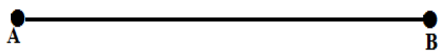
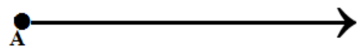
FUNDAMENTAL CONCEPTS OF GEOMETRY

Point: It is an exact location. It is a fine dot which has neither length nor breadth nor thickness but has position i.e., it has no magnitude.

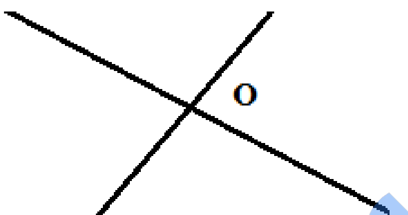
Line segment: The straight path joining two points A and B is called a line segment points and a definite length.



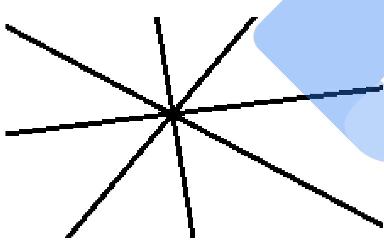
Ray: A line segment which can be extended in only one direction is called a ray.



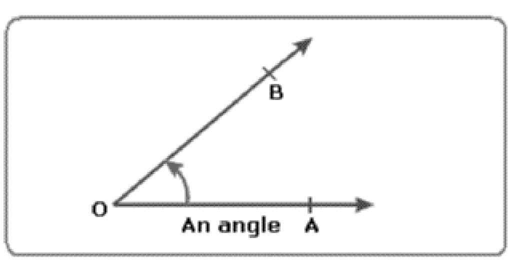
Intersecting lines: Two lines having a common point are called intersecting lines. The common point is known as the point of intersection.



Concurrent lines: If two or more lines intersect at the same point, then they are known as concurrent lines.

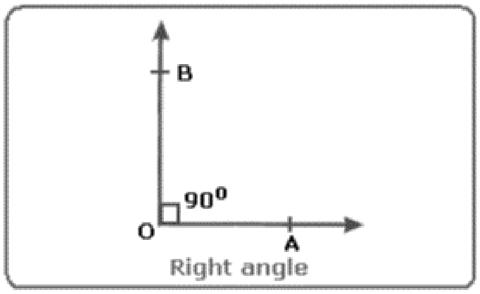


Angles: When two straight lines meet at a point they form an angle.

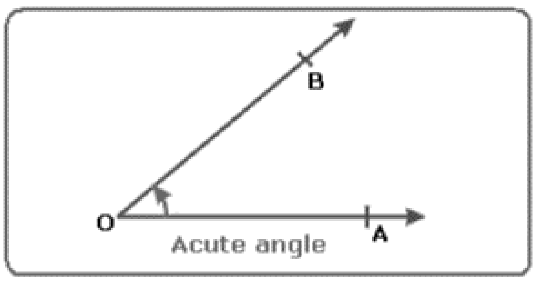


In the figure above, the angle is represented as ∠AOB. OA and OB are the arms of ∠AOB. Point O is the vertex of ∠AOB. The amount of turning from one arm (OA) to other (OB) is called the measure of the angle (Δ AOB).

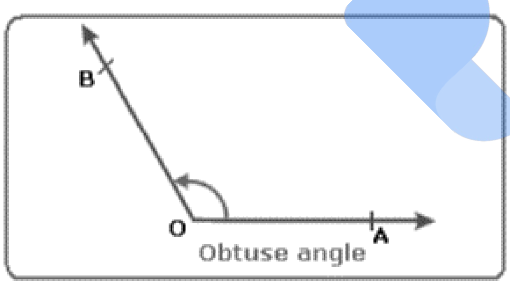
Right angle: An angle whose measure is 90 is called a right angle.



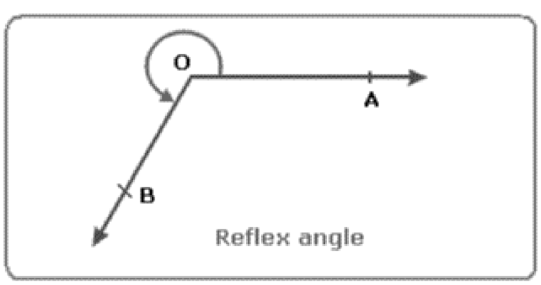
Acute angle: In angle whose measure is less than one right angle (i.e., less than 90), is called an acute angle.



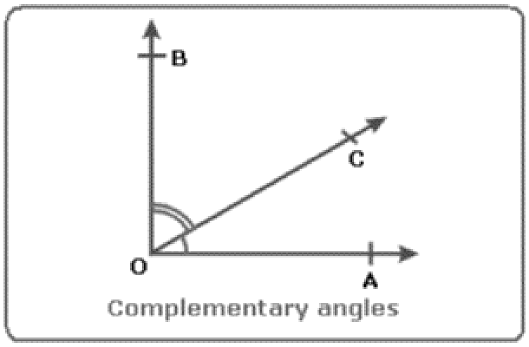
Obtuse angle: An angle whose measure is more than one right angle and less than two right angles (i.e., less than 180 and more than 90) is called an obtuse angle.



Reflex angle: An angle whose measure is more than 180 and less than 360 is called a reflex angle.

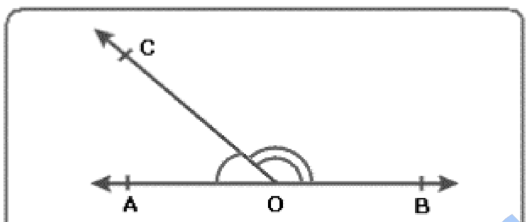


Complementary angles: If the sum of the two angles is one right angle (i.e., 90), they are called Complementary angles. Therefore, the complement of an angle θ is equal to 90° − θ.



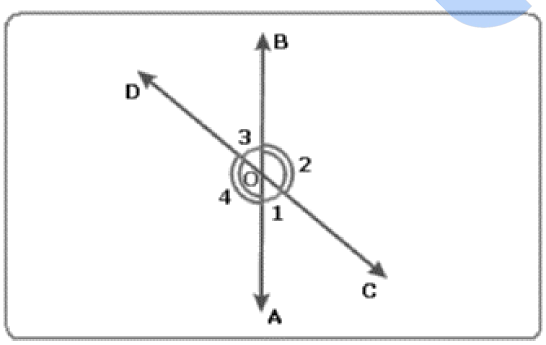
Complementary angles

Supplementary angles: Two angles are said to be supplementary, if the sum of their measures is 180. Example: Angles measuring 130 and 50 are supplementary angles. Two supplementary angles are the supplement of each other. Therefore, the supplement of an angle θ is equal to 180° − θ.



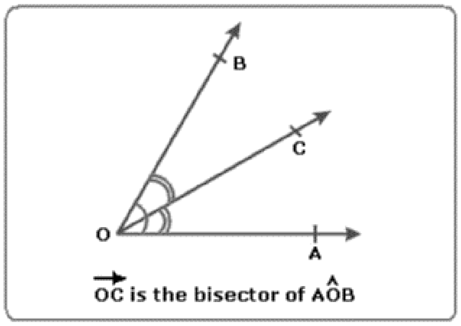
Supplementary angles

Vertically opposite angles: When two straight lines intersect each other at a point, the pairs of opposite angles so formed are called vertically opposite angles.

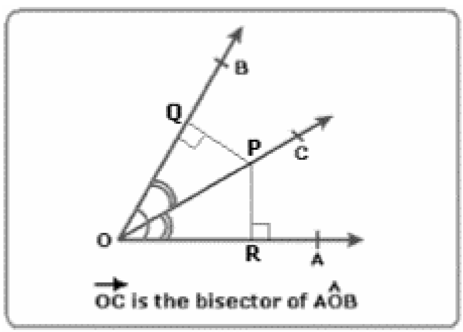


In the above figure, ∠1 and ∠3 and angles ∠2 and ∠4 are vertically opposite angles.
Note: Vertically opposite angles are always equal.

Bisector of an angle: If a ray or a straight line passing through the vertex of that angle, divides the angle into two angles of equal measurement, then that line is known as the Bisector of that angle.



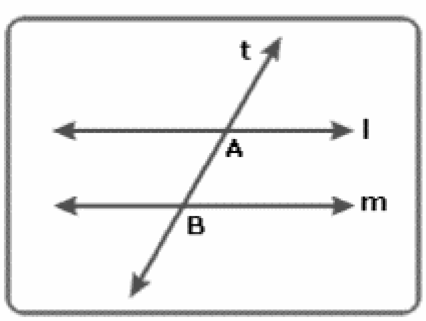
A point on an angle is equidistant from both the arms.



In the figure above, Q and R are the feet of perpendiculars drawn from P to OB and OA. It follows that PQ = PR.

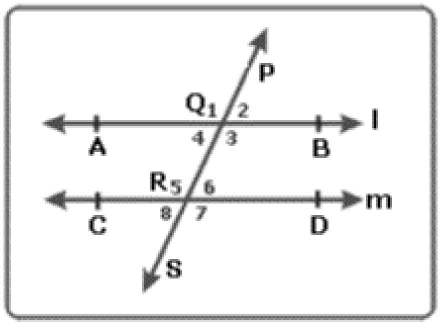
Parallel lines: Two lines are parallel if they are coplanar and they do not intersect each other even if they are extended on either side.

Transversal: A transversal is a line that intersects (or cuts) two or more coplanar lines at distinct points.



In the above figure, a transversal t is intersecting two parallel lines, l and m, at A and B, respectively.

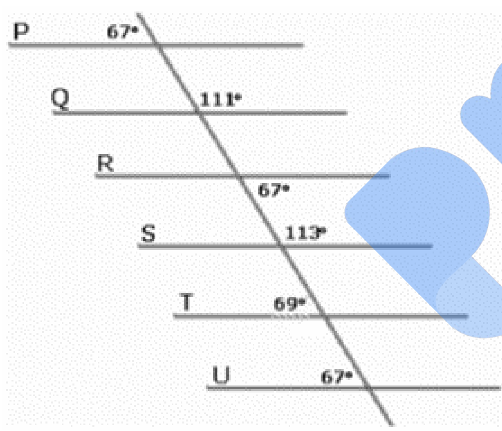
Angles formed by a transversal of two parallel lines:



In the above figure, l and m are two parallel lines intersected by a transversal PS. The following properties of the angles can be observed:

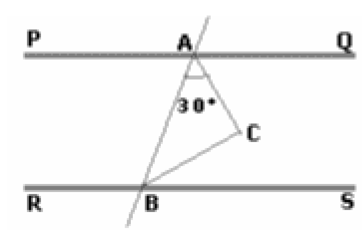
$\angle 3 = \angle 5$ and $\angle 4 = \angle 6$ [Alternate angles]
$\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 4 = \angle 8$, $\angle 3 = \angle 7$ [Corresponding angles]
$\angle 4 + \angle 5 = \angle 3 + \angle 6 = 180°$ [Supplementary angles]

In the figure given below, which of the lines are parallel to each other?



Answer: As 67° + 113° = 180°, lines P and S, R and S, and S and U are parallel. Therefore, lines P, R, S and U are parallel to each other. Similarly, lines Q and T are parallel to each other.

Example: In the figure given below, PQ and RS are two parallel lines and AB is a transversal. AC and BC are angle bisectors of $\angle BAQ$ and $\angle ABS$, respectively. If $\angle BAC = 30°$, find $\angle ABC$ and $\angle ACB$.

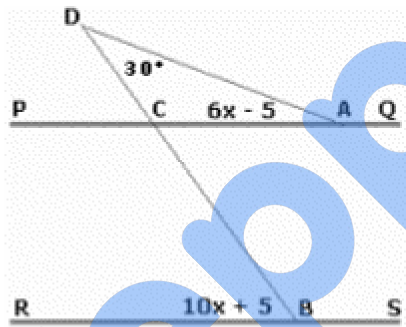


Answer:
$\angle BAQ + \angle ABS = 180°$ [Supplementary angles]

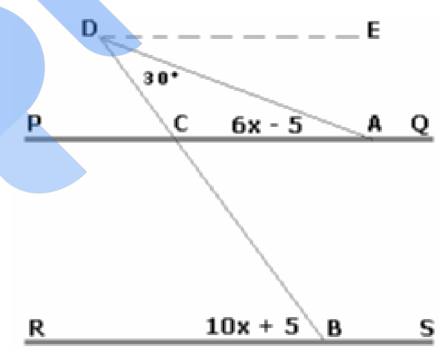
$\frac{\angle BAQ}{2} + \frac{\angle ABS}{2} = \frac{180}{2} = 90° \Rightarrow \angle BAC = \angle ABC = 90°$

Therefore, $\angle ABC = 60°$ and $\angle ACB = 90°$.

Example: For what values of x in the figure given below are the lines P-A-Q and R–B-S parallel, given that AD and BD intersect at D?

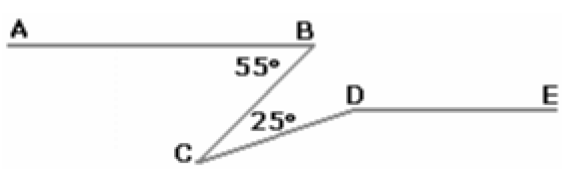


Answer: We draw a line DE, parallel to RS, as shown in the figure below:

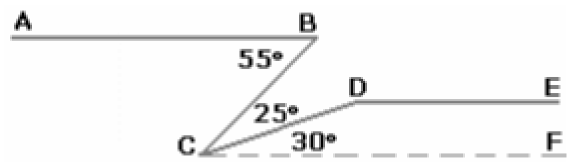


In the above figure, $\angle CDE = \angle RBD = 10x + 5 \Rightarrow \angle CDA = 10x + 5 - 30 = 10x - 25$.
Let the line PQ and RS be parallel. Therefore, PQ // DE $\Rightarrow$
$\angle EDA = \angle CAD = 10x - 25 = 6x - 5 \Rightarrow x = 5$

Example: In the figure given below, lines AB and DE are parallel. What is the value of ∠CDE?



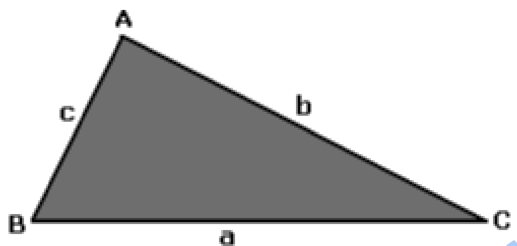
Answer: We draw a line CF // DE at C, as shown in the figure below.



∠BCF = ∠ABC = 55° ⇒ ∠DCF = 30°.
⇒ CDE = 180° − 30° = 150°.

TRIANGLE

Triangle is closed figures containing three angles and three sides.



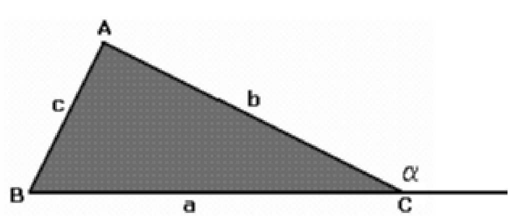
General Properties of Triangles:

1. **The sum of the two sides is greater than the third side: a + b > c, a + c > b, b + c > a**

Example: The two sides of a triangle are 12 cm and 7 cm. If the third side is an integer, find the sum of all the values of the third side.

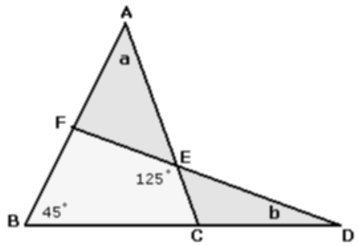
Answer: Let the third side be of x cm. Then, x + 7 > 12 or x > 5. Therefore, minimum value of x is 6. Also, x < 12 + 7 or x < 19. Therefore, the highest value of x is 18. The sum of all the integer values from 6 to 18 is equal to 156.

2. **The sum of the three angles of a triangle is equal to 180°: In the triangle below ∠A + ∠B + ∠C = 180°**



Also, the exterior angle α is equal to sum the two opposite interior angle A and B, i.e., $\alpha = \angle A + \angle B$.

Example: Find the value of a + b in the figure given below:



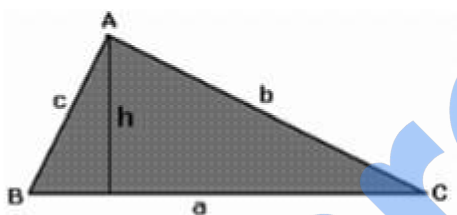
Answer: In the above figure, $\angle CED = 180° - 125° = 55°$.

$\angle ACD$ is the exterior angle of ΔABC. Therefore,

$\angle ACD = a + 45°$. In ΔCED, $a + 45° + 55° + b = 180°$

$\Rightarrow a + b = 80°$

3. Area of a Triangle:



Area of a triangle = ½ × base × height = ½ × a × h

Area of a triangle = ½ bc sin A = ½ ab sin C = ½ ac sin B

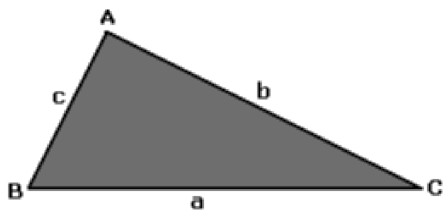
Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

Where $s = \frac{a+b+c}{2}$

Area of triangle = $\frac{abc}{4R}$ where R = circumradius

Area of a triangle = r × s where r = in radius and s = $\frac{a+b+c}{2}$

4. More Rules:



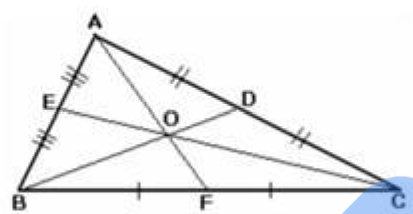
Sine Rule:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosine Rule:

$$\cos C = \frac{b^2 + c^2 + a^2}{2bc}$$

5. Medians of a triangle:



The medians of a triangle are lines joining a vertex to the midpoint of the opposite side. In the figure, AF, BD and CE are medians. The point where the three medians intersect is known as the centroid. O is the centroid in the figure.

The medians divide the triangle into two equal areas. In the figure, area ΔABF = area ΔAFC = area ΔBDC = area ΔBDA = area ΔCBE = area ΔCEA = $\frac{\text{Area } \Delta ABC}{2}$

The centroid divides a median internally in the ratio 2: 1. In the figure, $\frac{AO}{OF} = \frac{BO}{OD} = \frac{CO}{OE}$

Apollonius Theorem:

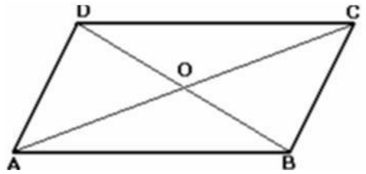
$AB^2 + AC^2 = 2(AF^2 + BF^2)$

$BC^2 + BA^2 = 2(BD^2 + DC^2)$

$BC^2 + AC^2 = 2(EC^2 + AE^2)$

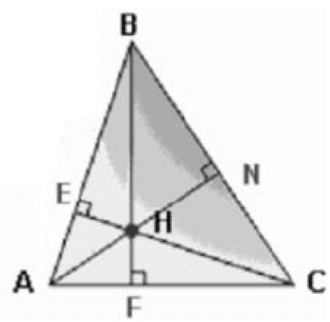
Example: ABCD is a parallelogram with AB = 21 cm, BC = 13 cm and BD= 14 cm. Find the length of AC.

Answer: The figure is shown below. Let AC and BD intersect at O. O bisects AC and BD. Therefore, OD is the median in triangle ADC



$\Rightarrow AD^2 + CD^2 = 2(AO^2 + DO^2) \Rightarrow AO = 16$. Therefore, AC = 32.

6. Altitudes of a Triangle:

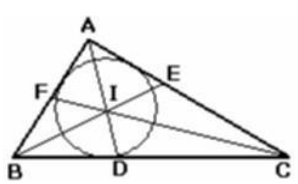


The altitudes are the perpendiculars dropped from a vertex tothe opposite side. In the figure, AN, BF, and CE are the altitudes, and their point of intersection, H, is known as the orthocenter.

Triangle ACE is a right-angled triangle. Therefore, $\angle ECA = 90° - \angle A$. Similarly in triangle CAN, $\angle CAN = 90° - \angle C$. In triangle AHC, $\angle CHA = 180° - (\angle HAC + \angle HCA) = 180° - (90° - \angle A + 90° - \angle C) = \angle A + \angle C = 180° - \angle B$.

Therefore, ∠AHC and ∠B are supplementary angles.

7. Internal Angle Bisectors of a Triangle:



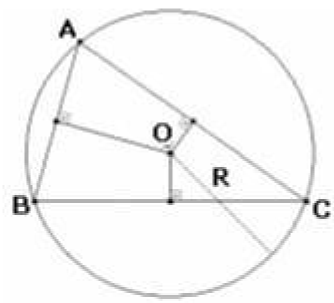
In the figure above, AD, BE and CF are the internal angle bisectors of triangle ABC. The point of intersection of these angle bisectors, I, is known as the incentre of the triangle ABC, i.e., centre of the circle touching all the sides of a triangle.

$\angle BIC = 180° - (\angle IBC + \angle ICB)$

$= \left(\frac{B}{2} + \frac{C}{2}\right) = 180 - \left(\frac{B+C}{2}\right) = 180 - \left(\frac{180-A}{2}\right) = 90 + \frac{A}{2}$

$\frac{AB}{AC} = \frac{BD}{CD}$ (internal bisector theorem)

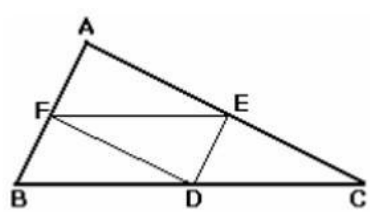
8. Perpendicular Side Bisectors of a Triangle:



In the figure above, the perpendicular bisectors of the sides AB, BC and CA of triangle ABC meet at O, the circumcentre (centre of the circle passing through the three vertices) of triangle ABC. In Above figure, O is the centre of the circle and BC is a chord. Therefore, the angle subtended at the centre by BC will be twice the angle subtended anywhere else in the same segment.

Therefore, ∠BOC = 2∠BAC.

9. Line Joining the Midpoints:



In the figure above, D, E and F are midpoints of the sides of triangle ABC. It can be proved that:

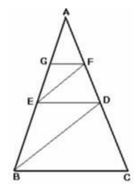
FE // BC, DE // AB and DF // AC.

$FE = \frac{BC}{2}$, $DE = \frac{AB}{2}$, $FD = \frac{AC}{2}$

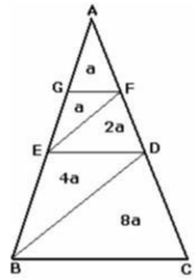
Area ΔDEF = Area Δ AFE = Area Δ BDF = Area Δ DEC

$= \frac{\text{Area } \Delta ABC}{4}$

Example: In the figure given below: AG = GE and GF || ED, EF || BD and ED || BC. Find the ratio of the area of triangle EFG to trapezium BCDE.

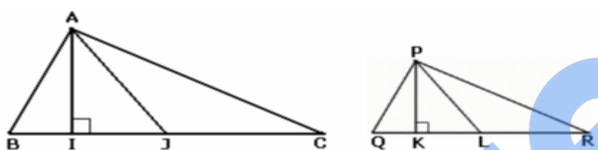


Answer: We know that line parallel to the base and passing through one midpoint passes through another midpoint also. Using this principle, we can see that G, F, E and D are midpoints of AE, AD, AB, and AC respectively. Therefore, GF, EF, ED, and BD are medians in triangles AFE, AED, ADB and ABC.



We know that medians divide the triangle into two equal areas.
Let the area of triangle AGF = a.
Therefore, the areas of the rest of the figures are as shown above.
The required ratio = a/12a = 1/12.

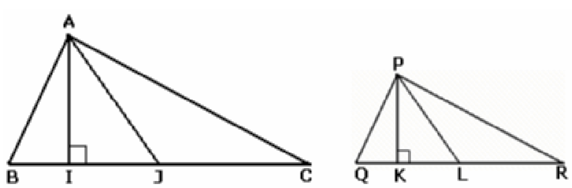
Similarity of Triangles



Two triangles are similar if their corresponding angles are equal or corresponding sides are in proportion.
In the figure given above, triangle ABC is similar to triangle PQR.
Then ∠A = ∠P, ∠B = ∠Q and ∠C = ∠R and
$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP} = \frac{AI}{PK}$ (altitudes) $= \frac{AJ}{PL}$ (medians)

Therefore, if you need to prove two triangles similar, prove their corresponding angles to be equal or their corresponding sides to be in proportion.

Ratio of Areas



If two triangles are similar, the ratio of their areas is the ratio of the squares of the length of their corresponding sides. Therefore,

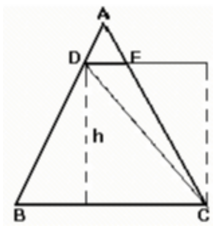
$$\frac{\text{Area of triangle ABC}}{\text{Area of triangle PQR}} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{CA^2}{RP^2}$$

Example: In triangle AC, shown above, DE || BC and DE/BC = ¼. If area of triangle ADE is 10, find the area of the trapezium BCED and the area of the triangle CED.

Answer: ΔADE and ΔABC are similar. Therefore,

$$\frac{\text{Area of triangle ABC}}{\text{Area of triangle ADE}} = \frac{BC^2}{DE^2}$$

Area of triangle ABC = 160 => Area of trapezium BCDE = Area Δ ABC – Area Δ ADE = 160 – 10 = 150



To find the area of triangle CDE, we draw altitudes of triangle BDC and CDE, as shown above. Let the length of the altitudes be h.

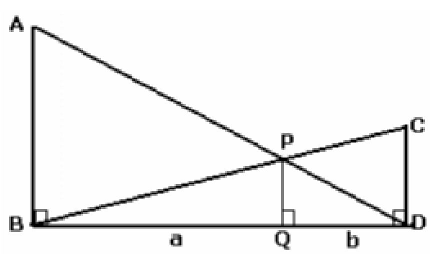
Area of triangle BCD = ½ x BC x h and area of triangle CDE = ½ x DE x h

- ⇨ $\frac{\text{Area of triangle BCD}}{\text{Area of triangle CDE}} = \frac{BC}{DE} = 4$

Therefore, we divide the area of the trapezium BCED in the ratio 1: 4 to find the area of triangle CDE.

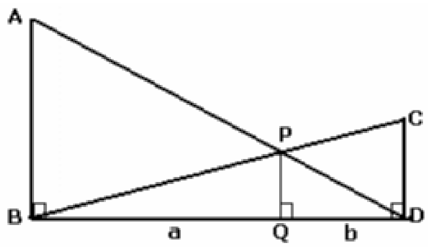
The required area = 1/5 x 150 = 30

Example: In the diagram given below, ∠ABD = ∠CDB = ∠PQD = 90°. If AB: CD = 3: 1, the ratio of CD: PQ is-



(A) 1 : 0.6 (B) 1 : 0.75
(C) 1: 0.72 (D) 1: 0.77

Answer: Let BQ = a and DQ = b, as shown in the figure below.



Triangle ABD and triangle PQD are similar. Therefore,

$\frac{PQ}{AB} = \frac{b}{a+b}$

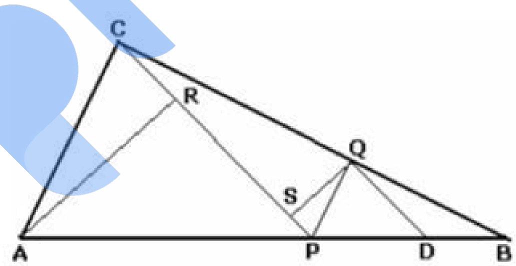
Also, triangle CBD and triangle PBQ are similar, therefore

$\frac{PQ}{CD} = \frac{a}{a+b}$

Dividing the second equality by the first, we get, $\frac{AB}{CD} = \frac{a}{b} = 3,$

Therefore, $\frac{CD}{PQ} = \frac{a+b}{a} = \frac{4}{3} = 1 : 0.75$

Example:



In the figure (not drawn to scale) given below, P is a point on AB such that AP: PB = 4: 3. PQ is parallel to AC and QD is parallel to CP. In ∆ARC, ∠ARC = 90°, and in ∆PQS, ∠PSQ = 90°. The length of QS is 6 cm. What is ratio AP: PD?

(A) 10 : 3 (B) 2 : 1

(C) 7 : 3 (D) 8 : 3

Answer: PQ is parallel to AC

$\frac{AP}{PB} = \frac{CQ}{QB} = \frac{4}{3}$

Let AP = 4x and PB = 3x.

QD is parallel CP ⇒ $\frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$ ⇒ PD = $\frac{4PB}{7} = \frac{12x}{7}$

⇒ AP: PD = 4x: $\frac{12x}{7}$ = 7 : 3

Example: In the figure (not drawn to scale given below, P is a point on AB such that AP : PB = 4 : 3. PQ is parallel to AC and QD is parallel to CP. In Δ ARC, ∠ARC = 90°, and in Δ PQS, ∠PSQ = 90°. The length of QS is 6 cm. What is ratio AP : PD?

(A) 10 : 3 (B) 2 : 1
(C) 7 : 3 (D) 8 : 3

Answer: PQ is parallel to AC ⇒ $\frac{AP}{PB} = \frac{CQ}{QB} = \frac{4}{3}$

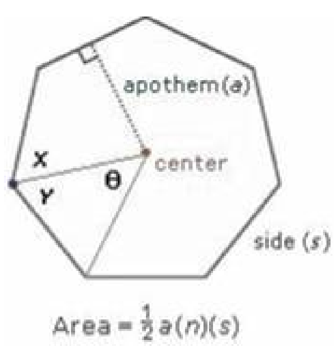
Let AP = 4 × and PB = 3x.

QD is parallel CP ⇒ $\frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$

PD = $\frac{4PB}{7} = \frac{12x}{7}$ ⇒ 4x : $\frac{12x}{7}$ = 7 : 3

Regular Polygon

A regular polygon is a polygon with all its sides equal and all its interior angles equal. All vertices of a regular lie on a circle whose centre is the center of the polygon.



Each side of a regular polygon subtends an angle ϴ = $\frac{360}{n}$ at the centre, as shown in the figure.

Also, X = Y = $\frac{180-\frac{360}{n}}{2} = \frac{\{180\ (n-2)\}}{2n}$

Therefore, interior angle of a regular polygon = x + y = $180(n\ -\ 2)/2n$

Sum of all angles of a regular polygon = $n\ x \frac{\{180\ (n-2)\}}{n} = 180\ (n\ -\ 2)$

Example: What is the interior angle of a regular octagon?

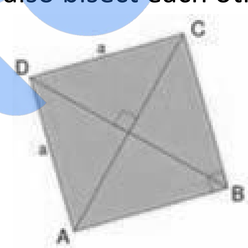
Answer: The interior angle of a regular octagon = n x 180 (n-2)/n = 180 (n-2)

Note: The formula for sum of all the angle of a regular polygon, i.e., 180(n-2), is true for all n-sided convex simple polygons.

Let's look at some polygons, especially quadrilaterals:

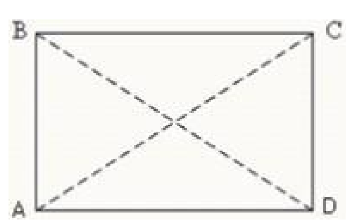
Quadrilateral: A quadrilateral is any closed shape that has four side. the sum of the measures of the angle is 3600. Some of the known quadrilaterals are square, rectangle, trapezium, parallelogram and rhombus.

Square: A square is regular quadrilateral that has four right angles and parallel sides. The sides of a square meet at right angles. The diagonal also bisect each other perpendicularly.



If the side of the square is a, then its perimeter = 4a, area = a^2 and the length of the diagonal = $\sqrt{2}$a

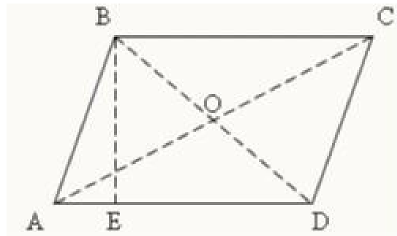
Rectangle: a rectangle is a parallelogram with all its angles equal to right angles.



Properties of a rectangle:
- Sides of rectangle are its heights simultaneously.
- Diagonals of a rectangle are equal: AC = BD.

- A square of a diagonal length is equal to a sum of squares of its side's lengths, i.e. $AC^2 = AD^2 + DC^2$.
- Area of a rectangle = length × breadth

Parallelogram: A parallelogram is a quadrangle in which opposite sides are equal and parallel.

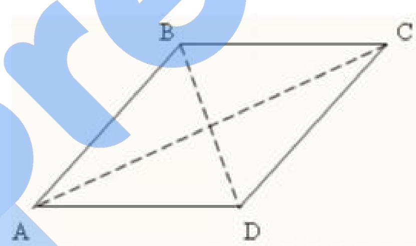


- Any two opposite sides of a parallelogram are called bases, a distance between them is called a height.
- Area of a parallelogram = base × height
- Perimeter = 2(sum of two consecutive sides)

Properties of a parallelogram:

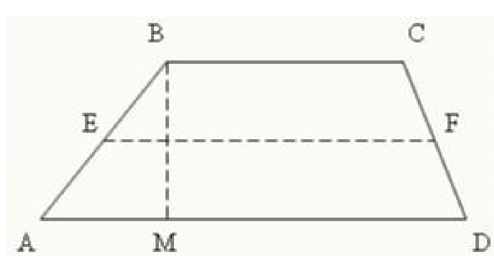
- Opposite side of a parallelogram are equal (AB = CD, AD = BC).
- Opposite angles of a parallelogram are equal (∠A = ∠C, ∠B = ∠D).
- Diagonals of a parallelogram are divided in their intersection point into two (AO = OC, BO = OD).
- A sum of squares of diagonals is equal to a sum of squares of four sides:
  ACs + BDs = ABs + BCs + CDs + ADs.

Rhombus: If all sides of parallelogram are equal, then this parallelogram is called a rhombus.



- Diagonals of a rhombus are mutually perpendicular (AC ⊥ BD) and divide its angle into two (∠DCA = ∠BCA, ∠ABD = ∠CBD etc.).
- Area of a rhombus = ½ × product of diagonals
  = ½ × AC × BD

Trapezoid: Trapezoid is a quadrangle two opposite sides of which are parallel.



Here AD || BC. Parallel sides are called bases of a trapezoid, the two others (AB and CD) are called lateral sides. A distance between bases (BM) is a height. The segment EF, joining midpoints E and F of the lateral sides, is called a midline of a trapezoid. A midline of a trapezoid is equal to a half-sum of bases:

$$EF = \frac{AD + BC}{2}$$

and parallel to them: EF || AD and EF || BC.

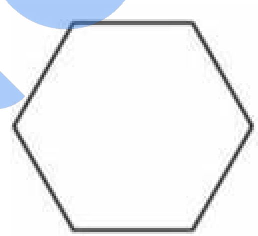
A trapezoid with equal lateral sides (AB = CD) is called an isosceles trapezoid. In an isosceles trapezoid angle by each base, are equal ($\angle A = \angle D$, $\angle B = \angle C$).

Area of a trapezoid = $\frac{\text{Sum of parallel sides}}{2}$ x height = $\frac{AD + BC}{2}$ x BM

In a trapezium ABCD with bases AB and CD, the sum of the squares of the lengths of the diagonals is equal to the sum of the squares of the lengths of the non-parallel sides and twice the product of the lengths of the parallel sides: AC2 + BD2 = AD2 + BC2 + 2.AB.CD

Here is one more polygon, a regular hexagon:

Regular Hexagon: A regular hexagon is a closed figure with six equal sides.

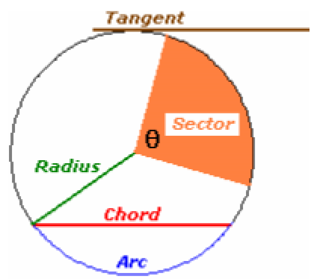


If we join each vertex to the centre of the hexagon, we get 6 equilateral triangles. Therefore, if the side of the hexagon is a, each equilateral triangle has a side a. Hence, area of the regular hexagon:

$$6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3\sqrt{3}}{2} a^2$$

Circle

A circle is a set of all points in a plane that lie at a constant distance from a fixed point. The fixed point is called the center of the circle and the constant distance is known as the radius of the circle.



Arc: An arc is a curved line that is part of the circumference of a circle. A minor arc is an arc less than the semicircle and a major arc is an arc greater than the semicircle.

Chord: A chord is a line segment within a circle that touches points on the circle.

Diameter: The longest distance from one end of a circle to the other is known as the diameter. It is equal to twice the radius.

Circumference: The perimeter of the circle is called the circumference. The value of the circumference = $2\pi r$, where r is the radius of the circle.

Area of a circle sector = $\pi \times (\text{radius})^2 = \pi r^2$

Sector: A sector is like a slice of pie (a circular wedge).
Area of circle sector (with central angle Θ);

$$\text{Area} = \frac{\Theta}{360} \times \pi \times r^2$$

Length of a circular Arc: (with central angle Θ) = $\frac{\Theta}{360} \times 2 \pi \times r$

Tangent of circle: A line perpendicular to the radius that touches ONLY one point on the circle.

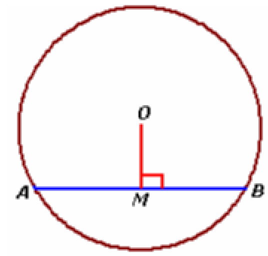
Example: If 45° arc of circle A has the same length as 60° arc of circle B, find the ratio of the areas of circle A and circle B.

Answer: Let the radius of circle A be r_1 and that of circle B be r_2.

- ⇨ $\frac{45}{360} \times 2 \pi \times r_1 = \frac{60}{360} \times 2 \pi \times r^2$
- ⇨ Ratio of areas = $\frac{\pi r_1^{\,2}}{\pi r_2^{\,2}} = \frac{16}{9}$

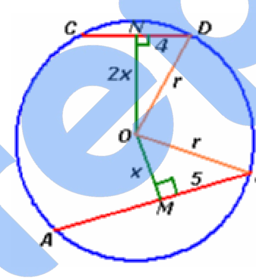
Rule:

The perpendicular from the center of a circle to a chord of the circle bisects the chord. In the figure below, O is the centre of the circle and OM ⊥ AB, Then, AM = MB.



Conversely, the line joining the center of the circle and the midpoint of a chord is perpendicular to the chord.

Example: In a circle, a chord of length 8 cm is twice as far from the center as a chord of length 10 cm. Find the circumference of the circle.



Answer: Let AB and CD be two chords of the circle such that AB = 10 and CD = 8.

Let O be the center of the circle and M and N be the midpoints of AB and CD.

Therefore OM ⊥ AB, ON ⊥ CD, and if ON = 2x then OM = x.

$BM^2 + OM^2 = OB^2$ and $DN^2 + ON^2 = OD^2$.

OB = OD = r ⟶ $(2x)^2 + 4^2\ r^2$ and $x^2 + 5^2 = r^2$.

Equating both the equations we get, $4x^2 + 16 = x^2 + 25$

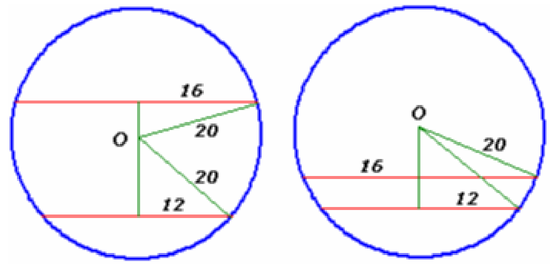
Or $x\sqrt{3} \longrightarrow 2\sqrt{7}$

Therefore, circumference is = 2 πr = 4 π$\sqrt{7}$

Example: What is the distance in cm between two parallel chords of length 32 cm and 24 cm in a circle of radius 20 cm?

(A) 1 or 7 (B) 2 or 14
(C) 3 or 21 (4) 4 or 28

Answer: The figures are shown below:



The parallel chords can be on the opposite side or the same side of the centre O. The perpendicular (s) dropped on the chords from the centre bisect (s) the chord into segments of 16 cm and 12 cm, as shown in the figure. From the Pythagoras theorem, the distances of the chords from the centre are –

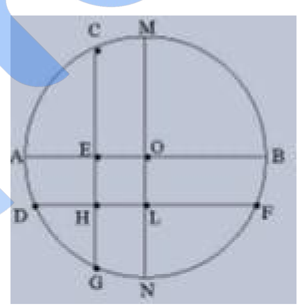
$\sqrt{}$ = 12 $\sqrt{}$ = 16, respectively.

$20^2 - 12^2$

$20^2 - 16^2$

Therefore, the distances between the chords can be 16 + 12 = 28 cm or 16 – 12 = 4 cm.

Example: In the following figure, the diameter of the circle is 3 cm. AB and MN are two diameters such that MN is perpendicular to AB. In addition, CG is perpendicular to AB such that AE:EB = 1:2, and DF is perpendicular to MN such that NL:LM = 1:2. The length of DH in cm is



Answer: In the above figure, AB = MN = 3 cm and AE: EB = NL: LM = 1: 2

⇒ AE = NL = 1 cm. Now AO = NO = 1.5 cm

⇒ OE = HL = OL

⇒ 0.5 cm. Join O and D

⇒ $OD^2 = OL^2 + DL^2$

⇒$\sqrt{}$ $\sqrt{}$ $\sqrt{}$ = =

2

$1.5^2 - 0.5^2$

$OD^2 - OL^2$

⇒$\sqrt{}$DH = DL – HL = $- \frac{1}{2} =$ $\sqrt{}$ 2 2 - 1

2

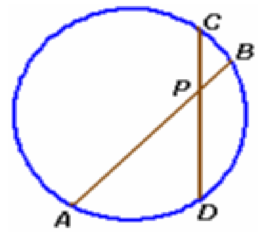
Some Important Rules

Rule #1

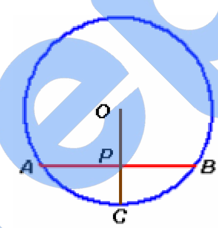
Equal chords are equidistant from the center. Conversely, if two chords are equidistant from the center of a circle, they are equal.

Rule #2

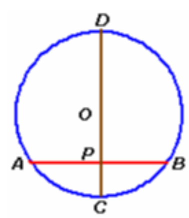
In the following figure, two chords of a circle, AB and CD, intersect at point P. Then, AP × PB = CP × PD.



Example: In the following figure, length of chord AB = 12. O-P-C is a perpendicular drawn to AB from center O and intersecting AB and the circle at P and C respectively. If PC = 2, find the length of OB.



Answer: Let us extend OC till it intersects the circle at some point D.



D is the diameter of the circle. Since OP is perpendicular to AB, P is the midpoint of AB.

Hence, AP = PB = 6.

Now DP × PC = AP × PB

DP = 18. Therefore, CD = 20, OC = 10

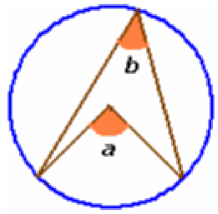
OB = OC = radius of the circle = 10.

Rule #3

In a circle, equal chords subtend equal angles at the center.

Rule #4

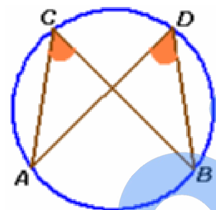
The angle subtended by an arc of a circle at the center is double the angle subtended by it at any point on the remaining part of the circumference.



In the figure shown above, a = 2b.

Rule #5

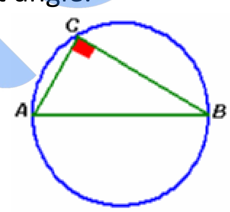
Angles inscribed in the same arc are equal.



In the figure angle ACB = angle ADB.

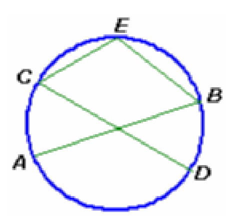
Rule #6

An angle inscribed in a semi-circle is a right angle.



Let angle ACB be inscribed in the semi-circle ACB; that is, let AB be a diameter and let the vertex C lie on the circumference; then angle ACB is a right angle.

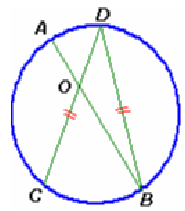
Example: In the figure AB and CD are two diameters of the circle intersecting at an angle of 48°. E is any point on arc CB. Find angle CEB.



Answer: Join E and D. Since arc BD subtends an angle of 48° at the center, it will subtend half as many degrees on the remaining part of circumference as it subtends at the center. Hence, angle DEB = 24°.

Since angle CED is made in a semicircle, it is equal to 90°. Hence, angle CEB = angle CED + angle DEB = 90° + 24° = 114°.

Example:



In the above figure, AB is a diameter of the circle and C and D are such points that CD = BD. AB and CD intersect at O. If angle AOD = 45°, find angle ADC.

Answer: Draw AC and CB.

$CD = BD \Rightarrow \angle DCB = \angle DBC = \theta$ (say).

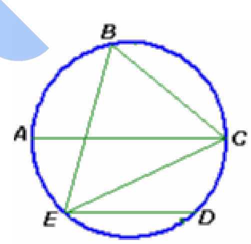
$\angle ACB = 90° \Rightarrow \angle ACD = 90° - \theta$.

$\angle ABD = \angle ACD = 90° - \theta \Rightarrow \angle ABC = \theta - (90° - \theta) = 2\theta - 90$.

In ΔOBC, $45° + 2\theta - 90 + \theta = 180° \Rightarrow 3\theta = 225° \Rightarrow \theta = 75°$.

$\angle ADC = \angle ABC = 2\theta - 90 = 60°$.

Example: In the adjoining figure, chord ED is parallel to the diameter AC of the circle. If angle CBE = 65°, then what is the value of angle DEC?



(A) 35° (B) 55°
(C) 45° (D) 25°

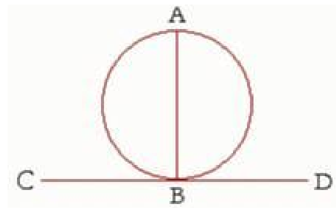
Answer: $\angle ABC = 90° \Rightarrow \angle ABE = 90 - \angle EBC = 25°$.

$\angle ABE = \angle ACE = 25°$

$\angle ACE = \angle CED = 25°$ (alternate angles)

Rule #7

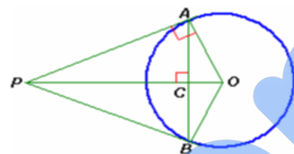
The straight line drawn at right angles to a diameter of a circle from its extremity is tangent to the circle. Conversely, if a straight line is tangent to a circle, then the radius drawn to the point of contact will be perpendicular to the tangent.



Let AB be a diameter of a circle, and let the straight line CD be drawn at right angles to AB from its extremity B; then the straight line CD is tangent to the circle.

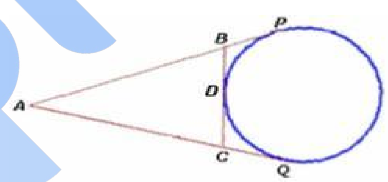
Rule #8

If two tangents are drawn to a circle from an exterior point, the length of two tangent segments are equal. Also, the line joining the exterior point to the centre of the circle bisects the angle between the tangents.



In the above figure, two tangents are drawn to a circle from point P and touching the circle at A and B. Then, PA = PB. Also, ∠APO = ∠BPO. Also, the chord AB is perpendicular to OP.

Example: In the following figure, lines AP, AQ and BC are tangent to the circle. The length of AP = 11. Find the perimeter of triangle ABC.

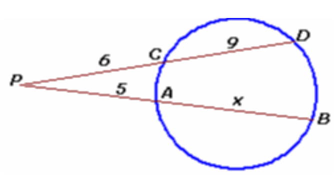


Answer: Let AB = x and BP = y. Then, BD = BP because they are tangents drawn from a same point B.
Similarly, CD = CQ and AP = AQ.
Now perimeter of triangle ABC = AB + BC + CA = AB + BD + DC + AC
= AB + BP + CQ + AC = AP + AQ = 2AP = 22.

Rule #9

From an external point P, a secant P-A-B, intersecting the circle at A and B, and a tangent PC are drawn.
Then, PA × PB = PC2.

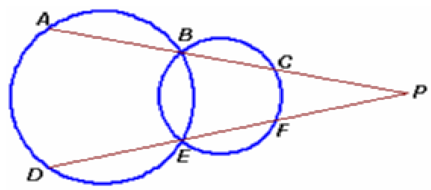
Example: In the following figure, if PC = 6, CD = 9, PA = 5 and AB = x, find the value of x.



Answer: Let a tangent PQ be drawn from P on the circle.
Hence, PC × PD = PQ = PA2 × PB = 6 × 15 = 5× (5 + x)

⇨ x = 13

Example: In the following figure, PC = 9, PB = 12, PA = 18, and PF = 8. Then, find the length of DE.



Answer: In the smaller circle
PC × PB = PF × PE

$PE = 12 \times \frac{9}{8} = \frac{27}{2}$

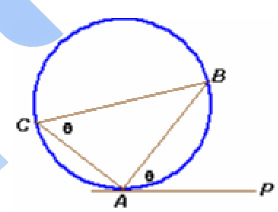
In the larger circle, PB × PA = PE × PD

$PD = 12 \times 18 \times \frac{2}{27} = 16$

Therefore, DE = PD – PE = 16 – 13.5 = 2.5

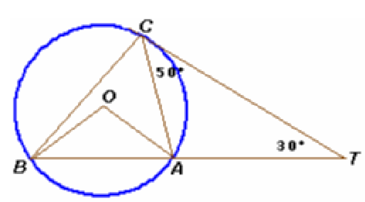
Rule #10

The angle that a tangent to a circle makes with a chord drawn from the point of contact is equal to the angle subtended by that chord in the alternate segment of the circle.



In the figure above, PA is the tangent at point A of the circle and AB is the chord at point A. Hence, angle BAP = angle ACB.

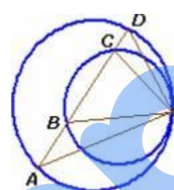
Example: In the figure given below, A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If ∠ATC = 30° and ∠ACT = 50°, then the angle ∠BOA is –



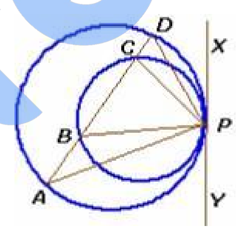
(A) 100°		(B) 150°
(C) 80°		(D) Not possible to determine

Answer: Tangent TC makes an angle of 50° with chord AC.
Therefore, ∠TBC = 50°.
In triangle TBC,
∠BCT = 180° − (30° + 50°) = 100°.
Therefore,
∠BCA = ∠BCT − ∠ACT = 100° − 50° = 50°.
∠BOA =2∠BCA = 100°.

Example: Two circles touch internally at P. The common chord AD of the larger circle intersects the smaller circle in B and C, as shown in the figure. Show that, ∠APB = ∠CPD.

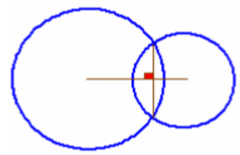


Answer: Draw the common tangent XPY at point P.



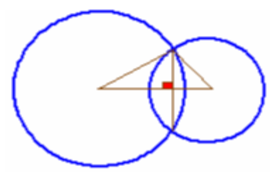
Now, for chord DP, ∠DPX = ∠DAP, and for chord PC, ∠CPX = ∠CBP.
⇒ ∠CPD = ∠CPX − ∠DPX = ∠CBP − ∠DAP.
In triangle APB, ∠CBP is the exterior angle
⇒ ∠CBP = ∠CAP + ∠APB
⇒ ∠CBP − ∠CAP = ∠APB
⇒ ∠CPD = ∠CPX − ∠DPX = ∠CBP − ∠DAP = ∠APB

Rule #11
When two circles intersect each other, the line joining the centers bisects the common chord and is perpendicular to the common chord.



In the figure given above, the line joining the centers divides the common chord in two equal parts and is also perpendicular to it.

Example: Two circles, with diameters 68 cm and 40 cm, intersect each other and the length of their common chord is 32 cm. Find the distance between their centers.



Answer: In the figure given above, the radii of the circles are 34 cm and 20 cm, respectively. The line joining the centers bisects the common chord. Hence, we get two right triangles: one with hypotenuse equal to 34 cm and height equal to 16 cm, and the other with hypotenuse equal to 20 cm and height equal to 16 cm. Using Pythagoras theorem, we get the bases of the two right triangles equal to 30 cm and 12 cm. Hence, the distance between the centers = 30 + 12 = 42 cm.