DU MA MSc Mathematics

Topic:- MATHS MA S2

1) Let $\{x_n\}$ and $\{y_n\}$ be sequences of real numbers such that $x_n \le y_n$ for all $n \ge N$, where N is some positive integer. Consider the following statements:

(a) $\lim_{n\to\infty} \inf x_n \le \lim_{n\to\infty} \inf y_n$

(b) $\lim_{n\to\infty} \sup x_n \le \lim_{n\to\infty} \sup y_n$

Which of the above statements is(are) correct?

[Question ID = 5742]

1. Neither (a) nor (b)

   [Option ID = 22962]
2. Only (a)

   [Option ID = 22963]
3. Only (b)

   [Option ID = 22964]
4. Both (a) and (b)

   [Option ID = 22965]

Correct Answer :-

- Both (a) and (b)

  [Option ID = 22965]

2) Which of the sequences $\{a_n\}$ and $\{b_n\}$ of real numbers with $n-th$ terms

$$a_n = \frac{(n^2+20n+35)\sin n^3}{n^2+n+1},$$

$$b_n = 2\cos n - \sin n$$

has(have) convergent subsequences?

[Question ID = 5743]

1. Neither $\{a_n\}$ nor $\{b_n\}$

   [Option ID = 22966]
2. Only $\{a_n\}$

   [Option ID = 22967]
3. Only $\{b_n\}$

   [Option ID = 22968]
4. Both $\{a_n\}$ and $\{b_n\}$

   [Option ID = 22969]

Correct Answer :-

- Both $\{a_n\}$ and $\{b_n\}$

  [Option ID = 22969]

3) Consider the following series:

(a) $\sum_{n=1}^{\infty} \frac{x^n}{n!}, x \in \mathbb{R}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n+\sin n}$

(c) $\sum_{n=1}^{\infty} \frac{1}{2^n\sqrt{n}}$

(d) $\sum_{n=1}^{\infty} \sin n$

Which of the above series is (are) convergent?

[Question ID = 5744]
1. All of (a), (b), (c) and (d)

   [Option ID = 22970]
2. Only (a), (c) and (d)

   [Option ID = 22971]
3. Only (a) and (c)

   [Option ID = 22972]
4. Only (c)

   [Option ID = 22973]

Correct Answer :-
- Only (a) and (c)

  [Option ID = 22972]

4) The union of infinitely many closed subsets of the real line is
[Question ID = 5745]
1. uncountable [Option ID = 22974]
2. finite [Option ID = 22975]
3. always closed [Option ID = 22976]
4. need not be closed [Option ID = 22977]

Correct Answer :-
- need not be closed [Option ID = 22977]

5) Consider the series $\sum_{n=1}^{\infty} a_n$ where $a_n = \left(2 + sin\frac{n\pi}{2}\right)r^n, r > 0$. What are the values of $\liminf_{n\to\infty}\frac{a_{n+1}}{a_n}$ and $\limsup_{n\to\infty}\frac{a_{n+1}}{a_n}$?

[Question ID = 5746]
1. r/2 and 2r [Option ID = 22978]
2. r/3 and r [Option ID = 22979]
3. 2r/3 and 3r/2 [Option ID = 22980]
4. 0 and 1 [Option ID = 22981]

Correct Answer :-
- r/2 and 2r [Option ID = 22978]

6) Consider the following series:

(a) $\sum_{n=1}^{\infty} 3^{-n} \sin 3^n x$ on $\mathbb{R}$

(b) $\sum_{n=1}^{\infty} 2^{-n} x^n$ on $(-2,2)$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$ on $\mathbb{R}$

Which of the above series converge uniformly on the indicated domain?

[Question ID = 5747]
1. Only (a) and (b)

   [Option ID = 22982]
2. Only (b) and (c)

   [Option ID = 22983]
3. Only (a) and (c)

   [Option ID = 22984]
4. All of (a), (b) and (c)

   [Option ID = 22985]

Correct Answer :-
- Only (a) and (c)

  [Option ID = 22984]

7) Let $\{f_n\}$ be a sequence of continuous functions on $[a,b]$ converging uniformly to the function f. Consider the following statements:

(a) f is bounded on $[a,b]$

(b) $\lim_{n\to\infty} \int_a^{-} f_n(t)\,dt = \int_a^{-} f(t)\,dt$

(c) If each f_n is differentiable, then the sequence $\{f'_n\}$ converges uniformly to f' on $[a,b]$, f' is the derivative of f

Which of the following statements is(are) correct?

[Question ID = 5748]
1. Only (a) and (b)

   [Option ID = 22986]
2. Only (a) and (c)

   [Option ID = 22987]
3. Only (c)

   [Option ID = 22988]
4. Only (b)

   [Option ID = 22989]

Correct Answer :-
- Only (a) and (b)

  [Option ID = 22986]

8) Let $G(x)$ be a real-valued function defined by $G(x) = \int_{x^2}^{4x^2} \cos\sqrt{t}\,dt$. If G′ is the derivative of G, then

[Question ID = 5749]
1. $G'(\pi/2) = -4\pi$

   [Option ID = 22990]
2. $G'(\pi/2) = -4\pi - 1$

   [Option ID = 22991]
3. $G'(\pi/2) = -\pi$

   [Option ID = 22992]
4. $G'(\pi/2) = 0$

   [Option ID = 22993]

Correct Answer :-
- $G'(\pi/2) = -4\pi$

  [Option ID = 22990]

9) Let $f(x) = \begin{cases} (4 - x^2)^{5/2}, & |x| < 2 \\ 0, & |x| \geq 2 \end{cases}$

Consider the following statements:

a. f is not continuous on $\mathbb{R}$
b. f is continuous on $\mathbb{R}$ but not differentiable at $x = 2, -2$
c. f is differentiable on $\mathbb{R}$ but f' is not continuous on $\mathbb{R}$
d. f is differentiable on $\mathbb{R}$ and f' is continuous on $\mathbb{R}$

Which of the above statements is(are) correct?

[Question ID = 5750]
1. Only (a) and (d)

   [Option ID = 22994]
2. Only (b) and (c)

   [Option ID = 22995]
3. Only (c)

   [Option ID = 22996]
4. Only (d)

   [Option ID = 22997]

Correct Answer :-
- Only (d)

  [Option ID = 22997]

10) **The zero of the function $f(x) = -2x^3 + 5x - 5$ defined on $\mathbb{R}$ lie on the interval**

[Question ID = 5751]

1. (-1, 1)

   [Option ID = 22998]
2. [3, 4]

   [Option ID = 22999]
3. [-2, -1]

   [Option ID = 23000]
4. [-5, -3]

   [Option ID = 23001]

Correct Answer :-

- [-2, -1]

  [Option ID = 23000]

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11) The Wronskian of $\cos x, \sin x$ and e^{-x} at $x = 0$ is

[Question ID = 5752]

1. 1

   [Option ID = 23002]
2. 2

   [Option ID = 23003]
3. -1

   [Option ID = 23004]
4. -2

   [Option ID = 23005]

Correct Answer :-

- 2

  [Option ID = 23003]

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12) The solution of the initial value problem $y' = 1 + y^2, y(0) = 1,$ is:-

[Question ID = 5753]

1. $y = cosec(x + \pi/4)$

   [Option ID = 23006]
2. $y = tan(x + \pi/4)$

   [Option ID = 23007]
3. $y = sec(x + \pi/4)$

   [Option ID = 23008]
4. $y = cot(x + \pi/4)$

   [Option ID = 23009]

Correct Answer :-

- $y = tan(x + \pi/4)$

  [Option ID = 23007]

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13) How many solution(s) does the initial value problem $y' - \frac{2}{x}y = 0, y(0) = 0$ have?

[Question ID = 5754]

1. No solution

   [Option ID = 23010]
2. Unique solution

   [Option ID = 23011]
3. Two solutions

   [Option ID = 23012]
4. Infinitely many solutions

   [Option ID = 23013]

Correct Answer :-

- Infinitely many solutions

[Option ID = 23013]

14) The general solution of the equation $y'' + y = cosec\, x, (0 < x < \pi)$ is (c_1, c_2 are arbitrary constants)

[Question ID = 5755]

1. $c_1 \cos x + c_2 \sin x - x \cos x + \sin x \ln(\sin x)$

   [Option ID = 23014]
2. $c_1 \cos x + c_2 \sin x + x \cos x + \sin x \ln(\sin x)$

   [Option ID = 23015]
3. $c_1 \cos x + c_2 \sin x - x \sin x + \cos x \ln(\sin x)$

   [Option ID = 23016]
4. $c_1 \cos x + c_2 \sin x + x \sin x + \cos x \ln(\sin x)$

   [Option ID = 23017]

Correct Answer :-

- $c_1 \cos x + c_2 \sin x - x \cos x + \sin x \ln(\sin x)$

  [Option ID = 23014]

15) The particular integral of the differential equation is $y'' + y = x^3$ is

[Question ID = 5756]

1. $x^2 + 6x$

   [Option ID = 23018]
2. $x^2 - 6x$

   [Option ID = 23019]
3. $x^3 + 6x$

   [Option ID = 23020]
4. $x^3 - 6x$

   [Option ID = 23021]

Correct Answer :-

- $x^3 - 6x$

  [Option ID = 23021]

16) The complete integral of the partial differential equation $p^2z^2 + q^2 = 1$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ is

(a, b are arbitrary constants)

[Question ID = 5757]

1. $z + a^2 \ln\left(\frac{z + \sqrt{z^2 + a^2}}{a}\right) = 0$

   [Option ID = 23022]
2. $a^2 z + by + x^2 = 0$

   [Option ID = 23023]
3. $z\sqrt{z^2 + a^2} + a^2 \ln\left(\frac{z+\sqrt{z^2+a^2}}{a}\right) = 2x + 2ay + 2b$

   [Option ID = 23024]
4. $z^2 + y^2 = x^2 + 2x + 2ay + 2b$

   [Option ID = 23025]

Correct Answer :-

- $z\sqrt{z^2 + a^2} + a^2 \ln\left(\frac{z+\sqrt{z^2+a^2}}{a}\right) = 2x + 2ay + 2b$

  [Option ID = 23024]

17) The complete integral of the partial differential equation $z = px + qy - \sin(pq)$ where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ is

[Question ID = 5758]

1. $z = ax + by + \sin(ab)$

[Option ID = 23026]

2. $z = ax + by - \sin(ab)$

[Option ID = 23027]

3. $z = ax + y + \sin b$

[Option ID = 23028]

4. $z = x + by - \sin a$

[Option ID = 23029]

Correct Answer :-

- $z = ax + by - \sin(ab)$

[Option ID = 23027]

18) The partial differential equation $yu_{xx} + 2xyu_{xy} + xu_{yy} = u_x + u_y$ is

[Question ID = 5759]

1. Hyperbolic in $\{(x, y)|0 < xy < 1\}$

[Option ID = 23030]

2. Hyperbolic in $\{(x, y)|xy > 1\}$

[Option ID = 23031]

3. Elliptic in $\{(x, y)|xy > 1\}$

[Option ID = 23032]

4. Elliptic in $\{(x, y)|xy < 0\}$

[Option ID = 23033]

Correct Answer :-

- Hyperbolic in $\{(x, y)|xy > 1\}$

[Option ID = 23031]

19) The general solution of the equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$ is

[Question ID = 5760]

1. $\frac{1}{4}x(x-y)^2 + \emptyset_1(x^2+y) + \emptyset_2(x-y)$

[Option ID = 23034]

2. $\frac{1}{4}x(x-y)^2 + \emptyset_1(x+y) + \emptyset_2(x-y)$

[Option ID = 23035]

3. $\emptyset_1(x+y) + \emptyset_2(x^2-y)$

[Option ID = 23036]

4. $\emptyset_1(x^2+y) + \emptyset_2(x^2-y) - \frac{1}{4}x(x+y)$

[Option ID = 23037]

Correct Answer :-

- $\frac{1}{4}x(x-y)^2 + \emptyset_1(x+y) + \emptyset_2(x-y)$

[Option ID = 23035]

20) The general solution of $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with $u(0,t) = u(2,t) = 0,\ u(x,0) = \sin^3 \frac{\pi x}{2}$ *and* $u_t(x,0) = 0$ is

[Question ID = 5761]

1. $\frac{3}{4} sin\frac{\pi x}{2} sin\frac{\pi ct}{2}$

[Option ID = 23038]

2. $\frac{3}{4} sin\frac{\pi x}{2} cos\frac{\pi ct}{2} - \frac{1}{4}\sin\frac{3\pi x}{2} cos\frac{3\pi ct}{2}$

[Option ID = 23039]

3. $\frac{3}{4}\cos\frac{\pi x}{2} sin\frac{\pi ct}{2} - \frac{1}{4} sin\frac{3\pi x}{2}\sin\frac{3\pi ct}{2}$

[Option ID = 23040]

4. $\frac{3}{4} sin\frac{\pi x}{2} cos\frac{\pi ct}{2} - \frac{1}{4}\cos\frac{3\pi x}{2}$

[Option ID = 23041]

Correct Answer :-

- $\frac{3}{4} sin\frac{\pi x}{2} cos\frac{\pi ct}{2} - \frac{1}{4}\sin\frac{3\pi x}{2} cos\frac{3\pi ct}{2}$

  [Option ID = 23039]

21) Let $f: \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x) = \begin{cases}(x^2+y^2)\ln(x^2+y^2), if\ (x,y) \neq (0,0) \\ 0, \quad if\ (x,y) = (0,0)\end{cases}$$

Then,

[Question ID = 5762]

1. f_{xy} and f_{yx} are continuous at (0, 0), and $f_{xy}(0,0) = f_{yx}(0,0)$

   [Option ID = 23042]
2. f_{xy} and f_{yx} are discontinuous at (0, 0), but $f_{xy}(0,0) = f_{yx}(0,0)$

   [Option ID = 23043]
3. f_{xy} and f_{yx} are continuous at (0, 0), but $f_{xy}(0,0) \neq f_{yx}(0,0)$

   [Option ID = 23044]
4. f_{xy} and f_{yx} are discontinuous at (0, 0) and $f_{xy}(0,0) \neq f_{yx}(0,0)$

   [Option ID = 23045]

Correct Answer :-

- f_{xy} and f_{yx} are discontinuous at (0, 0), but $f_{xy}(0,0) = f_{yx}(0,0)$

  [Option ID = 23043]

22) The directional derivative of $f(x,y,z) = xy^2 + yz^2 + zx^2$ defined on $\mathbb{R}^3$ along the tangent to the curve $x = t, y = t^2, z = t^3$ at the point $(1,1,1)$ is

[Question ID = 5763]

1. $-\frac{18}{\sqrt{14}}$

   [Option ID = 23046]
2. $\frac{13}{\sqrt{14}}$

   [Option ID = 23047]
3. $-\frac{13}{\sqrt{14}}$

   [Option ID = 23048]
4. $\frac{18}{\sqrt{14}}$

   [Option ID = 23049]

Correct Answer :-

- $\frac{18}{\sqrt{14}}$

  [Option ID = 23049]

23) The unique polynomial of degree 2 passing through (1, 1), (3, 27) and (4, 64) obtained by Lagrange interpolation is
[Question ID = 5764]

1. $8x^2 - 17x + 12$

   [Option ID = 23050]
2. $8x^2 - 19x - 12$

   [Option ID = 23051]
3. $8x^2 + 14x - 12$

   [Option ID = 23052]
4. $8x^2 - 19x + 12$

   [Option ID = 23053]

Correct Answer :-

- $8x^2 - 19x + 12$

[Option ID = 23053]

24) **The approximate value of $\int_0^3 \frac{dx}{(1+x)^2}$ by Simpson's 1/3-rd rule, using the least number of equal subintervals, is**

[Question ID = 5765]

1. 0.8512

   [Option ID = 23054]
2. 0.8125

   [Option ID = 23055]
3. 0.7625

   [Option ID = 23056]
4. 0.6702

   [Option ID = 23057]

Correct Answer :-

- 0.8512

  [Option ID = 23054]

25) Consider the differential equation, $\frac{dy}{dx} = y - x, y(0) = 2.$ The absolute value of the difference in the solutions obtained by Euler method and Runge-Kutta second order method at $y(0.1)$ using step size 0.1 is

[Question ID = 5766]

1. 2.205 [Option ID = 23058]
2. 2.252 [Option ID = 23059]
3. 0.005 [Option ID = 23060]
4. 0.055 [Option ID = 23061]

Correct Answer :-

- 0.005 [Option ID = 23060]

26) **The approximate value of $(17)^{1/3}$ obtained after two iterations of Newton-Raphson method starting with initial approximation $x_0 = 2$ is**

[Question ID = 5767]

1. 2.7566

   [Option ID = 23062]
2. 2.5826

   [Option ID = 23063]
3. 2.6713

   [Option ID = 23064]
4. 2.4566

   [Option ID = 23065]

Correct Answer :-

- 2.5826

  [Option ID = 23063]

27) **For an infinite discrete metric space (X, d), which of the following statements is correct?**

[Question ID = 5768]

1. X is compact

   [Option ID = 23066]
2. For every $A \subseteq X$, $A^o \cup \bar{A} = X$, where $\bar{A}$ and A^o denote respectively the closure and interior of A in X

   [Option ID = 23067]
3. X is connected

   [Option ID = 23068]
4. X is not totally bounded

   [Option ID = 23069]

Correct Answer :-

- X is not totally bounded

[Option ID = 23069]

28) **Consider the metric space (l_2,d) of square summable sequences in $\mathbb{R}$ with the Euclidean metric. Let $Y = \{e_1, e_2, ...\} \subseteq l_2$ where e_i is the sequence of 1 at the $i-th$ place and 0 elsewhere. Then,**

[Question ID = 5769]

1. Y is not compact and has no limit point

   [Option ID = 23070]
2. Y is compact and each e_i is a limit point of Y

   [Option ID = 23071]
3. Y is not compact and has a limit point

   [Option ID = 23072]
4. Y is compact and has no limit point

   [Option ID = 23073]

Correct Answer :-

- Y is not compact and has no limit point

  [Option ID = 23070]

29) **Let $C[0,1]$ be the set of real valued continuous functions on [0, 1] with sup-metric. Let $A = \{f \in C[0,1] \mid f(0) = 0\}$ and $B = \{f \in C[0,1] \mid f(0) > 0\}$ be the subspaces of $C[0,1]$. Then,**

[Question ID = 5770]

1. Both A and B are complete

   [Option ID = 23074]
2. A is complete but B is incomplete

   [Option ID = 23075]
3. A is incomplete but B is complete

   [Option ID = 23076]
4. Neither A nor B is complete

   [Option ID = 23077]

Correct Answer :-

- A is complete but B is incomplete

  [Option ID = 23075]

30) **Let $(\mathbb{R}, d)$ and $(\mathbb{R}, u)$ be the metric spaces with the discrete metric space d and usual metric u respectively.**

Let $f: (\mathbb{R}, d) \to (\mathbb{R}, u)$ and $g: (\mathbb{R}, u) \to (\mathbb{R}, d)$ be the functions given by

$$f(x) = g(x) = \begin{cases} 0, & x \le 0 \\ x+1, & x > 0 \end{cases}$$

Then,

[Question ID = 5771]

1. Both f and g are continuous

   [Option ID = 23078]
2. Neither f nor g is continuous

   [Option ID = 23079]
3. f is continuous but g is not

   [Option ID = 23080]
4. g is continuous but f is not

   [Option ID = 23081]

Correct Answer :-

- f is continuous but g is not

  [Option ID = 23080]

31) Let $Y_1 = \{(x,y) \in \mathbb{R}^2 | y = sin\frac{1}{x}, 0 < x \le \pi\}$ and $Y_2 = \{(0,y) \in \mathbb{R}^2 | y \in [-2,2]\}$ be subspaces of the

metric space $(\mathbb{R}^2, d)$ being the Euclidean metric. For any $A \subseteq \mathbb{R}^2, \bar{A}$ denotes the closure of A in $\mathbb{R}^2$. Which of the following statements is correct?

[Question ID = 5772]
1. $\overline{Y_1} \cup Y_2$ is connected

[Option ID = 23082]
2. $Y_1 \cup \overline{Y_2}$ is connected

[Option ID = 23083]
3. $\overline{Y_1} \cap Y_2$ is disconnected

[Option ID = 23084]
4. $\overline{Y_1 \cap Y_2}$ is a non-empty bounded subset of $\mathbb{R}^2$

[Option ID = 23085]

Correct Answer :-
- $\overline{Y_1} \cup Y_2$ is connected

[Option ID = 23082]

32) **Let be the set of all real-valued Riemann integrable functions on and let be the function given by**

$$f(x) = \begin{cases} 0, & if\ x = 0 \\ \frac{1}{n}, & if\ \frac{1}{n+1} < x \leq \frac{1}{n}\ for\ n \in \mathbb{N} \end{cases}$$

Which of the following statements is correct?

[Question ID = 5773]
1. f is monotonically decreasing on $[0, 1]$ but $f \notin R[0, 1]$

[Option ID = 23086]
2. f is monotonically decreasing on $[0, 1]$ and $f \in R[0, 1]$

[Option ID = 23087]
3. f is discontinuous at infinitely many points in $[0, 1]$ but $f \notin R[0, 1]$

[Option ID = 23088]
4. f is discontinuous at infinitely many points in $[0, 1]$ and $f \in R[0, 1]$

[Option ID = 23089]

Correct Answer :-
- f is discontinuous at infinitely many points in $[0, 1]$ and $f \in R[0, 1]$

[Option ID = 23089]

33) **The improper integral**

$$\int_{-\infty}^{\infty} \frac{dx}{x^2 + 1}$$

[Question ID = 5774]
1. Converges to π

[Option ID = 23090]
2. Converges to $\pi/2$

[Option ID = 23091]
3. Converges to 0

[Option ID = 23092]
4. Diverges

[Option ID = 23093]

Correct Answer :-
- Converges to π

[Option ID = 23090]

34) **Consider the functions** $f(x) = \frac{x^2-1}{x-1}$ **and** $g(x) = \frac{|x^2-1|}{x-1}, x \neq 1.$ **Then**

[Question ID = 5775]
1. Both f and g have removable discontinuity at $x = 1$

[Option ID = 23094]

2. Both f and g have jump discontinuity at $x = 1$

[Option ID = 23095]

3. f has a removable discontinuity at $x = 1$, while g has a jump discontinuity at $x = 1$

[Option ID = 23096]

4. f has a jump discontinuity at $x = 1$ while g has a removable discontinuity at $x = 1$

[Option ID = 23097]

Correct Answer :-

- f has a removable discontinuity at $x = 1$, while g has a jump discontinuity at $x = 1$

[Option ID = 23096]

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35) What is the length of the interval on which the function $f(x) = x^3 - 6x^2 - 15x + 8$ is decreasing?

[Question ID = 5776]

1. 8

[Option ID = 23098]

2. 6

[Option ID = 23099]

3. 4

[Option ID = 23100]

4. 2

[Option ID = 23101]

Correct Answer :-

- 6

[Option ID = 23099]

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36) Let $f: [a, b] \to \mathbb{R}$ be a monotonic function. Consider the following statements:

a. The function f obeys the maximum principle

b. The function f is Riemann integrable on [a, b]

Which of the above statement(s) is(are) true?

[Question ID = 5777]

1. Only (a)

[Option ID = 23102]

2. Only (b)

[Option ID = 23103]

3. Both (a) and (b)

[Option ID = 23104]

4. Neither (a) nor (b)

[Option ID = 23105]

Correct Answer :-

- Both (a) and (b)

[Option ID = 23104]

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37) Consider the following:

a. $\langle (a,b),(c,d) \rangle = ac - bd, (a,b),(c,d) \in \mathbb{R}^2$

b. $\langle f(x), g(x) \rangle = \int_0^1 f'(x)g(x)\,dx$, **where $f(x)$, $g(x)$ are polynomials over $\mathbb{R}$**

Which of the above is(are) an inner product?

[Question ID = 5778]

1. Neither (a) nor (b)

[Option ID = 23106]

2. Both (a) and (b)

[Option ID = 23107]

3. Only (a)

[Option ID = 23108]

4. Only (b)

[Option ID = 23109]

Correct Answer :-

- Neither (a) nor (b)

  [Option ID = 23106]

38) **Let $T = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{pmatrix}$. Then $T^3 + 4T^2 + 5T - 2I$ is equal to**

[Question ID = 5779]

1. $10T + 4I$

   [Option ID = 23110]
2. $10T - 4I$

   [Option ID = 23111]
3. $-10T + 4I$

   [Option ID = 23112]
4. $-10T - 4I$

   [Option ID = 23113]

Correct Answer :-

- $10T - 4I$

  [Option ID = 23111]

39) **Let V be an infinite dimensional vector space over a field F.**

Consider the following statements:

a. Any one-one linear transformation from V to itself is onto

b. Any onto linear transformation from V to itself must be one-one

Which of the above statements is (are) correct?

[Question ID = 5780]

1. Both (a) and (b)

   [Option ID = 23114]
2. Only (a)

   [Option ID = 23115]
3. Only (b)

   [Option ID = 23116]
4. Neither (a) nor (b)

   [Option ID = 23117]

Correct Answer :-

- Neither (a) nor (b)

  [Option ID = 23117]

40) **Let $P_n(\mathbb{R})$ be the set of all polynomials over $\mathbb{R}$ of degree at most n. Let $T: P_n(\mathbb{R}) \longrightarrow P_{n+1}(\mathbb{R})$ be given by $T(f(x)) = xf(x)$. Then**

[Question ID = 5781]

1. T is one-one and onto linear transformation

   [Option ID = 23118]
2. T is an onto function but neither a linear transformation nor one-one

   [Option ID = 23119]
3. T is not onto but a one-one linear transformation

   [Option ID = 23120]
4. T is one-one but neither a linear transformation nor onto

   [Option ID = 23121]

Correct Answer :-

- T is not onto but a one-one linear transformation

  [Option ID = 23120]

41) Let $(\mathbb{Z},*)$ be a group, where $a*b=a+b-2$ and $\mathbb{Z}$ is the set of integers. The inverse of a is

[Question ID = 5782]

1. $a-6$

   [Option ID = 23122]
2. $a-4$

   [Option ID = 23123]
3. $4-a$

   [Option ID = 23124]
4. $6-a$

   [Option ID = 23125]

Correct Answer :-

- $4-a$

  [Option ID = 23124]

42) Let G be a group of even order. Suppose that exactly half of G consists of elements of order 2 and the rest forms a subgroup H of G. Which of the following statements is incorrect?

[Question ID = 5783]

1. H is a normal subgroup of G

   [Option ID = 23126]
2. Order of H is even

   [Option ID = 23127]
3. H is abelian

   [Option ID = 23128]
4. $|G:H|=2$

   [Option ID = 23129]

Correct Answer :-

- Order of H is even

  [Option ID = 23127]

43) Let G and K be finite groups such that $|G|=21$ and $|K|=49$. Suppose G does not have a normal subgroup of order 3. Let L be the set of all group homomorphism from G to K. Then the number of elements in L is

[Question ID = 5784]

1. 1

   [Option ID = 23130]
2. 3

   [Option ID = 23131]
3. 5

   [Option ID = 23132]
4. 7

   [Option ID = 23133]

Correct Answer :-

- 1

  [Option ID = 23130]

44) Let G be a finite group of $a\in G$ has exactly two conjugates. Suppose that $C(a)=\{x^{-1}ax|x\in G\}$ and $N(a)=\{x\in G|ax=xa\}$.

Which of the following statements is incorrect?

[Question ID = 5785]

1. The number of elements in $C(a)$ is a prime number

   [Option ID = 23134]
2. G is a simple group

   [Option ID = 23135]

3. $N(a) \neq G$

[Option ID = 23136]

4. $N(a)$ is a normal subgroup of G

[Option ID = 23137]

Correct Answer :-

- G is a simple group

[Option ID = 23135]

45) Let G be a finite group of order 385. Let H, K and L be p-Sylow subgroups of G for $p = 5, 7$ and 11, respectively. Which of the following statements is incorrect?

[Question ID = 5786]

1. K is a normal subgroup of G

[Option ID = 23138]

2. L is normal subgroup of G

[Option ID = 23139]

3. HK is a non-abelian subgroup of G

[Option ID = 23140]

4. $G = HKL$

[Option ID = 23141]

Correct Answer :-

- HK is a non-abelian subgroup of G

[Option ID = 23140]

46) The remainder when 2020^{2020} is divided by 12 is

[Question ID = 5787]

1. 0 [Option ID = 23142]
2. 2 [Option ID = 23143]
3. 4 [Option ID = 23144]
4. 8 [Option ID = 23145]

Correct Answer :-

- 4 [Option ID = 23144]

47) The smallest integer $a > 2$ such that $2|a, 3|(a+1), 4|(a+2), 5|(a+3)$ and $6|(a+4)$ is

[Question ID = 5788]

1. 14

[Option ID = 23146]

2. 56

[Option ID = 23147]

3. 122

[Option ID = 23148]

4. 62

[Option ID = 23149]

Correct Answer :-

- 62

[Option ID = 23149]

48) Let $R = \left\{\begin{pmatrix} a & b \\ b & a \end{pmatrix} \middle| a, b \in \mathbb{Z}\right\}$ be a ring and $f: R \to \mathbb{Z}$ be given by $\emptyset\left(\begin{pmatrix} a & b \\ b & a \end{pmatrix}\right) = a - b$. Which of the following statements is incorrect?

[Question ID = 5789]

1. $\emptyset$ is a ring homomorphism

[Option ID = 23150]

2. $\ker \emptyset$ is a prime ideal but not maximal

[Option ID = 23151]

3. $\ker \emptyset$ is maximal ideal

[Option ID = 23152]

4. $\emptyset$ is surjective

[Option ID = 23153]

Correct Answer :-

- $\ker\emptyset$ is maximal ideal

[Option ID = 23152]

49) **Consider the following statements**

a. A polynomial is irreducible over a field F if it has no zeros in F

b. Let $f(x) \in \mathbb{Z}[x]$. If $f(x)$ is reducible over $\mathbb{Q}$, then it is reducible over $\mathbb{Z}$

c. For any prime p, the polynomial $x^{p-1} + x^{p-2} + \cdots + x + 1$ is irreducible over $\mathbb{Q}$

Which of the above statements is (are) correct?

[Question ID = 5790]

1. Only (a) and (b)

[Option ID = 23154]

2. Only (a) and (c)

[Option ID = 23155]

3. Only (b) and (c)

[Option ID = 23156]

4. All of (a), (b) and (c)

[Option ID = 23157]

Correct Answer :-

- Only (b) and (c)

[Option ID = 23156]

50) **Which of the following is a Euclidean domain?**

[Question ID = 5791]

1. $\mathbb{Q}[x]/\langle x^3 - 2\rangle$

[Option ID = 23158]

2. $\mathbb{Z}[x]$

[Option ID = 23159]

3. $\mathbb{Q}[x, y]$

[Option ID = 23160]

4. None of these

[Option ID = 23161]

Correct Answer :-

- $\mathbb{Q}[x]/\langle x^3 - 2\rangle$

[Option ID = 23158]