QUESTION PAPER CODE 65/1

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\lim_{x\to 0} \frac{\sin\frac{3x}{2}}{2} = \lim_{x\to 0} \frac{3}{2}\cdot\frac{\sin\frac{3x}{2}}{\frac{3x}{2}} = \frac{3}{2}$ $\frac{1}{2}$

$\Rightarrow \quad k = \frac{3}{2}$ $\frac{1}{2}$

2. $|A^{-1}| = \frac{1}{|A|}$ $\frac{1}{2}$

$= \frac{1}{4}$ $\frac{1}{2}$

3. $(\vec{a}\times\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 = 225 \Rightarrow |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 225$ $\frac{1}{2}$

$\Rightarrow \quad (5)^2\,|\vec{b}|^2 = 225 \Rightarrow |\vec{b}| = 3$ $\frac{1}{2}$

4. $\int \frac{3x}{3x-1}dx = \int \frac{3x-1+1}{3x-1}dx$ $\frac{1}{2}$

$= x + \frac{1}{3}\log|3x-1| + C$ $\frac{1}{2}$

SECTION B

5. Getting $\begin{pmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$ 1

$2x + 3 = 7$ and $2y - 4 = 14$

$\Rightarrow \quad x = 2,\ y = 9$ 1

6. $f(x) = \sin 2x - \cos 2x$

$\Rightarrow \quad f'(x) = 2\cos 2x + 2\sin 2x$ 1

$f'\left(\frac{\pi}{6}\right) = 2\left[\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right] = (1+\sqrt{3})$ 1

7. $6y = x^3 + 2 \Rightarrow 6\frac{dy}{dt} = 3x^2\frac{dx}{dt}$ 1

$\frac{dy}{dt} = 2\frac{dx}{dt} \Rightarrow 12 = 3x^2 \Rightarrow x = \pm 2$ $\frac{1}{2}$

$\therefore$ The points are (2, 5/3), (–2, –1) $\frac{1}{2}$

8. $\int \frac{x}{\sqrt{32 - x^2}}dx = -\int 1.dt$ where $32 - x^2 = t^2$ 1

$= -t + C = -\sqrt{32 - x^2} + C$ 1

9. $\log\left(\frac{dy}{dx}\right) = 3x + 4y \Rightarrow \frac{dy}{dx} = e^{3x}.e^{4y}$ $\frac{1}{2}$

$\Rightarrow \int e^{-4y}dy = \int e^{3x}dx$ $\frac{1}{2}$

$\Rightarrow -\frac{1}{4}e^{-4y} = \frac{1}{3}e^{3x} + C$ 1

10. Given differential equation can be written as

$\frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$ 1

Getting integrating factor $= e^{x - \log x}$ or $\frac{e^x}{x}$ 1

11. For coplanarity of vectors $\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ \lambda & 7 & 3 \end{vmatrix} = 0$ 1

Solving to get $\lambda = 0$ 1

12. Let, Number of executive class tickets be x and economy class tickets be y.

$\therefore$ LPP is Maximise Profit $P = 1500x + 1000y$ 1

Subject to: $x + y \leq 250,\ x \geq 25,\ y \geq 3x$ 1

SECTION C

13. Let the award for regularily be ₹ x and for hard work be ₹ y.

$\therefore \quad x + y = 6000$ and 1

$x + 3y = 11000$

$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6000 \\ 11000 \end{pmatrix}$ or $A.X = B$ 1

$\therefore \quad X = A^{-1}B$ as $|A| = 2 \neq 0.$

$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 6000 \\ 11000 \end{pmatrix}$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3500 \\ 2500 \end{pmatrix} \quad \therefore \quad x = ₹\,3500,\ y = ₹\,2500$ 1

Any two values like obedience, respect for elders,... 1

14. Given equation can be written as $\tan^{-1}(1) - \tan^{-1}x = \frac{1}{2}\tan^{-1}x$ $1\frac{1}{2}$

$\Rightarrow \frac{3}{2}\tan^{-1}x = \frac{\pi}{4}$ or $\tan^{-1}x = \frac{\pi}{6}$ $1\frac{1}{2}$

$\Rightarrow \quad x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ 1

15. Let $u = (\cos x)^x \Rightarrow \log u = x.\log\cos x$ $\frac{1}{2}$

$\Rightarrow \frac{du}{dx} = (\cos x)^x.[-x\tan x + \log\cos x]$ $1\frac{1}{2}$

$\therefore \quad y = (\cos x)^x + \sin^{-1}\sqrt{3x} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-3x}}.\frac{\sqrt{3}}{2\sqrt{x}}$ $1\frac{1}{2}$

$\therefore \quad \frac{dy}{dx} = (\cos x)^x[-x\tan x + \log\cos x] + \frac{\sqrt{3}}{2\sqrt{x}}.\frac{1}{\sqrt{1-3x}}$ $\frac{1}{2}$

OR

$$y = (\sec^{-1}x)^2 \quad \Rightarrow \quad \frac{dy}{dx} = 2\sec^{-1}x.\frac{1}{x\sqrt{x^2-1}}$$ 1

$$\therefore \quad x\sqrt{x^2-1}\cdot\frac{dy}{dx} = 2\sec^{-1}x$$ $\frac{1}{2}$

$$\Rightarrow \quad x\sqrt{x^2-1}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(\frac{x^2}{\sqrt{x^2-1}}+\sqrt{x^2-1}\right) = \frac{2}{x\sqrt{x^2-1}}$$ $1\frac{1}{2}$

$$\Rightarrow \quad x^2(x^2-1)\frac{d^2y}{dx^2}+\frac{dy}{dx}\cdot x(2x^2-1) = 2$$ 1

$$\text{i.e.,} \quad x^2(x^2-1)\frac{d^2y}{dx^2}+(2x^3-x)\frac{dy}{dx} = 2$$

16. $f'(x) = 6x^2 - 6x - 36$ $\frac{1}{2}$

$= 6(x^2 - x - 6) = 6(x-3)(x+2)$

$f'(x) = 0 \Rightarrow x = -2, x = 3$ 1

$\therefore$ the intervals are $(-\infty, -2), (-2, 3), (3, \infty)$ $\frac{1}{2}$

getting $f'(x)$ +ve in $(-\infty, -2) \cup (3, \infty)$
and –ve in $(-2, 3)$ $1\frac{1}{2}$

$\therefore$ f(x) is strictly increasing in $(-\infty, -2) \cup (3, \infty)$, and $\frac{1}{2}$

strictly decreasing in $(-2, 3)$

17. $I = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx = \int_0^{\pi} \frac{(\pi - x)\sin(\pi - x)}{1+\cos^2(\pi - x)} dx = \int_0^{\pi} \frac{(\pi - x)\sin x}{1+\cos^2 x} dx$ $1\frac{1}{2}$

$\Rightarrow \quad 2I = \pi\int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$ $\frac{1}{2}$

$\Rightarrow \quad I = \frac{-\pi}{2}\int_1^{-1} \frac{dt}{1+t^2} = \frac{\pi}{2}\int_{-1}^{1} \frac{dt}{1+t^2}$, where $\cos x = t$ 1

$I = \frac{\pi}{2}[\tan^{-1} t]_{-1}^{1} = \frac{\pi}{2}\left[\frac{\pi}{4} + \frac{\pi}{4}\right] = \frac{\pi^2}{4}$ 1

18. For $\int \frac{x^2 + x + 1}{(x+1)^2(x+2)} dx = \int \left[\frac{3}{x+2} - \frac{2}{x+1} + \frac{1}{(x+1)^2}\right] dx$ $2\frac{1}{2}$

$= 3\log|x+2| - 2\log|x+1| - \frac{1}{x+1} + C$ $1\frac{1}{2}$

OR

$I = \int (x-3)\sqrt{3-2x-x^2}\,dx = \int \left[-\frac{1}{2}(-2-2x) - 4\right]\sqrt{3-2x-x^2}\,dx$ 1

$= -\frac{1}{2}\int (-2-2x)\sqrt{3-2x-x^2}\,dx - 4\int \sqrt{4-(x+1)^2}\,dx$ $\frac{1}{2}+1$

$= -\frac{1}{3}(3-2x-x^2)^{3/2} - 4\left[\frac{(x+1)}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right)\right] + C$ $\frac{1}{2}+1$

19. Given differential equation can be written as $\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$

$\frac{y}{x} = v \quad \Rightarrow \quad y = vx \quad \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{1+v^2}{2v} \quad \Rightarrow \quad x\frac{dv}{dx} = \frac{1-v^2}{2v}$

$\Rightarrow \quad \int \frac{2v}{v^2 - 1} dv = -\int \frac{dx}{x}$ 1

$\Rightarrow \quad \log|v^2 - 1| + \log|x| = \log C$ 1

$\Rightarrow \quad x(v^2 - 1) = C$

$\Rightarrow \quad y^2 - x^2 = Cx$ 1

20. Let the vector $\vec{p} = (2\vec{a} + \vec{b} + 2\vec{c})$ makes angles α, β, γ respectively with the vector $\vec{a}, \vec{b}, \vec{c}$

Given that $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}\cdot\vec{b} = \vec{c}\cdot\vec{a} = 0$ 1

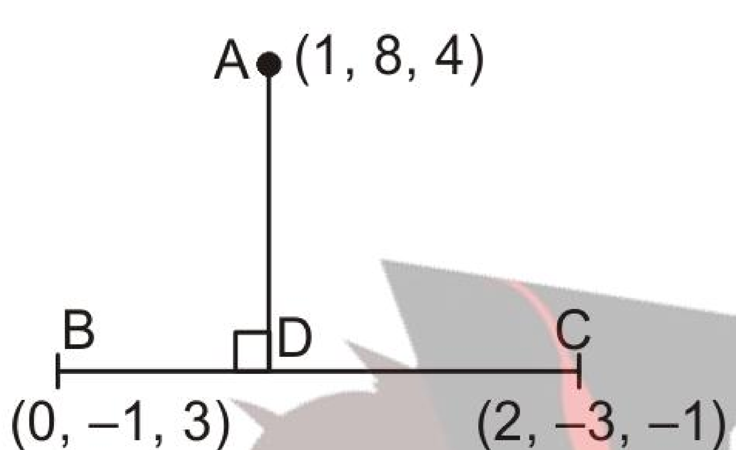
$$\cos\alpha = \frac{(2\vec{a} + \vec{b} + 2\vec{c})\cdot\vec{a}}{|2\vec{a} + \vec{b} + 2\vec{c}||\vec{a}|}$$ $\frac{1}{2}$

$$= \frac{2|a|^2}{3|\vec{a}||\vec{a}|} = \frac{2}{3} \Rightarrow \alpha = \cos^{-1}\frac{2}{3}$$ 1

$$\cos\beta = \frac{(2\vec{a} + \vec{b} + 2\vec{c})\cdot\vec{b}}{|2\vec{a} + \vec{b} + 2\vec{c}||\vec{b}|} = \frac{|\vec{b}|^2}{3|\vec{b}||\vec{b}|} = \frac{1}{3} \Rightarrow \beta = \cos^{-1}\frac{1}{3}$$ 1

$$\cos\gamma = \frac{(2\vec{a} + \vec{b} + 2\vec{c})\cdot\vec{c}}{|2\vec{a} + \vec{b} + 2\vec{c}||\vec{c}|} = \frac{2|\vec{c}|^2}{3|\vec{c}||\vec{c}|} = \frac{2}{3} \Rightarrow \gamma = \cos^{-1}\frac{2}{3}$$ $\frac{1}{2}$

21.



Equation of line passing through B and C is

$$\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} \text{ or } \frac{x}{1} = \frac{y+1}{-1} = \frac{z-3}{-2}$$ 1

Any point D on BC can be

$[\lambda, -\lambda - 1, -2\lambda + 3]$ for some value of λ. 1

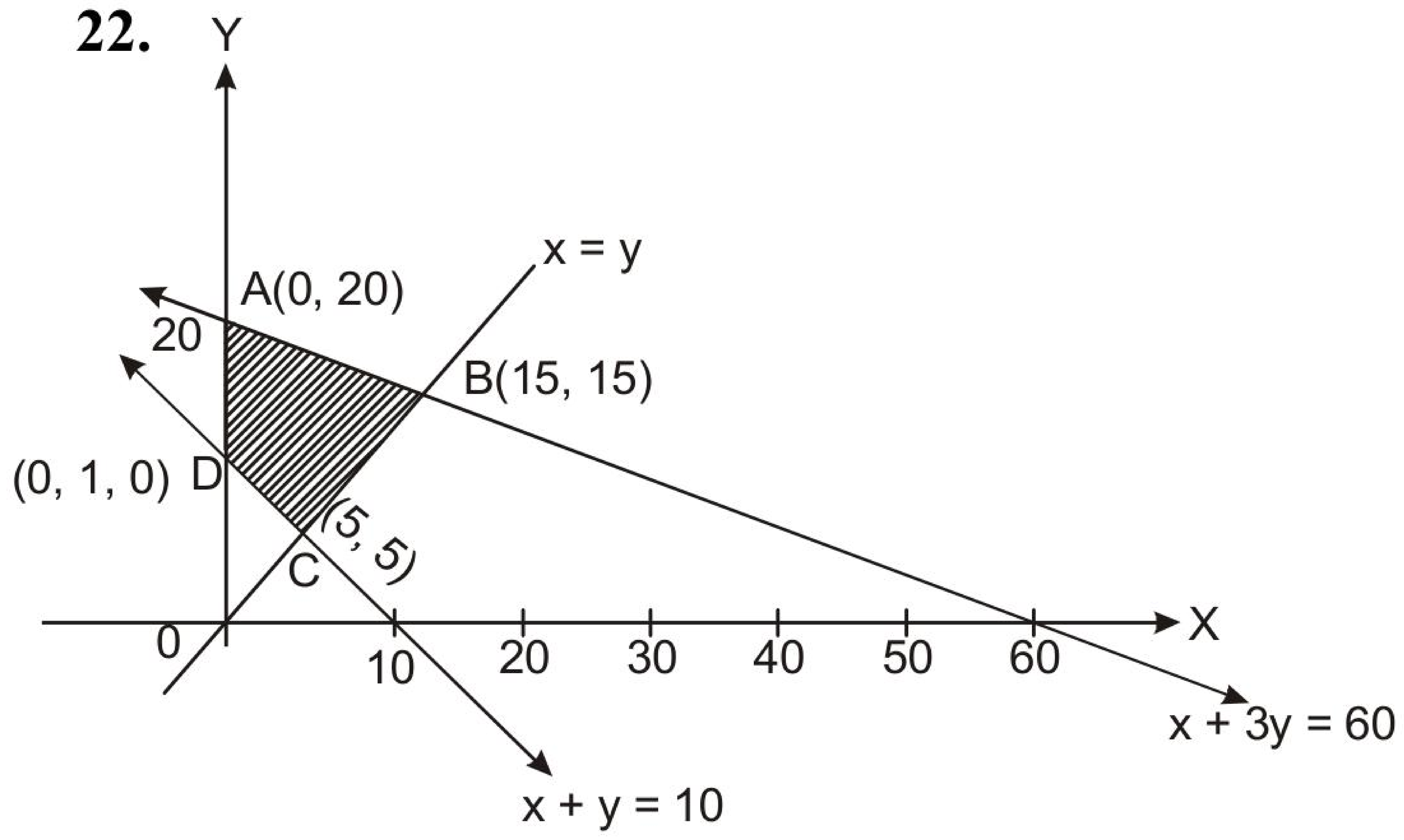
$\therefore$ Direction ratios of AD are $<\lambda - 1, -\lambda - 9, -2\lambda - 1>$ $\frac{1}{2}$

$AD \perp BC \Rightarrow 1(\lambda - 1) - 1(-\lambda - 9) - 2(-2\lambda - 1) = 0$ $\frac{1}{2}$

$\Rightarrow \lambda = -\frac{5}{3}$ $\frac{1}{2}$

$\therefore$ D is $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ $\frac{1}{2}$

22.



Correct graph of three lines $1\frac{1}{2}$

Correct shading 1

Vertices of feasible region are

A(0, 20), B(15, 15), C(5, 5), D(0, 10)

$Z(A) = 180$

$Z(B) = 180$

$Z(C) = 60$ 1

$Z(D) = 90$

$\therefore$ Z = 60 is minimum at x = 5, y = 5 $\frac{1}{2}$

23. Let the events be

E_1: tansferring a red ball from A to B

E_2: transferring a black ball from A to B

A: Getting a red ball from bag B $\frac{1}{2}$

$P(E_1) = \frac{3}{5},\ P(E_2) = \frac{2}{5}$ $\frac{1}{2}$

$P(A/E_1) = \frac{1}{2},\ P(A/E_2) = \frac{1}{3}$ 1

$P(E_1/A) = \dfrac{P(E_1)\cdot P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$ $\frac{1}{2}$

$= \dfrac{\frac{3}{5}\cdot\frac{1}{2}}{\frac{3}{5}\cdot\frac{1}{2} + \frac{2}{5}\cdot\frac{1}{3}} = \dfrac{9}{13}$ $1+\frac{1}{2}$

OR

Required probability $= P(A \cup B)$ 1

$= P(A) + P(B) - P(A)\cdot P(B)$ $\frac{1}{2}$

$= P(A)\,[1 - P(B)] + 1 - P(B')$ $\frac{1}{2}$

$= P(A)\,P(B') - P(B') + 1$ 1

$= (1 - P(B')\,(1 - P(A)) = 1 - P(A')\,P(B')$ 1

SECTION D

24. $C_1 \to C_1 + C_2 + C_3$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$ 1

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0$$ 1+1

$\Rightarrow -(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$ 1

$\Rightarrow \frac{-1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2] = 0$ 1

$\Rightarrow a - b = 0 = b - c = c - a$ as $a + b + c \neq 0$ 1

$\Rightarrow a = b = c$

25. (i) for any $A, B \in P(X)$, $A*B = A \cap B$ and $B*A = B \cap A$

as $A \cap B = B \cap A \therefore A*B = B*A$ 2

$\Rightarrow$ * is commutative

(ii) for any $A, B, C \in P(X)$

$(A*B)*C = (A \cap B)*C = (A \cap B) \cap C$

and $A*(B*C) = A*(B \cap C) = A \cap (B \cap C)$

Since $(A \cap B) \cap C = A \cap (B \cap C) \Rightarrow$ * is associative 2

(iii) for every $A \in P(X)$, $A*X = A \cap X = A$

$X*A = X \cap A = A$ 1

$\Rightarrow$ X is the identity element

(iv) $X*X = X \cap X = X \Rightarrow X$ is the only invertible element. $\because$ it is true only for X. 1

OR

$$f(x) = \frac{4x}{3x+4}$$

for $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$, $f(x_1) = f(x_2) \Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$

$\therefore \quad 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$

$\Rightarrow \quad x_1 = x_2$ 2

$\therefore \quad$ f is a 1 – 1 function.

for $y = \frac{4}{3}$, there is no x such that $f(x) = \frac{4}{3}$

$\therefore \quad$ f is not invertible 1

But $f : R - \left\{-\frac{4}{3}\right\} \to$ Range of f is ONTO so invertible. 1

and $f^{-1}(y) = \frac{4y}{4-3y}$ 2

26. Let given volume of cone be, $V = \frac{1}{3}\pi r^2 h$ ...(i) $\frac{1}{2}$

$\therefore \quad$ Surface area (curved) $S = \pi rl = \pi r\sqrt{r^2 + h^2}$ $\frac{1}{2}$

or $\quad A = S^2 = \pi r^2(r^2 + h^2)$

$$A = S^2 = \pi^2 r^2\left[r^2 + \left(\frac{3V}{\pi r^2}\right)^2\right] \quad \text{[using (i)]}$$

$$= \pi^2\left[r^4 + \frac{9V^2}{\pi^2 r^2}\right]$$ $1\frac{1}{2}$

$$\frac{dA}{dr} = \pi^2\left[4r^3 - \frac{18V^2}{\pi^2 r^3}\right]$$ 1

$$\frac{dA}{dr} = 0 \Rightarrow 4\pi^2 r^6 = 18 \cdot \frac{1}{9}\pi^2 r^4 h^2$$

$\Rightarrow \quad 2r^2 = h^2$ or $h = \sqrt{2}r$ $\quad 1\frac{1}{2}$

$$\frac{d^2A}{dr^2} = \pi^2\left[12r^2 + \frac{54V^2}{\pi^2 r^4}\right] > 0 \quad 1$$

$\Rightarrow$ for least curved surface area, height = $\sqrt{2}$ (radius)

OR

$$x = a\cos\theta + a\theta\sin\theta \Rightarrow \frac{dx}{d\theta} = -a\sin\theta + a\sin\theta + a\theta\cos\theta$$

$= a\theta\cos\theta$ $\quad 1$

$$y = a\sin\theta - a\theta\cos\theta \Rightarrow \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + a\theta\sin\theta$$

$= a\theta\sin\theta$ $\quad 1$

$$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta \quad \frac{1}{2}$$

Equation of tangent is

$y - (a\sin\theta - a\theta\cos\theta) = \tan\theta(x - a\cos\theta - a\theta\sin\theta)$ $\quad 1$

Equation of normal is

$$y - (a\sin\theta - a\theta\cos\theta) = -\frac{\cos\theta}{\sin\theta}(x - a\cos\theta - a\theta\sin\theta) \quad 1$$

$\Rightarrow \quad y\sin\theta + x\cos\theta = a$ $\quad \frac{1}{2}$

distance of normal from origin $= \dfrac{|-a|}{\sqrt{\sin^2\theta + \cos^2\theta}} = |a| =$ constant $\quad 1$

27. Equation of plane through A(2, –2, 1), B(4, 1, 3) and C(–2, –2, 5) is

$$\begin{vmatrix} x-2 & y+2 & z-1 \\ 2 & 3 & 2 \\ -4 & 0 & 4 \end{vmatrix} = 0 \Rightarrow 3x - 4y + 3z - 17 = 0$$ 2+1

For the given line $3(3) - 4(3) + 3(1) = 0$ 1

$\Rightarrow$ line is parallel to the plane

$\therefore$ Distance, $d = \dfrac{|3(5) - 4(4) + 3(8) - 17|}{\sqrt{9+16+9}} = \dfrac{6}{\sqrt{34}}$ 2

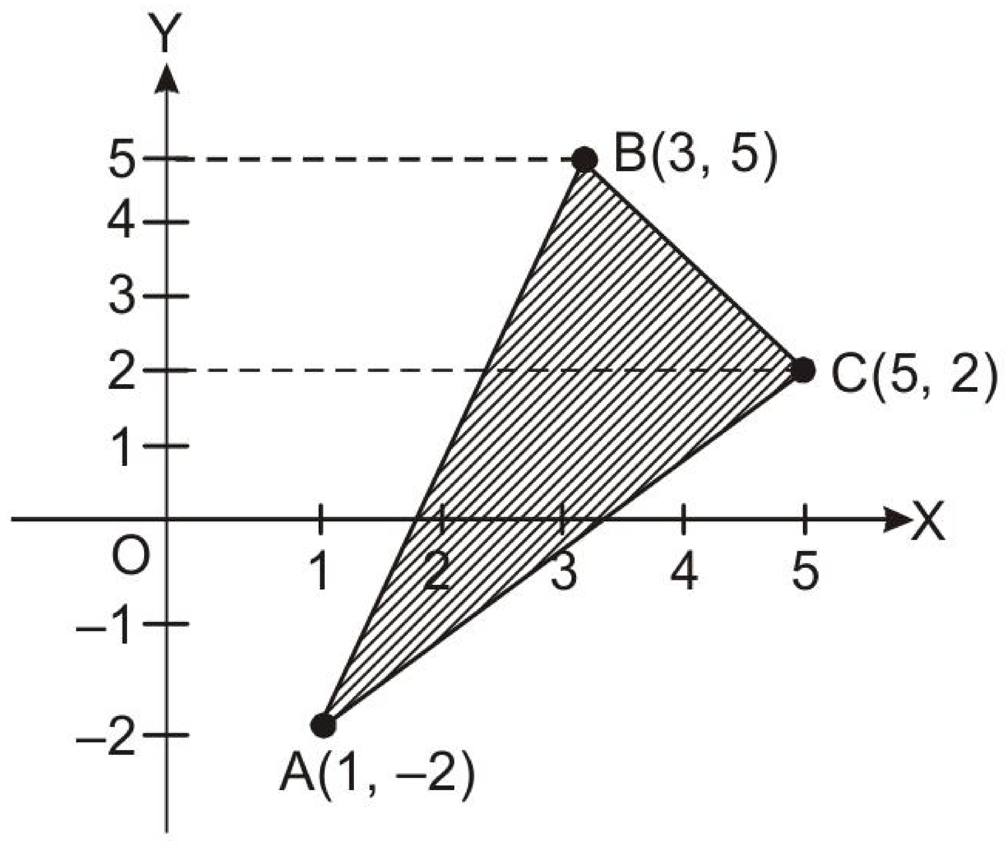
28. $P(\text{Head}) = 4P(\text{Tail}) \Rightarrow P(H) = \dfrac{4}{5}, P(T) = \dfrac{1}{5}$ 1

X (Number of tails)	0	1	2	3	$\frac{1}{2}$
P(X)	$\left(\frac{4}{5}\right)^3$	$3\left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right)$	$3\left(\frac{4}{5}\right)\left(\frac{1}{5}\right)^2$	$\left(\frac{1}{5}\right)^3$	
	$= \frac{64}{125}$	$= \frac{48}{125}$	$= \frac{12}{125}$	$= \frac{1}{125}$	2
XP(X):	0	$\frac{48}{125}$	$\frac{24}{125}$	$\frac{3}{125}$	$\frac{1}{2}$
$X^2P(X)$:	0	$\frac{48}{125}$	$\frac{48}{125}$	$\frac{9}{125}$	$\frac{1}{2}$

$\text{Mean} = \Sigma XP(X) = \dfrac{75}{125} = \dfrac{3}{5}$ $\frac{1}{2}$

Variance: $\Sigma X^2P(X) - [\Sigma XP(X)]^2 = \dfrac{105}{125} - \dfrac{9}{25} = \dfrac{60}{125} = \dfrac{12}{25}$ 1

29. **For Correct Figure** 1



Equation of AB: $x = \frac{1}{7}(2y + 11)$

Equation of BC: $x = \frac{1}{3}(19 - 2y)$ $1\frac{1}{2}$

Equation of AC: $x = y + 3$

Required area $= \int_{-2}^{2}(y+3)dy + \frac{1}{3}\int_2^5 (19-2y)dy - \frac{1}{7}\int_{-2}^{5}(2y+11)dy$ $1\frac{1}{2}$

$\Rightarrow A = \left.\frac{(y+3)^2}{2}\right]_{-2}^{2} + \frac{1}{3}\left.\frac{(19-2y)^2}{-4}\right]_2^5 - \frac{1}{7}\left.\frac{(2y+11)^2}{4}\right]_{-2}^{5}$ 1

$= \frac{1}{2}(25-1) - \frac{1}{12}(81-225) - \frac{1}{28}(441-49) = 10$ sq.units 1

OR

Here $h = \frac{4}{n}$ or $nh = 4$, $f(x) = 3x^2 + 2x + 1$ $\frac{1}{2}$

$\int_0^4 (3x^2 + 2x + 1)dx = \lim_{h \to 0} h[f(0) + f(0+h) + f(0+2h) + ... + f(0 + \overline{n-1}h)]$ 1

$= \lim_{h \to 0} h[(1) + (3h^2 + 2h + 1) + (3.2^2h^2 + 2.2h + 1) + ... + (3(n-1)^2h^2 + 2(n-1)h + 1)]$ $1\frac{1}{2}$

$= \lim_{h \to 0} h\left[n + 3h^2 \frac{n(n-1)(2n-1)}{6} + 2h\frac{n(n-1)}{2}\right]$ 1

$= \lim_{h \to 0}\left[nh + \frac{(nh)(nh-h)(2nh-h)}{2} + (nh)(nh-h)\right]$ 1

$= 4 + 64 + 16 = 84$ 1