JEE-Main-25-06-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: $A=\{x\in R:|x+1|<2\}, B=\{x\in R:|x-1|\geq 2\}$ then:

Options:

(a) $A\cup B=R-[1,3]$

(b) $A\cap B=(-1,1)$

(c) $A\cap B=(-3,-1]$

(d) $B-A=R-(-3,1)$

Answer: (c)

Solution:

$A:|x+1|<2\Rightarrow x\in(-3,1)$

$B:|x-1|\geq 2\Rightarrow x\in(-\infty,-1]\cup[3,\infty)$

$\therefore\ A\cup B=(-\infty,1]\cup[3,\infty)$

$A\cap B=(-3,-1]$

$B-A=(-\infty,-3]\cup[3,\infty)$

Question: $y=y(x)$ is the solution of the differential equation $2x^2\frac{dy}{dx}-2xy+8y^2=0,\ y(e)=\frac{e}{3}$, then $y(1)$ is equal to:

Options:

(a) $\frac{2}{3}$

(b) 3

(c) $\frac{3}{2}$

(d) -1

Answer: (d)

Solution:

$2x^2\cdot\frac{dy}{dx}-2xy=-8y^2$

$\Rightarrow\frac{dy}{dx}-\frac{y}{x}=\frac{-4y^2}{x^2}$

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$$\frac{-1}{y^2}\cdot\frac{dy}{dx}+\frac{1}{xy}=\frac{4}{x^2}$$

Put $\frac{1}{y}=t;\ \frac{-1}{y^2}\frac{dy}{dx}=\frac{dt}{dx}$

$$\Rightarrow \frac{dt}{dx}+\frac{t}{x}=\frac{4}{x^2}$$

$$\Rightarrow I.F.=e^{\int\frac{1}{x}dx}=x$$

$$\therefore t(x)=\int\frac{4}{x}dx=4\ln x+c$$

$$\Rightarrow \frac{x}{y}=4\ln x+c$$

$$\therefore\ y(e)=\frac{e}{3}\Rightarrow c=-1$$

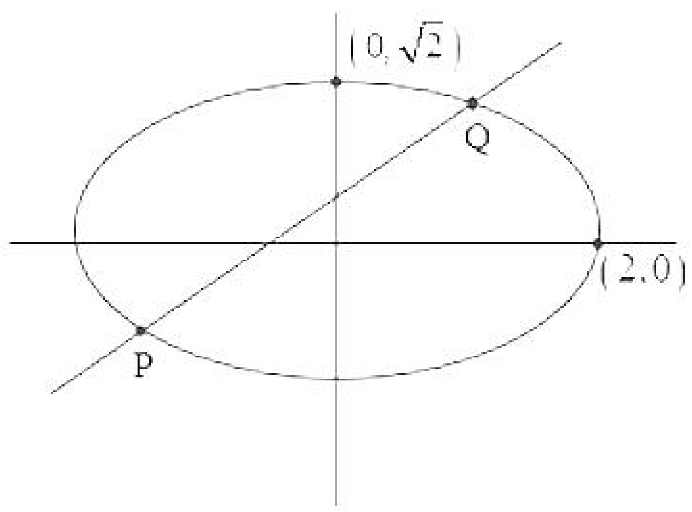
$$\therefore\ \frac{x}{y}=4\ln x-1$$

$$\Rightarrow y(1)=-1$$

Question: $y=x+1$ intersect the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$ at the point P and Q. PQ is the diameter of circle then the value of $(3r)^2$

Answer: 20.00

Solution:



$$\frac{x^2}{4}+\frac{y^2}{2}=1;\ y=x+1$$

$$\Rightarrow x^2+2(x+1)^2=4$$

$$\Rightarrow 3x^2+4x-2=0$$

$$x=\frac{-4\pm\sqrt{16+24}}{6}=\frac{-4\pm2\sqrt{10}}{6}$$

$$x_1=\frac{-2+\sqrt{10}}{3};\ x_2=\frac{-2-\sqrt{10}}{3}$$

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$y_1 = \frac{1+\sqrt{10}}{3}$; $y_2 = \frac{1-\sqrt{10}}{3}$

$\therefore PQ = 2r = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{\frac{40+10}{9}} = \frac{2}{3}\sqrt{20}$

$\therefore r = \frac{\sqrt{20}}{3}$

$\Rightarrow (3r)^2 = 20$

Question: $1 + 2.3 + 3.3^2 + ... + 10.3^9 =$

Answer: $\frac{1+19.3^{10}}{4}$

Solution:

$s = 1 + 2.3 + 3.3^2 + 4.3^3 + ... + 10.3^9$

$-(3s = \quad 1.3 + 2.3^2 + 3.3^3 + ... + 9.3^9 + 10.3^{10})$

$-2s = 1 + 3 + 3^2 + 3^3 + ...3^9 - 10.3^{10}$

$= \frac{3^{10}-1}{2} - 10.3^{10}$

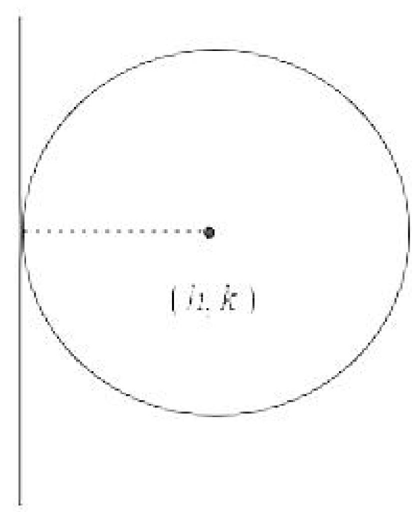
$-2s = \frac{-1-19.3^{10}}{2}$

$\Rightarrow s = \frac{1+19.3^{10}}{4}$

Question: A circle touching y-axis and the line $x + y = 0$ find the locus of the centre.

Answer: $x^2 - y^2 = 2xy$

Solution:



$h = r$

Also, $x + y = 0$ is tangent

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$\therefore \left|\frac{h+k}{\sqrt{2}}\right| = r = h$

$x^2 + y^2 + 2xy - 2x^2 = 0$

$\Rightarrow x^2 - y^2 - 2xy = 0$

Question: Find coefficient of x^{101} in $(5+x)^{500} + x.(5+x)^{499} + x^2.(5+x)^{498} + ... + x^{500}$

Answer: ${}^{501}C_{101}(5)^{399}$

Solution:

$(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ... + x^{500}(x+5)^0$

$$\Rightarrow (5+x)^{500} \frac{\left[\left(\frac{x}{5+x}\right)^{501} - 1\right]}{\left(\frac{x}{5+x} - 1\right)}$$

$$\Rightarrow (x+5)^{500}\left[\frac{(5+x)^{501} - x^{501}}{5(x+5)^{500}}\right]$$

$$\Rightarrow \frac{1}{5}\left[(x+5)^{501} - x^{501}\right]$$

$$\Rightarrow \text{Coeff. of } x^{101} = \frac{1}{5}\cdot {}^{501}C_{101}(5)^{400} = {}^{501}C_{101}(5)^{399}$$

Question:

$x = 12(1 + \sin t \cos t)$

$y = 12(1 + \sin t)^2$

If at $P(x_0, y_0)$ the tangent makes angle $\frac{\pi}{3}$ with +ve direction of x-axis then $y_0 = ?$

Answer: 3.00

Solution:

$$\frac{dy}{dx} = \sqrt{3} = \frac{12 \times 2(1+\sin t)\cos t}{12\left(\frac{2\cos 2t}{2}\right)}$$

$\sqrt{3}\cos 2t = 2\cos t(1+\sin t)$

$\sqrt{3}\cos 2t - \sin 2t = 2\cos t$

$\frac{\sqrt{3}}{2}\cos 2t - \frac{1}{2}\sin 2t = \cos t$

$\cos\left(2t + \frac{\pi}{6}\right) = \cos t$

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$2t+\frac{\pi}{6}$ may be equal to t

$2t+\frac{\pi}{6}=t$

$t=\frac{-\pi}{6}$

$y_0=12\left(1-\frac{1}{2}\right)^2=3$

Question: $\lim_{x\to\frac{\pi}{2}}\tan^2 x\left(\left(2\sin^2 x+4\sin x+3\right)^{\frac{1}{2}}-\left(\sin^2 x+6\sin x+2\right)^{\frac{1}{2}}\right)$

Answer: 0.00

Solution:

Given, $\lim_{x\to\frac{\pi}{2}}\tan^2 x\left(\left(2\sin^2 x+4\sin x+3\right)^{\frac{1}{2}}-\left(\sin^2 x+6\sin x+2\right)^{\frac{1}{2}}\right)$

$\lim_{x\to\frac{\pi}{2}}\tan^2 x\left[\sqrt{2\sin^2 x+4\sin x+3}-\sqrt{\sin^2 x+6\sin x+2}\right]$

$\Rightarrow \lim_{x\to\frac{\pi}{2}}\tan^2 x\left[\frac{\sin^2 x-2\sin x+1}{\sqrt{2\sin^2 x+4\sin x+3}+\sqrt{\sin^2 x+6\sin x+2}}\right]$

$\Rightarrow \lim_{x\to\frac{\pi}{2}}\frac{1}{6}\frac{(\sin x-1)^2}{\cot^2 x}$

$\Rightarrow \lim_{x\to\frac{\pi}{2}}\frac{2(\sin x-1).\cos x}{2\cot x\cdot\left(-\operatorname{cosec}^2 x\right)}\cdot\frac{-1}{6}$

$\Rightarrow \lim_{x\to\frac{\pi}{2}}\frac{-1}{6}(\sin x-1)\sin^3 x$

$\Rightarrow 0$

Question: Find b if $12\times\int_3^b\frac{1}{\left(x^2-4\right)\left(x^2-1\right)}=\ln\left(\frac{49}{40}\right)$ & $b>3$.

Answer: 6.00

Solution:

$12\times\int_3^b\frac{1}{\left(x^2-4\right)\left(x^2-1\right)}=\ln\left(\frac{49}{40}\right)$

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$$4\times\int_3^b\left(\frac{1}{x^2-4}-\frac{1}{x^2-1}\right)dx=\ln\left(\frac{49}{40}\right)$$

$$4\left[\frac{1}{2(2)}\ln\left|\frac{x-2}{x+2}\right|-\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|\right]_3^b=\ln\left(\frac{49}{40}\right)$$

$$\ln\left|\frac{b-2}{b+2}\right|-2\ln\left|\frac{b-1}{b+1}\right|-\ln\frac{1}{5}+2\ln\frac{1}{2}=\ln\left(\frac{49}{40}\right)$$

As $b>3$

$$\ln\left[\left(\frac{b-2}{b+2}\right)\left(\frac{b+1}{b-1}\right)^2\right]=\ln\left(\frac{49}{40}\times\frac{1}{5}\times 4\right)$$

$$\left(\frac{b-2}{b+2}\right)\left(\frac{b+1}{b-1}\right)^2=\frac{49}{50}=\frac{1}{2}\left(\frac{7}{5}\right)^2$$

$$=\left(\frac{6-2}{6+2}\right)\left(\frac{6+1}{6-1}\right)^2$$

$$\Rightarrow b=6$$

Question: Find slope of VP where V is vertex of $y=x-x^2$ & P is point of tangency of $y=4+kx$ & $y=x-x^2$.

Answer: $\frac{-3}{2},\frac{5}{2}$

Solution:

$y=x-x^2$

$\frac{dy}{dx}=1-2x$

Vertex $v\left(\frac{1}{2},\frac{1}{4}\right)$

Let point of tangency is (x_1, y_1)

$\Rightarrow y_1=x_1-x_1^2 \quad ....(1)$

Tangent $y-y_1=(1-2x_1)(x-x_1)$

It passes through $(0,4)$

$4-y_1=(1-2x_1)(-x_1)$

$4-x_1+x_1^2=-x_1+2x_1^2$ (From eqn. (1))

$x_1^2=4$

$x_1=\pm 2$

So, r can be $(2,-2)$ or $(-2,-6)$

Slope of VP $= \dfrac{-2-\frac{1}{4}}{2-\frac{1}{2}}$ or $\dfrac{-6-\frac{1}{4}}{-2-\frac{1}{2}}$

$= \dfrac{-1}{4} \times \dfrac{2}{3}$ or $\dfrac{25}{4} \times \dfrac{2}{5}$

$= \dfrac{-3}{2}$ or $\dfrac{5}{2}$

Question: $ax^2 - 2bx + 15 = 0$ has a repeated root α . $x^2 - 2bx + 22$ has roots α and β then $\alpha^2 + \beta^2 = ?$

Answer: $\dfrac{137}{2}$

Solution:

$ax^2 - 2bx + 15 = 0 \Rightarrow \alpha = \dfrac{b}{a}; \alpha^2 = \dfrac{15}{a}$

$\Rightarrow b^2 = 15a$

$x^2 - 2bx + 22 = 0$

$\Rightarrow \alpha + \beta = 2b; \alpha\beta = 22$

$\therefore \beta = 2b - \alpha = 2b - \dfrac{15}{b}; \alpha = \dfrac{15}{b}$

$\therefore \alpha\beta = \dfrac{15}{b}\left(2b - \dfrac{15}{b}\right) = 30 - \dfrac{225}{b^2} = 22$

$\Rightarrow b = \dfrac{15}{2\sqrt{2}}$

$\therefore \alpha = 2\sqrt{2},\ \beta = \dfrac{11\sqrt{2}}{2}$

$\Rightarrow \alpha^2 + \beta^2 = 8 + \dfrac{121}{2} = \dfrac{137}{2}$

Question: If there is a biased dice with number 2, 4, 8, 16, 32, 32. Probability of appearing of number n is $\dfrac{1}{n}$. Then the probability of sum 48 in 3 throws is

Answer: $\dfrac{7}{16^3}$

Solution:

$s = \{2, 4, 8, 16, 32, 32\}$

$P(2) = \dfrac{1}{2}, P(4) = \dfrac{1}{4}, P(8) = \dfrac{1}{8}, P\left(\dfrac{1}{16}\right) = \dfrac{1}{16}, P(32) = \dfrac{1}{32}$

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For sum = 48 in 3 throw, possible cases are

$\{8,8,32\},\{16,16,16\}$

$\therefore$ Required probability $=\frac{1}{8}\times\frac{1}{8}\times\frac{1}{32}\times 3+\frac{1}{16}\times\frac{1}{16}\times\frac{1}{16}=\frac{7}{16^3}$

Question: If mean deviation of first n natural numbers from mean is $\frac{5(n+1)}{n}$, where n is odd then $n=?$

Answer: 21.00

Solution:

Mean deviation about mean of set of first n natural number when n is odd $=\frac{n^2-1}{4n}=\frac{5(n+1)}{n}$

$\Rightarrow n^2-1=20n+20$

$\Rightarrow n^2-20n-21=0$

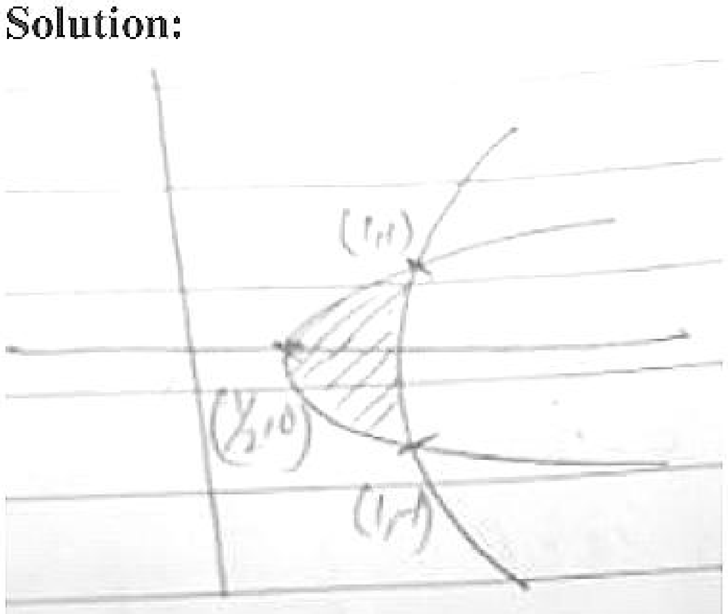
$\Rightarrow (n-21)(n+1)=0$

$\Rightarrow n=21$

Question: Find area bounded by $y^2=2x-1$ and $y^2=4x-2$

Answer: $\frac{1}{3}$

Solution:



$y^2=2\left(x-\frac{1}{2}\right);\ y^2=4\left(x-\frac{3}{4}\right)$

Area $=\int_{-1}^{1}\left(\frac{y^2+3}{4}\right)-\left(\frac{y^2+1}{2}\right)dy$

$=\left\{\frac{1}{4}\left[\frac{y^3}{3}+3y\right]-\frac{1}{2}\left[\frac{y^3}{3}+y\right]\right\}_{-1}^{1}$

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$$= \frac{1}{4}\left[\left(\frac{10}{3} + \frac{10}{3}\right) - 2\left(\frac{4}{3} + \frac{4}{3}\right)\right]$$

$$= \frac{1}{4}\left[\frac{20}{3} - \frac{16}{3}\right] = \frac{1}{3}$$

Question: Find value of $2\sin 12 - \sin 72$

Answer: $\sqrt{3}\left(\frac{1-\sqrt{5}}{4}\right)$

Solution:

$$2\sin 12 - \sin 72 = \sin 12 + \sin 12 - \sin 72$$

$$= \sin 12 + 2\sin(-30)\cos(42) = \sin 12 - \cos 42$$

$$= \sin 12 - \sin 48 = 2\sin(-18)\cos(30)$$

$$= -\sqrt{3}\sin 18 = \sqrt{3}\left(\frac{1-\sqrt{5}}{4}\right)$$

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