#### 65(B)

### QUESTION PAPER CODE 65(B)

# EXPECTED ANSWER/VALUE POINTS

#### **SECTION A**

1. 
$$|A| = |3B| = 3^3 \times |B| = 27 \times 2 = 54$$
  $\frac{1}{2} + \frac{1}{2}$ 

2. Put 
$$\sin^{-1} x = t \Rightarrow \frac{1}{\sqrt{1 - x^2}} dx = dt$$

$$I = \int t \, dt = \frac{t^2}{2} + c = \frac{1}{2} (\sin^{-1} x)^2 + c$$

3. 
$$u = \tan x \Rightarrow \frac{du}{dx} = \sec^2 x$$
  
 $v = \sec x \Rightarrow \frac{dv}{dx} = \sec x \tan x$  (any one correct)

$$\therefore \frac{du}{dv} = \frac{\sec^2 x}{\sec x \cdot \tan x} \text{ or cosec } x$$

4. 
$$|\hat{a} - \hat{b}| = 1 \Rightarrow |\hat{a} - \hat{b}|^2 = 1 \Rightarrow |\hat{a}|^2 + |\hat{b}|^2 - 2|\hat{a}||\hat{b}|\cos\theta = 1$$

$$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

#### **SECTION B**

5. Put 
$$\cot^{-1}(-x) = \theta \Rightarrow x = -\cot \theta = \cot (\pi - \theta)$$

$$\Rightarrow \pi - \theta = \cot^{-1} x$$

$$\Rightarrow \theta = \pi - \cot^{-1} x$$

$$\Rightarrow \cot^{-1}(-x) = \pi - \cot^{-1}x$$

 $65(B) \tag{1}$ 



6. 
$$\sin^{-1}\frac{2}{7} + \cos^{-1}2x = \sin^{-1}1 = \frac{\pi}{2}$$

$$\Rightarrow \frac{2}{7} = 2x \Rightarrow x = \frac{1}{7}$$

7. 
$$2X = \begin{pmatrix} 7 & 4 \\ -1 & 10 \end{pmatrix} - 3 \begin{pmatrix} 3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} -2 & -2 \\ -4 & -2 \end{pmatrix}$$

$$X = \begin{pmatrix} -1 & -1 \\ -2 & -1 \end{pmatrix}$$

f being polynomial is continuous in [1, 3] and differentiable in (1, 3) with  $f'(x) = 3x^2 - 10x - 3$ 8.

$$f'(c) = \frac{f(3) - f(1)}{3 - 1} \Rightarrow 3c^2 - 10c - 3 = -10$$

$$\Rightarrow 3c^2 - 10c + 7 = 0$$

$$\therefore f'(c) = \frac{f(3) - f(1)}{3 - 1} \Rightarrow 3c^2 - 10c - 3 = -10$$

$$\Rightarrow 3c^2 - 10c + 7 = 0$$

$$\Rightarrow c = 1, \frac{7}{3}$$
Since  $1 \notin (1, 3), \frac{7}{3} \in (1, 3)$  and  $1 = 1$ 

$$\therefore \text{ Theorem verified.}$$

Theorem verified.

9. 
$$6y = x^3 + 2 \Rightarrow 6\frac{dy}{dt} = 3x^2 \cdot \frac{dx}{dt}$$
 ...(1)

Given: 
$$\frac{dy}{dt} = 2 \cdot \frac{dx}{dt}$$
 ...(2)

from (1) and (2), 
$$2\left(2\frac{dx}{dt}\right) = x^2 \cdot \frac{dx}{dt} \implies x = \pm 2$$

when 
$$x = 2$$
,  $y = \frac{5}{3}$ ; when  $x = -2$ ,  $y = -1$ 

$$\therefore \quad \text{Points are } \left(2, \frac{5}{3}\right) \text{ and } (-2, -1)$$

65(B) **(2)** 



10. 
$$I = \int \frac{\sin(x+a-2a)}{\sin(x+a)} dx$$

$$= \int \frac{\sin(x+a)\cos 2a - \cos(x+a) \cdot \sin 2a}{\sin(x+a)} dx$$

$$= x \cos 2a - \sin 2a \cdot \log |\sin (x + a)| + c$$

11. Diagonals of parallelogram are  $\vec{a} + \vec{b}$  and  $\vec{b} - \vec{a}$  [or  $\vec{a} - \vec{b}$ ]

$$\vec{a} + \vec{b} = 3\hat{i} - 8\hat{j} + 4\hat{k}$$
 and  $\vec{b} - \vec{a} = \hat{i} - 6\hat{j} - 2\hat{k}$ 

Req. area = 
$$\frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -8 & 4 \\ 1 & -6 & -2 \end{vmatrix} = \frac{1}{2} |40\hat{i} + 10\hat{j} - 10\hat{k}|$$

$$= \frac{1}{2}\sqrt{1800} = 15\sqrt{2} \text{ sq. units}$$

12. P (getting an odd no. atleast once)

$$= 1 - P(\text{even no. } \& \text{ even no.})$$

$$=1-\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{7}{8}$$

#### **SECTION C**

13.  $R_1 \rightarrow x \cdot R_1, R_2 \rightarrow y \cdot R_2, R_3 \rightarrow z \cdot R_3$ 

LHS = 
$$\frac{1}{\text{xyz}}\begin{vmatrix} x^2y & x^2z & x(x^2+1) \\ y(y^2+1) & y^2z & xy^2 \\ yz^2 & z(z^2+1) & xz^2 \end{vmatrix}$$

taking y, z and x respectively common from  $C_1$ ,  $C_2$ ,  $C_3$ 

$$= \frac{xyz}{xyz} \begin{vmatrix} x^2 & x^2 & x^2 + 1 \\ y^2 + 1 & y^2 & y^2 \\ z^2 & z^2 + 1 & z^2 \end{vmatrix}$$



$$R_1 \rightarrow R_1 + R_2 + R_3$$

$$= \begin{vmatrix} 1+x^2+y^2+z^2 & 1+x^2+y^2+z^2 & 1+x^2+y^2+z^2 \\ y^2+1 & y^2 & y^2 \\ z^2 & z^2+1 & z^2 \end{vmatrix}$$

$$= (1+x^{2}+y^{2}+z^{2})\begin{vmatrix} 1 & 1 & 1 \\ y^{2}+1 & y^{2} & y^{2} \\ z^{2} & z^{2}+1 & z^{2} \end{vmatrix}$$

$$C_1 \to C_1 - C_3, C_2 \to C_2 - C_3$$

$$= (1+x^2+y^2+z^2) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 0 & y^2 \\ 0 & 1 & z^2 \end{vmatrix} = 1+x^2+y^2+z^2$$

=RHS

$$LHS = A^2 - 7A + 10I_3$$

$$= \begin{pmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{pmatrix} - \begin{pmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{pmatrix} + \begin{pmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{pmatrix}$$

$$1\frac{1}{2} + \frac{1}{2} +$$

Now,  $A^2 - 7A + 10I = O$ 

$$\Rightarrow A^{-1}(A^2 - 7A + 10I) = A^{-1}. O \Rightarrow A^{-1} = \frac{1}{10}(7I - A)$$

$$= \frac{1}{10} \begin{pmatrix} 4 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

(4)



14. 
$$k = \lim_{x \to 0} \frac{(3^{\sin x} - 1)^2}{x \log (1 + 5x)}$$

$$= \lim_{x \to 0} \frac{(3^{\sin x} - 1)^2}{\sin^2 x} \times \lim_{x \to 0} \frac{\sin^2 x}{x^2} \times \frac{1}{5} \lim_{x \to 0} \frac{1}{\frac{\log (1 + 5x)}{5x}}$$
1+1+1

$$=\frac{1}{5}(\log 3)^2$$

OR

Getting

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x} (1+x)^{3/2}}$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x}(1+x)^{3/2}}$$

$$\therefore LHS = (1-x^2) \cdot \left(\frac{-1}{\sqrt{1-x}(1+x)^{3/2}}\right) + \sqrt{\frac{1-x}{1+x}}$$

$$= -\sqrt{\frac{1-x}{1+x}} + \sqrt{\frac{1-x}{1+x}} = 0 = R.H.S.$$
1

15.  $y \cdot \log(\sin x) = \log(x+y)$ 

$$\Rightarrow y \cdot \frac{\cos x}{\sin x} + \log(\sin x) \cdot \frac{dy}{dx} = \frac{1}{x+y} \left(1 + \frac{dy}{dx}\right)$$
2

$$= -\sqrt{\frac{1-x}{1+x}} + \sqrt{\frac{1-x}{1+x}} = 0 = \text{R.H.S.}$$

15. 
$$y \cdot \log(\sin x) = \log(x + y)$$

$$\Rightarrow y \cdot \frac{\cos x}{\sin x} + \log(\sin x) \cdot \frac{dy}{dx} = \frac{1}{x + y} \left( 1 + \frac{dy}{dx} \right)$$

$$\Rightarrow \left(y \cot x - \frac{1}{x+y}\right) = \left(\frac{1}{x+y} - \log(\sin x)\right) \cdot \frac{dy}{dx}$$

$$\Rightarrow [(x+y) \cdot y \cot x - 1] = [1 - (x+y) \log (\sin x)] \cdot \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{1 - (x + y) \cdot y \cot x}{(x + y) \log(\sin x) - 1}$$

16. Let 
$$\frac{x^2 + x}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx + C}{x^2 + 1}$$

Getting 
$$A = 1$$
,  $B = 0$ ,  $C = 1$ 



$$I = \int \frac{1}{x - 1} dx + \int \frac{1}{x^2 + 1} dx$$

$$= \log |x - 1| + \tan^{-1} x + C$$
 1+1

OR

$$I = \int \cos(3x + 1) \cdot e^{2x} dx$$

$$= \cos (3x+1) \cdot \frac{e^{2x}}{2} - \int -3\sin (3x+1) \cdot \frac{e^{2x}}{2} dx$$

$$= \frac{1}{2} \cdot e^{2x} \cos (3x+1) + \frac{3}{2} \int \sin (3x+1) \cdot e^{2x} dx$$

$$= \frac{1}{2}e^{2x}\cos(3x+1) + \frac{3}{2}\left[\sin(3x+1)\cdot\frac{e^{2x}}{2} - \int 3\cos(3x+1)\cdot\frac{e^{2x}}{2}dx\right]$$

$$= \frac{1}{2}e^{2x}\cos(3x+1) + \frac{3}{4}\sin(3x+1)\cdot e^{2x} - \frac{9}{4}I$$

$$\Rightarrow I = \frac{e^{2x}}{13} [2\cos(3x+1) + 3\sin(3x+1)] + C$$

$$= \frac{1}{2}e^{2x}\cos(3x+1) + \frac{3}{4}\sin(3x+1) \cdot e^{2x} = \frac{9}{4}I$$

$$\Rightarrow I = \frac{e^{2x}}{13}[2\cos(3x+1) + 3\sin(3x+1)] + C$$

$$1$$

$$1 = \int_{0}^{\pi/2} \frac{\sin^{2}x}{\sin x + \cos x} dx = \int_{0}^{\pi/2} \frac{\sin^{2}(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)} dx$$

$$1$$

$$I = \int_{0}^{\pi/2} \frac{\cos^{2}x}{\sin x + \cos^{2}x} dx$$

$$I = \int_{0}^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x} dx$$

$$\therefore 2I = \int_{0}^{\pi/2} \frac{1}{\sin x + \cos x} dx$$

$$2I = \frac{1}{\sqrt{2}} \int_{0}^{\pi/2} \frac{1}{\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x} dx = \frac{1}{\sqrt{2}} \int_{0}^{\pi/2} \csc (\pi/4 + x) dx$$

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$$I = \frac{1}{2\sqrt{2}} \log|\csc(\pi/4 + x) - \cot(\pi/4 + x)|_0^{\pi/2}$$

$$= \frac{1}{2\sqrt{2}} \left\{ \log \left( \sqrt{2} + 1 \right) - \log \left( \sqrt{2} - 1 \right) \right\}$$

18. I.F. = 
$$e^{\int -3\cot x \, dx = e^{-3\log(\sin x)}} = \csc^3 x$$

solution is given by:

$$y \cdot \csc^3 x = \int \sin 2x \cdot \csc^3 x \, dx + c$$

$$= 2 \int \cot x \cdot \csc x \, dx + c$$

$$=$$
  $-2$  cosec x + c

$$y = -2\sin^2 x + c\sin^3 x$$

when 
$$y = 2$$
,  $x = \frac{\pi}{2} \Rightarrow c = 4$ 

$$y = -2 \sin^2 x + c \sin^3 x$$
when  $y = 2$ ,  $x = \frac{\pi}{2} \implies c = 4$ 

$$y = -2 \sin^2 x + 4 \sin^3 x$$

$$\frac{1}{2}$$
19. Equation of ellipse is  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ;  $a > b$ 

$$\Rightarrow \frac{2x}{2} + \frac{2y \cdot y'}{2} = 0$$

19. Equation of ellipse is 
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
;  $a > b$ 

$$\Rightarrow \frac{2x}{a^2} + \frac{2y \cdot y'}{b^2} = 0$$

$$\Rightarrow \frac{yy'}{x} = -\frac{b^2}{a^2}$$

differentiating again,

$$\Rightarrow \frac{x[y \cdot y'' + y' \cdot y'] - yy' \cdot 1}{x^2} = 0$$

$$\Rightarrow xy\cdot y'' + x(y')^2 - yy' = 0$$

or 
$$xy \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^2 - y \left(\frac{dy}{dx}\right) = 0$$

**65(B) (7)** 



20. For coplanarity 
$$[\overrightarrow{AB} \ \overrightarrow{AC} \ \overrightarrow{AD}] = 0$$

 $\overrightarrow{AB} = [\hat{i} + (x - 2)\hat{j} + 4\hat{k}]$ 

$$\overrightarrow{AC} = \hat{i} - 3\hat{k}$$

$$\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\Rightarrow \begin{vmatrix} 1 & x - 2 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$

$$\Rightarrow x = 5$$

**21.** Here, 
$$\vec{a}_2 - \vec{a}_1 = 3\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 2 \\ 2 & 3 & 1 \end{vmatrix} = -9\hat{i} + 3\hat{j} + 9\hat{k}$$

$$|\vec{\mathbf{b}}_1 \times \vec{\mathbf{b}}_2| = \sqrt{81 + 9 + 81} = \sqrt{171}$$

S.D. = 
$$\frac{\left| (\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) \right|}{\left| \vec{b}_1 \times \vec{b}_2 \right|}$$

$$= \frac{3(-9) + 3(3) + 3(9)}{\sqrt{171}} = \frac{3}{\sqrt{19}}$$
 units

# 22. E<sub>1</sub>: Bag A is selected; E<sub>2</sub>: Bag B is selected

$$P(E_1) = P(E_2) = P(E_3) = 1/3$$

$$P(A/E_1) = 1/2, P(A/E_2) = 3/8, P(A/E_3) = 5/8$$

(8)



Using Bayes' theorem, 
$$P(E_2/A) = \frac{P(E_2) \cdot P(A/E_2)}{P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3) \cdot P(A/E_3)}$$

$$= \frac{\frac{1}{3} \times \frac{3}{8}}{\frac{1}{3} \times \frac{4}{9} + \frac{1}{3} \times \frac{3}{9} + \frac{1}{3} \times \frac{5}{9}}$$

$$1\frac{1}{2}$$

$$=\frac{1}{4}$$

Let X represent the no. of kings

$$\therefore X = 0, 1, 2$$

$$P(X=0) = \frac{48}{52} \times \frac{47}{51} = \frac{564}{663}$$

$$P(X = 0) = \frac{1}{52} \times \frac{1}{51} = \frac{1}{663}$$

$$P(X = 1) = 2 \times \frac{4}{52} \times \frac{48}{51} = \frac{96}{663}$$

$$P(X = 2) = \frac{4}{52} \times \frac{3}{51} = \frac{3}{663}$$
Probability distribution table is:

$$P(X = 2) = \frac{4}{52} \times \frac{3}{51} = \frac{3}{663}.$$

Probability distribution table is:

X	0	1	2
	564	96	3
P(X)	663	663	$\frac{3}{663}$

Mean = 
$$\sum X \cdot P(X) = 1 \times \frac{96}{663} + 2 \times \frac{3}{663} = \frac{2}{13}$$

#### **SECTION D**

**24.** Let 
$$x_1, x_2 \in [-1, 1]$$

Let 
$$f(x_1) = f(x_2)$$

$$\Rightarrow \frac{x_1}{x_1 + 2} = \frac{x_2}{x_2 + 2} \Rightarrow x_1 x_2 + 2x_1 = x_1 x_2 + 2x_2$$



$$\Rightarrow$$
 x<sub>1</sub> = x<sub>2</sub>  $\Rightarrow$  f is 1 – 1 function

For, f:  $[-1, 1] \rightarrow R_f$ 

Given, co-domain = Range  $\Rightarrow$  f is onto

 $\Rightarrow$  f is invertible

To find  $f^{-1}$ : Let  $y = f(x) \Rightarrow x = f^{-1}(y)$ 

Now, 
$$y = \frac{x}{x+2} \implies x = \frac{2y}{1-y}$$
;  $y \ne 1$ 

$$f^{-1}(x) = \frac{2x}{1-x}; x \neq 1$$

getting 
$$f^{-1}\left(\frac{-1}{3}\right) = \frac{-1}{2}$$

getting 
$$f^{-1}\left(\frac{-1}{3}\right) = \frac{-1}{2}$$

$$f^{-1}\left(\frac{1}{5}\right) = \frac{1}{2}$$

$$b * a = \frac{b+a}{2} = \frac{a+b}{2} = a * b \ \forall \ a, \ b \in \mathbb{R}$$

$$\therefore * \text{ is commutative.}$$

$$0 * a = 2$$

Let a, b,  $c \in R$ 

Consider (a \* b) \* c = 
$$\left(\frac{a+b}{2}\right)$$
 \* c =  $\frac{a+b+2c}{2}$ 

and, 
$$a * (b * c) = a * \left(\frac{b+c}{2}\right) = \frac{2a+b+c}{2}$$

clearly,  $(a * b) * c \neq a * (b * c)$ 

$$\Rightarrow$$
 \* is not associative. [Can be shown by example]

Let  $e \in R$  be identity (if exists)

then, 
$$a * e = a = e * a$$

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$$\Rightarrow \frac{a+e}{2} = a = \frac{e+a}{2} \Rightarrow a+e=2a$$

 $\Rightarrow$  e = a, which is not unique

∴ e does not exist.

### Given system can be written as

$$\begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

i.e, AX = B

 $|A| = 4 \neq 0 \Rightarrow A^{-1}$  exists.

Now, 
$$A_{11} = 7$$
,  $A_{12} = -19$ ,  $A_{13} = -11$   
 $A_{21} = 1$ ,  $A_{22} = -1$ ,  $A_{23} = -1$ 

$$A_{21} = 1$$
,  $A_{22} = -1$ ,  $A_{23} = -1$   
 $A_{31} = -3$ ,  $A_{32} = 11$ ,  $A_{33} = 7$   
 $A_{31} = -3$   $A_{32} = 11$   $A_{33} = 7$   
 $A_{31} = -3$   $A_{32} = 11$   $A_{33} = 7$   
 $A_{31} = -3$   $A_{32} = 11$   $A_{33} = 7$   
 $A_{33} = 7$   $A_{33} = 7$   $A_{34} = 7$   $A_{35} = 7$ 

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 8\\4\\12 \end{bmatrix} = \begin{bmatrix} 2\\1\\3 \end{bmatrix} \implies x = 2, y = 1, z = 3$$

$$1\frac{1}{2}$$

26. 
$$y^2 = 4x \Rightarrow 2y \frac{dy}{dx} = 4 \Rightarrow \frac{dy}{dx} = \frac{2}{y}$$

$$\therefore \frac{dy}{dx} \Big|_{(x_1, y_1)} = \frac{2}{y_1}$$

$$\Rightarrow$$
 slope of normal =  $-\frac{y_1}{2}$ 

Equation of normal: 
$$y - y_1 = -\frac{y_1}{2}(x - x_1)$$

Normal passes through (0, 3)

$$\therefore 3 - y_1 = -\frac{y_1}{2}(0 - x_1) \implies 6 - 2y_1 = x_1 y_1 \dots (1)$$

also,  $(x_1, y_1)$  lies on  $y^2 = 4x \Rightarrow y_1^2 = 4x_1$  ...(2)

Solving (1) and (2), 
$$x_1 = 1$$
,  $y_1 = 2$  :  $(x_1, y_1) = (1, 2)$ 

Slope of normal =  $\frac{-y_1}{2} = \frac{-2}{2} = -1$ 

Equation of normal is 
$$y-2=-(x-1) \Rightarrow x+y=3$$

OR

Let side of square is a and radius of circle is r, then, 
$$a = \frac{x}{4}$$
,  $r = \frac{28 - x}{2\pi}$ 

Let perimeter of square be x cm, then circumference of circle is 
$$(28 - x)$$
 cm.

Let side of square is a and radius of circle is r, then,  $a = \frac{x}{4}$ ,  $r = \frac{28 - x}{2\pi}$ 

1

Now,  $A = \left(\frac{x}{4}\right)^2 + \pi \left(\frac{28 - x}{2\pi}\right)^2$ 

$$\therefore A = \frac{x^2}{16} + \frac{1}{4\pi} (28 - x)^2$$

$$\Rightarrow \frac{dA}{dx} = \frac{x}{8} - \frac{1}{2\pi} (28 - x)$$

$$\frac{dA}{dx} = 0 \Rightarrow x = \frac{112}{\pi + 4} \text{ cm}$$

$$\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{2\pi} > 0 \implies \text{Area is minimum}$$

other length = 
$$28 - x = 28 - \frac{112}{\pi + 4} \text{cm} = \frac{28\pi}{\pi + 4} \text{cm}$$

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27. 
$$A = \int_{2}^{4} 3\sqrt{x} dx = 3 \times \frac{2}{3} [x^{3/2}]_{2}^{4}$$

$$= 2(8-2^{3/2})$$
 sq. units

OR

$$a = 2, b = 5, nh = 3$$

Let  $f(x) = 2x^2 + 3x + 1$ 

$$\int_{2}^{5} (2x^{2} + 3x + 1)dx = \lim_{h \to 0} h \cdot [f(2) + f(2+h) + f(2+2h) + ... + f(2+(n-1)h)]$$

here,  $f(2) = 2(2)^2 + 3(2) + 1 = 15$ 

$$f(2 + h) = 2(2 + h)^2 + 3(2 + h) + 1 = 2h^2 + 11h + 15$$

$$f(2 + 2h) = 2[2 + 2h]^2 + 3[2 + 2h] + 1 = 2.2^2h^2 + 22h + 15$$

$$f(2 + 2h) = 2[2 + 2h]^{2} + 3[2 + 2h] + 1 = 2 \cdot 2^{2}h^{2} + 22h + 15$$

$$f(2 + (n - 1)h) = 2[2 + (n - 1)h]^{2} + 3[2 + (n - 1)h] + 1$$

$$= 2(n - 1)^{2}h^{2} + 11(n - 1)h + 15$$

$$\therefore \int_{2}^{5} (2x^{2} + 3x + 1)dx$$

$$- \lim_{} h \cdot [15 + (2h^{2} + 11h + 15) + ... + (2(n - 1)^{2}h^{2} + 11(n - 1)h + 15)]$$

$$= 2(n-1)^2h^2 + 11(n-1)h + 15$$

$$\therefore \int_{2}^{5} (2x^2 + 3x + 1) dx$$

$$= \lim_{h \to 0} h \cdot [15 + (2h^2 + 11h + 15) + ... + (2(n-1)^2 h^2 + 11(n-1)h + 15)]$$

$$= \lim_{h\to 0} h[15n + 2h^2 \cdot (1^2 + 2^2 + ... + (n-1)^2) + 11h(1 + 2 + ... + (n-1))]$$

$$= \lim_{h \to 0} \left( 15h + 2 \cdot \frac{(h)(h - h)(2h - h)}{6} + \frac{11 \cdot (h)(h - h)}{2} \right)$$

$$= \lim_{h \to 0} \left( 45 + \frac{1}{3} \times 3(3 - h) (6 - h) + \frac{11}{2} \times 3(3 - h) \right)$$

$$=45+18+\frac{99}{2}=\frac{225}{2}$$

**65(B)** (13)



28. Any point on given line is  $(3\lambda + 2)\hat{i} + (4\lambda - 1)\hat{j} + (2\lambda + 2)\hat{k}$ 

1

If this line and given plane intersect, then

$$1(3\lambda + 2) - 1(4\lambda - 1) + 1(2\lambda + 2) = 5 \Rightarrow \lambda = 0$$

2

 $\therefore$  Point of intersection is (2, -1, 2)

1

:. the distance = 
$$\sqrt{(2+1)^2 + (-1+5)^2 + (2+10)^2} = 13$$
 units

2

29. Let x kg of food X and y kg of food Y are mixed then,

minimum cost, 
$$Z = 16x + 20y$$

1

subject to following constraints:

$$x + 2y \ge 10$$

$$2x + 2y \ge 12 \text{ or } x + y \ge 6$$

4

 $3x + y \ge 8$ 

$$x \ge 0, y \ge 0$$

X = 0, y = 0

Value: Any relevant value.

Largest Student Re.

(14)

