f(g(x - 1))$$

In case (2),

$$x+1>x$$

$$g(x+1)>g(x)$$

$$f(g(x+1))<f(g(x))$$

$$f(g(x))>f(g(x+1))$$

In case (3),

$$x>x-1$$

$$f(x)>f(x-1)$$

$$g(f(x))<g(f(x-1))$$

In case (4),

$$x+1>x$$

$$f(x+1)>f(x)$$

$$g(f(x+1))<g(f(x))$$

$$g(f(x))>g(f(x+1))$$

73. Sol:

Write the given equation.

$$1+x^4+x^5=\sum_{i=0}^{5} a_i(1+x)^i$$

$$=\begin{bmatrix} a_0+a_1(1+x)+a_2(1+x)^2+ \\ a_3(1+x)^3+a_4(1+x)^4+ \\ a_5(1+x)^5 \end{bmatrix}$$

$$=\begin{bmatrix} a_0+a_1(1+x)+a_2(1+x^2+2x)+ \\ a_3(1+3x+3x^2+x^3)+a_4(1+4x+6x^2+4x^3+x^4) \\ +a_5(1+5x+10x^2+10x^3+5x^4+x^5) \end{bmatrix}$$

Compare the coefficients of x^5.

$$a_5=1$$

Compare the coefficients of x^4.

$$a_4+5a_5=1$$
$$a_4+5=1$$
$$a_4=-4$$

Compare the coefficients of x^3.

$$a_3+4a_4+10a_5=0$$
$$a_3-16+10=0$$
$$a_3=6$$

Compare the coefficients of x^2.

$$a_2 + 3a_3 + 6a_4 + 10a_5 = 0$$

$$a_2 + 18 - 24 + 10 = 0$$

$$a_2 = -4$$

74. Sol:

Write the given integral.

$$\int \frac{\sin^2 x \cos^2 x}{\left(\sin^3 x + \cos^3 x\right)^2} dx = \int \frac{\sin^2 x \cos^2 x}{\cos^6 x \left(\frac{\sin^3 x}{\cos^3 x} + \frac{\cos^3 x}{\cos^3 x}\right)^2} dx$$

$$= \int \frac{\tan^2 x \sec^2 x}{\left(\tan^3 x + 1\right)^2} dx$$

Let, the term $\tan^3 x + 1$ is,

$$\tan^3 x + 1 = p$$

$$\left(3\tan^2 x \sec^2 x\right) dx = dp$$

$$dx = \frac{dp}{3\tan^2 x \sec^2 x}$$

The integral is,

$$\int \frac{\tan^2 x \sec^2 x}{\left(\tan^3 x+1\right)^2} dx = \int \frac{\tan^2 x \sec^2 x}{(p)^2} \frac{dp}{3\tan^2 x \sec^2 x}$$

$$= \frac{1}{3}\int \frac{1}{(p)^2} dp$$

$$= \frac{-1}{3\left(1+\tan^3 x\right)} + c$$

75. Sol:

Write the given integral.

$$\int_0^{\pi} [\cos x] dx = \int_0^{\pi/2} 0 dx + \int_{\pi/2}^{\pi} -1 dx$$

$$= -(x)_{\pi/2}^{\pi}$$

$$= -\left[\pi - \frac{\pi}{2}\right]$$

$$= -\frac{\pi}{2}$$

76. Sol:

Write the given continuous function.

$$\int_{-\pi}^{t}\left(f(x)+x\right)dx=\pi^2-t^2$$

$$\int_{-\pi}^{t} f(x)dx+\int_{-\pi}^{t} xdx=\pi^2-t^2$$

$$\int_{-\pi}^{t} f(x)dx+\left(\frac{t}{2}-\frac{-\pi}{2}\right)=\pi^2-t^2$$

$$\int_{-\pi}^{t} f(x)dx=\frac{3}{2}\left(\pi^2-t^2\right)$$

Write the expression of the function.

$$\int_{-\pi}^{t} f(x)dx=\int_{-\pi}^{t} -3xdx$$

Thus, the value of $f(x)$ is,

$$f(x)=-3x$$

The value of $f\left(-\frac{\pi}{3}\right)$ is,

$$f\left(-\frac{\pi}{3}\right)=-3\left(-\frac{\pi}{3}\right)$$

$$=\pi$$

77. Sol:

Write the given differential equation.

$$\sin 2x\left(\frac{dy}{dx}-\sqrt{\tan x}\right)-y=0$$

$$\frac{dy}{dx}-\frac{y}{\sin 2x}=\sqrt{\tan x}$$

Compute the integration factor of the above equation as,

$$IF=e^{\int P(x)dx}$$

$$=e^{-\int \frac{1}{\sin 2x}dx}$$

$$=e^{-\frac{1}{2}\ln \tan x}$$

$$=\frac{1}{\sqrt{\tan x}}$$

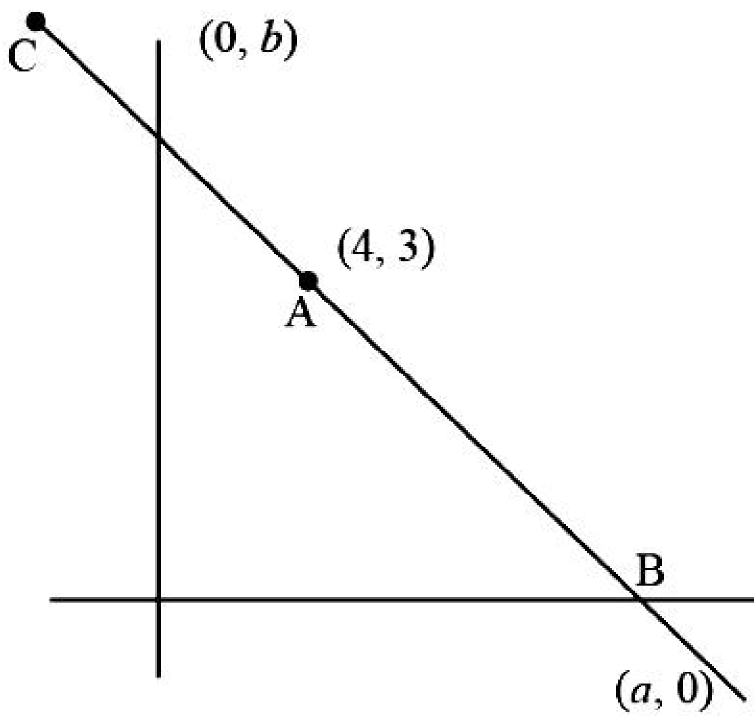
The expression for the general solution of the equation is,

$$y\cdot\frac{1}{\sqrt{\tan x}}=\int\sqrt{\tan x}\cdot\frac{1}{\sqrt{\tan x}}+c$$

$$y\sqrt{\cot x}=x+c$$

78. Sol:

The below figure represents the diagram of the line,



Write the value of the abscissa at point B.

$$\frac{a}{3} = 4$$

$$a = 12$$

Write the value of the ordinate at point C.

$$\frac{2b}{3} = 3$$

$$b = \frac{9}{2}$$

Write the equation of the line.

$$\frac{x}{a} + \frac{y}{b} = 1$$

$$\frac{x}{12} + \frac{2y}{9} = 1$$

$$3x + 8y = 36$$

79. Sol:

Write the matrix form of the given equations.

$$\begin{bmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4a & a \end{bmatrix} = 0$$

By gauss elimination, apply the first row transformation $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$.

$$\begin{bmatrix} 1 & 2a & a \\ 0 & 3b-2a & b-a \\ 0 & 2a & 0 \end{bmatrix} = 0$$

Compute the determinant of the above matrix as,

$$1\left[-2a\left(b-a\right)\right] = 0$$

$$a = b$$

The locus of (a, b) lies at x = 0 or y = x. Hence, the point (a, b) will lie on a straight line.

80. Sol:

The distance between their centers is 1 unit and sum of their radii is 3 from the two given equations of the circle. Thus, one of them lie completely inside the other. So, there will be no common tangent.

81. Sol:

Write the equation of the tangent.

$$y-(-1)=m(x-(-2))$$

$$y+1=m(x+2)$$

$$y=mx+(2m-1)$$

Write the condition of tangency.

$$C=\frac{a}{m}$$

$$2m-1=\frac{1}{m}$$

$$2m^2-m-1=0$$

$$(2m+1)(m-1)=0$$

Compute the roots of the above equation as,

$$(2m+1)(m-1)=0$$

$$m=-\frac{1}{2}\text{ and }1$$

Compute the value of the $\tan\alpha$ is,

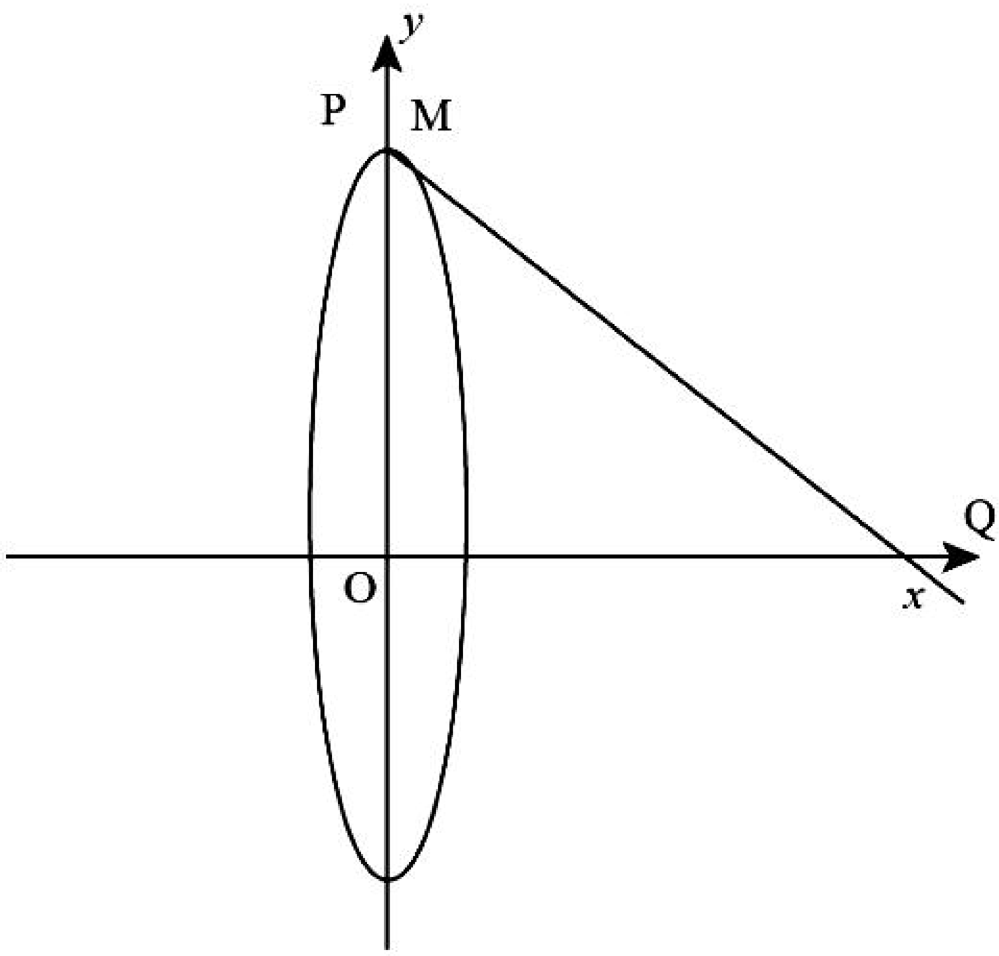
$$|\tan\alpha| = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$= \left|\frac{-\frac{1}{2} - 1}{1 + \left(-\frac{1}{2}\right)1}\right|$$

$$= 3$$

82. Sol:

Let the point on an ellipse is M having coordinates $(4\cos\theta, 9\sin\theta)$ as shown in figure.



Write the equation of the tangent.

$$\frac{x}{4}\cos\theta + \frac{y}{9}\sin\theta = 1$$

Let the points P and Q intersect the tangent at M with coordinate axis.

Write the coordinates of point P.

$$P=\left(\frac{4}{\cos\theta},0\right)$$

Write the coordinates of point Q.

$$P=\left(0,\frac{9}{\sin\theta}\right)$$

Compute the area of the triangle.

$$A=\frac{1}{2}\left(\frac{4}{\cos\theta}\right)\left(\frac{9}{\sin\theta}\right)$$
$$=\frac{36}{\sin 2\theta}$$

Compute the minimum area of the triangle as,

$$A_{\min}=\frac{36}{1}$$
$$=36$$

83. Sol:

Write the first equation.

$$x=ay+b$$
$$y=\frac{x-b}{a}$$

Write the second equation.

$$z = cy + d$$

$$y = \frac{z - d}{c}$$

Equate the both equations.

$$\frac{x - b}{a} = y = \frac{z - d}{c}$$

$$\frac{x - b}{a} - 1 = y - 1 = \frac{z - d}{c} - 1$$

$$\frac{x - b - a}{a} = \frac{y - 1}{1} = \frac{z - d - c}{c}$$

84. Sol:

The expression for the distance between the two lines is,

$$\left| \frac{d - 1}{\sqrt{4^2 + 2^2 + 4^2}} \right| = 7$$

$$\left| \frac{d - 1}{6} \right| = 7$$

$$d - 1 = \pm 42$$

$$d = +43 \text{ and } -41$$

85. Sol:

Write the given expression.

$$|\hat{x}+\hat{y}|^2+|\hat{y}+\hat{z}|^2+|\hat{z}+\hat{x}|^2=\begin{bmatrix}\hat{x}^2+\hat{y}^2+2\hat{x}\hat{y}+\hat{y}^2+\\ \hat{z}^2+2\hat{y}\hat{z}+\hat{z}^2+\hat{x}^2+2\hat{z}\hat{x}\end{bmatrix}$$

$$=\begin{bmatrix}2\hat{x}^2+2\hat{y}^2+2\hat{z}^2+\\ 2\hat{x}\cdot\hat{y}+2\hat{y}\cdot\hat{z}+2\hat{z}\cdot\hat{x}\end{bmatrix}$$

$$=\begin{bmatrix}2\cdot 1+2\cdot 1+2\cdot 1+\\ 2\left(\cos\theta_1+\cos\theta_2+\cos\theta_3\right)\end{bmatrix}$$

$$=6+2\left(\cos\theta_1+\cos\theta_2+\cos\theta_3\right)$$

The minimum value of given expression lies at

$$\theta_1=\theta_2=\theta_3=\frac{2\pi}{3}.$$

The minimum value of the given expression is,

$$|\hat{x}+\hat{y}|^2+|\hat{y}+\hat{z}|^2+|\hat{z}+\hat{x}|^2=6+2\left(\cos\frac{2\pi}{3}+\cos\frac{2\pi}{3}+\cos\frac{2\pi}{3}\right)$$

$$=6-3$$

$$=3$$

86. Sol:

The expression for the mean of the n observation is,

$$\bar{X}=\frac{a_1+a_2+a_3+\ldots+a_n}{n}$$

The new mean after the increment 5 in each observation is given as,

$$\bar{X}_{new}=\frac{(a_1+5)+(a_2+5)+(a_3+5)+\ldots+(a_n+5)}{n}$$

$$=\bar{X}+5$$

The expression for the mean deviation of the n observation is,

$$\text{M.D.}=\frac{(a_1-\bar{X})+(a_2-\bar{X})+\ldots+(a_n-\bar{X})}{n}$$

The new mean deviation after the increment 5 in each observation is given as,

$$(\text{M.D.})_{new}=\frac{\left[\begin{array}{l}((a_1+5)-(\bar{X}+5))+((a_2+5)-(\bar{X}+5))+\\ \ldots+((a_n+5)-(\bar{X}+5))\end{array}\right]}{n}$$

$$=\frac{(a_1-\bar{X})+(a_2-\bar{X})+\ldots+(a_n-\bar{X})}{n}$$

$$=\text{M.D.}$$

87. Sol:

Write the given expression.

$$\frac{(x-10)(x-50)}{(x-30)}\geq 0$$

Here, the range of x lies

$x\in\{10,11,12,13\ldots29\}\cup\{50,51,52\ldots100\}$.

The total value of x is 71.

Thus, the value of $P(A)$ is,

$$P(A) = \frac{71}{100}$$
$$= 0.71$$

88. Sol:

Write the given equation.

$$\left(\sin^{-1}x\right)^3 + \left(\cos^{-1}x\right)^3 - a\pi^3 = 0$$

$$\left(\sin^{-1}x\right)^3 + \left(\cos^{-1}x\right)^3 = a\pi^3$$

$$\left(\sin^{-1}x + \cos^{-1}x\right)^3 - 3\sin^{-1}x\cos^{-1}x\left(\sin^{-1}x + \cos^{-1}x\right) = a\pi^3$$

$$\left(\frac{\pi}{2}\right)^3 - 3\left(\frac{\pi}{2} - \cos^{-1}x\right)\cos^{-1}x\left(\frac{\pi}{2}\right) = a\pi^3$$

Solve the above equation.

$$\frac{\pi^3}{8} - \frac{3\pi^2}{4}\cos^{-1}x + \frac{3\pi}{2}\left(\cos^{-1}x\right)^2 = a\pi^3$$

$$\frac{3\pi}{2}\left[\left(\cos^{-1}x\right)^2 - \frac{\pi}{2}\cos^{-1}x\right] + \frac{\pi^3}{8} = a\pi^3$$

$$\frac{3\pi}{2}\left[\left(\cos^{-1}x - \frac{\pi}{4}\right)^2 - \frac{\pi^2}{16}\right] + \frac{\pi^3}{8} = a\pi^3$$

$$\left(\cos^{-1}x - \frac{\pi}{4}\right)^2 = \left(\frac{2a}{3} - \frac{1}{48}\right)\pi^2$$

Since, the value of $\cos^{-1} x$ lies,

$$0 \le \cos^{-1} x \le \pi$$

$$-\frac{\pi}{4} \le \left(\cos^{-1} x - \frac{\pi}{4}\right) \le \frac{3\pi}{4}$$

$$0 \le \left(\cos^{-1} x - \frac{\pi}{4}\right)^2 \le \frac{9\pi^2}{16}$$

Thus, statement I is false.

For statement II, the given equation is not applicable for any $x \in \mathrm{R}$. So, statement II is false.

89. Sol:

The determinant of the given matrix is calculated as,

$$f(\theta) = 1(1 + \sin\theta\cos\theta) - \cos\theta(-\sin\theta - \cos\theta) + 1(-\sin^2\theta + 1)$$

$$= 1 + \sin\theta\cos\theta + \sin\theta\cos\theta + \cos^2\theta - \sin^2\theta + 1$$

$$= 1 + \sin 2\theta + (1 + \cos 2\theta)$$

$$= 2 + \sin 2\theta + \cos 2\theta$$

The value of $\sin 2\theta + \cos 2\theta$ lies between $-\sqrt{2}$ and $\sqrt{2}$.

The maximum and minimum value of $\sin 2\theta + \cos 2\theta$ is,

$$2 + \sin 2\theta + \cos 2\theta = 2 + \left[\sqrt{2}\left(\sin\left(\frac{\pi}{4} + 2\theta\right)\right)\right]$$

$$= 2 \pm \sqrt{2}$$

The maximum value of A is $2+\sqrt{2}$ and the minimum value of B is $2-\sqrt{2}$.

90. Sol:

Simplify the given statement as,

$p \Rightarrow (q \vee r)$

$\sim p \vee (q \vee r)$

$(\sim p \vee q) \vee (\sim p \vee r)$

$(p \Rightarrow q) \vee (p \Rightarrow r)$