17 — MATHEMATICS

(Answer ALL questions)

56. Which one is not a group?
1. $(R,+)$
2. $(N,.)$
3. $(c,+)$
4. $(I,+)$

57. Which one is correct?
1. They may be non-abelian subgroup of an abelian group
2. A non-abelian group has abelian subgroup
3. A subgroup can be defined as a subset
4. Every subset of a group is subgroup

58. A ring $(R,+,.)$ is called commutative ring if for $a,b \in R$
1. $(ab)c = a(bc)$
2. $ab = ba$
3. $a+b = b+a$
4. None of these

59. Which integral domain in the following is not an ordered integral domain?
1. $(I, +,.)$
2. $(Q, +,.)$
3. $(R, +,.)$
4. $(C, +,.)$

60. Let G be a finite group of order 200, then the number of subgroups of G of order 25 is
1. 1
2. 4
3. 5
4. 20

61. For $0 < \theta < \pi$, the matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
1. has no real eigen value
2. is orthogonal
3. is symmetric
4. is skew symmetric

62. Let U be a 3×3 complex Hermitian matr which is unitary. Then the distinct eig values of U are
1. $\pm i$
2. $1 \pm i$
3. ± 1
4. $\frac{1}{2}(1 \pm i)$

63. Let A be a 3×3 matrix with eigen valu $1, -1, 0$. Then the determinant of $I + A^{100}$ is
1. 6
2. 8
3. 9
4. 100

64. If a quadratic form is a diagonal form then contains
1. no cross product terms
2. cross product terms
3. product terms
4. None of the above

65. Let A be a non zero upper triangular matr and all of whose eigen values are 0. The $I + A$ is
1. Invertible
2. Singular
3. Idempotent
4. Nilpotent

66. If R is relation on a set A such that $R = R^{-}$ then R is
1. Reflexive
2. Symmetric
3. Transitive
4. Anti-Symmetric

67. Every non-empty set of real numbers tha has a lower bounded has
1. A supremum
2. An infimum
3. Neither infimum nor supremum
4. Both infimum and supremum

68. The series $\sum_{n=1}^{\infty}\frac{z^n}{n\sqrt{n+1}}$, $|z|\le 1$ is

1. uniformly but not absolutely convergent
2. uniformly and absolutely convergent
3. absolutely convergent but not uniformly convergent
4. convergent but not uniformly convergent

69. Let V be the volume of a region bounded by a smooth closed surface S. Let r denote the position vector and $\hat{n}$ denote the outward unit normal to S. Then the integral $\iint_S r.\hat{n}\,ds$ equals

1. V
2. $V/3$
3. $3V$
4. 0

70. Let $X=\{x \text{ in } Q/0<x<1\}$ be the metric space with standard metric from R. The completion of X is

1. $\{x \text{ in } Q/0<x<1\}$
2. $\{x \text{ in } R/0<x<1\}$
3. $\{x \text{ in } Q/0\le x\le 1\}$
4. $\{x \text{ in } R/0\le x\le 1\}$

71. A necessary and sufficient condition for a monotonic sequence to be convergent is that it

1. is bounded
2. is unbounded
3. may be bounded or unbounded
4. None of the above

72. $f(z)=\sin\frac{1}{z}, z=0$ is a

1. removable singularity
2. simple pole
3. branch point
4. essential singularity

73. If $u-v=e^x(\cos y-\sin y)$. Find $w=f(z)$

1. $e^{-z}+c$
2. $e^{z}+c$
3. $\frac{1}{e^{1/z}}+c$
4. e^{-z+c}

74. Let r denote the boundary of square whose sides lie along $x=\pm 1$ and $y=\pm 1$ where r is described in the positive sense, then the value of $\int_r \frac{z^2}{2z+3}dz$ is

1. $\frac{\pi i}{4}$
2. $2\pi i$
3. 0
4. $-2\pi i$

75. The bilinear transform $w=\frac{2z}{z-2}$ maps $\{z:|z-1|<1\}$ on to

1. $\{\omega:\text{Re}\,\omega<0\}$
2. $\{\omega:\text{Re}\,\omega>0\}$
3. $\{\omega:|w+2|>1\}$
4. $\{\omega:|\omega+2|<1\}$

76. Let $f(z)=u(x,y)+iv(x,y)$ be an entire function having Taylor's expansion $\sum_{n=0}^{\infty}a_n z^n$. If $f(x)=u(x,0)$ and $f(iy)=iv(0,y)$ then

1. $a_{2n}=0\,\forall n$
2. $a_0=a_1=a_2=a_3=0, a_n\ne 0$
3. $a_{2n+1}=0\,\forall n$
4. $a_0\ne 0$ but $a_2=0$

77. A subset A of topological space X is closed set if

1. $(X-A)$ is open
2. A is open
3. A is closed
4. None of the above

78. Let X and Y be topological spaces. The constant function $f:X\to Y$, is

1. Continuous
2. Inverse function
3. Discontinuous
4. None of the above

79. Every closed subset of a compact space is

1. Compact space
2. Null set
3. Open set
4. None of the above

80. The Cartesian product of a connected topological space is
1. Connected
2. Separable
3. Disconnected
4. None of the above

81. A countable product of first countable spaces is
1. First countable
2. Third countable
3. Second countable
4. Fourth countable

82. Banach space is a
1. Complete normed vector space
2. Complete vector space
3. Normed vector space
4. None of the above

83. A dual of normed vector space is
1. Bounded space
2. Banach space
3. Unbounded space
4. Not a Banach space

84. X is called Hilbert space if
1. X is complete under the norm obtained from its inner product
2. X is complete space
3. X is not complete under the norm obtained from its inner product
4. None of the above

85. Let T be a bounded linear operator on Hilbert space H then, T is unitary if
1. $T*T = I = TT*, I$ being an identity operator on H
2. $T* = T$
3. $T*T = T*T*$
4. None of the above

86. x is almost convergent if
1. $p'(x) = p(x)$
2. $p'(x) > p(x)$
3. $p'(x) < p(x)$
4. None of the above

87. Solution of $p^2 + q^2 = x + y$ is
1. $z = (a+x)^{3/2} + \frac{3}{2}(y-a)^{3/2} + b$
2. $z = \frac{3}{2}(a+x)^{3/2} + \frac{3}{2}(y-a)^{3/2} + b$
3. $z = \frac{2}{3}(a+x)^{3/2} + \frac{2}{3}(y-a)^{3/2} + b$
4. None of the above

88. The integrating factor for the differential equation $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$ is
1. $\frac{1}{x+1}$
2. $x+1$
3. $\frac{1}{x^2+1}$
4. x^2+1

89. The solution of the differential equation $x^2\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} - 6y = x$ is
1. $y = c_1x + c_2x^2 + \frac{x}{2}$
2. $y = c_1x^2 + c_2x^3 + \frac{x}{2}$
3. $y = c_1x + c_2x^3 + \frac{x^2}{2}$
4. $y = c_1x^2 + c_2x^3 + \frac{x^2}{2}$

90. $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0$ has the solution
1. $y = c_1e^{-x} + xc_2e^{-x} + x^2c_3e^{-x}$
2. $y = c_1e^{-x} + c_2\cos 2x + c_3\sin 2x$
3. $y = c_1\cos 2x + c_2\sin 2x$
4. None of the above

91. The equation of the envelope of the family of curves represented by the general solution of the differential equation is called
1. Complementary solution
2. Singular solution
3. Particular solution
4. None of the above

92. Given $\sin x, \cos x, \sin 2x$, then
1. The Wronskian of given equation $3\sin 2x$
2. The Wronskian of given equation $3\cos 2x$
3. The Wronskian of given equation $3\sin 3x$
4. The Wronskian of given equation $3\cos 3x$

93. The function $u(r, \theta)$ satisfying the Laplace equation $\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=0, e<r<e^2$ subject to the conditions $u(e, \theta)=1, u(e^2, \theta)=0$ is

1. $\ln(e/r)$
2. $\ln(e/r^2)$
3. $\ln(e^2/r)$
4. $\sum_{n=1}^{\infty}\left(\frac{r-e^2}{e-e^2}\right)\sin n\theta$

94. Solution of the diffusion equation $\frac{\partial^2 u}{\partial x^2}=\frac{\partial u}{\partial t}$, $u=u(x,t)$, with $u(0,t)=0=u(\pi,t)$, $u(x,0)=\cos x \sin 5x$ is

1. $\frac{e^{-36t}}{2}(\sin 6x+e^{20t}\sin 4x)$
2. $\frac{e^{-20t}}{2}(\sin 3x+e^{15t}\sin 5x)$
3. $\frac{e^{-36t}}{2}(\sin 4x+e^{20t}\sin 6x)$
4. $\frac{e^{-36t}}{2}(\sin 5x+e^{20t}\sin x)$

95. Heat flux F at $(1/4, t)$ of one dimensional heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, 0<x<1, t>0, u(x,0)=2\cos^2\pi x$, $u_t(0,t)=0=u_x(1,t)$, is

1. $F=2\pi e^{-4\pi^2 t}$
2. $F=-2\pi e^{-4\pi^2 t}$
3. $F=4\pi e^{-2\pi^2 t}$
4. $F=-4\pi e^{-2\pi^2 t}$

96. The Fourier transform $F(\omega)$ of $f(x), -\infty<x<\infty$ is defined by $F(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty} f(x)e^{-i\omega x}dx$. The Fourier transform with respect to x of the solution $u(x,y)$ of the boundary value problem $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, -\infty,<x<\infty$ which remains bounded for large y is given by $U(\omega,y)=F(\omega)e^{-|\omega|y}$. Then, the solution $u(x, y)$ is given by

1. $u(x,y)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(x-z)}{y^2+z^2}dz$
2. $u(x,y)=\frac{y}{\pi}\int_{-\infty}^{\infty}\frac{f(x-z)}{y^2+z^2}dz$
3. $u(x,y)=\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(x+z)}{y^2+z^2}dz$
4. $u(x,y)=\frac{y}{\pi}\int_{-\infty}^{\infty}\frac{f(x+z)}{y^2+z^2}dz$

97. The PDE

$x\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y\frac{\partial^2 u}{\partial y^2}+x\frac{\partial u}{\partial y}+y\frac{\partial u}{\partial x}=0$ is

1. Elliptic in the region $x<0, y<0, xy>1$
2. Elliptic in the region $x>0, y>0, xy>1$
3. Parabolic in the region $x<0, y<0, xy>1$
4. Hyperbolic in the region $x<0, y<0, xy>1$

98. If $f(x)$ and $g(x)$ are arbitrary functions, then the general solution of the PDE $\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}=0$ is given by

1. $u(x,y)=f(x)+g(x)$
2. $u(x,y)=f(x)g(y)$
3. $u(x,y)=f(x+y)+g(x-y)$
4. $u(x,y)=x\,g(y)+yf(x)$

99. The solution to Euler's characteristic equation is referred as

1. Extremals
2. Zeros
3. Nulls
4. Residues

100. Let the functional form of F be given, then the integral $I=\int_a^b F(x,y,y_1,y_2,y_3,.......y_n)dx$ is stationary when y satisfies the equation

1. $(-1)^n\frac{d^n}{dx^n}\left(\frac{\partial F}{\partial y_n}\right)=0$
2. $\frac{\partial F}{\partial x}-\frac{d}{dx}\left(\frac{\partial F}{\partial y_1}\right)-\frac{d^2}{dx^2}\left(\frac{\partial F}{\partial y_2}\right)-......+(-1)^n\frac{d^n}{dx^n}\left(\frac{\partial F}{\partial y_n}\right)=0$
3. $\frac{\partial F}{\partial x}-\frac{d}{dx}\left(\frac{\partial F}{\partial y_1}\right)-\frac{d^2}{dx^2}\left(\frac{\partial F}{\partial y_2}\right)-......+\frac{d^n}{dx^n}\left(\frac{\partial F}{\partial y_n}\right)=0$
4. None of the above

101. The integral I has strong maximum if

1. the equation of Γ_e, the arc integration satisfies $\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$
2. the equation of Γ_e, the arc integration satisfies $\frac{\partial F}{\partial y} - \frac{\partial F}{\partial y'} = 0$
3. the equation of Γ_e, the arc integration satisfies $\frac{\partial F}{\partial y} = 0$
4. the equation of Γ_e, the arc integration satisfies $\frac{\partial F}{\partial y} - \frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right) = 0$

102. Any solution of homogeneous Volterra integral equation of second kind $\varphi(x) - \lambda\int_0^x k(x,y)\varphi(x)dy = 0$ in L_2 -space is

1. necessarily a non-zero function
2. absolute function
3. necessarily a zero function
4. none of the above

103. Any L_2-function $w(x)$ is orthogonal to all the eigen functions $\phi_n(x)$ of symmetric kernel $k(x, y)$ if

1. $\int k(x, y)w(x)\, dy > 0$
2. $\int k(x, y)w(x)\, dy = 0$
3. $\int k(x, y)w(x)\, dy \neq 0$
4. $\int k(x, y)w(x)\, dy < 0$

104. The daily wages (in Rupees) of workers in two cities are as follows

	Size of the sample	S.D. of wages
City A	16	25
City B	13	32

Test at 5% level, the equality of variances of the wage distribution in the two cities. [Tabulated value of F for (12, 15) degree of freedom at 5% LOS is 2.48]

1. Variances of the wage distribution in the two cities may be equal
2. Variances of the wage distribution in the two cities may not be equal
3. Data in sufficient
4. None of the above

105. A simple random sample of size 'n' is to b drawn from a large population to estimat the population proportion θ. Let p be th sample proportion. Using the norma approximation, determine which of th following sample size value will ensur $|p-\theta| \leq 0.02$ with probability at least 0.95 irrespective of the true value of θ? [You ma assume, $\varphi(1.96) = 0.975$, $\varphi(1.64) = 0.95$ where φ denotes the cumulative distributio function of the standard normal distribution]

1. $n = 1000$
2. $n = 1500$
3. $n = 1200$
4. $n = 2500$

106. The random variable X has t-distribution with υ degree of freedom. The the probability distribution of X^2 is

1. Chi-square distribution with 1 degre of freedom
2. Chi-square distribution with υ degre of freedom
3. F-distribution with $(1, \upsilon)$ degree freedom
4. F-distribution with $(\upsilon, 1)$ degree freedom

107. The objective function of the dual problem f the following primal LPP :

$Max\, f = 2x_1 + x_2$ subject to

$x_1 - 2x_2 \geq 2$

$x_1 + 2x_2 = 8$

$x_1 - x_2 \leq 11$ with $x_1 \geq 0$ and x_2 unrestrict in sign, is given by

1. $\min z = 2y_1 - 8y_2 + 11y_3$
2. $\min z = 2y_1 - 8y_2 - 11y_3$
3. $\min z = 2y_1 + 8y_2 + 11y_3$
4. $\min z = 2y_1 + 8y_2 - 11y_3$

108. Let 'x' be a non-optimal feasible solution of a linear programming maximization problem and 'y' a dual feasible solution. Then

1. The primal objective value at x is greater than the dual objective value at y
2. The primal objective value at x could equal the dual objective value at y
3. The primal objective value at x is less than the dual objective value at y
4. The dual could be unbounded

109. Consider the following LPP

$Min\, z = 2x_1 + 3x_2 + x_3$ subject to constraints

$x_1 + 2x_2 + 2x_3 - x_4 + x_5 = 3$

$2x_1 + 3x_2 + 4x_3 + x_6 = 6,\ x_i \geq 0,\ i = 1, 2, \ldots, 6$.

A non degenerate basic feasible solution $(x_1, x_2, \ldots, x_6)$ is

1. (1, 0, 1, 0, 0, 0)
2. (0, 0, 0, 0, 3, 6)
3. (1, 0, 0, 0, 0, 7)
4. (3, 0, 0, 0, 0, 0)

110. The singular points of the resolvent kernel H corresponding to a symmetric L_2 kernel $k(x,y)$ are

1. simple poles
2. not a simple pole
3. double poles
4. pole of order greater than 2

111. In a hypothesis testing problem, which of the following is NOT required in order to compute the p-value?

1. Value of the test statistic
2. Distribution of the test statistic under the null hypothesis
3. The level of significance
4. Weather the test is one tailed (or) two tailed

112. Suppose person A and person B draw random samples of sizes 15 and 20 respectively from $N(\mu, \sigma^2), \sigma^2 > 0$ for testing $H_0 : \mu = 2$ against $H_1 : \mu > 2$. In both cases, the observed sample means and sample S.Ds are same with $\bar{x}_1 = \bar{x}_2 = 1.8,\ s_1 = s_2 = s$. Both of them use the usual t-test and state the $p-$values p_A and p_B respectively. Then which of the following is correct?

1. $p_A > p_B$
2. $p_A < p_B$
3. $p_A = p_B$
4. relation between p_A and p_B depends on the value of s.

113. In a clinical trail 'n' randomly chosen persons were enrolled to examine whether two different skin creams A and B have different effects on the human body. Cream A was applied to one of the randomly chosen arms of each person, cream B to the other arm. Which statistical test is to be used to examine the difference? Assume that the response measured is a continuous variable.

1. Two sample t-test if normality can be assumed
2. Paired t-test if normality can be assumed
3. Two sample Kolmogorov-Smirnov test
4. Test for randomness

114. Let T be the matrix (occurring in a typical transportation problem) given by $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$. Then

1. Rank $T = 4$ and T is unimodular
2. Rank $T = 3$ and T is unimodular
3. Rank $T = 4$ and T is not unimodular
4. Rank $T = 3$ and T is not unimodular

115. Consider the linear programming formulation ($P2$) of optimally assigning 'n' men to 'n' jobs with respect to some costs $\{c_{ij}\}\, i, j = 1, \ldots .n$. Let A denote the coefficient matrix of the constraint set. Then

1. rank of A is $2n-1$ and every basic feasible solution of $P2$ is integer valued
2. rank of A is $2n-1$ and every basic feasible solution of $P2$ is non integer valued
3. rank of A is $2n$ and every basic feasible solution of $P2$ is integer valued
4. rank of A is $2n$ and every basic feasible solution of $P2$ is non integer valued