QUESTION BOOKLET CODE **A**

2011 CA

Test Paper Code: CA

Time: 3 Hours **Max. Marks: 300**

INSTRUCTIONS

A. General:

1. This Booklet is your Question Paper. It contains **20** pages and has 100 questions.
2. The Question Booklet **Code** is printed on the right-hand top corner of this page.
3. The Question Booklet contains blank spaces for your rough work. No additional sheets will be provided for rough work.
4. **Clip board, log tables, slide rule, calculator, cellular phone or any other electronic gadget in any form are NOT allowed.**
5. Write your **Name** and **Registration Number** in the space provided at the bottom.
6. All answers are to be marked only on the machine gradable Objective Response Sheet **(ORS)** provided along with this booklet, as per the instructions therein.
7. The Question Booklet along with the Objective Response Sheet **(ORS)** must be handed over to the Invigilator before leaving the examination hall.
8. Refer to **Special Instruction/Useful Data** on reverse of this sheet.

B. Filling-in the ORS:

9. Write your Registration Number in the boxes provided on the upper left-hand-side of the **ORS** and darken the appropriate bubble under each digit of your Registration Number using a **HB pencil**.
10. Ensure that the **code** on the **Question Booklet** and the **code** on the **ORS** are the same. If the codes do not match, report to the Invigilator immediately.
11. On the lower-left-hand-side of the **ORS**, write your Name, Registration Number, and Name of the Test Centre and put your signature in the appropriate box with ball-point pen. Do not write these anywhere else.

C. Marking of Answers on the ORS:

12. Each question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer.
13. On the right-hand-side of **ORS**, for each question number, darken with a **HB Pencil** ONLY one bubble corresponding to what you consider to be the most appropriate answer, from among the four choices.
14. There will be **negative marking** for wrong answers.

MARKING SCHEME:

(a) For each correct answer, you will be awarded **3 (Three)** marks.
(b) For each wrong answer, you will be awarded **-1 (Negative one)** mark.
(c) Multiple answers to a question will be treated as a wrong answer.
(d) For each un-attempted question, you will be awarded **0 (Zero)** mark.

Name							
Registration Number							

Special Instructions/ Useful Data

N denotes the set of natural numbers {1, 2, 3, … }
Q denotes the set of rational numbers
R denotes the set of real numbers
$A \setminus B = \{x \in A | x \notin B\}$, for two sets A, B

f' denotes the first derivative of f
f'' denotes the second derivative of f
$f_x = \frac{\partial f}{\partial x}$ denotes the partial derivative of f with respect to x
f_{xx}, f_{xy}, f_{yx}, f_{yy} denote the usual second order partial derivatives of f
∇f denotes the gradient of f

P ($X = n$) denotes the probability of $X = n$

$\bar{x}$ denotes the complement of a Boolean variable x

LPP denotes Linear Programming Problem
max f denotes maximum of f
min f denotes minimum of f

For all C programs, assume that all standard library functions are accessible.

Q.1 Consider the following C program

```
#include <stdio.h>
int main() {
      int x = 01234;
      printf("%d", x);
      return 0;
}
```

The output of the program will be

(A) 01234 (B) 1234 (C) 567 (D) 668

Q.2 Consider the following C function

```
float f(float a, int m) {
    float x;
    if (m == 0) return 1;
    x = f(a, m/2);
    if (m%2 == 1) return x * x * a;
    else return x * x;
}
```

What will be the return value of the function `f(2,3)`?

(A) 20.0 (B) 16.0 (C) 12.0 (D) 8.0

Q.3 When a computer is switched on, the BIOS is loaded from

(A) Hard Disk (B) RAM (C) ROM (D) CD-ROM

Q.4 In a computer, TFT is related to

(A) Memory (B) Monitor (C) Input Device (D) Serial Port

Q.5 Consider the following lists:

List I	**List II**
1. Pen drive	P. Optical Memory
2. Hard disk	Q. Flash Memory
3. CD-ROM	R. Magnetic Memory
4. Floppy	S. Volatile Memory

The correct match is

(A) $1 \rightarrow P, 2 \rightarrow R, 3 \rightarrow P, 4 \rightarrow S$
(B) $1 \rightarrow Q, 2 \rightarrow R, 3 \rightarrow P, 4 \rightarrow R$
(C) $1 \rightarrow S, 2 \rightarrow P, 3 \rightarrow R, 4 \rightarrow Q$
(D) $1 \rightarrow Q, 2 \rightarrow R, 3 \rightarrow S, 4 \rightarrow R$

Q.6 Consider the following C program

```
#include <stdio.h>
int main(){
      int x = 5;
      int y = 2;
      while (x) {
          y += 2 * x;
          printf("%d", x);
          x--;
      }
      printf("%d", y);
      return 0;
}
```

What is printed when the above program is executed?

(A) 5432132 (B) 432132 (C) 5432129 (D) 432130

Q.7 If the speed of a computer is 2 GHz, then which one of the following statements must be TRUE?

(A) Its processor performs 2×10^9 operations per second
(B) Its clock cycles 2×10^9 times per second
(C) Its RAM stores 2×10^9 bytes per second
(D) Its printer prints 2×10^9 characters per second

Q.8 Consider the following lists:

List I	**List II**
1. Linux	P. Text File Editor
2. Mozilla	Q. Image File Format
3. Notepad	R. Operating System
4. JPEG	S. Web Browser

The correct match is

(A) $1 \rightarrow$R, $2 \rightarrow$S, $3 \rightarrow$P, $4 \rightarrow$Q (B) $1 \rightarrow$S, $2 \rightarrow$R, $3 \rightarrow$Q, $4 \rightarrow$P
(C) $1 \rightarrow$R, $2 \rightarrow$ P, $3 \rightarrow$S, $4 \rightarrow$Q (D) $1 \rightarrow$Q, $2 \rightarrow$S, $3 \rightarrow$P, $4 \rightarrow$R

Q.9 If $y = x\cos x$ is a solution of an n-th order linear differential equation

$$\frac{d^n y}{d x^n} + a_1 \frac{d^{n-1} y}{d x^{n-1}} + \cdots + a_{n-1} \frac{d y}{d x} + a_n y = 0$$

with real constant coefficients, then the least possible value of n is

(A) 1 (B) 2 (C) 3 (D) 4

Q.10 The general solution of the differential equation

$$\frac{dy}{dx} = (1+y^2)(e^{-x^2} - 2x\tan^{-1}y)$$

is

(A) $e^{x^2}\tan^{-1}y = x + c$ (B) $e^{-x^2}\tan y = x + c$

(C) $e^{x}\tan y = x^2 + c$ (D) $e^{-x}\tan^{-1}y = x^3 + c$

Q.11 If $g(x, y)\,dx + (x+y)\,dy = 0$ is an exact differential equation and if $g(x, 0) = x^2$, then the general solution of the differential equation is

(A) $2x^3 + 2xy + y^2 = c$ (B) $2x^3 + 6xy + 3y^2 = c$

(C) $2x + 2xy + y^2 = c$ (D) $x^2 + xy + y^2 = c$

Q.12 The value of $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$ is

(A) 0 (B) $\frac{\pi}{2}$ (C) π (D) 2π

Q.13 Let $f(x) = \int_0^x (t-1)(t^2-5t+6)\,dt$ for all $x \in \mathbf{R}$. Then

(A) f is continuous but not differentiable on $\mathbf{R}$

(B) f' is bounded on $\mathbf{R}$

(C) f' has exactly three zeroes

(D) f is continuous and bounded on $\mathbf{R}$

Q.14 If $f(x, y) = \frac{1}{x^2}\tan^{-1}\frac{x}{\sqrt{x^2+y^2}} + \frac{x^{10}}{y^{12}}\,e^{\frac{x^2}{y^2}}$ for $x > 1$, $y > \frac{\pi}{2}$,

then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} + 1000f$ equals

(A) $998f$ (B) $999f$ (C) $1000f$ (D) $1002f$

Q.15 The general solution of the differential equation

$$\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

is

(A) $x = c_1e^{-y} + c_2e^{y}$ (B) $x = c_1e^{y} + c_2$ (C) $x = c_1e^{-y} + c_2$ (D) $x = c_1e^{y} + c_2y$

Q.16 Let $f(x, y) = \begin{cases} xy\dfrac{x^4 - y^4}{x^4 + y^4} & \text{if} \quad (x, y) \neq (0,0), \\ 0 & \text{if} \quad (x, y) = (0,0). \end{cases}$

Which of the following is TRUE?

(A) $f_{xy}(0,0) \neq f_{yx}(0,0)$

(B) $f_{xy}(x, y) = f_{yx}(x, y)$ for all (x, y)

(C) $f_x(x,0)$ does not exist for any real x

(D) $\lim\limits_{(x,y)\to(0,0)} f(x, y) = 1$

Q.17 The value of $\int\limits_{1/2}^{2} \frac{1}{x}\sin\left(x - \frac{1}{x}\right) dx$ is

(A) 1 (B) $\frac{\pi}{2}$ (C) 0 (D) $\sin\left(\frac{3}{2}\right)$

Q.18 The area included between the curves $x^2 + y^2 = a^2$ and $b^2x^2 + a^2y^2 = a^2b^2$ $(a > 0, b > 0)$, is

(A) $\frac{\pi a}{2}|a - b|$ (B) $\pi|a^2 - 3ab + b^2|$

(C) $\pi a|a - b|$ (D) $\pi|a^2 - b^2|$

Q.19 Changing the order of integration of $\int\limits_{1}^{2}\int\limits_{0}^{x} f(x, y)\, dy\, dx$ gives

(A) $\int\limits_{0}^{1}\int\limits_{1}^{2} f(x, y)\, dx\, dy + \int\limits_{0}^{2}\int\limits_{0}^{1} f(x, y)\, dx\, dy$

(B) $\int\limits_{0}^{1}\int\limits_{1}^{2} f(x, y)\, dx\, dy + \int\limits_{1}^{2}\int\limits_{y}^{2} f(x, y)\, dx\, dy$

(C) $\int\limits_{0}^{1}\int\limits_{y/2}^{y} f(x, y)\, dx\, dy + \int\limits_{1}^{2}\int\limits_{y}^{2y} f(x, y)\, dx\, dy$

(D) $\int\limits_{0}^{1}\int\limits_{y}^{1} f(x, y)\, dx\, dy + \int\limits_{1}^{2}\int\limits_{1}^{y} f(x, y)\, dx\, dy$

Q.20 The volume of the closed region bounded by the surfaces $x^2 + y^2 = 2x$, $z = -1$ and $z = 1$ is

(A) 0 (B) $\frac{\pi}{2}$ (C) 2π (D) π

Q.21 Let $f(x) = \begin{cases} x+1 & \text{if } x < 0, \\ (x-1)^2 & \text{if } x \geq 0. \end{cases}$

Which one of the following is TRUE?

(A) f is differentiable on **R**
(B) f has neither a local maximum nor a local minimum in **R**
(C) f is bounded on **R**
(D) f is not differentiable at $x = 0$ but has a local maximum at $x = 0$

Q.22 If $p_{ij} = 1$ for $1 \leq i, j \leq m$, then the characteristic equation of the matrix $P = (p_{ij})$ is

(A) $\lambda^m - \lambda^{m-1} + 1 = 0$ (B) $\lambda^m - m = 0$ (C) $\lambda^m - m\lambda^{m-1} = 0$ (D) $\lambda^m + 1 = 0$

Q.23 If $P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, then P^{50} equals

(A) $\begin{bmatrix} 1 & 100 & 500 \\ 0 & 1 & 100 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 50 & 100 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 50 & 100 & 150 \\ 0 & 50 & 100 \\ 0 & 0 & 50 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 50 & 1275 \\ 0 & 1 & 50 \\ 0 & 0 & 1 \end{bmatrix}$

Q.24 The dimension of the subspace

$$W = \{(x, y, z, w) \in \mathbf{R}^4 \mid x + y + z + w = 0, \quad x + y + 2z = 0, \quad x + 3y = 0\}$$

is

(A) 1 (B) 2 (C) 3 (D) 4

Q.25 Let P be a matrix of size 3×3 with eigenvalues 1, 2 and 3. Then P is

(A) neither invertible nor diagonalizable
(B) both invertible and diagonalizable
(C) invertible but not diagonalizable
(D) not invertible but diagonalizable

Q.26 The integral $\int_{-1}^{1} |x| \, dx$ is computed by the trapezoidal rule with step length $h = 0.01$. The absolute error in the computed value is

(A) 0 (B) 0.0001 (C) 0.0025 (D) 0.005

Q.27 An iteration scheme generates a sequence $\{x_n\}$. For some $\alpha, c \in \mathbf{R}$, $\{x_n\}$ satisfies $|\alpha - x_{n+1}| \leq c\,|\alpha - x_n|$ for all $n \geq 0$. Which one of the following conditions on c ensures the convergence of $\{x_n\}$?

(A) $c = 1$ (B) $c > 1$ (C) $c > 0$ (D) $0 < c < 1$

Q.28 The integral $\int_0^1 f(x)\,dx$ is approximated by the formula

$$\int_0^1 f(x)\,dx \approx \alpha_1\, f(0) + \alpha_2\, f(1) + \alpha_3\, f'(0) + \alpha_4\, f'(1).$$

This approximation is exact for all the polynomials of degree ≤ 3. Then (α_3, α_4) is

(A) $\left(\frac{1}{6}, -\frac{1}{6}\right)$ (B) $\left(\frac{1}{12}, \frac{1}{12}\right)$ (C) $\left(\frac{1}{12}, -\frac{1}{12}\right)$ (D) $\left(\frac{1}{6}, \frac{1}{6}\right)$

Q.29 An approximate value of $\sqrt{3}$ is computed by the formula $x_{n+1} = x_n - \frac{1}{4}(x_n^2 - 3)$. If $x_0 = 1.75$, the value of x_1 correct to three decimal places is

(A) 1.734 (B) 1.733 (C) 1.732 (D) 1.731

Q.30 Consider the following table:

x	1	2	3
y	-10	-6	0

The roots of the corresponding interpolating quadratic polynomial are

(A) $-4, 3$ (B) $3, 4$ (C) $-2, 4$ (D) $-1, 3$

Q.31 The optimal solution of the LPP

$$\max f = 2x + 3y + 20$$

subject to

$$x + y \leq 1,$$
$$2x + 5y \leq 3,$$
$$x \geq 0, \quad y \geq 0,$$

is

(A) $\left(\frac{1}{3}, \frac{2}{3}\right)$ (B) $\left(\frac{2}{3}, \frac{1}{3}\right)$ (C) $\left(0, \frac{3}{5}\right)$ (D) $\left(\frac{3}{2}, 0\right)$

Q.32 The number of optimal solutions of the LPP

$$\max f = 2x + 3y$$

subject to

$$4x + 6y \le 5,$$
$$2x + 2y \ge 1,$$
$$x \ge 0, \quad y \ge 0,$$

is

(A) zero
(B) one
(C) two
(D) infinite

Q.33 The value of x in the sequence 2, 4, 10, 28, 82, x, … is

(A) 102 (B) 168 (C) 252 (D) 244

Q.34 Consider the following segment of a C program

```
int x = 2;
if (x = 3) printf("%d", x++);
else printf("%d", --x);
```

The output of the program segment will be

(A) 0 (B) 2 (C) 3 (D) 4

Q.35 Four different weights W_1, W_2, W_3, W_4 can take only integral values. They can be used on one or both the pans of a balance to weigh objects having all possible integral weights from unit weight to W, where, $W = W_1 + W_2 + W_3 + W_4$. The vector ($W_1$, W_2, W_3, W_4) which maximizes W is

(A) (1, 2, 5, 10) (B) (1, 3, 9, 27) (C) (1, 2, 4, 8) (D) (1, 3, 15, 25)

Q.36 In a C program, variables `x` and `y` are declared to be of type `int`. Consider the following four statements

S1: `y = x & 1;` S2: `y = x % 2;`
S3: `y = x / 2;` S4: `y = x << 1;`

Which of the statements will result in the same value of `y` for every value of `x`?

(A) S3 and S4 (B) S1 and S3 (C) S1 and S2 (D) S2 and S4

Q.37 IBM stands for

(A) Indian Business Machine
(B) International Business Manufacturer
(C) Indian Business Manufacturer
(D) International Business Machine

Q.38 Consider the following fragment of a C program

```
int  x = 20;
int  y = 25;
int  z = x ^ y;
```

where ^ denotes bit-wise XOR operation. Then the value assigned to z will be

(A) 20 (B) 25 (C) 23 (D) 13

Q.39 An ASCII code contains

(A) 8 bits (B) 4 bits (C) 7 bits (D) 6 bits

Q.40 Who among the following developed Linux ?

(A) Bill Gates (B) Sabeer Bhatia (C) Narayan Murthy (D) Linus Torvalds

Q.41 IPR stands for

(A) Intelligence Performance Ratio
(B) Intellectual Property Rights
(C) Intelligence Production Rights
(D) Intellectual Performance Research

Q.42 A software is termed an open source software if

(A) the developer company is open 24 hours
(B) its source code is available to share, study and modify
(C) it can be downloaded from the Internet
(D) it is available free of cost

Q.43 The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 7 \end{bmatrix}$ is

(A) 1 (B) 2 (C) 3 (D) 4

Q.44 Consider the following LPP

$$\min f = 4x + 3y$$

subject to

$$x + y \geq 12,$$
$$4x + 3y \geq 36,$$
$$x \geq 2, \quad y \geq 2.$$

The minimum value of f is

(A) 36 (B) 48 (C) 46 (D) 38

Q.45 An LPP has the following constraints:

$$2x + 5y \geq 10,$$
$$3x + 4y \leq 24,$$
$$x \geq y,$$
$$x \geq 0, \quad y \geq 0.$$

Which of the following is **NOT** a feasible solution to the LPP ?

(A) (8, 0) (B) (5, 0) (C) $\left(\frac{10}{7}, \frac{10}{7}\right)$ (D) (5, 3)

Q.46 Consider the following LPP:

$$\max f = 2x + 5y$$
subject to
$$5x + 6y \geq 15,$$
$$6x + 15y \leq 90,$$
$$x \leq 10,$$
$$x \geq 0, \quad y \geq 0.$$

The number of extreme points of the feasible region of the LPP is

(A) 3 (B) 4 (C) 5 (D) 6

Q.47 A particular integral of the differential equation $\frac{d^2y}{dx^2} - 16y = 4\sinh^2 2x$ is

(A) $\frac{1}{8}\left(xe^{4x} + xe^{-4x} - 1\right)$ (B) $\frac{1}{8}\left(xe^{4x} - xe^{-4x} + 1\right)$

(C) $\frac{1}{4}\left(e^{4x} - xe^{-4x} + \frac{1}{2}\right)$ (D) $\frac{1}{4}\left(xe^{4x} + e^{-4x} + \frac{1}{2}\right)$

Q.48 The general solution of the differential equation $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$ is

(A) $y = c_1e^x + c_2e^{2x} + c_3xe^{2x}$ (B) $y = c_1e^{-x} + c_2e^{2x} + c_3xe^{2x}$

(C) $y = c_1e^{-x} + c_2xe^{-x} + c_3e^{2x}$ (D) $y = c_1e^{-x} + c_2e^x + c_3e^{4x}$

Q.49 The area bounded by the curves $x^2 = 4 - 2y$ and $x^2 = y + 4$ is

(A) 16 (B) 24 (C) 30 (D) 36

Q.50 The volume of the region in $\mathbf{R}^3$ given by $3|x| + 4|y| + 3|z| \leq 12$ is

(A) 64 (B) 48 (C) 32

Q.51 Let $F(x, y, z) = x^2 + y^2 + z^2 + xy + yz + zx$. The value of $F_x + F_y + F_z$ at (1, 1, 1) is

(A) 12 (B) 10 (C) 16 (D) 8

Q.52 Three unbiased dice of different colours are rolled. The probability that the same number appears on at least two of the three dice is

(A) $\frac{5}{36}$ (B) $\frac{1}{2}$ (C) $\frac{5}{12}$ (D) $\frac{4}{9}$

Q.53 An unbiased coin is tossed eight times. The probability of obtaining at least one head and at least one tail is

(A) $\frac{255}{256}$ (B) $\frac{127}{128}$ (C) $\frac{63}{64}$ (D) $\frac{31}{32}$

Q.54 Suppose the sum and the product of the mean and the variance of a binomial random variable are 10 and 24 respectively. Then the probability of success in a single trial is

(A) $\frac{1}{4}$ (B) $\frac{3}{4}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

Q.55 A Poisson random variable X has unit mean. Then P (X = odd) is

(A) $\frac{1}{2}\left(1-\frac{1}{e}\right)$ (B) $1-\frac{1}{e^2}$ (C) $\frac{1}{2}-\frac{1}{e^2}$ (D) $\frac{1}{2}\left(1-\frac{1}{e^2}\right)$

Q.56 The order of the permutation (12)(546)(3978) in the symmetric group S_9 is

(A) 6 (B) 9 (C) 12 (D) 24

Q.57 If $\alpha = (13)(254)$ in the symmetric group S_5, then α^{65} equals

(A) (13)(254) (B) (12)(345) (C) (32)(154) (D) (31)(245)

Q.58 Let S be a set with 10 elements. The number of subsets of S having odd number of elements is

(A) 256 (B) 512 (C) 752 (D) 1024

Q.59 If $\vec{a}, \vec{b}, \vec{c}$ are three vectors in $\mathbf{R}^3$, then $(\vec{a}-\vec{b}+\vec{c})\cdot\left((\vec{b}-\vec{c}+\vec{a})\times(\vec{c}-\vec{a}+\vec{b})\right)$ equals

(A) 0 (B) $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$ (C) $4\,\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$ (D) $6\,\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

Q.60 If $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$, then $\nabla |\vec{r}|^4$ equals

(A) $4|\vec{r}|$ (B) $4|\vec{r}|^2\,\vec{r}$ (C) $4|\vec{r}|\,\vec{r}$ (D) $4|\vec{r}|^3$

Q.61 The area of the parallelogram in $\mathbf{R}^2$ whose diagonals are $3\hat{i} + \hat{j}$ and $\hat{i} - 3\hat{j}$ is

(A) 2.5 (B) 5 (C) $\sqrt{2.5}$ (D) $\sqrt{5}$

Q.62 Let $a_0 + a_1\,x + a_2\,x^2 + a_3\,x^3 + \cdots$ be the Taylor series for the function $\sin(x^2 + 3x)$ about $x = 0$. Then a_3 equals

(A) $-\frac{9}{2}$ (B) $\frac{9}{2}$ (C) $\frac{27}{2}$ (D) $-\frac{27}{2}$

Q.63 The number of real values of a for which the set $\{(a, a^2), (a^2, a)\}$ is **NOT** a basis of $\mathbf{R}^2$, is

(A) 1 (B) 2 (C) 3 (D) 4

Q.64 The set $(\mathbf{Q} \times \mathbf{Q}) \setminus (\mathbf{N} \times \mathbf{N})$ equals

(A) $(\mathbf{Q} \setminus \mathbf{N}) \times (\mathbf{Q} \setminus \mathbf{N})$
(B) $[(\mathbf{Q} \setminus \mathbf{N}) \times \mathbf{Q}] \cup [\mathbf{Q} \times (\mathbf{Q} \setminus \mathbf{N})]$
(C) $[(\mathbf{N} \times \mathbf{Q}) \setminus (\mathbf{Q} \times \mathbf{N})] \cup [(\mathbf{Q} \times \mathbf{N}) \setminus (\mathbf{N} \times \mathbf{Q})]$
(D) $(\mathbf{Q} \times \mathbf{N}) \setminus (\mathbf{N} \times \mathbf{Q})$

Q.65 Let $f(x) = \frac{2}{1+x^2}$ for all $x \in \mathbf{R}$. Then $\lim\limits_{n\to\infty} \frac{1}{n}\left(f'\left(\frac{1}{n}\right) + f'\left(\frac{2}{n}\right) + \cdots + f'\left(\frac{n}{n}\right)\right)$ equals

(A) -2 (B) -1 (C) 1 (D) 2

Q.66 Let $f(x) = \begin{cases} x + x^2 & \text{if } x \geq 0, \\ x^2 & \text{if } x < 0. \end{cases}$

Which one of the following is TRUE?

(A) $f'(0) = 1$ and $f''(0) = 2$
(B) $f'(0) = 1$ but $f''(0)$ is not defined
(C) $f'(0)$ does not exist
(D) f is not continuous at $x = 0$

Q.67 Let $f(x) = 2x^3 + 3x^2 - 12x + 4$ for all $x \in \mathbf{R}$. Then

(A) f is not one-one on $[-1, 1]$
(B) f is one-one on $[-1, 1]$ but not one-one on $[-2, 2]$
(C) f is one-one on $[0, 2]$ but not one-one on $[-2, 0]$
(D) f is one-one on $[-2, 2]$

Q.68 Let $f(x, y) = x^3 + y^3$ for all $(x, y) \in \mathbf{R}^2$. Then

(A) f has a local maximum at $(0, 0)$
(B) f has a local minimum at $(0, 0)$
(C) f has neither a local maximum nor a local minimum at $(0, 0)$
(D) f has both a local maximum and a local minimum at $(0, 0)$

Q.69 Let F be a field with five elements and let $K = \{(a,b) \mid a,b \in F\}$ with the binary operations defined component-wise. Then

(A) K is not a field
(B) K is a field with 5 elements
(C) K is a field with 25 elements
(D) K is a field with 32 elements

Q.70 Let $f(x, y) = \begin{cases} \frac{x}{|x|}\sqrt{x^2 + y^2} & \text{if} \quad x \neq 0, \\ 0 & \text{if} \quad x = 0. \end{cases}$

Then $f_x(0, 0) + f_y(0, 0)$ equals

(A) -2 (B) -1 (C) 0 (D) 1

Q.71 Let $a_1, a_2, \ldots, a_n$ be a finite sequence of numbers with the property $a_i \leq a_{i+2}$ for all $i \in \{1, \ldots, n-2\}$. Which one of the following is always TRUE?

(A) The sequence is sorted
(B) First $(n-2)$ elements of the sequence are sorted
(C) The first element of the sequence is the minimum
(D) Either a_{n-1} or a_n is the maximum of the sequence

Q.72 Order the following memory types in increasing order of access time

M1: Cache, M2: CD-ROM, M3: Hard disk, M4: RAM, M5: Register

(A) M5 M1 M4 M3 M2
(B) M5 M1 M3 M4 M2
(C) M1 M5 M4 M3 M2
(D) M1 M4 M5 M3 M2

Q.73 Consider the following statements about terminating (finite number of digits to the right of the point) representations

X: If the binary representation of a number terminates then its corresponding decimal representation also terminates.
Y: If the decimal representation of a number terminates then its corresponding binary representation also terminates.

Then

(A) X is true but Y is false
(B) Y is true but X is false
(C) both X and Y are true
(D) neither X nor Y is true

Q.74 The octal equivalent of decimal 204 is

(A) 304 (B) 306 (C) 314 (D) 316

Q.75 Consider the following C program

```
int main() {
      char str[] = "leap";
      int len = strlen(str)-1;
      int i = 0;
      while (i <= len) {
          str[i] = str[len-i];
          i++;
      }
      printf("%s", str);
      return 0;
}
```

The output of the program will be

(A) `paal` (B) `pael` (C) `papa` (D) `paap`

Q.76 Let $f(A,B,C,D) = ABC + B(\overline{C} + \overline{D})$ be a Boolean function. The complement of $f(A,B,C,D)$ is

(A) $\overline{B} + \overline{A}\,CD$
(B) $A\,\overline{B} + \overline{A}\,\overline{C}\,\overline{D} + \overline{B}\,\overline{C}$
(C) $B\,\overline{A} + \overline{B}\,\overline{A}\,\overline{D} + \overline{A}\,\overline{C}$
(D) $\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}$

Q.77 The number of three digit numbers greater than 100 in which digits appear in strictly increasing order is

(A) 36 (B) 84 (C) 90 (D) 120

Q.78 Consider the following C function

```
int oddeven(int n) {
    int i = 0;
    while(n>1) {
       if (n%2)
          n = 3*n+1;
       else
          n = n/2;
       i++;
    }
    return i;
}
```

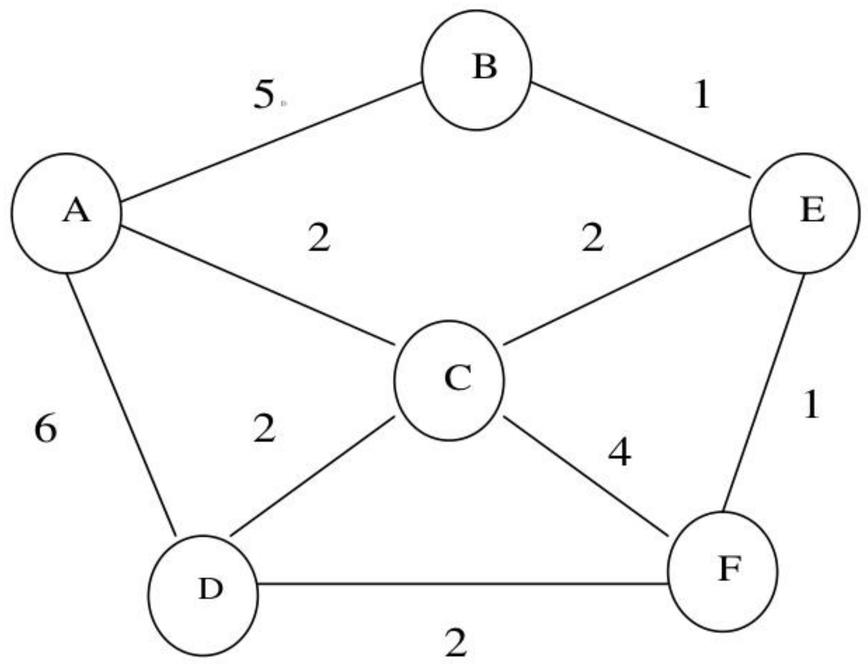
If `n = 12` is given as input, what is the return value?

(A) 8 (B) 9 (C) 10 (D) 12

Q.79 The next number in the sequence of binary numbers 0, 10, 100, 110, … is

(A) 101 (B) 1000 (C) 1001 (D) 1010

Q.80 Following graph shows distances between six cities A through F.



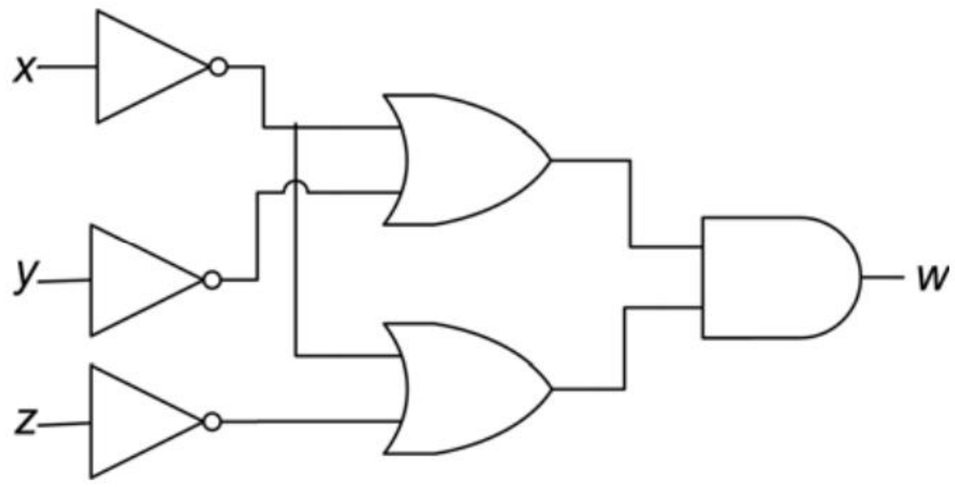
If x and y are minimum and maximum distances from A to F where no city is visited more than once, then (x, y) is

(A) (6, 11) (B) (5, 12) (C) (4, 13) (D) (6, 12)

Q.81 The number of reflexive relations on a set with four elements is

(A) 10 (B) 1024 (C) 4096 (D) 8192

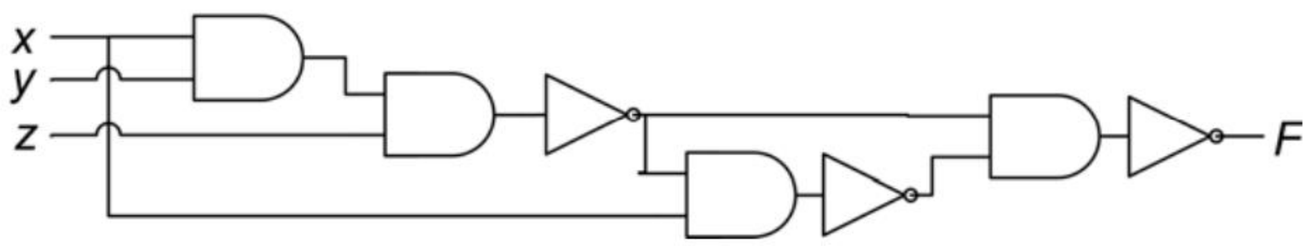
Q.82 Consider the following logic circuit:



The output w is

(A) $\overline{y} + \overline{x}\,\overline{z}$ (B) $\overline{x} + \overline{y}\,\overline{z}$ (C) $\overline{z} + \overline{x}\,\overline{y}$ (D) $\overline{x}\,(\overline{y} + \overline{z})$

Q.83 Consider the following logic circuit:



The output F is

(A) x (B) xy (C) $x + y$ (D) xz

Q.84 WWW stands for

(A) World Wide Wire
(B) World With Web
(C) World Wide Web
(D) World Wise Web

Q.85 The first Prime Minister of India was

(A) Indira Gandhi
(B) Lal Bahadur Shastri
(C) Rajendra Prasad
(D) Jawaharlal Nehru

Q.86 Consider the following two lists:

List I	List II
1. DOS	P. Sun Microsystems
2. P4	Q. Microsoft Corporation
3. Java	R. IBM
4. PC	S. Intel Corporation

The correct match is

(A) $1 \rightarrow Q, 2 \rightarrow S, 3 \rightarrow P, 4 \rightarrow R$
(B) $1 \rightarrow Q, 2 \rightarrow R, 3 \rightarrow S, 4 \rightarrow P$
(C) $1 \rightarrow S, 2 \rightarrow P, 3 \rightarrow Q, 4 \rightarrow R$
(D) $1 \rightarrow R, 2 \rightarrow P, 3 \rightarrow Q, 4 \rightarrow S$

Q.87 The song "Vande Mataram" was written by

(A) Bankim Chandra Chatterjee
(B) Rabindranath Tagore
(C) A. R. Rahman
(D) Satyajit Ray

Q.88 The number of gold medals won by India in the commonwealth games held in New Delhi in 2010 is

(A) 36 (B) 37 (C) 38 (D) 40

Q.89 When $28^{30} - 15^{30}$ is divided by 13, the remainder is

(A) 0 (B) 1 (C) 11 (D) 12

Q.90 Let H be a subgroup of order 60 of a group G of order 120. If $a \in G \setminus H$, then which of the following is **NOT** a subgroup of G ?

(A) $\{ah \mid h \in H\}$
(B) $\{h^{-1} \mid h \in H\}$
(C) $\{aha^{-1} \mid h \in H\}$
(D) $H \cup \{a^{-1}h \mid h \in H\}$

Q.91 Consider the following system of equations

$$\begin{aligned} 2x+3y+4z &= 13 \\ 5x+7y+7z &= 26 \\ 9x+13y+15z &= 13\lambda \end{aligned}$$

The value of λ for which the system has infinitely many solutions is

(A) 1 (B) 2 (C) 3 (D) 4

Q.92 Let $x * y = 3xy$ for all $x, y \in \mathbf{R} \setminus \{0\}$. The inverse of the element 2 in the group $(\mathbf{R} \setminus \{0\}, *)$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{18}$

Q.93 The number of subsets of {1, 2, ... , 10} which are disjoint from {3,7,8} is

(A) 128 (B) 1021 (C) 1016 (D) 7

Q.94 If Q and Q^+ denote the outputs during the current and the next clock cycles of a *JK* flip-flop, which one of the following is its characteristic equation ?

(A) $Q^+ = J\,\overline{Q} + \overline{K}\,Q$ (B) $Q^+ = J\,Q + \overline{K}\,\overline{Q}$ (C) $Q^+ = \overline{J}\,Q + \overline{K}\,Q$ (D) $Q^+ = \overline{J}\,\overline{Q} + K\,Q$

Q.95 The number of functions taking two Boolean variables as input and providing three Boolean variables as output is

(A) 12 (B) 32 (C) 4096 (D) 65536

Q.96 The Boolean expression $(X + Y)(\overline{X} + Z)$ equals

(A) $X\,Y + \overline{X}\,Z$ (B) $Z\,Y + Z\,\overline{X}$ (C) $\overline{X}\,Z + Y\,\overline{Z}$ (D) $X\,Z + \overline{X}\,Y$

Q.97 Consider the following algorithm

```
gcd(a, b)
begin
  if b equals 0 then return a
  else return gcd(b, X)
end
```

Which of the following expressions for X returns the gcd of positive integers a and b?

(A) a / b (B) b / a (C) a mod b (D) b mod a

Q.98 Let P, Q, R and S be statements, each of which can be either true or false. It is known that if P is true or Q is true then R is true and S is false. Suppose it is given that R is false. Then which one of the following will certainly be TRUE?

(A) Both P and Q are true
(B) P is true and Q is false
(C) P is false and Q is true
(D) Both P and Q are false

Q.99 A *JK* flip-flop runs on a clock of period 20 KHz. If we set $J = K = 1$, the output Q is a

(A) constant LOW
(B) constant HIGH
(C) 10 KHz wave
(D) 20 KHz wave

Q.100 HIV stands for

(A) Human Immunodeficiency Virus
(B) Hypersensitive Internal Vein
(C) Human Interactive Virus
(D) Human Immune Virus

A

SPACE FOR ROUGH WORK

A

SPACE FOR ROUGH WORK

A

SPACE FOR ROUGH WORK