IIT - JEE 2016 (Advanced)

P1-16-3-6

PAPER-1

CODE 6

Time : 3 Hours

Maximum Marks : 186

READ THE INSTRUCTIONS CAREFULLY

GENERAL

1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The paper CODE is printed on the right hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on the left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator for change of ORS.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name, roll number and sign in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at **9:00 am**, verify that the booklet contains **36** pages and that all the **54** questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). The Candidate's Sheet is a carbon-less copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work.**
14. Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

15. Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
16. Darken the bubble ◯ **COMPLETELY.**
17. The correct way of darkening a bubble is as : ●
18. The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
19. Darken the bubbles **ONLY IF** you are sure of the answer. There is **NO WAY** to erase or "un-darken" a darkened bubble.

Please see the last page of this booklet for rest of the instructions.

DO NOT BREAK THE SEALS WITHOUT BEING INSTRUCTED TO DO SO BY THE INVIGILATOR

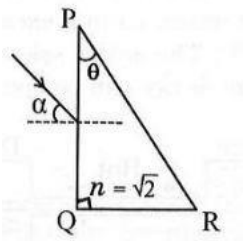
Solution to IIT JEE 2016 (Advanced) : Paper - I

PART I – PHYSICS

SECTION 1 (Maximum Marks:15)

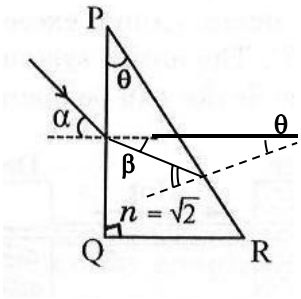
- This section contains **FIVE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases.

1. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index n = $\sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45°. The angle θ of the prism is



(A) 15° (B) 22.5° (C) 30° (D) 45°

1. (A)



$$\sin 45° = \sqrt{2}\sin\beta$$
$$\sin\beta = \frac{1}{2}$$
$$\beta = 30°$$
$$\sin\theta_c = \frac{1}{n} = \frac{1}{\sqrt{2}}$$
$$\theta_c = 45°$$
$$\theta + \beta = 45°$$
$$\theta = 15°$$

2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below:

λ (μm)	V_0(Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ m s^{-1} and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J s) found from such an experiment is

(A) 6.0×10^{-34} (B) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34}

2. (B)

$$KE_{max} = \frac{hc}{\lambda_0} - \phi = eV$$

$$\frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = e(V_1 - V_2)$$

$$hc\left(\frac{1}{0.3} - \frac{1}{0.4}\right) = 1.6\times10^{-19} \times 10^{-6}$$

$$hc\left(\frac{0.1}{0.12}\right) = 1.6\times10^{-25}$$

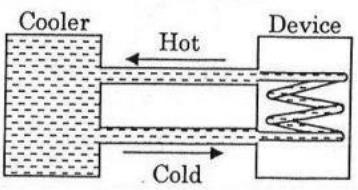
$$h = \frac{1.6\times10^{-25} \times 1.2}{3\times10^{8}} = 0.64\times10^{-33} = 6.4\times10^{-34}$$

$$hc\left(\frac{1}{0.4} - \frac{1}{}\right) = \left(1.6\times10^{-19}\right) \times 0.6\times10^{-6}$$

$$h = \left(0.96\times10^{-25}\right) \times \frac{0.20}{0.10} \times \frac{1}{3\times10^{8}}$$

$$h = \frac{1.92}{3}\times10^{-33} = 6.4\times10^{-34}$$

3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30°C and the entire stored 120 litres of water is initially cooled to 10°C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is 4.2 kJ kg^{-1} K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

3. (B)

Rate of heat generated $\frac{dQ}{dt} = 3KW$

Let at any time 't', temperature of cooler = T

Rate of cooling :

$$ms\frac{dT}{dt} = 3KW - P$$

$$\int_{10}^{30} dT = \frac{(3KW - P)}{ms}\int_{0}^{3} dt$$

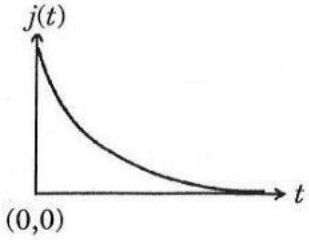
$$30 - 10 = \frac{(3KW - P) \times 3 \times 3600}{120 \times 4.2 \times 10^3}$$

$$3KW - P = \frac{20 \times 120 \times 42}{3 \times 36} = \frac{2800}{3}$$

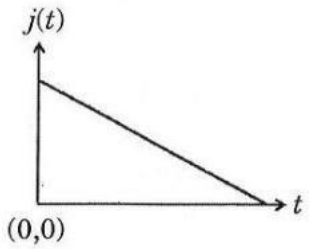
$$P = 3000 - 933 = 2067$$

4. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density *j(t)* at any point in the material?

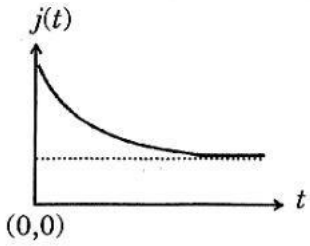
(A)



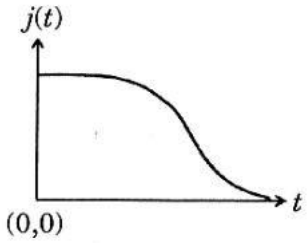
(B)



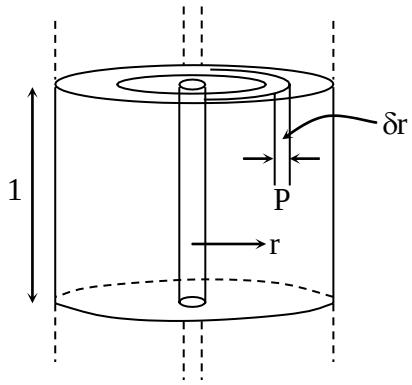
(C)



(D)



4. (A)



Let $\lambda(t)$ represent the linear density of charge as a function of time on the inner wire.

λ_0 be the charge density at t = 0 on the inner wire.

Let $\beta(t)$ represent the linear density on the outer cylinder.

$\lambda(t) + \beta(t) = \lambda_0$ by conservation of charge

The electric field at P as a function of time 't' be E(t) then

$$E(r, t) = \frac{\lambda(t)}{2\pi\varepsilon_0 \cdot \varepsilon r}$$

$$\therefore \quad j(t) = \sigma E(r, t) = \left(\frac{o}{2\pi\varepsilon_0 \varepsilon}\right)\frac{\lambda(t)}{r}$$

$$\therefore j(t) \propto \lambda(t)$$

Clearly $\lambda(t) \to 0$ as $t \to \infty$ as all the charge will eventually migrate to the outer surface.

Let the cross–sectional radius of the inner wire be 'r_0'.
Potential difference between the inner and outer cylinder

$$= \frac{\lambda(t)}{2\pi\varepsilon_0\varepsilon}\int_{r_0}^{R}\frac{dr}{r} = -\frac{\lambda(t)}{2\pi\varepsilon_0\varepsilon}\log\left(\frac{R}{r_0}\right)$$

Now consider the cylindrical shell of thickness 'dr'.
Let the length of the cylinder be unit.

elementary resistance $= \left(\frac{1}{\sigma}\right)\frac{dr}{2\pi r}$

Total resistance $= \frac{\log(R/r_0)}{2\pi\sigma}$ [per unit length]

$$-\frac{\lambda(t)}{2\pi\varepsilon_0\varepsilon}\log\left(\frac{R}{r_0}\right) = \frac{\log(R/r_0)}{2\pi\sigma}\cdot\dot{\lambda}(t) \Rightarrow \qquad \dot{\lambda}(t) = -c\,\lambda(t)$$

which is an exponential decay

$\square$ $j(t) \propto \lambda(t)$

j(t) decays exponentially.

5. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h($< \ell$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are ($g = 10\ m\ s^{-2}$)

(A) $\frac{h}{\ell} = \frac{\sqrt{3}}{16}$, $f = \frac{16\sqrt{3}}{3}$ N

(B) $\frac{h}{\ell} = \frac{3}{16}$, $f = \frac{16\sqrt{3}}{3}$ N

(C) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}$, $f = \frac{8\sqrt{3}}{3}$ N

(D) $\frac{h}{\ell} = \frac{3\sqrt{3}}{16}$, $f = \frac{16\sqrt{3}}{3}$ N

5. (D)

Force balance

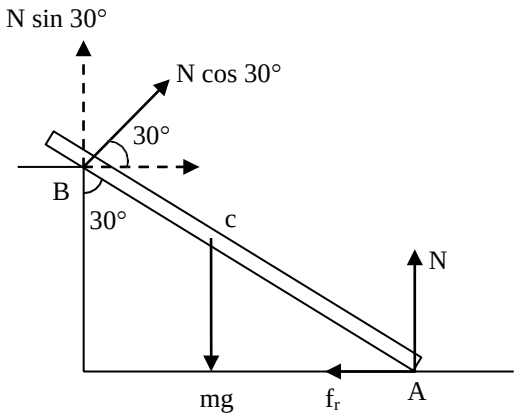
$N + N\sin 30° = mg$

$\frac{3}{2}N = mg$

$$\boxed{N = \frac{2}{3}mg}$$

$f_r = N\cos 30°$

$$f_r = \frac{mg}{\sqrt{3}} = \frac{16}{\sqrt{3}} = \frac{16\sqrt{3}}{3}$$



Torque balance (about A)

$$N\times\frac{h}{\cos 30°} = mg\times\frac{L}{2}\sin 30°$$

$$\frac{2}{3}mg\times\frac{2h}{\sqrt{3}} = mg\times\frac{L}{4}$$

$$\boxed{\frac{h}{L} = \frac{3\sqrt{3}}{16}}$$

SECTION 2 (Maximum Marks:32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Partial Marks	: +1	For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) result in –2 marks, as a wrong option is also darkened.

6. The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$$\vec{r}(t) = \alpha t^3 \hat{i} + \beta t^2 \hat{j},$$

Where $\alpha = 10/3$ m s^{-3}, $\beta = 5$ m s^{-2} and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle?

(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})$ ms^{-1}

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -(5/3)\hat{k}$ N m s

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$N

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{t} = -(20/3)\hat{k}$ N m

6. (A), (B), (D)

$\vec{r} = \alpha t^3 \hat{i} + \beta t^2 \hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = (3\alpha t^2)\hat{i} + (2\beta t)\hat{j}$

at t = 1 sec

$\boxed{\vec{v} = 10\hat{i} + 10\hat{j}}$

$\vec{L} = m(\vec{r} \times \vec{v})$

$\vec{L} = m\left[2\alpha\beta t^4(\hat{k}) + 3\alpha\beta t^4(-\hat{k})\right]$

$\vec{L} = m(\alpha\beta t^4)(-\hat{k})$

at t = 1 sec

$\boxed{\vec{L} = \frac{5}{3}(-\hat{k})}$

$\vec{a} = \frac{d\vec{v}}{dt} = (6\alpha t)\hat{i} + (2\beta)\hat{j}$

at t = 1 sec

$\vec{a} = 20\hat{i} + 10\hat{j}$

$\boxed{F = m\vec{a} = 2\hat{i} + \hat{j}}$

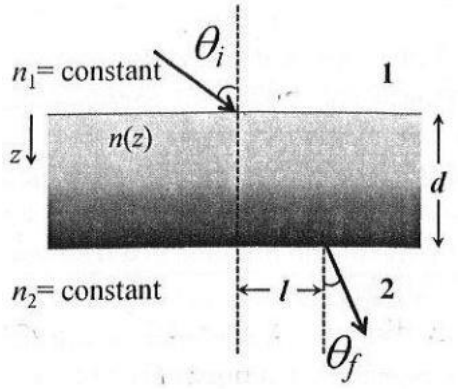
$$\vec{\tau} = \frac{d\vec{L}}{dt} = \frac{d}{dt}\left(m\alpha\beta t^4\right)(-\hat{k})$$

$$\vec{\tau} = \left(4m\alpha\beta t^3\right)(-\hat{k})$$

at t = 1 sec

$$\boxed{\vec{\tau} = \frac{20}{3}(-\hat{k})}$$

7. A transparent slab of thickness d has a refractive index n(z) that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and $n_2 (> n_1)$, as shown in the figure. A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction and θ_f with a lateral displacement ℓ



Which of the following statement(s) is(are) true?

(A) ℓ is independent of n_2

(B) $n_1 \sin\theta_i = n_2 \sin\theta_f$

(C) ℓ is dependent on n(z)

(D) $n_1 \sin\theta_i = (n_2 - n_1)\sin\theta_f$

7. (A), (B), (C)

The gradient in 'n' is along z-direction.

n sin θ remains invariant when θ is measured from z-direction.

Hence option (B) is correct.

It is also clear that □ is independent of n_2 & depends only on n(z).

8. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?

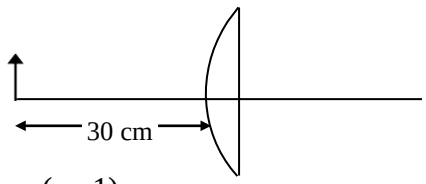
(A) The refractive index of the lens is 2.5

(B) The radius of curvature of the convex surface is 45 cm

(C) The faint image is erect and real

(D) The focal length of the lens is 20 cm

8. (A), (D)



$$\frac{1}{f} = \frac{(\mu - 1)}{R}$$ R = Radius of curvature

u = – 30 magnification magnitude = 2.

$\Rightarrow v = 60$

$\frac{1}{f} = \frac{1}{60} + \frac{1}{30} = \frac{1}{20}$

$\therefore \frac{\mu - 1}{R} = \frac{1}{20}$ ..... (i)

Consider reflection now :

The surface acts on a convex mirror. For the real object image is virtual.

$u = -30$ $v = +10$

$\frac{2}{R} = +\frac{1}{10} - \frac{1}{30}$ [focal length of the mirror = R/2]

$\Rightarrow$ R = 30 cm .....(ii)

From (i) & (ii) $\mu = 2.5$

9. Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n >>1. Which of the following statement(s) is(are) true?

(A) Relative change in the radii of two consecutive orbitals does not depend on Z

(B) Relative change in the radii of two consecutive orbitals varies as 1/n

(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(D) Relative change in the angular momenta of two consecutive orbitals varies as 1/n

9. (A), (B), (D)

Orbital radius $r_n = n^2c$ [c = constant]

Angular momentum = nh = L

$\frac{\Delta r}{r_n} = \frac{(n+1)^2 - n^2}{n^2} = \frac{2}{n}$ .....[B]; $\frac{\Delta L_n}{L_n} = \frac{1}{n}$ .... [D]

(A) is correct since it will get cancelled in calculation of relative charge.

10. A length-scale (ℓ) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for ℓ is(are) dimensionally correct?

(A) $\ell = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$

(B) $\ell = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$

(C) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$

(D) $\ell = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

10. (B), (D)

$[\varepsilon] = \frac{[AT]^2}{[MLT^{-2}][L^2]}$

$[\varepsilon] = [M^{-1}L^{-3}A^2T^4]$

$[k_B] = [M L^2 T^{-2} \theta^{-1}]$

$[T] = [\theta]$

$[n] = [L^{-3}]$

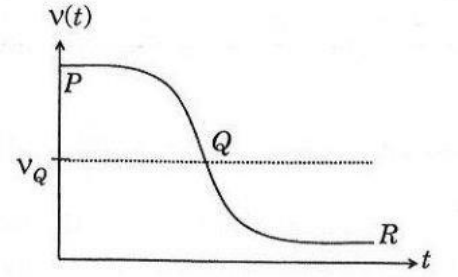
$[q] = [A\,T]$

(B) $\sqrt{\dfrac{\in k_B T}{nq^2}} = \sqrt{\dfrac{L^{-1}T^2A^2}{L^{-3}T^2A^2}} = [L]$

(D) $\sqrt{\dfrac{q^2}{\in n^{1/3} k_B T}} = \sqrt{\dfrac{A^2T^2}{\left[L^{-1}T^2A^2\right]\times\left[L^{-3}\right]^{1/3}}} = \sqrt{\dfrac{1}{\left[L^{-1}\right]\times\left[L^{-1}\right]}} = [L]$

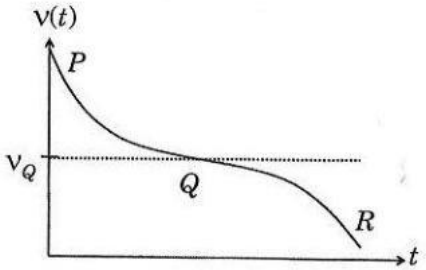
11. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let v(t) represent the beat frequency measured by a person sitting in the car at time t. Let v_P, v_Q and v_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 m/s. Which of the following statement(s) is(are) true regarding the sound heard by the person?

(A) The plot below represents schematically the variation of beat frequency with time



(B) $v_P + v_R = 2v_Q$

(C) The plot below represents schematically the variation of beat frequency with time



(D) The rate of change in beat frequency is maximum when the car passes through Q.

11. (A), (B), (D)

Frequency of M received by car

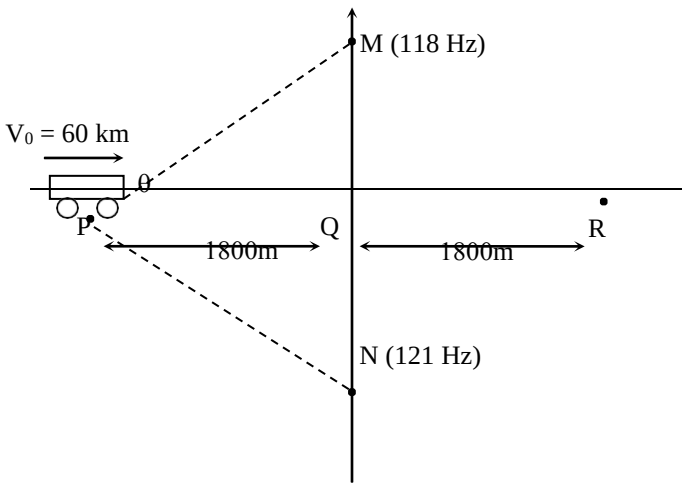
$f_1 = 118\left(\dfrac{V + V_0\cos\theta}{V}\right)$

$f^2 = 121\left(\dfrac{V + V_0\cos\theta}{V}\right)$

No. of beats $n = \Delta f = f_2 - f_1$

$n = 3\left(\dfrac{V + V_0\cos\theta}{V}\right)$

$n = 3\left(1 + \dfrac{V_0}{V}\cos\theta\right)$



As $\theta \uparrow$, $\cos\theta \downarrow$, $n \downarrow$

Rate of change of beat frequency

$$\frac{dn}{d\theta} = 3\left[\frac{V_0}{V}(-\sin\theta)\right]$$

$\frac{dn}{d\theta}$ is maximum when $\sin\theta = 1$

$$\theta = 90°$$

i.e. car is at point Q.

$$\nu_p = 3\left(1+\frac{V_0}{V}\cos\theta\right)$$

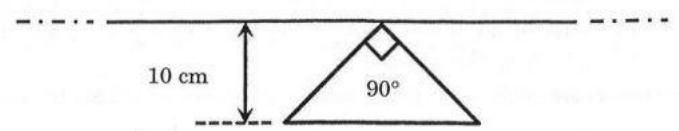
$$\nu_R = 3\left(1-\frac{V_0}{V}\cos\theta\right)$$

<u>at Q</u>

no. of beats $\nu_Q = 121-118 = 3$

$$\nu_{R\,Q} = \frac{\nu_P +}{2}$$

12. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rete of 10 A s^{-1}. Which of the following statement(s) is(are) true?



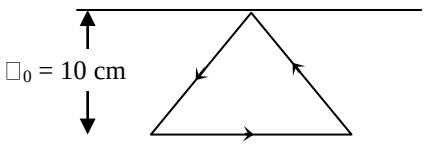
(A) The induced current in the wire is in opposite direction to the current along the hypotenuse

(B) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(C) There is a repulsive force between the wire and the loop

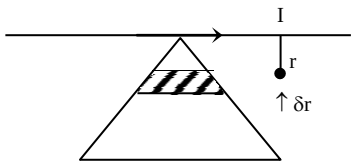
(D) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

12. (B), (C)



The induced current will be parallel to the current in the Hypotenuse.

Consider the reciprocal case.



Suppose the current I flows in the infinite wire

Flux $\delta\Phi$ through the shaded area is :

$$\delta\Phi = \frac{\mu_0 I}{2\pi r}\, 2r \cdot \delta r = \frac{\mu_0 I}{\pi} \cdot dr$$

The total flux = $\frac{\mu_0 I}{\pi}\int_0^{\ell} dr = \frac{\mu_0 I \ell}{\pi}$

$\therefore$ Mutual inductance M = $\frac{\Phi}{I} = \frac{\mu_0 \ell}{\pi}$

When the current flows through the triangle,

the flux associated with the wire $\phi = \frac{\mu_0 \ell}{\pi} I$

□ $E = -\frac{d\phi}{dt}$; we have $|E| = \frac{\mu_0 \ell}{\pi}\frac{dI}{dt}$

Putting □ = 10 cm & $\frac{dI}{dt}$ = 10 A/s

$\therefore$ Induced emf = $\left|\frac{\mu_0}{\pi}\right|$volt

Force is repulsive follows from Lenz Law.

(D) is correct because no extra motional emf is induced since such a emf = $\left(\vec{v} \times \vec{B}\right)$□.

Any element of the rotating wire will only have its velocity parallel or anti-parallel to the magnetic field (in the reciprocal case) of the straight wire.

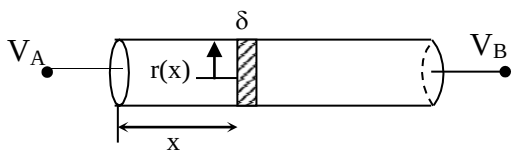
13. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true?

(A) The temperature distribution over the filament is uniform
(B) The resistance over small sections of the filament decreases with time
(C) The filament emits more light at higher band of frequencies before it breaks up
(D) The filament consumes less electrical power towards the end of the life of the bulb

13. (A), (D)

If the temperature distribution was uniform (assuming a uniform cross section for the filament initially) the rate of evaporation from the surface would be same everywhere. But because the filaments break at random locations; it follows that the cross–sections of various filaments are non-uniform.

$$\delta R(x) = \rho\frac{\delta x}{\pi\, r(x)^2}$$



The temperature of points A and B are decided by ambient temperature are identical. Then the average heat flow through the section S is O. After sufficiently long time, this condition implies that the temperature across the filament will be uniform.

If the instantaneous current is i(t) through the filament then by conservation of energy :

$$\frac{(V_B - V_A)^2}{R(t)^2} \times \frac{dx}{\kappa\pi r(x)^2} = e\sigma 2\pi r(x) \cdot \delta(x) T^4 + \rho\pi r(x)^2 \cdot dx L_v$$

in above κ = material conductivity

R(t) = Resistance of whole filament as a function of time

ρ = material density

L_v = Latent heat of vapourisation for the material at temperature T

Since R(t) increases with time

$P(t) = \frac{(V_B - V_A)^2}{R(t)}$ decreases

SECTION 3 (Maximum Marks:15)

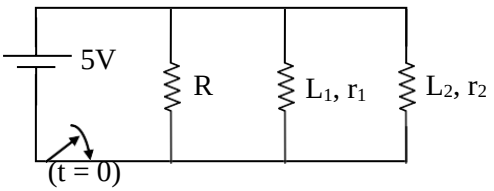
- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in <u>one of the following categories</u>:

  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

  Zero Marks : 0 If all other cases.

14. The inductor L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5 V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is

14. [8]



L_1 = 1 mH r_1 = 3Ω

L_2 = 2mH r_2 = 4Ω

R = 12Ω

Current through R = $\frac{5V}{12\Omega} = \frac{5}{12}$ A

at t = 0 current through both the inductors = 0

after a sufficiently long time, current in the inductors is stabilized. For $t \to \infty$

$i_1(\text{through } L_1) = \frac{5V}{3\Omega} = \frac{5}{3}$ A

$i_2(\text{through } L_2) = \frac{5V}{4\Omega} = \frac{5}{4}$ A

$$\frac{I_{max}}{I_{min}} = \frac{\left(\frac{5}{12} + \frac{5}{3} + \frac{5}{4}\right)}{\left(\frac{5}{12}\right)} = 1 + 4 + 3 = 8$$

15. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has scale that displays $\log_2(P/P_0)$, whre P_0 is a constant. When the metal surface is at a temperature of 487°C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767°C?

15. [9]

at $\left(T_1 = 487 + 273 = 760\,K\right)$ $\quad P_1 \propto (760)^4$

i.e. $P_1 = c\,(760)^4$ where c = constant

$\log_2 \frac{P_1}{P_0} = 1 \Rightarrow P_1 = 2P_0 \Rightarrow P_0 = \frac{P_1}{2}$

at $\left(T_2 = 2767 + 273 = 3040\right)$

$P_2 = c\,(3040)^4$

Reading of the sensor at $T_2 = \log_2\left(\frac{P_2}{P_0}\right)$

$= \log_2\left[2 \cdot \frac{P_2}{P_1}\right] = \log_2\left[2\left(\frac{3040}{760}\right)^4\right] = \log_2\left[2^1 \cdot 2^8\right] = 9.$ $\quad \therefore$ Reading of $T_2 = 9$

16. A hydrogen atom in its ground state is irradiated by light of wavelength 970A. Taking hc/e = 1.237×10^{-6} eVm and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is

16. [6]

Photon Energy = $\frac{hc}{\lambda} = \frac{1.237 \times 10^{-6}}{970 \times 10^{-10}} = \frac{1237}{970} \times 10$ eV

Absorption of this photon changes the energy to = – 13.6 + 12.75 = – 0.85 eV

Number of possible transitions from the 4th quantum state = ${}^4C_2 = 6$

17. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameters 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 08. gm cm^{-3} and viscosity η = 3 poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity η = 2 poiseulles. The ratio of the terminal velocities of P and Q is

17. [3]

It is known that the terminal speed v_T

that is attained is in accordance with $v_T \propto \frac{(\rho - \sigma)r^2}{\eta}$

ρ = density of sphere $\quad \sigma$ = density of medium.

$\therefore \frac{v_{T,P}}{v_{T,Q}} = \frac{(8-0.8) \times 1^2 \times 2}{3 \times (8-1.6) \times 0.5^2} = \frac{2 \times 4 \times 72}{3 \times 64} = 3$

18. The isotope ${}^{12}_{5}B$ having a mass 12.014 u undergoes β–decay to ${}^{12}_{6}C$. ${}^{12}_{6}C$ has an excited state of the nucleus (${}^{12}_{6}C^*$) at 4.041 MeV above its ground state. If ${}^{12}_{5}B$ decays to ${}^{12}_{6}C^*$, the (1 u = 931.5 MeV/c^2), where c is the speed of light in vacuum)

18. [9]

${}^{12}_{5}B \rightarrow {}^{12}_{6}C^* + e^- + \nu$

We take the mass of ${}^{12}_{6}C$ as 12 amu

Rest energy of ${}^{12}_{6}C^*$ = 12 × 931.5 MeV + 4.041 MeV

Energy of ${}^{12}_{5}B$ = 12 × 931.5 MeV + 0.014 × 931.5

$\therefore$ Value of the reaction = 13.041 MeV – 4.041 MeV = 9 MeV

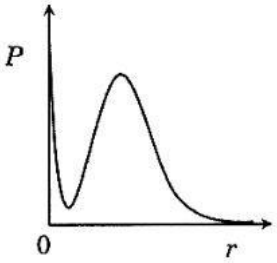
Maximum e^- energy = 9 MeV

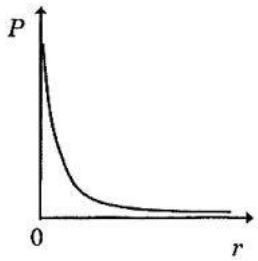
PART II : CHEMISTRY

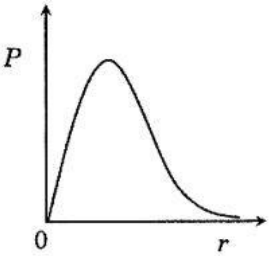
SECTION 1 (Maximum Marks: 15)

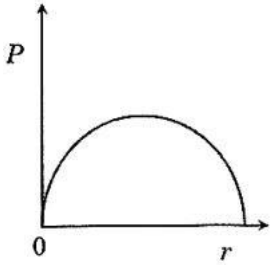
- This section contains **FIVE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases.

19. P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r from the nucleus. The volume of this shell is $4\pi r^2$ dr. The qualitative sketch of the dependence of P on r is

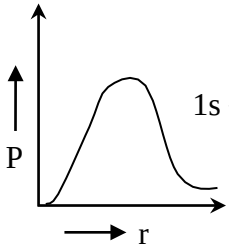
(A) 

(B) 

(C) 

(D) 

19. (C)



20. One mole of an ideal gas at 300 K in thermal contact with surroundings expands isothermally from 1.0 L to 2.0 L against a constant pressure of 3.0 atm. In this process, the change in entropy of surroundings (ΔS_{surr}) in JK^{-1} is (1 L atm = 101.3 J)

(A) 5.763 (B) 1.013 (C) –1.013 (D) –5.763

20. (C)

$\Delta E = q + w$

$0 = q - P_{ext}\,\Delta V$

$q = P_{ext}\,\Delta V = 3 \text{ atm } (2 - 1) \text{ L} = 3 \text{ atm L}$

$= (3 \times 101.3) \text{ Joule}$

$$\Delta S_{surr} = -\frac{q}{T} = \frac{3\times101.3}{300} = -1.013 \text{ Joule/K}$$

21. Among $[Ni(CO)_4]$, $[NiCl_4]^{2-}$, $[Co(NH_3)_4Cl_2]Cl$, $Na_3[CoF_6]$, Na_2O_2 and CsO_2, the total number of paramagnetic compounds is

(A) 2 (B) 3 (C) 4 (D) 5

21. (B)

$[Ni(CO)_4]$ – sp^3 – Diamagnetic

$[NiCl_4]^{2-}$ – sp^3 – Paramagnetic

$[Co(NH_3)_4Cl_2]Cl$ – d^2sp^3 – Diamagnetic

$Na_3[CoF_6]$ – sp^3d^2 – Paramagnetic

Na_2O_2 i.e. O_2^{2-} – Diamagnetic

CsO_2 i.e. O_2^- – Paramagnetic

22. The increasing order of atomic radii of the following Group 13 elements is

(A) Al < Ga < In < Tl (B) Ga < Al < In < Tl

(C) Al < In < Ga < Tl (D) Al < Ga < Tl < In

22. (B)

23. On complete hydrogenation, natural rubber produces

(A) ethylene–propylene copolymer (B) vulcanised rubber

(C) polypropylene (D) polybutylene

23. (A)

$$n\,CH_2=\underset{\underset{CH_3}{|}}{C}-CH=CH_2 \longrightarrow \left(CH_2-\underset{\underset{CH_3}{|}}{C}=CH-CH_2\right)_n$$

Natural Rubber

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in <u>one of the following categories</u>:

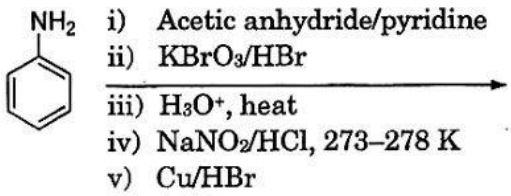
  Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

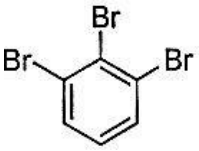
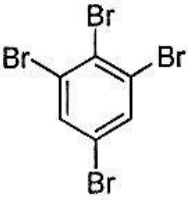
  Partial Marks : +1 For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

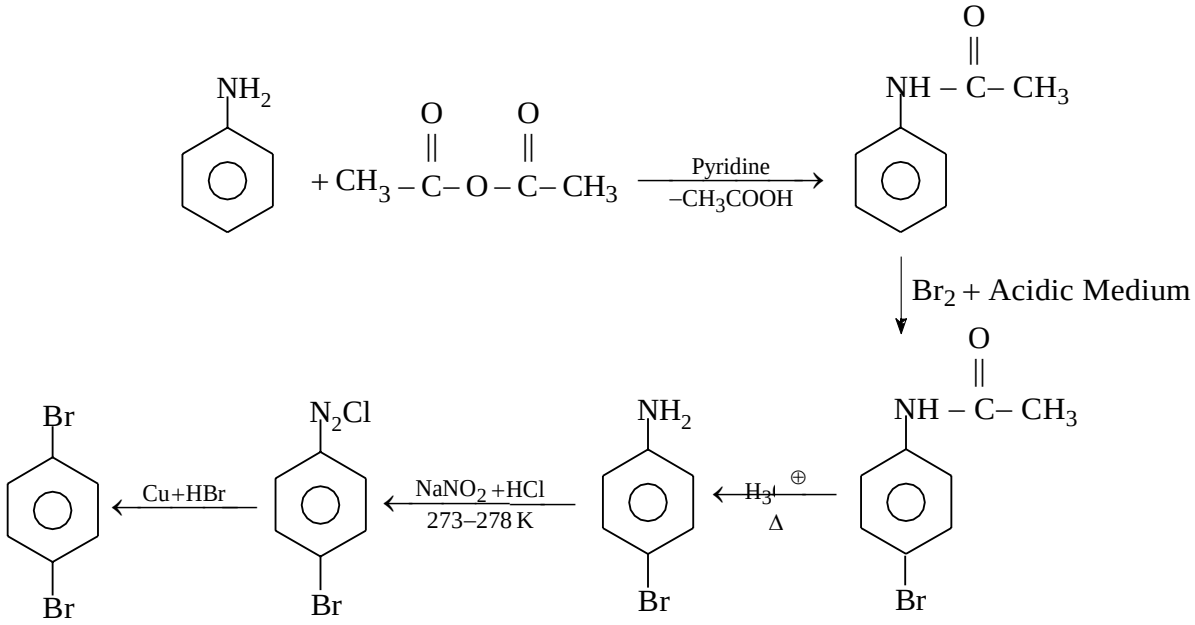
  Negative Marks : –2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) result in –2 marks, as a wrong option is also darkened.

24. The product (s) of the following reaction sequence is (are)



(A) 
(B) 
(C) 
(D) 

24. (B)

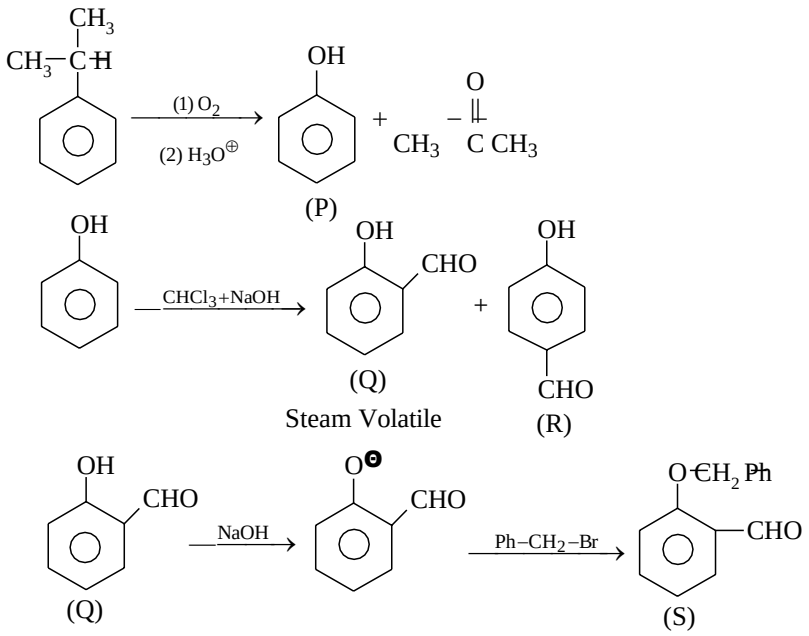


25. The correct statement(s) about the following reaction sequence is(are)

Cumene (C_9H_{12}) $\xrightarrow[\text{ii) } H_3O^+]{\text{i) } O_2}$ **P** $\xrightarrow{CHCl_3/NaOH}$ **Q**(major) + **R**(minor) **Q** $\xrightarrow[PhCH_2Br]{NaOH}$ **S**

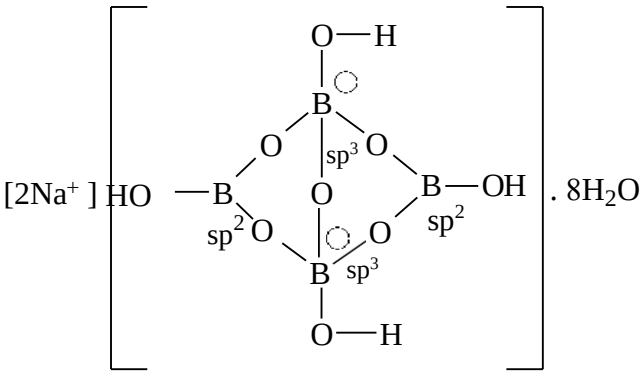
(A) **R** is steam volatile
(B) **Q** gives dark violet coloration with 1% aqueous $FeCl_3$ solution
(C) **S** gives yellow precipitate with 2, 4–dinitrophenylhydrazine
(D) **S** gives dark violet coloration with 1% aqueous $FeCl_3$ solution

25. (B), (C)



26. The crystalline form of borax has
(A) tetranuclear $[B_4O_5(OH)_4]^{2-}$ unit
(B) all boron atoms in the same plane
(C) equal number of sp^2 and sp^3 hybridized boron atoms
(D) one terminal hydroxide per boron atom

26. (A), (C), (D)



27. The reagent(s) that can selectively precipitate S^{2-} from a mixture of S^{2-} and SO_4^{2-} in aqueous solution is (are)
(A) $CuCl_2$ (B) $BaCl_2$ (C) $Pb(OOCCH_3)_2$ (D) $Na_2[Fe(CN)_5NO]$

27. (A) or (A), (C)

(A) $CuCl_2 + S^{-2} \rightarrow \underset{\text{Black ppt}}{CuS \downarrow} + 2Cl^-$

$CuCl_2 + SO_4^{2-} \rightarrow$ No. ppt.

(B) $BaCl_2 + S^{-2} \rightarrow \underset{\text{No. ppt.}}{BaS} + 2Cl^-$

$BaCl_2 + SO_4^{2-} \rightarrow \underset{\text{white ppt.}}{BaSO_4 \downarrow} + 2Cl^-$

(C) $Pb(OAc)_2 + S^{-2} \rightarrow \underset{\text{(Black ppt)}}{PbS \downarrow} + 2CH_3 - COO^-$

$Pb(OAc)_2 + SO_4^{2-} \rightarrow \underset{\text{(White ppt)}}{PbSO_4 \downarrow} + 2CH_3COO^-$

PbS can be selective ppt out first as K_{sp} is much less than K_{sp} of $PbSO_4$.

K_{sp} of PbS = 3×10^{-28}

K_{sp} of $PbSO_4$ = 25×10^{-8}

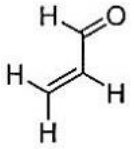
(D) $Na_2[Fe(CN)_5NO]^{2-} + S^{-2} \rightarrow \underset{\text{Purple Colour}}{[Fe(CN)_5NOS]}$

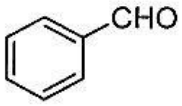
$Na_2[Fe(CN)_5NO] + SO_4^{2-} \rightarrow$ No. ppt.

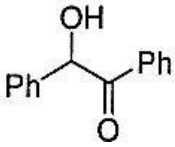
28. A plot of the number of neutrons (N) against the number of protons (P) of stable nuclei exhibits upward deviation from linearity for atomic number, Z > 20. For an unstable nucleus having N/P ratio less than 1, the possible mode(s) of decay is (are)
(A) β^- –decay (β emission) (B) orbital or K–electron capture
(C) neutron emission (D) β^+ –decay (positron emission)

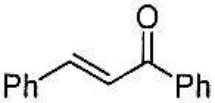
28. (B), (D)
Factual

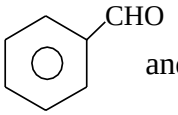
29. Positive Tollen's test is observed for

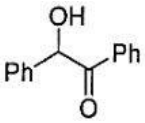
(A) 

(B) 

(C) 

(D) 

29. (A), (B), (C)
$CH_2 = CH - CHO$, 

and 

gives positive test with Tollen's reagent.

30. The compound(s) with TWO lone pairs of electrons on the central atom is (are)
(A) BrF_5 (B) ClF_3 (C) XeF_4 (D) SF_4

30. (B), (C)
BrF_5 = One lone pair + 5 bond pair
ClF_3 = 2 lone pair + 3 bond pair
XeF_4 = 2 lone pair + 4 bond pair
SF_4 = 1 lone pair + 4 bond pair.

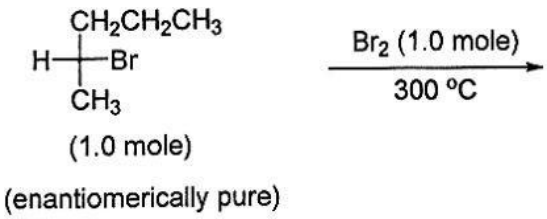
31. According to the Arrhenius equation,
(A) a high activation energy usually implies a fast reaction
(B) rate constant increases with increase in temperature. This is due to a greater number of collisions whose energy exceeds the activation energy
(C) higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant
(D) the pre–exponential factor is a measure of the rate at which collisions occur, irrespective of their energy.

31. (B), (C), (D)
A high activation energy usually implies a slow reaction.

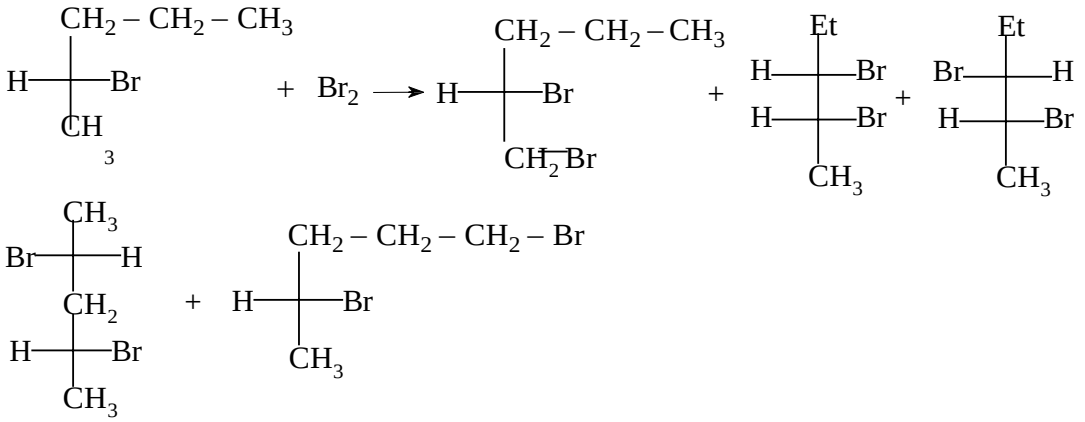
SECTION 3 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in <u>one of the following categories</u>:
  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 If all other cases.

32. In the following monobromination reaction, the number of possible chiral products is



32. [5]



33. The mole fraction of a solute in a solution is 0.1. At 298 K, molarity of this solution is the same as its molality. Density of this solution at 298 K is 2.0 g cm^{-3}. The ratio of the molecular weights of the solute and solvent, $\left(\frac{MW_{solute}}{MW_{solvent}}\right)$, is

33. [9]

$$\frac{X_{solute}}{X_{solvent}} = \frac{0.1}{0.9} = \frac{1}{9}$$

$$\Rightarrow \frac{W_{solute}}{W_{solvent}} \times \frac{M_{solvent}}{M_{solute}} = \frac{1}{9} \qquad \ldots (1)$$

$W_{solute} + W_{solvent} = W_{solution}$ = density × volume

Wsolute + Wsolvent = 2 × V … (2)

Molarity = molality

$$\frac{n_{solute}}{V_{solution}} = \frac{n_{solute}}{W_{solvent}}$$

$$W_{solvent} = V_{solution} = \frac{W_{solute} + W_{solvent}}{2}$$

$\Rightarrow 2W_{solvent} = W_{solute} + W_{solvent}$

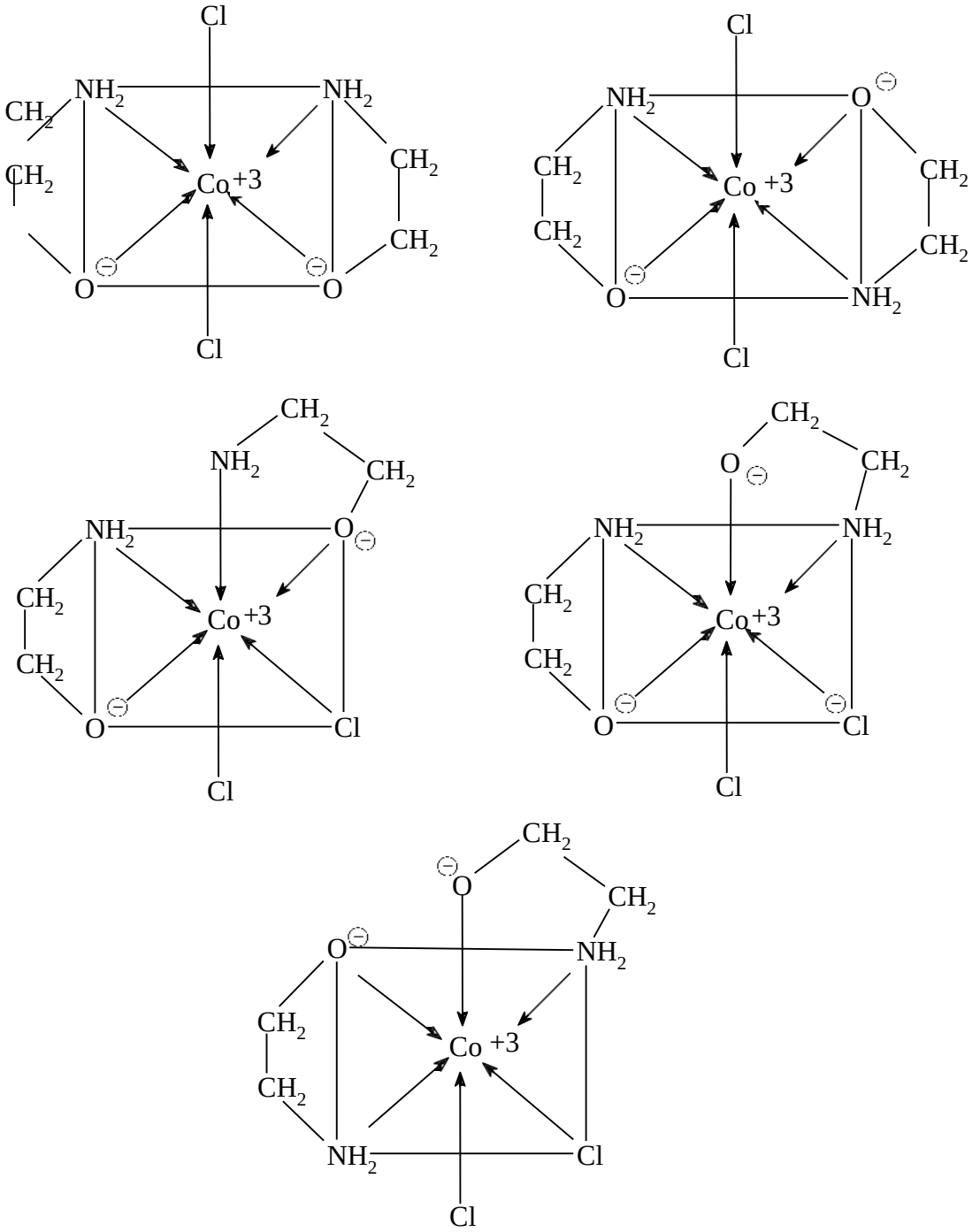
$\Rightarrow W_{solute} = W_{solvent}$ ... (3)

Using eq. (1) and (3), we get

$$\frac{M_{solute}}{M_{solvent}} = 9$$

34. The number of geometric isomers possible for the complex $[CoL_2Cl_2]^-$ ($L = H_2NCH_2CH_2O^-$) is

34. [5]



35. In neutral or faintly alkaline solution, 8 moles of permanganate anion quantitatively oxidize thiosulphate anions to produce X moles of a sulphur containing product. The magnitude of X is

35. [6]

$$8\,MnO_4^- + 3S_2O_3^{2-} + H_2O \rightarrow 8MnO_2 + 6SO_4^{2-} + 2OH^-$$

36. The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and its pressure is increased 2 times. As a result, the diffusion coefficient of this gas increases x times. The value of x is

36. [4]

Diffusion coefficient $\propto$ mean free path $\times$ mean speed

$D_1 \propto \lambda_1 C_1$

$C_2 = 2C_1$

$\lambda_2 = \frac{\lambda_1}{2} \times 4 = 2\lambda_1$

$D_2 \propto \lambda_2 C_2$

$\frac{D_2}{D_1} = \frac{\lambda_2 C_2}{\lambda_1 C_1} = 4$

PART III – MATHEMATICS

SECTION 1 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

  Negative Marks : –1 In all other cases.

37. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

P (computer turns out to be defective given that it is produced in plant T_1)

= 10 P (computer turns out to be defective given that it is produced in plant T_2),

where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

37. (C)

$P(T_1) = \frac{1}{5}$

$P(T_2) = \frac{4}{5}$

$P(D) = \frac{7}{100}$

$P\left(\frac{D}{T_1}\right) = 10.\ P\left(\frac{D}{T_2}\right)$. Let $P\left(\frac{D}{T_2}\right) = x$

Now, $P(T_1) \times P\left(\frac{D}{T_1}\right) + P(T_2) \cdot P\left(\frac{D}{T_2}\right) = \frac{7}{100}$

$= \frac{1}{5} \times 10x + \frac{4}{5} \times x = \frac{7}{100} \Rightarrow x = \frac{1}{40}$ $\therefore P\left(\frac{T_2}{D}\right) = \frac{\frac{4}{5} \times \frac{39}{40}}{\frac{93}{100}} = \frac{78}{93}$

38. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

(A) 380 (B) 320 (C) 260 (D) 95

38. (A)

$= {}^6C_3 \times {}^4C_1 \times 4 + {}^6C_4 \times 4 = 380$

39. The least value of $\alpha \in \square$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is

(A) $\frac{1}{64}$ (B) $\frac{}{32}$ (C) $\frac{}{27}$ (D) $\frac{}{25}$

39. (C)

$$\frac{4\alpha x^2 + \frac{1}{2x} + \frac{1}{2x}}{3} \geq \sqrt[3]{4\alpha x^2 \cdot \frac{1}{4x^2}}$$

$$4\alpha x^2 + \frac{1}{x} \geq 3 \sqrt[3]{\alpha}$$

If this is true for all $x > 0$, then

$$3\sqrt[3]{\alpha} \geq 1 \Rightarrow \alpha \geq \frac{1}{27}$$

40. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x\sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x\tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

40. (C)

From given conditions,

$\alpha_1 = \sec(\theta) - \tan(\theta)$

$\beta_1 = \sec(\theta) - \tan(\theta)$

$\alpha_2 = \sec(\theta) - \tan(\theta)$

$\beta_2 = -\sec(\theta) - \tan(\theta)$

$\alpha_1 + \beta_2 = -2\tan\theta$

41. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

41. (C)

$\sqrt{3}\sec(x) + \operatorname{cosec}(x) = 2(\cot x - \tan x)$

$\Rightarrow \frac{\square 3\sqrt{}}{\cos(x)} + \frac{\square 1}{\sin(x)} = 2\left(\frac{\cos x}{\sin x} - \frac{\sin x}{\cos x}\right)$

$\Rightarrow \sqrt{3}\sin(x) + \cos(x) = 2(\cos^2 x - \sin^2 x)$

$\Rightarrow \cos\left(x - \frac{\pi}{3}\right) = \cos(2x)$

$\Rightarrow 2x = 2n\pi \pm \left(x - \frac{\pi}{3}\right) \Rightarrow x = 2n\pi - \frac{\pi}{3}, \frac{2n\pi}{3} + \frac{\pi}{9}$

In $(-\pi, \pi)$, $x = \frac{-\pi}{3}, \frac{\pi}{9}, \frac{7\pi}{9}, \frac{-5\pi}{9}$ $\therefore \sum x_i = 0$

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Partial Marks	: +1	For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) result in –2 marks, as a wrong option is also darkened.

42. Let $f : (0, \infty) \to \square$ be a differentiable function such that $f'(x) = 2 - \frac{f(x)}{x}$ for all $x \in (0, \infty)$ and $f(1) \neq 1$. Then

(A) $\lim_{x\to 0+} f'\left(\frac{1}{x}\right) = 1$

(B) $\lim_{x\to 0+} x f\left(\frac{1}{x}\right) = 2$

(C) $\lim_{x\to 0+} x^2 f'(x) = 0$

(D) $|f(x)| \leq 2$ for all $x \in (0,2)$

42. (A)

$$f'(x) + \frac{f(x)}{x} = 2$$

$$\Rightarrow \quad xf'(x) + f(x) = 2x$$

$$\Rightarrow \int d(x \cdot f(x)) = \int 2x dx$$

$$\Rightarrow xf(x) = x^2 + c$$

$$f(x) = x + \frac{c}{x} \quad (c \neq 0 \text{ as } f(1) \neq 1)$$

For this function, only (A) is correct.

43. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then

(A) $Q_2Q_3 = 12$

(B) $R_2R_3 = 4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$

(D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

43. (A), (B), (C)

$x^2 + y^2 = 3$

$x^2 = 2y$

Intersection point is $P \equiv (\sqrt{2}, 1)$

Equation of tangent is $\sqrt{2}x + y = 3$

$\tan(\theta) = -\sqrt{2}$

$\tan(\alpha) = \tan(\theta - 90) = -\cot\theta = \frac{1}{\sqrt{2}}$

$\sin(\alpha) = \frac{1}{\sqrt{3}} = \frac{2\sqrt{3}}{Q_3T}$

$\Rightarrow Q_3T = 6$

$\therefore Q_2Q_3 = 2Q_3T = 12$

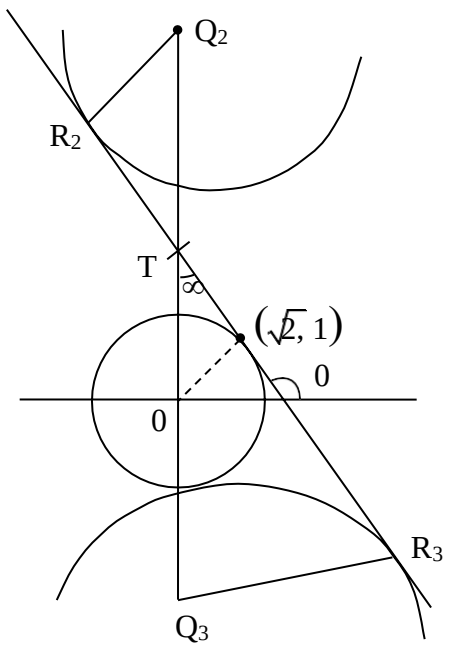
$\tan(\alpha) = \frac{1}{\sqrt{2}} = \frac{2\sqrt{3}}{R_3T} \Rightarrow R_3T = 2\sqrt{6}$

$\therefore R_2R_3 = 2R_3T = 4\sqrt{6}$

$\perp$ distance of o from R_2R_3 is $\left|\frac{3}{\sqrt{(\sqrt{2})^2 + 1^2}} = \sqrt{3}\right|$

$\therefore$ Area $(OR_2R_3) = \frac{1}{2} \times \sqrt{3} \times 4\sqrt{6} = 6\sqrt{2}$ square units

Similarly, Area $(PQ_2Q_3) = \frac{1}{2} \times \sqrt{2} \times 12 = 6\sqrt{2}$ square units



44. A solution curve of the differential equation $(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0, x > 0,$ passes through the point (1, 3). Then the solution curve

(A) intersects $y = x + 2$ exactly at one point.

(B) intersects $y = x + 2$ exactly at two points

(C) intersects $y = (x + 2)^2$

(D) does **NOT** intersect $y = (x + 3)^2$

44. (A), (C)

$(x + 2)^2 + y(x + 2) = y^2 . \frac{dx}{dy}$

$\frac{dx}{dy} = \frac{(x+2)^2}{y^2} + \frac{x+2}{y}$

$\frac{1}{(x+2)^2}\frac{dx}{dy} = \frac{1}{y^2} + \frac{1}{y(x+2)}$

$\therefore -\frac{1}{(x+2)^2}\frac{dx}{dy} - \frac{1}{(x+2)y} = \frac{1}{y^2}$

$\therefore$ Put $\frac{1}{x+2} = t$, $-\frac{1}{(x+2)^2}\frac{dx}{dy} = \frac{dt}{dy}$

$-\frac{dt}{dy} - \frac{t}{y} = \frac{1}{y^2}$

$\frac{dt}{dy} + \frac{t}{y} = -\frac{1}{y^2}$

I.F $= e^{\int \frac{1}{y} dy} = y$

$t.y = C + \int y\left(-\frac{1}{y^2}\right)dy$

$t.y = C - 1ogy$

$\therefore \frac{1}{x+2}.y = C - logy$

It passes (1, 3) $\Rightarrow 1 = C + 1og3 \quad \Rightarrow C = 1 + \log(3)$

$\frac{y}{x+2} = 1 + \log 3 - \log y$

[A] option is correct.

For Option (C)

$\frac{(x+2)^2}{x+2} = 1 - \log\left(\frac{y}{3}\right)$

$x + 1 = \log\left(\frac{3}{y}\right)$

$\therefore y = 3e^{-x-1}$

$\Rightarrow$ Intersect

For Option (D)

$\frac{(x+3)^2}{4+2} - 1 = -\log\left(\frac{(x+3)^2}{3}\right)$

$\therefore \frac{(x+3)^2 - 1}{x+2} = -\log\left\{\frac{(x+3)^2}{3}\right\}$

$3e\left(\frac{(x+3)^2 - 1}{-x-2}\right) = (x+3)^2$

$\Rightarrow$ will intersect.

$\Rightarrow$ (D) is not correct.

45. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and 2s = x + y + z. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then

(A) area of the triangle XYZ is $6\sqrt{6}$

(B) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(C) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2} = \frac{4}{35}$

(D) $\sin^2\left(\frac{X+Y}{2}\right) = \frac{3}{5}$

45. (A), (C), (D)

$\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2} = \frac{3s-(x+y+z)}{9} = \frac{s}{9}$

$\therefore\ x = \frac{5s}{9},\ y = \frac{2s}{3},\ z = \frac{7s}{9}$

$A = \pi r^2 = \frac{8\pi}{3}$

$\Rightarrow\ \frac{\Delta}{s} = \sqrt{\frac{8}{3}} \quad \Rightarrow\ \Delta^2 = \frac{8s^2}{3}$

$\Rightarrow\ s(s-x)(s-y)(s-z) = \frac{8s^2}{3}$

$\Rightarrow s \cdot \frac{4s}{9} \cdot \frac{s}{3} \cdot \frac{2s}{9} = \frac{8}{3}s^2$

$\Rightarrow s = 9$

$\therefore\ \Delta = \sqrt{\frac{8}{3}} \times 9 = 6\sqrt{6}$ square units

$R = \frac{xyz}{4\Delta} = \frac{\frac{5s}{9} \cdot \frac{2s}{3} \cdot \frac{7s}{9}}{4 \times 6\sqrt{6}} = \frac{35}{24}\sqrt{6}$

$\sin\left(\frac{x}{2}\right)\sin\left(\frac{y}{2}\right)\sin\left(\frac{z}{2}\right) = \frac{r}{4R} = \frac{\sqrt{\frac{8}{3}}}{4 \times \frac{35\sqrt{6}}{24}} = \frac{4}{35}$

$\sin^2\left(\frac{x+y}{z}\right) = \cos^2\left(\frac{z}{2}\right) = \frac{1+\cos(z)}{2} = \frac{3}{5}$ (Using cosine rule)

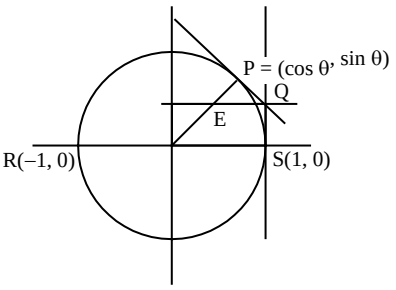
46. Let RS be the diameter of the circle $x^2 + y^2 = 1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s)

(A) $\left(\frac{1}{3}, \frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (C) $\left(\frac{1}{3}, -\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4}, -\frac{1}{2}\right)$

46. (A), (C)

Tangent at P $\quad x\cos\theta + y\sin\theta = 1$

$Q = \left(1, \frac{1-\cos\theta}{\sin\theta}\right)$



Normal at P, $\quad y = \tan\theta\, x$

Equation of QE, $\quad y - \frac{1-\cos\theta}{\sin\theta} = 0$

$y = \frac{1-\cos\theta}{\sin\theta}$

$y = \frac{1 - \frac{x}{\sqrt{x^2+y^2}}}{\frac{x}{\sqrt{x^2+y^2}}} = \frac{\sqrt{x^2+y^2} - x}{y}$

$y^2 + x = \sqrt{x^2+y^2}$

Hence option [A] and [C] satisfy.

47. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in \square$. Suppose $Q = [q_{ij}]$ is a matrix such that $PQ = kI$, where $k \in \square$, $k \neq 0$ and I is the identity matrix of order 3. If $q_{23} = -\frac{k}{8}$ and det (Q) = $\frac{k^2}{2}$, then

(A) $\alpha = 0, k = 8$
(B) $4\alpha - k + 8 = 0$
(C) det (P adj (Q)) = 2^9
(D) det (Q adj (P)) = 2^{13}

47. (B), (C)

$$\left(\frac{P}{K}\right)\cdot Q = I$$

$$\therefore \quad Q = \left(\frac{P}{K}\right)^{-1}$$

Comparing q_{23}, we get

$$\frac{-K}{8} = \frac{-K(3\alpha+4)}{(12\alpha+20)}$$

$$\alpha = -1$$

Also, $|P| \cdot |Q| = K^3$

$$\therefore (12\alpha + 20)\frac{K^2}{2} = K^3$$

$$K = 6\alpha + 10 = 4$$

Hence (B), (C) are correct.

48. Let $f : \square \to \square$, $g : \square \to \square$ and $h : \square \to \square$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in \square$. Then

(A) $g'(2) = \frac{1}{15}$
(B) $h'(1) = 666$
(C) $h'(0) = 16$
(D) $h(g(3)) = 36$

48. (B),(C)

$f(x) = x^3 + 3x + 2$, $\quad f(1) = 6$, $\quad g(6) = 1$

$g(f(x)) = x \quad \Rightarrow \quad g'(f(x)) \times f'(x) = 1$

put $x = 0$, $\quad g'(f(0)).\ f'(0) = 1$

$$g'(2) = \frac{1}{f'(0)} = \frac{1}{3}$$

$f(3) = 38$

$\therefore \quad g(38) = 3$

$\therefore h(g(3)) = h(g(g(38))) = 38$

$f(2) = 16 \quad \Rightarrow g(16) = 2$

$\therefore h(g(g(16)) = h(g(2)) = h(0)$

$\therefore 16 = h(g(g(16)) = h(0)$

$\therefore$ (C) is correct.

$f'(x) = 3x^2 + 3$

$f(6) = 111, \quad f(1) = 6 \quad \Rightarrow g'(6) = \frac{1}{6}$

$h(g(g(x))) = x$

$\Rightarrow h'(g(g(x))) \times g'(g(x)) \times g'(x) = 1$

Put x = 236, $h'(g(g(236))) \times g'(g(236)) \times g'(236) = 1 \Rightarrow h'(g(6))\, g'(6) \times \dfrac{1}{f'(6)} = 1$

$\Rightarrow h'(1) = 666$ But $g(1) \neq 1$

49. Consider a pyramid OPQRS located in the first octant ($x \geq 0$, $y \geq 0$, $z \geq 0$) with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the midpoint T of diagonal OQ such that TS = 3. Then

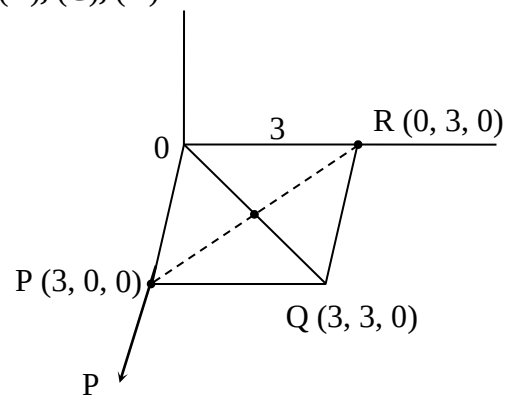
(A) the acute angle between OQ and OS is $\dfrac{\pi}{3}$

(B) the equation of the plane containing the triangle OQS is $x - y = 0$

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\dfrac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\dfrac{15}{2}}$

49. (B)**,** (C)**,** (D)



$S \equiv \left(\dfrac{3}{2}, \dfrac{3}{2}, 3\right)$

$S \equiv \left(\dfrac{3}{2}, \dfrac{3}{2}, 3\right)$

Direction of OQ $\equiv (3, 3, 0)$

Direction of OS $\equiv \left(\dfrac{3}{2}, \dfrac{3}{2}, 3\right)$

$$\cos\theta = \frac{3 \times \frac{3}{2} + 3 \times \frac{3}{2}}{\sqrt{3^2 + 3^2}\sqrt{\left(\frac{3}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + 3^2}}$$

$$= \frac{1}{\sqrt{3}}$$

$\therefore$ Hence (A) wrong.

For option B

$$\text{Normal of plane } \overrightarrow{OQ} \times \overrightarrow{OS} = \pm \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 0 \\ \frac{3}{2} & \frac{3}{2} & 3 \end{vmatrix}$$

$$= \pm\left(9\hat{i} - 9\hat{j}\right)$$

Equation of plane passing origin is $\vec{r}\cdot\vec{\eta} = 0$

$$\therefore \quad \left(x\hat{i}+y\hat{j}+z\hat{k}\right)\cdot\left(9\hat{i}-9\hat{j}\right)=0$$

$$\Rightarrow x - y = 0$$

For (C)

Perpendicular from P(3, 0, 0) to x – y = 0

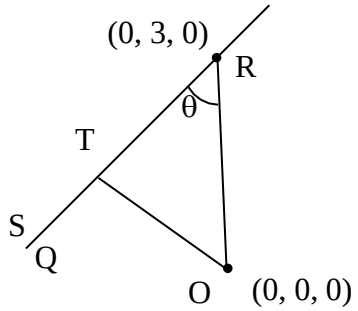
$$= \left|\frac{3-0}{\sqrt{1^2+1^2}}\right| = \frac{3}{\sqrt{2}}$$

Equation of RS is $\dfrac{x-0}{\frac{3}{2}-0} = \dfrac{y-3}{\frac{3}{2}-3} = \dfrac{z-0}{3-0}$

$$\frac{x}{\frac{3}{2}} = \frac{y-3}{-\frac{3}{2}} = \frac{z}{3}$$

Angle between line RS and OR

$$\cos\theta = \frac{0+3\left(-\frac{3}{2}\right)+0}{\sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2+3^2}\sqrt{3^2}} = -\frac{1}{\sqrt{6}}$$



Distance = OT = OR sin θ

$$= 3\sqrt{1-\frac{1}{6}} = 3\sqrt{\frac{5}{6}} = \sqrt{\frac{15}{2}}$$

SECTION 3 (Maximum Marks: 15)

- This section contains **FIVE** questions.
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in <u>one of the following categories</u>:
  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 If all other cases.

50. Let $z = \dfrac{-1+\sqrt{3}i}{2}$, where $i = \sqrt{-1}$, and r, s $\in$ {1, 2, 3}. Let $P = \begin{bmatrix} (-z)^r & z^{2s} \\ z^{2s} & z^r \end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r, s) for which $P^2 = -I$ is

50. [1]

$$Z = \frac{-1+i\sqrt{3}}{2} = \omega$$

$$P = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$$P^2 = \begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}\begin{bmatrix} (-\omega)^r & \omega^{2s} \\ \omega^{2s} & \omega^r \end{bmatrix}$$

$$= \begin{bmatrix} (-\omega)^{2r} + \left(\omega^{2s}\right)^2 & \omega^{2s}(-\omega)^r + \omega^r\omega^{2s} \\ \omega^{2s}(-\omega)^r + \omega^r\omega^{2s} & \omega^{4s} + \omega^{2r} \end{bmatrix}$$

$$= \begin{bmatrix} \omega^{4s} + \omega^{2r} & \omega^{2s}\left(\omega^r + (-\omega)^r\right) \\ \omega^{2s}\left(\omega^r + (-\omega)^r\right) & \omega^{4s} + \omega^{2r} \end{bmatrix}$$

$= -I$ (Given)

$\omega^{4s} + \omega^{2r} = -1$ and $\omega^{2s}(\omega^r + (-\omega)^r) = 0$

$\omega^r + (-\omega)^r = 0$

r	s	r	s
1	1	1	1
2	2	3	3

Total no. pairs = 1

51. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1 + x)^2 + (1 + x)^3 + \ldots + (1 + x)^{49} + (1 + mx)^{50}$ is $(3n + 1)\,{}^{51}C_3$ for some positive integer n. Then the value of n is

51. [5]

Coeff. of x^2 in expansion is

$= 1 + {}^3C_2 + {}^4C_2 + {}^5C_2 + \ldots + {}^{49}C_2 + {}^{50}C_2.m^2$ [as ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$= {}^3C_3 + {}^3C_2 + {}^4C_2 + \ldots.. + {}^{50}C_2\, m^2$

$= {}^4C_3 + {}^4C_2 + \ldots. + {}^{50}C_2\, m^2$

$= {}^5C_3 + \ldots + {}^{49}C_2 + {}^{50}C_2\, m^2$

$= {}^{50}C_3 + {}^{50}C_2\, m^2 + {}^{50}C_2 - {}^{50}C_2$

$= {}^{51}C_3 + {}^{50}C_2\,(m^2 - 1)$ …(1)

As given $(3x + 1)\,{}^{57}C_3 = (3x + 1) \cdot \frac{51}{3} \cdot {}^{50}C_2$ …(2)

From (1) and (2) $3x \cdot {}^{51}C_3 = {}^{50}C_2\,(m^2 - 1)$

$3x \cdot \frac{51}{3}\,{}^{50}C_2 = {}^{50}C_2\,(m^2 - 1)$

$\frac{m^2 - 1}{51} = n$

$\therefore$ Then Value of n is 5.

52. The total number of distinct $x \in [0, 1]$ for which $\int_0^x \frac{t^2}{1+t^4}dt = 2x - 1$ is

52. [1]

Let $f(x) = \int_0^x \frac{t^2 dt}{1+t^4} - 2x + 1$

$f'(x) = \frac{x^2}{1+x^4} - 2$

$$\Rightarrow \frac{-2x^4 + x^2 - 2}{x^4 + 1} < 0 \quad \forall x \in R$$

$f(0) > 0, \quad f(1) < 0$

$\therefore$ One solution in (0, 1)

53. The total number of distinct $x \in \square$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

53. [2]

$$x^3 \begin{vmatrix} 1 & 1 & 1+x^3 \\ 2 & 4 & 1+8x^3 \\ 3 & 9 & 1+27x^3 \end{vmatrix} = 10$$

$$x^3 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} + x^6 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} = 10$$

$$x^3 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 3 & 6 & -2 \end{vmatrix} + x^6 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & 6 \\ 3 & 6 & 24 \end{vmatrix} = 10$$

$6x^3 + x^3 - 5 = 0 \quad \Rightarrow 6x^6 + 6x^3 - 5x^3 - 5 = 0$

$(6x^3 - 5)(x^3 + 1) = 0$

$x^3 = \frac{5}{6}$ or $x^3 = -1$. Two real distinct values of x.

54. Let $\alpha, \beta \in \square$ be such that $\lim_{x \to 0} \frac{x^2 \sin(\beta x)}{ax - \sin x} = 1$. Then $6(\alpha + \beta)$ equals

54. [7]

$$\lim_{x \to 0} \frac{x^2 \left\{ \beta x - \frac{(\beta x)^3}{3!} + \ldots \right\}}{\alpha x - \left(\square^3 x - \frac{x}{3!} + \ldots \right)} = 1$$

$$\lim_{x \to 0} \frac{x^3 \left(\beta - \frac{\beta^3 x^2}{3!} + \ldots \right)}{(\alpha - 1)x + \frac{x^3}{3!} - \frac{x^5}{5!} + \ldots} = 1$$

As limit 1 $\Rightarrow$ $\boxed{\alpha = 1}$

$$\lim_{x \to 0} \frac{\beta - \frac{\beta^3}{3!} x^2 + \ldots}{\frac{1}{3!} - \frac{x^2}{5!} + \ldots} = 1$$

$\therefore \beta = \frac{1}{3!} = \frac{1}{6} \quad \therefore 6(\alpha + \beta) = 6\left(1 + \frac{1}{6}\right) = 7$

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