65/3

65/3 VALUE POINTS
SECTION A

1. $[\hat{i}\ \hat{k}\ \hat{j}] = \hat{i}\cdot(\hat{k}\times\hat{j}) = -\hat{i}\cdot(\hat{j}\times\hat{k})$ $\frac{1}{2}$

$= -1$ $\frac{1}{2}$

2. $\frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$ $\quad$ $\frac{1}{2}$ for any one of $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ $\quad$ $\frac{1}{2}+\frac{1}{2}$

3. Writing $\frac{3ae}{2} = a$ and finding $e = \frac{2}{3}$ $\quad$ $\frac{1}{2}+\frac{1}{2}$

4. $A' = \begin{pmatrix} 1 & 2 & -2 \\ 2 & 1 & 2 \\ 2 & x & -1 \end{pmatrix}$ and getting $x = -2$ $\quad$ $\frac{1}{2}+\frac{1}{2}$

SECTION B

5. $R'(x) = 6x + 36.$ $\quad$ 1

$R'(5) = 66$ $\quad$ 1

6. Let $y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right) = \tan^{-1}\left(\frac{1-\tan x}{1+\tan x}\right)$ $\frac{1}{2}$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right)$ $\frac{1}{2}$

$= \frac{\pi}{4} - x$ $\frac{1}{2}$

$\Rightarrow \frac{dy}{dx} = -1$ $\frac{1}{2}$

7. $P(A/B) = \frac{P(A\cap B)}{P(B)}$ gives $P(A\cap B) = \frac{2}{13}$ $\quad$ 1

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$= \frac{5}{26} + \frac{5}{13} - \frac{2}{13} = \frac{11}{26}$ $\quad$ 1

8. $\vec{a}+\vec{b}+\vec{c}=0$

$\vec{a}+\vec{b} = -\vec{c}$

$\vec{a}^2+\vec{b}^2+2\vec{a}\cdot\vec{b} = \vec{c}^2$

$\Rightarrow \vec{a}\cdot\vec{b} = \frac{|\vec{c}|^2-|\vec{a}|^2-|\vec{b}|^2}{2}$ $\frac{1}{2}$

$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{|\vec{c}|^2-|\vec{a}|^2-|\vec{b}|^2}{2|a||b|}$ $\frac{1}{2}$

$= \frac{9^2-5^2-6^2}{2(5)(6)}$

$\cos\theta = \frac{81-25-36}{60} = \frac{1}{3}$ $\frac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{3}\right)$ $\frac{1}{2}$

9. $\frac{dy}{dx} = \cos^{-1}a \Rightarrow \int dy = \cos^{-1}a\cdot\int dx$ $\frac{1}{2}+\frac{1}{2}$

$y = x\cos^{-1}a + c$ 1

10. $\frac{3-5\sin x}{\cos^2 x}dx = 3\int \sec^2 x\,dx - 5\int \sec x\tan x\,dx$ 1

$= 3\tan x - 5\sec x + C$ $\frac{1}{2}+\frac{1}{2}$

11. Finding $A^{-1} = \frac{-1}{19}\begin{bmatrix}-2 & -3\\ -5 & 2\end{bmatrix}$ 1

$\Rightarrow \frac{-1}{19}\begin{bmatrix}-2 & -3\\ -5 & 2\end{bmatrix} = \begin{bmatrix}2k & 3k\\ 5k & -2k\end{bmatrix}$ $\frac{1}{2}$

$\Rightarrow k = \frac{1}{19}$ $\frac{1}{2}$

12. Put $x = \cos\theta$ in R.H.S $\frac{1}{2}$

as $\frac{1}{2} \le x \le 1$, RHS $= \cos^{-1}(4\cos^3\theta - 3\cos\theta) = \cos^{-1}(\cos 3\theta) = 3\theta$ $\frac{1}{2}+\frac{1}{2}$

$= 3\cos^{-1} x = \text{LHS}$ $\frac{1}{2}$

13. Getting $\overrightarrow{AB} = (5-4)\hat{i} + (x-4)\hat{j} + (8-4)\hat{k} = \hat{i} + (x-4)\hat{j} + 4\hat{k}$

$\overrightarrow{AC} = \hat{i} + 0\hat{j} - 3\hat{k}$ and $\overrightarrow{AD} = 3\hat{i} + 3\hat{j} - 2\hat{k}$ $1\frac{1}{2}$

for coplanarity $[\overrightarrow{AB} \quad \overrightarrow{AC} \quad \overrightarrow{AD}] = 0$ $\frac{1}{2}$

$$\Rightarrow \begin{vmatrix} 1 & x-4 & 4 \\ 1 & 0 & -3 \\ 3 & 3 & -2 \end{vmatrix} = 0$$ $\frac{1}{2}$

$\Rightarrow x = 7$ $1\frac{1}{2}$

SECTION C

14. $\dfrac{4}{(x-2)(x^2+4)} = \dfrac{A}{x-2} + \dfrac{Bx+C}{x^2+4}$ 1

$4 = A(x^2+4) + (Bx+C)(x-2)$

gives $A = \frac{1}{2}, B = -\frac{1}{2}. C = 1$ $\frac{1}{2}\times 3$

$$\int \frac{4\,dx}{(x-2)(x^2+4)} = \frac{1}{2}\int \frac{dx}{x-2} - \int \frac{(x+2)}{2(x^2+4)}dx$$

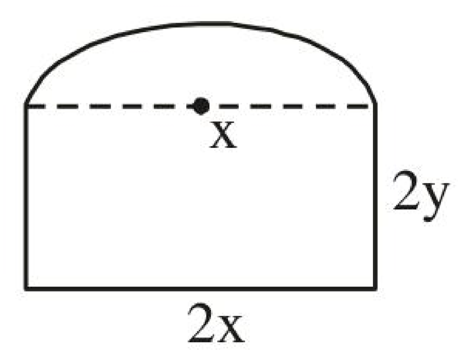
$= \frac{1}{2}\log|x-2| - \frac{1}{4}\log|x^2+4| - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$ $\frac{1}{2}+\frac{1}{2}+\frac{1}{2}$

15. $\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = 1$ 1+1

$\sqrt{(b_1c_2 - b_2c_1)^2 + (a_2c_1 - a_1c_2)^2 + (a_1b_2 - a_2b_1)^2} = \sqrt{1+4+1} = \sqrt{6}$ $1+\frac{1}{2}$

$d = \frac{1}{\sqrt{6}}$ $\frac{1}{2}$

16. Let the dimensions of window be 2x and 2y



$2x + 4y + \pi x = 10$ $\frac{1}{2}$

$A = 4xy + \frac{1}{2}\pi x^2 = 4x\left(\frac{10 - \pi x - 2x}{4}\right) + \frac{1}{2}\pi x^2$ 1

$= 10x - \frac{\pi x^2}{2} - 2x^2 \Rightarrow \frac{dA}{dx} = 10 - (\pi + 4)x$

$\frac{dA}{dx} = 0 \Rightarrow x = \frac{10}{\pi + 4}$ $\frac{1}{2}$

$\frac{d^2A}{dx^2} = -(\pi + 4) < 0$ $\frac{1}{2}$

Getting, $y = \frac{5}{\pi + 4}$, so the dimensions are $\frac{20}{\pi + 4}$ m and $\frac{10}{\pi + 4}$ m $\frac{1}{2}$

Any relevant explanation. 1

17. Let X denote the number of defective bulbs. $\frac{1}{2}$

$X = 0, 1, 2, 3$ $\frac{1}{2}$

$$P(X=0)=\left(\frac{15}{20}\right)^3=\frac{27}{64}$$

$$P(X=1)=3\left(\frac{5}{20}\right)\left(\frac{15}{20}\right)^2=\frac{27}{64}$$

$$P(X=2)=3\left(\frac{5}{20}\right)^2\left(\frac{15}{20}\right)=\frac{9}{64}$$

$$P(X=3)=\left(\frac{5}{20}\right)^3=\frac{1}{64}$$

$\frac{1}{2}\times 4$

$$\text{Mean}=\sum XP(X)=\frac{27}{64}+\frac{18}{64}+\frac{3}{64}=\frac{3}{4}$$ 1

18. $$\frac{dy}{dx}=\frac{y^2-x^2}{2xy}=\frac{\frac{y^2}{x^2}-1}{\frac{2y}{x}}$$ 1

Put $\frac{y}{x}=v \Rightarrow y=vx$ and so $\frac{dy}{dx}=v+x\frac{dv}{dx}$ 1

$$v+x\frac{dv}{dx}=\frac{v^2-1}{2v} \Rightarrow \frac{xdv}{dx}=-\frac{(1+v^2)}{2v}$$ $\frac{1}{2}$

$$\int\frac{dx}{x}=-\int\frac{2vdv}{1+v^2} \Rightarrow \log x=-\log(1+v^2)+\log C$$ $\frac{1}{2}+\frac{1}{2}$

$$\Rightarrow x(1+v^2)=C \text{ so } x\left(1+\frac{y^2}{x^2}\right)=C \text{ or } x^2+y^2=Cx$$ $\frac{1}{2}$

OR

$$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{1}{(1+x^2)^2}$$ 1

$$\text{I.F.}=e^{\int\frac{2x}{1+x^2}dx}=e^{\log(1+x^2)}=(1+x^2)$$ 1

Solution is $y(1+x^2) = \int \frac{1}{1+x^2} dx = \tan^{-1} x + C$ 1

getting $C = -\frac{\pi}{4}$ $\frac{1}{2}$

$\therefore\ y(1+x^2) = \tan^{-1} x - \frac{\pi}{4}$

or $y = \frac{\tan^{-1} x}{1+x^2} - \frac{\pi}{4(1+x^2)}$ $\frac{1}{2}$

19. Let E_1 = First group wins, E_2 = Second group wins 1

H = Introduction of new product.

$P(E_1) = 0.6,\ P(E_2) = 0.4,$ $\frac{1}{2}$

$P(H/E_2) = 0.3,\ P(H/E_1) = 0.7$ $\frac{1}{2}$

Now, $P(E_2/H) = \frac{P(E_2)\,P(H/E_2)}{P(E_2)\,P(H/E_2) + P(E_1)\,P(H/E_1)}$ $\frac{1}{2}$

$= \frac{0.4 \times 0.3}{0.4 \times 0.3 + 0.6 \times 0.7} = \frac{2}{9}$ $1+\frac{1}{2}$

20. $x = \frac{\sin y}{\cos(a+y)}$ gives $\frac{dx}{dy} = \frac{\cos(a+y)\cos y + \sin y \sin(a+y)}{\cos^2(a+y)}$ $\frac{1}{2}+1$

$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\cos(a+y-y)} = \frac{\cos^2(a+y)}{\cos a}$ $1+\frac{1}{2}$

Hence $\frac{dy}{dx} = \cos a$ when $x = 0$ i.e. $y = 0$ 1

21. $C_1 \to C_1 + C_2 + C_3$ gives L.H.S. as

$$\begin{vmatrix} a+b+c & -2a+b & -2a+c \\ a+b+c & 5b & -2b+c \\ a+b+c & -2c+b & 5c \end{vmatrix}$$ 1

$$= (a+b+c)\begin{vmatrix} 1 & -2a+b & -2a+c \\ 1 & 5b & -2b+c \\ 1 & -2c+b & 5c \end{vmatrix}$$ $\frac{1}{2}$

$R_2 \to R_2 - R_1, R_3 \to R_3 - R_1$ gives

$$= (a+b+c)\begin{vmatrix} 1 & -2a+b & -2a+c \\ 0 & 2a+4b & 2a-2b \\ 0 & 2a-2c & 4c+2a \end{vmatrix}$$ 1

$$= (a+b+c)\begin{vmatrix} 2a+4b & 2a-2b \\ 2a-2c & 4c+2a \end{vmatrix}$$ $\frac{1}{2}$

$$= 4(a+b+c)\begin{vmatrix} a+2b & a-b \\ a-c & 2c+a \end{vmatrix} = 4(a+b+c)\,3(ab+bc+ac)$$ $\frac{1}{2}+\frac{1}{2}$

$= 12(a+b+c)\,(ab+bc+ac)$

22. Point of intersection $= (1, \sqrt{3})$ 1

$$x^2+y^2=4 \Rightarrow 2x+2y\frac{dy}{dx}=0 \quad \frac{dy}{dx}_{(1,\sqrt{3})} \quad -\frac{1}{\sqrt{3}} \; m_1$$ $\frac{1}{2}+\frac{1}{2}$

$$(x-2)^2+y^2=4 \Rightarrow 2(x-2)+2y\frac{dy}{dx}=0 \Rightarrow \left.\frac{dy}{dx}\right]_{[1,\sqrt{3}]} = \frac{1}{\sqrt{3}} = m_2$$ $\frac{1}{2}+\frac{1}{2}$

$$\text{So, } \tan\phi = \frac{\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{3}}}{1-1/3} = \sqrt{3} \quad \Rightarrow \quad \phi = \frac{\pi}{3}$$ 1

OR

$f'(x) = -6(x+1)(x+2)$ 1

$f'(x) = 0 \Rightarrow x = -2, x = -1$ $\frac{1}{2}$

$\Rightarrow$ Intervals are $(-\infty, -2)$, $(-2, -1)$ and $(-1, \infty)$ $\frac{1}{2}$

Getting $f'(x) > 0$ in $(-2, -1)$ and $f'(x) < 0$ in $(-\infty, -2) \cup (-1, \infty)$ 1

$\left.\begin{array}{l}\Rightarrow f(x) \text{ is strictly increasing in } (-2, -1) \\ \text{and strictly decreasing in } (-\infty, 2) \cup (-1, \infty)\end{array}\right\}$ 1

23. Writing $\frac{dy}{d\theta} = 3a\tan^2\theta\sec^2\theta$ 1

$\frac{dx}{d\theta} = 3a\sec^3\theta\tan\theta$ 1

$\frac{dy}{dx} = \frac{\tan\theta}{\sec\theta} = \sin\theta$ $\frac{1}{2}$

$\frac{d^2y}{dx^2} = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\frac{d\theta}{dx} = \cos\theta \times \frac{1}{3a\sec^3\theta\tan\theta}$ 1

$\left.\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{3}} = \frac{\frac{1}{2}}{3a \times 8 \times \sqrt{3}} = \frac{1}{48\sqrt{3}a}$ $\frac{1}{2}$

OR

$y = e^{\tan^{-1}x}$

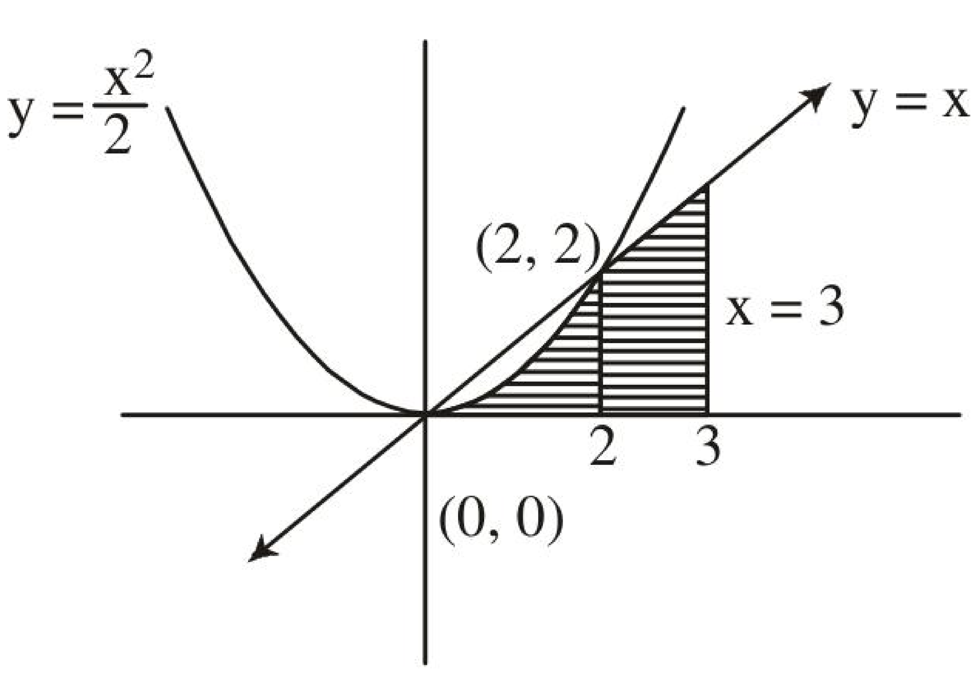
$\frac{dy}{dx} = e^{\tan^{-1}x}\left(\frac{1}{1+x^2}\right) = \frac{y}{1+x^2}$ $1+\frac{1}{2}$

$(1+x^2)\frac{dy}{dx} = y \Rightarrow (1+x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} = \frac{dy}{dx}$ 1+1

$\Rightarrow (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$ $\frac{1}{2}$

SECTION D

24.



1

Point of intersection of $x^2 = 2y$ and $y = x$ are (0, 0) and (2, 2). 2

Required area $= \int_0^2 \frac{x^2}{2} dx + \int_2^3 x\, dx$ 2

$= \frac{8}{6} + \frac{5}{2} = \frac{23}{6}$ 1

25. $|A| = 5(-1) + 4(1) = -1$ 1

$C_{11} = -1 \quad C_{21} = 8 \quad C_{31} = -12$
$C_{12} = 0 \quad C_{22} = 1 \quad C_{32} = -2$
$C_{13} = 1 \quad C_{23} = -10 \quad C_{33} = 15$ 2

$A^{-1} = \begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$ 1

$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & -8 & 12 \\ 0 & -1 & 2 \\ -1 & 10 & -15 \end{bmatrix}$ 1

$= \begin{bmatrix} -2 & 19 & -27 \\ -2 & 18 & -25 \\ -3 & 29 & -42 \end{bmatrix}$ 1

OR

$\begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_1 \to R_1 + 2R_3$

$\begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$ 1

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} A \qquad 1$$

$R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A \qquad 1$$

$R_1 \to R_1 + 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A \qquad 1$$

So, $A^{-1} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$ 1

26. $(x - x) = 0$ is divisible by 3 for all $x \in z$. So, $(x, x) \in R$ 1

$\therefore$ R is reflexive.

$(x - y)$ is divisible by 3 implies $(y - x)$ is divisible by 3.

So $(x, y) \in R$ implies $(y, x) \in R$, $x, y \in z$ $1\frac{1}{2}$

$\Rightarrow$ R is symmetric.

$(x - y)$ is divisible by 3 and $(y - z)$ is divisble by 3.

So $(x - z) = (x - y) + (y - z)$ is divisible by 3. $1+1+\frac{1}{2}$

Hence $(x, z) \in R \Rightarrow$ R is transitive

$\Rightarrow$ R is an equivalence relation 1

OR

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Table Format 1

Values of each correct row, $\frac{1}{2} \times 6 = 3$

$a * 0 = a + 0 = a \; \forall \; a \in A \Rightarrow 0$ is the identifty for *. $\frac{1}{2}$

Let $b = 6 - a$ for $a \neq 0$ $\frac{1}{2}$

Since $a + b = a + 6 - a < 6$

$\Rightarrow a * b = b * a = a + 6 - a - 6 = 0$ $\frac{1}{2}$

Hence $b = 6 - a$ is the inverse of a. $\frac{1}{2}$

27. Since the line is parallel to the two planes.

$\therefore$ Direction of line $\vec{b} = (\hat{i} - \hat{j} + 2\hat{k}) \times (3\hat{i} + \hat{j} + \hat{k})$ 1

$= -3\hat{i} + 5\hat{j} + 4\hat{k}$ 1

$\therefore$ Equation of required line is

$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(-3\hat{i} + 5\hat{j} + 4\hat{k}) \quad ...(i)$ 1

Any point on line (i) is $(1 - 3\lambda, 2 + 5\lambda, 3 + 4\lambda)$ 1

For this line to intersect the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 4$

we have $(1 - 3\lambda)2 + (2 + 5\lambda)1 + (3 + 4\lambda)1 = 4$

$\Rightarrow \lambda = -1$ 1

$\therefore$ Point of intersection is $(4, -3, -1)$ 1

28. Let number of units of type A be x and that of type B be y

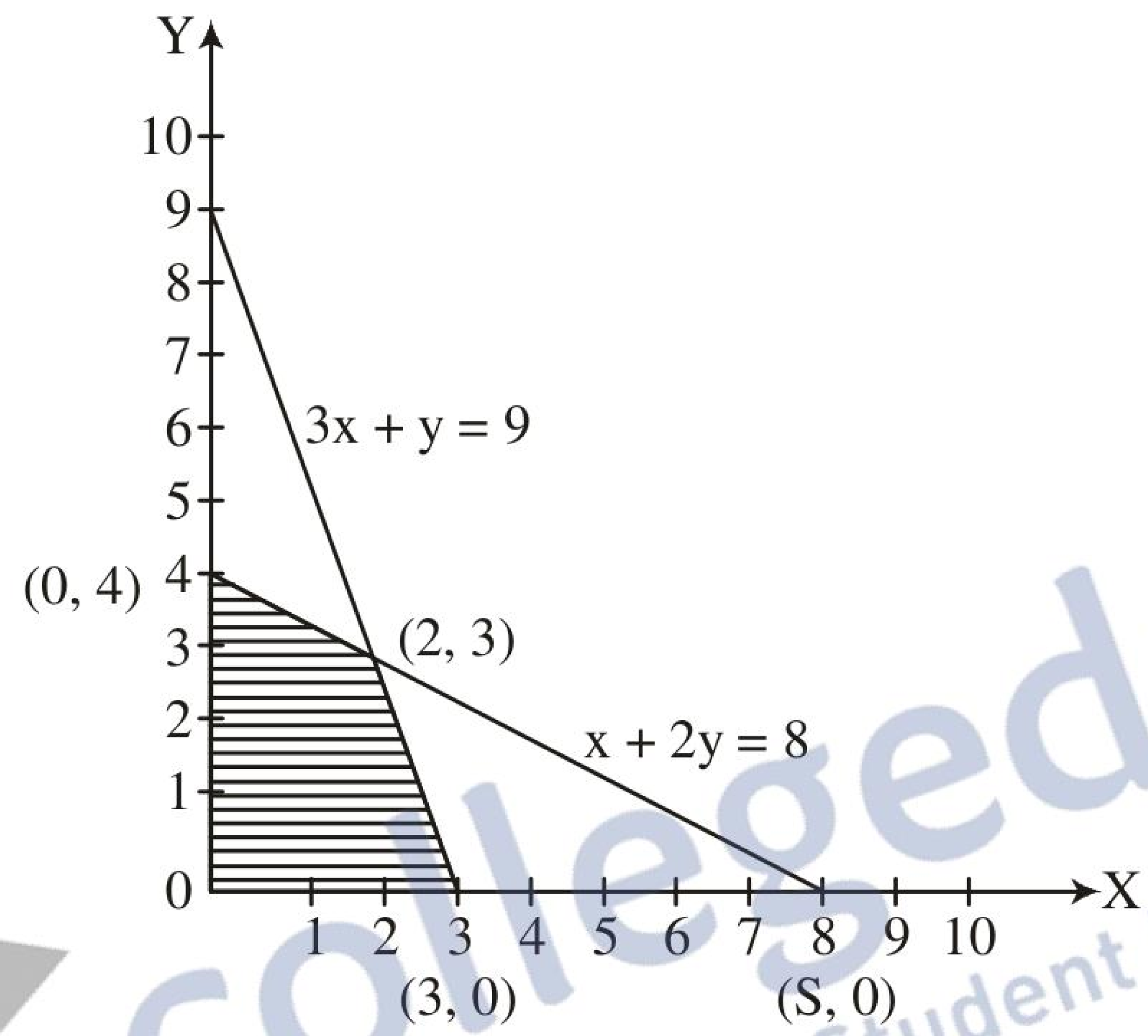
LPP is Maximize $P = 40x + 50y$ 1

subject to constraints

$3x + y \leq 9$ 2

$x + 2y \leq 8$

$x, y \geq 0$



2

$P(3, 0) = 120$

$P(2, 3) = 230$

$P(0, 4) = 200$

$\therefore$ Max profit = ₹ 230 at (2, 3) 1

So to maximise profit, number of units of A = 2 and number of units of B = 3

29. $$I = \int_0^{\pi/2} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx$$

$$= \int_0^{\pi/2} \frac{(\pi/2 - x) \sin (\pi/2 - x) \cos (\pi/2 - x)}{\sin^4 (\pi/2 - x) + \cos^4 (\pi/2 - x)} dx = \int_0^{\pi/2} \frac{(\pi/2 - x) \cos \times \sin x}{\cos^4 x + \sin^4 x} dx$$ 1

$$2I = \pi/2 \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + \cos^4 x} dx = \frac{\pi}{2} \int_0^{\pi/2} \frac{\sin x \cos x}{\sin^4 x + (1 - \sin^2 x)^2} dx$$ 1

Let $\sin^2 x = t \Rightarrow \sin x \cos x\, dx = \frac{1}{2} dt$ $\frac{1}{2}$

$$2I = \frac{\pi}{2}\frac{1}{2}\int_0^1 \frac{dt}{t^2 + (1-t)^2}$$ 1

$$\Rightarrow I = \frac{\pi}{8}\int_0^1 \frac{dt}{2t^2 - 2t + 1} = \frac{\pi}{16}\int_0^1 \frac{dt}{(t - 1/2)^2 + (1/2)^2}$$ 1

$$I = \frac{\pi}{16}\frac{2}{1}\cdot \tan^{-1}(2t-1)\Big]_0^1 = \frac{\pi}{8}\cdot\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \frac{\pi^2}{16}$$ $1+\frac{1}{2}$

OR

$$a = 1,\ b = 3,\ h = \frac{2}{n} \quad \Rightarrow \quad nh = 2$$ 1

$$\int_1^3 (3x^2 + 2x + 1)dx = \lim_{h\to 0} h[f(1) + f(1+h) + f(1+2h) + \ldots + f(1+(n-1)h)]$$ 1

$$= \lim_{h\to 0} h[6 + \{3(1+h^2+2h) + 2(1+h) + 1\} + \{3(1+4h^2+4h) + 2(1+2h) + 1\}$$

$$+ \ldots \{3(1+(n-1)^2h^2 + 2(n-1)h + 2(1+(n-1)h) + 1\}]$$ 1

$$= \lim_{h\to 0} h[6n + 8h(1+2+\ldots(n-1)) + 3h^2(1^2 + 2^2 + \ldots(n-1)^2]$$ $\frac{1}{2}$

$$= \lim_{h\to 0} 6hn + \frac{8(nh-h)(nh)}{2} + \frac{3(nh-h)(nh)(2hn-h)}{6}$$ $1\frac{1}{2}$

$$= 6(2) + \frac{8(2)(2)}{2} + \frac{3(2-0)(2)(4)}{6}$$ $\frac{1}{2}$

$$= 12 + 16 + 8 = 36$$ $\frac{1}{2}$