QUESTION PAPER CODE 65/3

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\int \frac{3x}{3x-1}dx = \int \frac{3x-1+1}{3x-1}dx$ $\frac{1}{2}$

$= x + \frac{1}{3}\log|3x-1| + C$ $\frac{1}{2}$

2. $(\vec{a}\times\vec{b})^2 + (\vec{a}\cdot\vec{b})^2 = 225 \Rightarrow |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = 225$ $\frac{1}{2}$

$\Rightarrow \quad (5)^2|\vec{b}|^2 = 225 \Rightarrow |\vec{b}| = 3$ $\frac{1}{2}$

3. $|A^{-1}| = \frac{1}{|A|}$ $\frac{1}{2}$

$= \frac{1}{4}$ $\frac{1}{2}$

4. $\lim_{x\to 0} \frac{\sin\frac{3x}{2}}{2} = \lim_{x\to 0} \frac{3}{2}\cdot\frac{\sin\frac{3x}{2}}{\frac{3x}{2}} = \frac{3}{2}$ $\frac{1}{2}$

$\Rightarrow \quad k = \frac{3}{2}$ $\frac{1}{2}$

SECTION B

5. Let, Number of executive class tickets be x and economy class tickets be y.

$\therefore$ LPP is Maximise Profit $P = 1500x + 1000y$ 1

Subject to: $x + y \le 250,\ x \ge 25,\ y \ge 3x$ 1

6. Given differential equation can be written as

$\frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$ 1

Getting integrating factor $= e^{x - \log x}$ or $\frac{e^x}{x}$ 1

7. $\int \frac{x}{\sqrt{32-x^2}}dx = -\int 1.dt$ where $32 - x^2 = t^2$ 1

$= -t + C = -\sqrt{32-x^2} + C$ 1

8. $f(x) = \sin 2x - \cos 2x$

$\Rightarrow \quad f'(x) = 2\cos 2x + 2\sin 2x$ 1

$f'\left(\frac{\pi}{6}\right) = 2\left[\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\right] = (1+\sqrt{3})$ 1

9. Getting $\begin{pmatrix} 2x+3 & 6 \\ 15 & 2y-4 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 15 & 14 \end{pmatrix}$ 1

$2x + 3 = 7$ and $2y - 4 = 14$

$\Rightarrow \quad x = 2, y = 9$ 1

10. For three vectors to be coplanar

$\begin{vmatrix} 1 & -1 & 1 \\ 3 & 1 & 2 \\ 1 & \lambda & -3 \end{vmatrix} = 0$ 1

$\Rightarrow \quad \lambda = 15$ 1

11. For $\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$ $\frac{1}{2}$

Integrating, we get

$\tan^{-1}y = \tan^{-1}x + C.$ $\frac{1}{2}$

As $x = 0$, $y = \sqrt{3}$ so $\tan^{-1}\sqrt{3} = C \quad \Rightarrow \quad C = \frac{\pi}{3}$ $\frac{1}{2}$

Solution is $\tan^{-1}y = \tan^{-1}x + \frac{\pi}{3}$ $\frac{1}{2}$

12. Radius $= \frac{1}{3}(3x+1)$ $\frac{1}{2}$

$$V = \frac{4}{3}\pi\frac{(3x+1)^3}{27}$$ $\frac{1}{2}$

$$\frac{dV}{dx} = \frac{12\pi\times 3}{81}(3x+1)^2 = \frac{4\pi}{9}(3x+1)^2$$ 1

SECTION C

13. Let the award for regularily be ₹ x and for hard work be ₹ y.

$\therefore$ $x + y = 6000$ and 1

$x + 3y = 11000$

$$\Rightarrow \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 6000 \\ 11000 \end{pmatrix} \text{ or } A.X = B$$ 1

$\therefore$ $X = A^{-1}B$ as $|A| = 2 \neq 0$.

$$\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 6000 \\ 11000 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3500 \\ 2500 \end{pmatrix} \quad \therefore \quad x = ₹\,3500,\ y = ₹\,2500$$ 1

Any two values like obedience, respect for elders,... 1

14. Let the events be

E_1: tansferring a red ball from A to B

E_2: transferring a black ball from A to B

A: Getting a red ball from bag B $\frac{1}{2}$

$$P(E_1) = \frac{3}{5},\ P(E_2) = \frac{2}{5}$$ $\frac{1}{2}$

$$P(A/E_1) = \frac{1}{2},\ P(A/E_2) = \frac{1}{3}$$ 1

$$P(E_1/A) = \frac{P(E_1)\cdot P(A/E_1)}{P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)}$$ $\frac{1}{2}$

$$= \frac{\frac{3}{5}\cdot\frac{1}{2}}{\frac{3}{5}\cdot\frac{1}{2}+\frac{2}{5}\cdot\frac{1}{3}} = \frac{9}{13}$$ $1+\frac{1}{2}$

OR

Required probability $= P(A \cup B)$ 1

$= P(A) + P(B) - P(A) \cdot P(B)$ $\frac{1}{2}$

$= P(A)\,[1 - P(B)] + 1 - P(B')$ $\frac{1}{2}$

$= P(A)\,P(B') - P(B') + 1$ 1

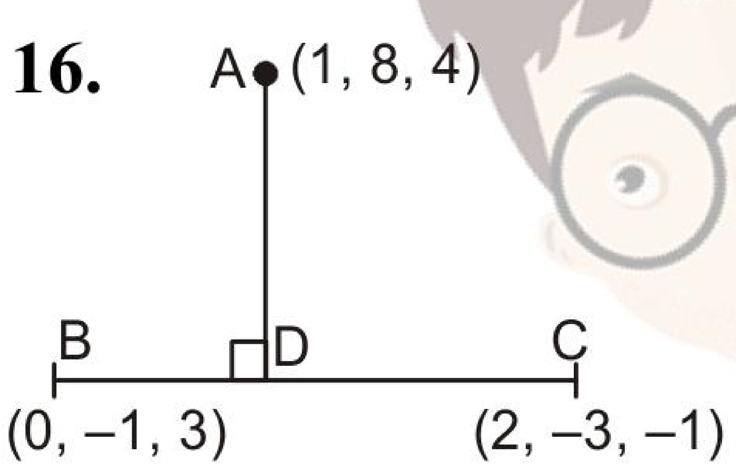
$= (1 - P(B')\,(1 - P(A)) = 1 - P(A')\,P(B')$ 1

15. Given equation can be written as $\tan^{-1}(1) - \tan^{-1}x = \frac{1}{2}\tan^{-1}x$ $1\frac{1}{2}$

$\Rightarrow \quad \frac{3}{2}\tan^{-1}x = \frac{\pi}{4} \quad \text{or} \quad \tan^{-1}x = \frac{\pi}{6}$ $1\frac{1}{2}$

$\Rightarrow \quad x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ 1

16.



Equation of line passing through B and C is

$$\frac{x}{2} = \frac{y+1}{-2} = \frac{z-3}{-4} \quad \text{or} \quad \frac{x}{1} = \frac{y+1}{-1} = \frac{z-3}{-2}$$ 1

Any point D on BC can be

$[\lambda, -\lambda - 1, -2\lambda + 3]$ for some value of λ. 1

$\therefore$ Direction ratios of AD are $\langle \lambda - 1, -\lambda - 9, -2\lambda - 1 \rangle$ $\frac{1}{2}$

$AD \perp BC \Rightarrow 1(\lambda - 1) - 1(-\lambda - 9) - 2(-2\lambda - 1) = 0$ $\frac{1}{2}$

$\Rightarrow \quad \lambda = -\frac{5}{3}$ $\frac{1}{2}$

$\therefore$ D is $\left(-\frac{5}{3}, \frac{2}{3}, \frac{19}{3}\right)$ $\frac{1}{2}$

17. Let the vector $\vec{p} = (2\vec{a} + \vec{b} + 2\vec{c})$ makes angles α, β, γ respectively with the vector $\vec{a}, \vec{b}, \vec{c}$

Given that $|\vec{a}| = |\vec{b}| = |\vec{c}|$ and $\vec{a}\cdot\vec{b} = \vec{c}\cdot\vec{a} = 0$ 1

$$\cos\alpha = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{a}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{a}|}$$ $\frac{1}{2}$

$$= \frac{2|a|^2}{3|\vec{a}||\vec{a}|} = \frac{2}{3} \Rightarrow \alpha = \cos^{-1}\frac{2}{3}$$ 1

$$\cos\beta = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{b}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{b}|} = \frac{|\vec{b}|^2}{3|\vec{b}||\vec{b}|} = \frac{1}{3} \Rightarrow \beta = \cos^{-1}\frac{1}{3}$$ 1

$$\cos\gamma = \frac{(2\vec{a}+\vec{b}+2\vec{c})\cdot\vec{c}}{|2\vec{a}+\vec{b}+2\vec{c}||\vec{c}|} = \frac{2|\vec{c}|^2}{3|\vec{c}||\vec{c}|} = \frac{2}{3} \Rightarrow \gamma = \cos^{-1}\frac{2}{3}$$ $\frac{1}{2}$

18. $f'(x) = 6x^2 - 6x - 36$ $\frac{1}{2}$

$= 6(x^2 - x - 6) = 6(x-3)(x+2)$

$f'(x) = 0 \Rightarrow x = -2, x = 3$ 1

∴ the intervals are $(-\infty, -2), (-2, 3), (3, \infty)$ $\frac{1}{2}$

getting $f'(x)$ +ve in $(-\infty, -2)$ U$(3, \infty)$
and –ve in $(-2, 3)$ } $1\frac{1}{2}$

∴ f(x) is strictly increasing in $(-\infty, -2)$ U$(3, \infty)$, and $\frac{1}{2}$

strictly decreasing in $(-2, 3)$

19. For $\int \frac{x^2+x+1}{(x+1)^2(x+2)}dx = \int\left[\frac{3}{x+2} - \frac{2}{x+1} + \frac{1}{(x+1)^2}\right]dx$ $2\frac{1}{2}$

$= 3\log|x+2| - 2\log|x+1| - \frac{1}{x+1} + C$ $1\frac{1}{2}$

OR

$$I = \int (x-3)\sqrt{3-2x-x^2}dx = \int\left[-\frac{1}{2}(-2-2x)-4\right]\sqrt{3-2x-x^2}dx \qquad 1$$

$$= -\frac{1}{2}\int(-2-2x)\sqrt{3-2x-x^2}dx - 4\int\sqrt{4-(x+1)^2}dx \qquad \frac{1}{2}+1$$

$$= -\frac{1}{3}(3-2x-x^2)^{3/2} - 4\left[\frac{(x+1)}{2}\sqrt{3-2x-x^2} + 2\sin^{-1}\left(\frac{x+1}{2}\right)\right] + C \qquad \frac{1}{2}+1$$

20. Let $u = (\cos x)^x \Rightarrow \log u = x.\log\cos x$ $\qquad \frac{1}{2}$

$$\Rightarrow \frac{du}{dx} = (\cos x)^x.[-x\tan x + \log\cos x] \qquad 1\frac{1}{2}$$

$$\therefore \quad y = (\cos x)^x + \sin^{-1}\sqrt{3x} \quad \Rightarrow \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{1}{\sqrt{1-3x}}.\frac{\sqrt{3}}{2\sqrt{x}} \qquad 1\frac{1}{2}$$

$$\therefore \quad \frac{dy}{dx} = (\cos x)^x[-x\tan x + \log\cos x] + \frac{\sqrt{3}}{2\sqrt{x}}.\frac{1}{\sqrt{1-3x}} \qquad \frac{1}{2}$$

OR

$$y = (\sec^{-1}x)^2 \quad \Rightarrow \quad \frac{dy}{dx} = 2\sec^{-1}x.\frac{1}{x\sqrt{x^2-1}} \qquad 1$$

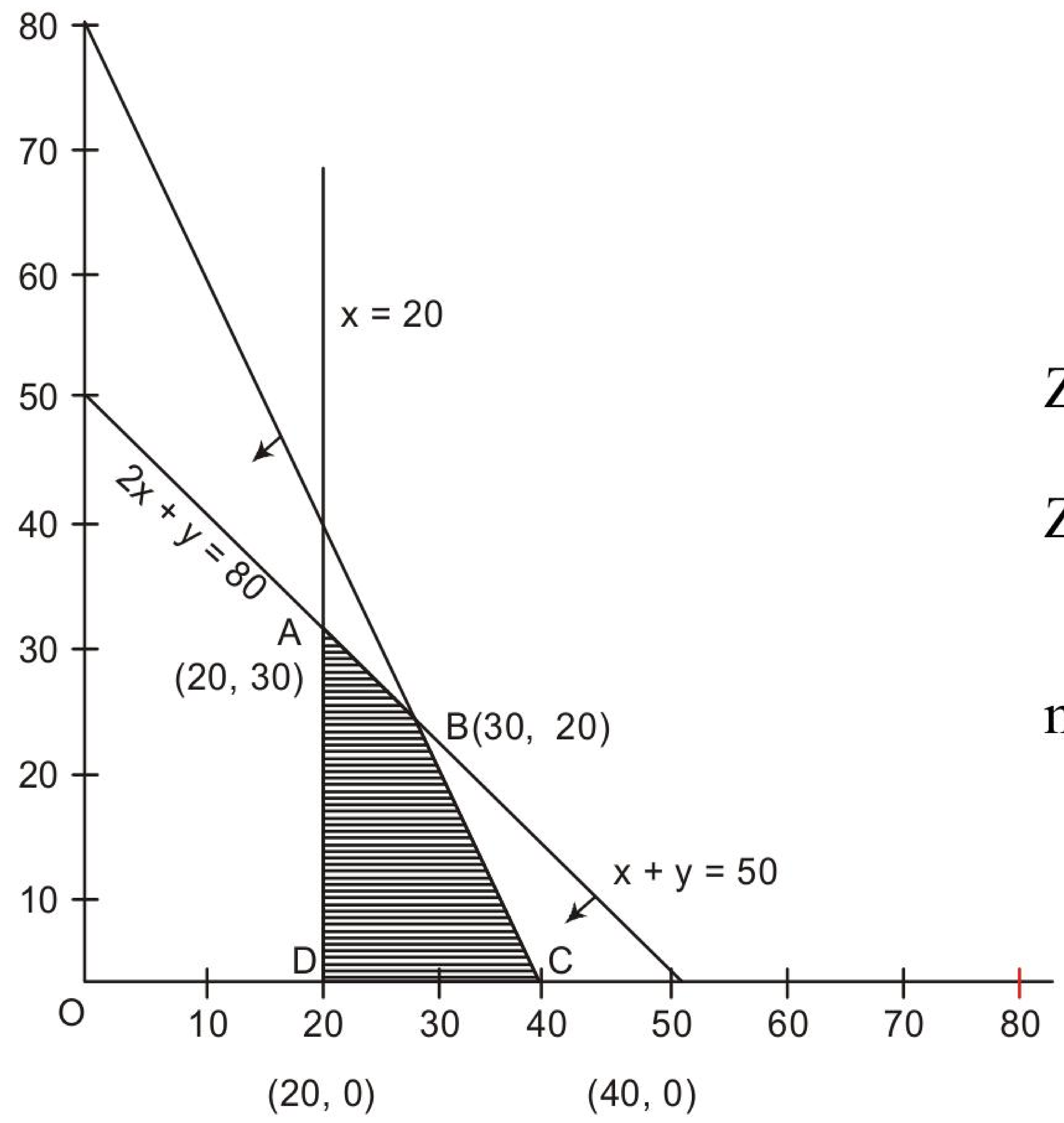
$$\therefore \quad x\sqrt{x^2-1}\cdot\frac{dy}{dx} = 2\sec^{-1}x \qquad \frac{1}{2}$$

$$\Rightarrow \quad x\sqrt{x^2-1}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\left(\frac{x^2}{\sqrt{x^2-1}} + \sqrt{x^2-1}\right) = \frac{2}{x\sqrt{x^2-1}} \qquad 1\frac{1}{2}$$

$$\Rightarrow \quad x^2(x^2-1)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot x(2x^2-1) = 2 \qquad 1$$

i.e., $\quad x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3-x)\frac{dy}{dx} = 2$

21.



Correct lines $1\frac{1}{2}$

Correct shading 1

Z(20, 0) = 2100, Z(40, 0) = 4200, Z(20, 30) = 4800 1

Z(30, 20) = 4950

max. value of Z is 4950 when x = 30, y = 20 $\frac{1}{2}$

22. Given differential equation can be written as,

$$\frac{dy}{dx} = \frac{2xy + y^2}{2x^2}$$ $\frac{1}{2}$

Put $y = vx, \ \frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

so, $v + x\frac{dv}{dx} = \frac{2v + v^2}{2}$ $\frac{1}{2}$

$$\int \frac{2}{v^2} dv = \int \frac{dx}{x}$$ $\frac{1}{2}$

$$\frac{-2}{v} = \log|x| + C \Rightarrow \frac{-2x}{y} = \log|x| + C$$ $\frac{1}{2}$

$y = 2, x = 1$ gives $C = -1$ $\frac{1}{2}$

Solution is $\frac{2x}{y} = 1 - \log|x|$ or $y = \frac{2x}{1 - \log|x|}$ where, $x \neq 0, e$ $\frac{1}{2}$

23. $I = \int_0^{\pi} \frac{x}{1+\sin x} dx = \int_0^{\pi} \frac{(\pi - x)}{1+\sin(\pi - x)} dx$ $\quad \frac{1}{2}$

$\Rightarrow \quad I = \pi \int_0^{\pi} \frac{dx}{1+\sin x} - I$ $\quad 1$

$\Rightarrow \quad 2I = \pi \int_0^{\pi} \frac{dx}{1+\sin x}$ $\quad \frac{1}{2}$

$= \pi \int_0^{\pi} \frac{(1-\sin x)}{\cos^2 x} dx = \pi \left[\int_0^{\pi} \sec^2 x \, dx - \int_0^{\pi} \sec x \tan x \, dx \right]$ $\quad \frac{1}{2}$

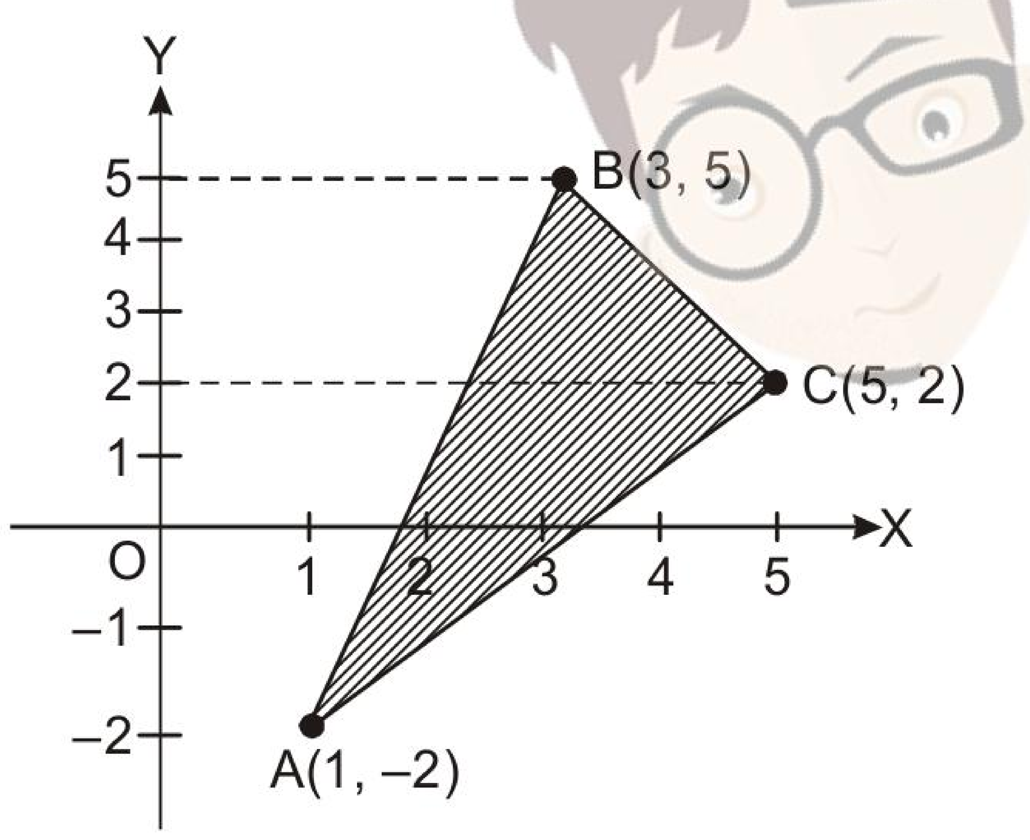
$= \pi | \tan x |_0^{\pi} - \pi | \sec x |_0^{\pi}$ $\quad \frac{1}{2}$

$= 0 - \pi(-2)$ $\quad \frac{1}{2}$

$\Rightarrow \quad I = \pi$ $\quad \frac{1}{2}$

SECTION D

24. **For Correct Figure** $\quad 1$



Equation of AB: $x = \frac{1}{7}(2y + 11)$

Equation of BC: $x = \frac{1}{3}(19 - 2y)$ $\quad 1\frac{1}{2}$

Equation of AC: $x = y + 3$

Required area $= \int_{-2}^{2} (y+3)dy + \frac{1}{3}\int_2^5 (19-2y)dy - \frac{1}{7}\int_{-2}^{5} (2y+11)dy$ $\quad 1\frac{1}{2}$

$\Rightarrow A = \left. \frac{(y+3)^2}{2} \right]_{-2}^{2} + \frac{1}{3} \left. \frac{(19-2y)^2}{-4} \right]_2^5 - \frac{1}{7} \left. \frac{(2y+11)^2}{4} \right]_{-2}^{5}$ $\quad 1$

$= \frac{1}{2}(25-1) - \frac{1}{12}(81-225) - \frac{1}{28}(441-49) = 10$ sq.units $\quad 1$

OR

Here $h = \frac{4}{n}$ or $nh = 4$, $f(x) = 3x^2 + 2x + 1$ $\frac{1}{2}$

$$\int_0^4 (3x^2 + 2x + 1)dx = \lim_{h \to 0} h[f(0) + f(0+h) + f(0+2h) + ... + f(0 + \overline{n-1}h)]$$ 1

$$= \lim_{h \to 0} h[(1) + (3h^2 + 2h + 1) + (3.2^2h^2 + 2.2h + 1) + ... + (3(n-1)^2h^2 + 2(n-1)h + 1)]$$ $1\frac{1}{2}$

$$= \lim_{h \to 0} h\left[n + 3h^2 \frac{n(n-1)(2n-1)}{6} + 2h\frac{n(n-1)}{2}\right]$$ 1

$$= \lim_{h \to 0}\left[nh + \frac{(nh)(nh-h)(2nh-h)}{2} + (nh)(nh-h)\right]$$ 1

$= 4 + 64 + 16 = 84$ 1

25. Let given volume of cone be, $V = \frac{1}{3}\pi r^2 h$ ...(i) $\frac{1}{2}$

$\therefore$ Surface area (curved) $S = \pi rl = \pi r\sqrt{r^2 + h^2}$ $\frac{1}{2}$

or $A = S^2 = \pi r^2(r^2 + h^2)$

$$A = S^2 = \pi^2 r^2\left[r^2 + \left(\frac{3V}{\pi r^2}\right)^2\right]$$ [using (i)]

$$= \pi^2\left[r^4 + \frac{9V^2}{\pi^2 r^2}\right]$$ $1\frac{1}{2}$

$$\frac{dA}{dr} = \pi^2\left[4r^3 - \frac{18V^2}{\pi^2 r^3}\right]$$ 1

$$\frac{dA}{dr} = 0 \Rightarrow 4\pi^2 r^6 = 18 \cdot \frac{1}{9}\pi^2 r^4 h^2$$

$\Rightarrow$ $2r^2 = h^2$ or $h = \sqrt{2}r$ $1\frac{1}{2}$

$$\frac{d^2A}{dr^2} = \pi^2\left[12r^2 + \frac{54V^2}{\pi^2 r^4}\right] > 0$$ 1

$\Rightarrow$ for least curved surface area, height $= \sqrt{2}$ (radius)

OR

$$x = a\cos\theta + a\theta\sin\theta \Rightarrow \frac{dx}{d\theta} = -a\sin\theta + a\sin\theta + a\theta\cos\theta$$

$= a\theta\cos\theta$ 1

$$y = a\sin\theta - a\theta\cos\theta \Rightarrow \frac{dy}{d\theta} = a\cos\theta - a\cos\theta + a\theta\sin\theta$$

$= a\theta\sin\theta$ 1

$$\frac{dy}{dx} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$ $\frac{1}{2}$

Equation of tangent is

$y - (a\sin\theta - a\theta\cos\theta) = \tan\theta(x - a\cos\theta - a\theta\sin\theta)$ 1

Equation of normal is

$$y - (a\sin\theta - a\theta\cos\theta) = -\frac{\cos\theta}{\sin\theta}(x - a\cos\theta - a\theta\sin\theta)$$ 1

$\Rightarrow$ $y\sin\theta + x\cos\theta = a$ $\frac{1}{2}$

distance of normal from origin $= \frac{|-a|}{\sqrt{\sin^2\theta + \cos^2\theta}} = |a| =$ constant 1

26. (i) for any A, B $\in$P(X), $A*B = A\cap B$ and $B*A = B\cap A$

as $A\cap B = B\cap A$ $\therefore$ $A*B = B*A$ 2

$\Rightarrow$ * is commutative

(ii) for any A, B, C $\in$P(X)

$(A*B)*C = (A\cap B)*C = (A\cap B)\cap C$

and $A*(B*C) = A*(B\cap C) = A\cap(B\cap C)$

Since $(A\cap B)\cap C = A\cap(B\cap C) \Rightarrow$ * is associative 2

(iii) for every $A \in P(X)$, $A*X = A\cap X = A$

$X*A = X\cap A = A$ 1

$\Rightarrow$ X is the identity element

(iv) $X*X = X\cap X = X \Rightarrow$ X is the only invertible element. $\because$ it is true only for X. 1

OR

$$f(x) = \frac{4x}{3x+4}$$

for $x_1, x_2 \in R - \left\{-\frac{4}{3}\right\}$, $f(x_1) = f(x_2) \Rightarrow \frac{4x_1}{3x_1+4} = \frac{4x_2}{3x_2+4}$

$\therefore \quad 12x_1x_2 + 16x_1 = 12x_1x_2 + 16x_2$

$\Rightarrow \quad x_1 = x_2$ 2

$\therefore \quad$ f is a 1 – 1 function.

for $y = \frac{4}{3}$, there is no x such that $f(x) = \frac{4}{3}$

$\therefore \quad$ f is not invertible 1

But $f : R - \left\{-\frac{4}{3}\right\} \rightarrow$ Range of f is ONTO so invertible. 1

and $f^{-1}(y) = \frac{4y}{4-3y}$ 2

27. $C_1 \rightarrow C_1 + C_2 + C_3$

$$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0 \Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$ 1

$R_2 \rightarrow R_2 - R_1, R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \quad (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0$$ 1+1

$\Rightarrow \quad -(a+b+c)(a^2+b^2+c^2-ab-bc-ca) = 0$ 1

$\Rightarrow \quad \frac{-1}{2}(a+b+c)[(a-b)^2+(b-c)^2+(c-a)^2] = 0$ 1

$\Rightarrow \quad a-b = 0 = b-c = c-a$ as $a+b+c \neq 0$ 1

$\Rightarrow \quad a = b = c$

28. Equation of plane passing through A(1, –2, 2), B(4, 2, 3) and C(3, 0, 2) is

$$[\vec{r}-(\hat{i}-2\hat{j}+2\hat{k})]\cdot[(3\hat{i}+4\hat{j}+\hat{k})\times(2\hat{i}+2\hat{j})]=0$$ $1\frac{1}{2}$

$\Rightarrow$ $\vec{r}\cdot(\hat{i}-\hat{j}+\hat{k})=5$ ...(i) $1\frac{1}{2}$

Any point on the given line is $(2+3\lambda, -1+4\lambda, 2+2\lambda)$ 1

$\Rightarrow$ When the line intersects the plane

$$((2+3\lambda)\hat{i}+(-1+4\lambda)\hat{j}+(2+2\lambda)\hat{k})\cdot(\hat{i}-\hat{j}+\hat{k})=5$$ 1

$\Rightarrow$ $\lambda=0$ $\frac{1}{2}$

$\Rightarrow$ The required point is (2, –1, 2) $\frac{1}{2}$

29. P(probability of getting 4) = $\frac{1}{10}$

P(probability of not getting 4) = $\frac{9}{10}$ 1

X	0	1	2	$\frac{1}{2}$
P(X)	$\left(\frac{9}{10}\right)^2=\frac{81}{100}$	$2\times\frac{9}{10}\times\frac{1}{10}=\frac{18}{100}$	$\left(\frac{1}{10}\right)^2=\frac{1}{100}$	$1\frac{1}{2}$
XP(X)	0	$\frac{18}{100}$	$\frac{2}{100}$	1
$X^2P(X)$	0	$\frac{18}{100}$	$\frac{4}{100}$	1

Variance $=\Sigma X^2P(X)-[\Sigma XP(X)]^2$

$$=\frac{22}{100}-\left(\frac{20}{100}\right)^2=\frac{18}{100}=0.18$$ 1