

Candidates must write the Code on the title page of the answer-book.



## NOTE

- |       |  |
|-------|--|
| (I)   | Please check that this question paper contains 15 printed pages.   |
| (II)  | Code number given on the right hand side of the question paper should be written on the title page of the answer -book by the candidate.   |
| (III) | Please check that this question paper contains 36 questions.   |
| (IV)  | Please write down the Serial Number of the question in the answer -book before attempting it.  |
| (V)   | 15 minute time has been allotted to read this question paper. The question paper will be distributed at 10.15 a.m. From 10.15 a.m. to 10.30 a.m., the students will read the question paper only and will not write any answer on the answer -book during this period. |



Time allowed : 3 hours

Maximum Marks : 80

## General Instructions :

Read the following instructions very carefully and strictly follow them :

- (i) This question paper comprises four Sections A, B, C and D. This question paper carries 36 questions. All questions are compulsory.
- (ii) Section A – Questions no. 1 to 20 comprises of 20 questions of 1 mark each .
- (iii) Section B – Questions no. 21 to 26 comprises of 6 questions of 2 marks each .
- (iv) Section C – Questions no. 27 to 32 comprises of 6 questions of 4 marks each .
- (v) Section D – Questions no. 33 to 36 comprises of 4 questions of 6 marks each .
- (vi) There is no overall choice in the question paper. However, an internal choice has been provided in 3 questions of one mark, 2 questions of two marks, 2 questions of four marks and 2 questions of six marks. Only one of the choices in such questions have to be attempted.
- (vii) In addition to this, separate instructions are given with each section and question, wherever necessary.
- (viii) Use of calculators is not permitted.

### SECTION A

Question numbers 1 to 20 carry 1 mark each.

Question numbers 1 to 10 are multiple choice type questions. Select the correct option.

1. The relation R in the set  $\{1, 2, 3\}$  given by  $R = \{(1, 2), (2, 1), (1, 1)\}$  is
  - (A) symmetric and transitive, but not reflexive
  - (B) reflexive and symmetric, but not transitive
  - (C) symmetric, but neither reflexive nor transitive
  - (D) an equivalence relation
  
2.  $\tan^{-1} 3 + \tan^{-1} \lambda = \tan^{-1} \frac{3 + \lambda}{1 - 3\lambda}$  is valid for what values of  $\lambda$  ?
  - (A)  $-\frac{1}{3}, \frac{1}{3}$
  - (B)  $> \frac{1}{3}$
  - (C)  $< \frac{1}{3}$
  - (D) All real values of

3. If  $A$  is a non-singular square matrix of order 3 such that  $A^2 = 3A$ , then value of  $|A|$  is
- (A)  $-3$   
 (B)  $3$   
 (C)  $9$   
 (D)  $27$
4. The function  $f: \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = -|x - 1|$  is
- (A) continuous as well as differentiable at  $x = 1$   
 (B) not continuous but differentiable at  $x = 1$   
 (C) continuous but not differentiable at  $x = 1$   
 (D) neither continuous nor differentiable at  $x = 1$
5. Let  $A = \{1, 3, 5\}$ . Then the number of equivalence relations in  $A$  containing  $(1, 3)$  is
- (A)  $1$   
 (B)  $2$   
 (C)  $3$   
 (D)  $4$
6. The interval in which the function  $f$  given by  $f(x) = x^2 e^{-x}$  is strictly increasing, is
- (A)  $(-\infty, 2)$   
 (B)  $(-\infty, 0)$   
 (C)  $(2, \infty)$   
 (D)  $(0, 2)$
7. If  $|a| = 4$  and  $-3 \leq a \leq 2$ , then  $|a|$  lies in
- (A)  $[0, 12]$   
 (B)  $[2, 3]$   
 (C)  $[8, 12]$   
 (D)  $[-12, 8]$
8. The vectors  $3\hat{i} - \hat{j} + 2\hat{k}$ ,  $2\hat{i} + \hat{j} + 3\hat{k}$  and  $\hat{i} + \hat{j} - \hat{k}$  are coplanar if value of  $\lambda$  is
- (A)  $-2$   
 (B)  $0$   
 (C)  $2$   
 (D) Any real number

9. The area of a triangle formed by vertices O, A and B, where  $OA = \hat{i} + 2\hat{j} + 3\hat{k}$  and  $OB = -3\hat{i} - 2\hat{j} + \hat{k}$  is
- (A)  $3\sqrt{5}$  sq. units  
 (B)  $5\sqrt{5}$  sq. units  
 (C)  $6\sqrt{5}$  sq. units  
 (D) 4 sq. units
10. The coordinates of the foot of the perpendicular drawn from the point (2, -3, 4) on the y-axis is
- (A) (2, 3, 4)  
 (B) (-2, -3, -4)  
 (C) (0, -3, 0)  
 (D) (2, 0, 4)

Fill in the blanks in question numbers 11 to 15.

11. The range of the principal value branch of the function  $y = \sec^{-1} x$  is \_\_\_\_\_.

OR

The principal value of  $\cos^{-1} -\frac{1}{2}$  is \_\_\_\_\_.

12. Given a skew-symmetric matrix  $A = \begin{pmatrix} - & & 1 \\ & - & \\ & & 0 \end{pmatrix}$ , the value of  $(a + b + c)^2$  is \_\_\_\_\_.
13. The distance between parallel planes  $2x + y - 2z - 6 = 0$  and  $4x + 2y - 4z = 0$  is \_\_\_\_\_ units.

OR

If  $P(1, 0, -3)$  is the foot of the perpendicular from the origin to the plane, then the cartesian equation of the plane is \_\_\_\_\_.

14. If the radius of the circle is increasing at the rate of 0.5 cm/s, then the rate of increase of its circumference is \_\_\_\_\_.
15. The corner points of the feasible region of an LPP are (0, 0), (0, 8), (2, 7), (5, 4) and (6, 0). The maximum profit  $P = 3x + 2y$  occurs at the point \_\_\_\_\_.

Question numbers 16 to 20 are very short answer type questions.

16. Differentiate  $\sec^{-2}(x^2)$  with respect to  $x^{-2}$ .

OR

If  $y = f(x^{-2})$  and  $f'(x) = e^{\sqrt{x}}$ , then find  $\frac{dy}{dx}$ .

17. Find the value of k, so that the function  $f(x) = \begin{cases} kx^2 - 5 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ & \text{if } x > 1 \end{cases}$  is continuous at  $x = 1$ .

18. Evaluate :

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \cos^2 x \, dx$$

19. Find the general solution of the differential equation  $e^y - x \frac{dy}{dx} = 1$ .

20. Find the coordinates of the point where the line  $\frac{x-1}{3} = \frac{y-4}{7} = \frac{z-4}{2}$  cuts the xy-plane.

## SECTION B

Question numbers 21 to 26 carry 2 marks each.

21. If  $A = \begin{pmatrix} - & 2 \\ 1 & -1 \end{pmatrix}$  and  $I = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ , find scalar  $k$  so that  $A^2 + I = kA$ .

22. If  $f(x) = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$ , find  $f\left(\frac{\pi}{3}\right)$ .

OR

Find  $f'(x)$  if  $f(x) = (\tan x)^{\tan x}$ .

23. Find :

$$\frac{\tan^3 x}{\cos^3 x} dx$$

24. Find a vector  $r$  equally inclined to the three axes and whose magnitude is  $3\sqrt{3}$  units.

OR

Find the angle between unit vectors  $a$  and  $b$  so that  $\sqrt{3}a - b$  is also a unit vector.

25. Find the points of intersection of the line  $r = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$  and the plane  $r \cdot (\hat{i} - \hat{j} + \hat{k}) = 5$ .

26. A purse contains 3 silver and 6 copper coins and a second purse contains 4 silver and 3 copper coins. If a coin is drawn at random from one of the two purses, find the probability that it is a silver coin.

## SECTION C

Question numbers 27 to 32 carry 4 marks each.

27. Check whether the relation  $R$  in the set  $N$  of natural numbers given by

$$R = \{(a, b) : a \text{ is divisor of } b\}$$

is reflexive, symmetric or transitive. Also determine whether  $R$  is an equivalence relation.

OR

Prove that  $\tan^{-1} \frac{1}{4} + \tan^{-1} \frac{2}{9} = \frac{1}{2} \sin^{-1} \frac{4}{5}$ .

28. If  $\tan^{-1} \frac{y}{x} = \log \sqrt{y^2}$ , prove that  $\frac{dy}{dx} = \frac{y}{x-y}$ .

OR

If  $y = e^{a \cos^{-1} x}$ ,  $-1 < x < 1$ , then show that

$$(1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2y = 0$$

29. Find :

$$\int \frac{1}{x^3 - x} dx$$

30. Solve the following differential equation :

$$e^x dy + e^x \left(1 - \frac{y}{x}\right) dx = 0 \quad (x > 0).$$

31. Find the shortest distance between the lines

$$r = 2\hat{i} - \hat{j} + \hat{k} + \lambda(3\hat{i} - 2\hat{j} + 5\hat{k})$$

$$r = 3\hat{i} + 2\hat{j} - 4\hat{k} + \mu(4\hat{i} - \hat{j} + 3\hat{k})$$

32. A cottage industry manufactures pedestal lamps and wooden shades. Both the products require machine time as well as craftsman time in the making. The number of hour(s) required for producing 1 unit of each and the corresponding profit is given in the following table :

Item	Machine Time	Craftsman time	Profit (in ₹)
Pedestal lamp	1.5 hours	3 hours	30
Wooden shades	3 hours	1 hour	20

In a day, the factory has availability of not more than 42 hours of machine time and 24 hours of craftsman time.

Assuming that all items manufactured are sold, how should the manufacturer schedule his daily production in order to maximise the profit? Formulate it as an LPP and solve it graphically.

## SECTION D

Question numbers 33 to 36 carry 6 marks each.

33. If  $A = \begin{pmatrix} 5 & - & 4 \\ & 5 & \\ 5 & - & 6 \end{pmatrix}$ , find  $A^{-1}$  and use it to solve the following system of equations :

$$\begin{aligned} 5x - y + 4z &= 5 \\ 2x + 3y + 5z &= 2 \\ 5x - 2y + 6z &= -1 \end{aligned}$$

OR

If  $x, y, z$  are different and  $\begin{vmatrix} 2 & x^3 \\ 2 & y^3 \\ 2 & z^3 \end{vmatrix} = 0$ , then using properties of determinants show that  $1 + xyz = 0$ .

34. Amongst all open (from the top) right circular cylindrical boxes of volume  $125 \text{ cm}^3$ , find the dimensions of the box which has the least surface area.
35. Using integration, find the area lying above  $x$ -axis and included between the circle  $x^2 + y^2 = 8x$  and inside the parabola  $y^2 = 4x$ .

OR

Using the method of integration, find the area of the triangle ABC, coordinates of whose vertices are  $A(2, 0)$ ,  $B(4, 5)$  and  $C(6, 3)$ .

36. Find the probability distribution of the random variable  $X$ , which denotes the number of doublets in four throws of a pair of dice. Hence, find the mean of the number of doublets ( $X$ ).