## PERCENTAGE

The Percentage is a fraction whose denominator is always 100 . The sign of percentage is $\%$.

## Example:

$10 \%$ can be converted to a fraction as $10 / 100=0.1$
If, we want to calculate $y \%$ of $x$, then

Percentage Formula: $\mathrm{y} \%$ of $\mathrm{x}=\mathrm{x} \times \frac{\mathrm{y}}{100}$

## Question.

If $40 \%$ of $P=100$, then find the value of $P$.
Ans. $\quad P \times 40 / 100=100$

$$
\begin{aligned}
& \Rightarrow P=100 \times 100 / 40 \\
& \Rightarrow P=250 .
\end{aligned}
$$

## Fractions and Percentages

## To express $\mathbf{x} \%$ as a fraction

$$
X \%=\frac{x}{100}
$$

Thus, $30 \%=30 / 100=3 / 10$

$$
20 \%=20 / 100=1 / 5
$$

To express $\frac{a}{b}$ as a percent
We have $\quad \frac{a}{b}=\left(\frac{a}{b} \times 100\right) \%$
Thus $\frac{1}{5}=\left(\frac{1}{5} \times 100\right) \%=20 \%$


Fig: Representation and Interpretation of \% in the form of Fraction.
Important Fraction to Percentage Conversions

| $\frac{1}{2}$ | $\underline{\mathbf{5 0 \%}}$ |
| :---: | :--- |
| $\frac{1}{3}$ | $\underline{\mathbf{3 3 . 3 3 \%}}$ |
| $\frac{1}{4}$ | $\underline{25 \%}$ |
| $\frac{1}{5}$ | $\underline{\mathbf{2 0 \%}}$ |
| $\frac{1}{6}$ | $\underline{16.66 \%}$ |
| $\frac{1}{7}$ | $\underline{14.28 \%}$ |
| $\frac{1}{8}$ | $\underline{12.5 \%}$ |
| $\frac{1}{9}$ | $\underline{11.11 \%}$ |
| $\frac{1}{10}$ | $\underline{10 \%}$ |
| $\frac{1}{11}$ | $\underline{9.09 \%}$ |
| $\frac{1}{12}$ | $\underline{8.33 \%}$ |
| $\frac{1}{13}$ | $\underline{7.69 \%}$ |
| $\frac{1}{14}$ | $\underline{\mathbf{7 . 1 4 \%}}$ |
| $\frac{1}{15}$ | $\underline{6.66 \%}$ |
| $\frac{1}{16}$ | $\underline{6.25 \%}$ |
| $\frac{1}{20}$ | $\underline{5 \%}$ |
| $\frac{1}{25}$ | $\underline{4 \%}$ |

## Basic Concepts of Percentages

Expressing One Quantity as a Percent with respect to the other:
To express a quantity as a percent with respect to other quantity, the following formula is used:
$\left(\frac{\text { The quantity to be expressed in percent }}{2 \text { nd quantity (in respect of which the percent has to be obtained })} \times 100\right) \%$

## Calculation of Percentage

1. To express $\mathrm{x} \%$ as a fraction:

We know
$\mathrm{x} \%=\mathrm{x} / 100$
Thus $10 \%=10 / 100$ (means 10 parts out of 100 parts)
$=1 / 10$ (means 1 part out of 10 parts)
2. To express $x / y$ as a percentage:

We know that $\mathrm{x} / \mathrm{y}=(\mathrm{x} / \mathrm{y} \times 100)$
Thus $1 / 4=(1 / 4 \times 100) \%=25 \%$
and $0.8=(8 / 10 \times 100) \%=80 \%$
3. To increase a number by a given percentage( $x \%$ ): Multiply the number by the following factor $=\left(\frac{100+x}{100}\right)$
4. To decrease a number by a given percentage(x\%): Multiply the number by the following factor $=\left(\frac{100-x}{100}\right)$
5. To find the \% increase of a number:

Percent Increment $=\left(\frac{\text { Final Value-Initial Value }}{\text { Initial value }} \times 100\right)$
6. To find the \% decrease of a number:

Percent Decrement $=\left(\frac{\text { Initial Value-Final Value }}{\text { Initial value }} \times 100\right)$

## Some Observations

(1) If $\mathbf{2 0 \%}$ candidate failed in an exam then observations are

- $80 \%$ represent passed in exam
- 100\% represent total appeared in exam
$\cdot(80 \%-20 \%)=60 \%$ represent difference between passed and failed candidate in exam
(2) If a number is increased by $\mathbf{2 5 \%}$ then observations are
- $100 \%$ represent the old number
- $125 \%$ represent the new number.
(3) Remember that Base in the given sentence (Question) is always $\mathbf{1 0 0 \%}$
E.g., Income of Ram is increased by 20\%

In this sentence
$100 \%$ - represent the income of Ram
20\% - represent increment
$120 \%$ - represent new income of Ram.
(4) If of $A$ is equal to $y \%$ of $B$ then -
$Z \%$ of $\mathrm{A}=\left(\frac{y z}{x}\right) \%$ of B
(5) If $\mathbf{A}$ is more than $\mathbf{B}$, then $\mathbf{B}$ is $\left(\frac{X}{100+X} \times 100\right) \%$ less than $\mathbf{A}$.

If A is $\mathrm{X} \%$ less than B , then B is $\left(\frac{X}{100-X} \times 100\right) \%$ more than A .
(6) If the passing marks in an examination is P\%. If a candidate scores S marks and fails by F marks then
$\mathrm{MM}=\frac{100 \times[R+5]}{P}$
(7) If a candidate scores marks and fails by a mark while another candidate scores y\% marks and gets $b$ marks more than minimum passing marks, then-
Maximum Marks $=\frac{\text { Sum of Scores }}{\text { Difference in } \% \text { marks }} \times 100$
(8) If due to decrement in the price of an item, a person can buy Kg more in y rupees, then actual price of that item -
$=\frac{(\text { Rate }) \times y}{(100-\text { Rate }) \times X} \operatorname{per} K g$
(9) If in an election, a candidate got of total votes cast and still lose by $y$ votes, the total number of votes cast -
$=\frac{100 \times X}{100-2 X}$
(10) If the population of a town is $P$ and it increases or decreases at the rate of $\mathrm{R} \%$ per annum then -
I. Population after ' $n$ ' years:
$=P \times\left(1 \pm \frac{R}{100}\right)^{2}$
II. Population ' $n$ ' years ago:
$=\frac{p}{\left[1 \pm \frac{R}{100}\right]^{n}}$

## PROBLEM TYPE-1:

Example: A reduction of $\mathbf{2 1 \%}$ in the price of an item enables a person to buy $\mathbf{3} \mathbf{~ k g}$ more for $\mathbf{1 0 0}$. The reduced price of item per kg is?
(a) Rs. 5.50
(b) Rs. 7.50
(c) Rs. 10.50
(d) Rs. 7.00

## Solution:(d)

Reduced price will be:
Rp/100y per kg
In our case $R=$ Rs. $100, x=21 \%, y=3 \mathrm{~kg}$
$\{(100 \times 21) /(100 \times 3)\}=$ Rs. 7
PROBLEM TYPE-2:

## PROBLEM TYPE-2

## QUESTIONS BASED ON MIXTURES

Example: A vessel has 60 L of solution of acid and water having $\mathbf{8 0 \%}$ acid. How much water is to be added to make a solution in which acid forms $60 \%$ ?
(a) 48 L
(b) 20 L
(c) 36 L
(d) None of these

Solution: (b)
Given, percentage of acid $=80 \%$
Then, percentage of water $=20 \%$
In 60 L of solution, water $=(60 \times 20) / 100=12 \mathrm{~L}$
Let p liter of water be added.
According to the question, $\Rightarrow>\{(12+p) /(60+p)\} \times 100=40(\because 100-60=40 \%$ water $)$
$=>1200+100 p=2400+40 p$
$\Rightarrow 60 \mathrm{p}=1200$
$\mathrm{p}=2 \mathrm{~L}$

## PROBLEM TYPE-3:

## QUESTIONS BASED ON RATIOS AND FRACTIONS

Example: If the numerator of a fraction is increased by $\mathbf{2 0 \%}$ and the denominator is decreased by $5 \%$, the value of the new fraction becomes $5 / 2$. The original fraction is:
(a)24/19
(b) $3 / 18$
(c) $95 / 48$
(d)48/95

Solution: (c)
Let original fraction be $\mathrm{p} / \mathrm{y}$
According to the question, $\{(120 / 100) p /(95 / 100) y\}=5 / 2$
$120 p / 95 y=5 / 2=>p / y=(5 / 2) \times(95 / 120)=95 / 48$.

## PROBLEM TYPE-4:

QUESTIONS BASED ON INCOME, SALARY, EXPENDITURE
Example: The monthly income of a person was Rs 13500 and his monthly expenditure was Rs 9000. Next year's income increased by $14 \%$ and his expenditure increased by $7 \%$. The percent increase in his savings was:
(a) $7 \%$
(b) $21 \%$
(c) $28 \%$
(d) $35 \%$

Solution: (c)
Given, monthly income $=13500$ and expenditure $=9000$
Then, original savings=Rs. (13500-9000) $=$ Rs 4500
New income $=114 \%$ of Rs. $13500=$ Rs 15390
New expenditure $=107 \%$ of Rs $9000=$ Rs 9630
New saving = Rs. $(15390-9630)=$ Rs 5760
NS = new savings, OS = Original savings

Percentage increase in savings $=\{(\mathrm{NS}-\mathrm{OS}) / \mathrm{OS}\} \times 100$
$\{(5760-4500) / 4500\} \times 100=(1260 / 4500) \times 100=28 \%$

## Practice Questions:

Q1. A man distributes $10 \%, 18 \%$ and $22 \%$ of his salary into his three children who spend $40 \%, 60 \%$ and $\mathbf{2 5 \%}$ of that amount respectively. The difference between the total amount left with the children and man is Rs. 1015. What is the salary of the man?
A. Rs. 6000
B. Rs. 4200
C. Rs. 4800
D. Rs. 5000
E. Rs. 5600

Ans- (d)

Q2. Salary of $A$ is $\mathbf{3 7 . 5 \%}$ of the total salary of $A$ and $B$. $B$ saves $60 \%$ of his salary and total savings of $A$ and $B$ is $50 \%$ of their total income. Their average expenditure is Rs $\mathbf{1 6 0 0 0}$. What is the total salary of $A$ and $B$ ?
A. Rs. 96000
B. Rs. 54000
C. Rs. 72000
D. Rs. 64000
E. Rs. 48000

Ans- (D)

Q3. In a class 25\% of the students passed in both English and Hindi. 37.5\% of the students failed in both the subjects while $60 \%$ students failed in Hindi. The difference between the students who passed in English and those who passed in Hindi is $\mathbf{1 5}$. What is the total number of students in class?
A. 180
B. 420
C. 360
D. 200
E. 240

Ans- (D)

Q4. Out of total students $\mathbf{1 0 0 / 3} \%$ are in hostel $A$ and remaining are in hostel $B$. If $\mathbf{2 0}$ students from hostel B are shifted to hostel A, then total students in hostel A becomes $50 \%$ of total students. If $\mathbf{2 0}$ students from hostel $A$ are shifted to hostel $B$, then the total students in hostel $A$ becomes what percent of total students?
A. $26.34 \%$
B. $16.67 \%$
C. $12.75 \%$
D. 20.67
E. None of these

Ans-(B)

Q5. AB de Villiers smashes 86 runs against Australia in 16 balls. If he only scored in boundaries (fours and sixes) only, then find the maximum percent of runs he scored by hitting fours.
A.23.25\%
B. $26.4 \%$
C. 74.5\%
D. $28 \%$
E. None of these

Ans-(A)

Q6. On a Big Billion-day sale, Google flagship mobile phone was available at a discount of $\mathbf{2 0 \%}$ on Flipkart. The customers who are purchasing for the first time on Flipkart will get additional cashback of $10 \%$ on the billing amount. Suraj being the 1st time user of Flipkart purchases the mobile phone for Rs. 36000, finds the actual cost price of the mobile phone.
A. Rs. 50000
B. Rs. 45000
C. Rs. 52250
D. Rs. 47250
E. None of these

Ans- (A)

Q7. As per a company policy only $\mathbf{2 5 \%}$ of the female employees and $\mathbf{2 0 \%}$ of the male employees can hold the positions higher than level 2. If the ratio of female and male employees in the company is 3: 2, then find the percentage of employees which are working below level 2.
A. $75 \%$
B. $77 \%$
C. $70 \%$
D. $72 \%$
E. 79\%

Ans- (B)

Q8. A dishonest salesman buys $x \%$ more grains than what he pays for, while selling he uses counterfeit weight which measures $\mathbf{8 0 0}$ grams for every 1000 grams. If he sells the item at $\mathbf{1 0 \%}$ above the cost price and earn an overall profit of $65 \%$, then find the value of $x$.
A. $20 \%$
B.25\%
C. $35 \%$
D. $15 \%$
E. None of these

Ans- (A)

Q9. In an exam minimum qualifying marks for class $I X$ and $X$ are $30 \%$ and $45 \%$ respectively. It is known that total marks of each class are the same and a boy of class $X$ scored 1225; thereby failing by 125 marks. Find passing marks for class IX.
A. 900
B. 1200
C. 1500
D. 925
E. None of these

Ans-(A)

Q10. ABC publication started with 2000 novels. The printing cost, packaging cost and delivery cost of each novel is Rs. 150, Rs. 20 and Rs. 50 respectively. If $40 \%$ of the novels are sold at 3/4th of the cost price, then how much percent above the cost price should the remaining novels be sold to get $\mathbf{2 0 \%}$ profit on total expenditure?
A. $25 \%$
B. $20 \%$
C. $30 \%$
D. $40 \%$
E. 50\%

Ans-(E)

Q11. A pickpocket stole the wallet of Mr. Jittu. Jittu remembers that before he lost his wallet, he bought a notebook and a marker. He pays $1 / 5$ th of his money for buying the notebook, and of the remaining, he spends $\mathbf{2 5 \%}$ on buying marker which is equal to Rs. 12. Find the amount of money lost by Mr. Jittu.
A. Rs. 125
B. Rs. 75
C. Rs. 100
D. Rs. 60
E. None of these

Ans-(E)

Q12. A survey was conducted in a village to know the reason of Deaths due to Critical Diseases. Number of people who died due to Diabetes were $\mathbf{2 0 \%}$ of the total population. It was found that $\mathbf{2 0 0 0}$ people died due to lung cancer. The people who died of Diabetes were $\mathbf{1 2 0 0}$ more than those who died of Lung Cancer. If the people who died of lung cancer were $33.33 \%$ of the people who smoke, then what percent of the total population were smokers?
A. $40 \%$
B. $62.5 \%$
C. $37.5 \%$
D. $28.50 \%$
E. $32.50 \%$

Ans- (C)

Q13. Rakul spent $10 \%$ of his yearly income on house rent, $14 \%$ on buying a new car, $12 \%$ on kids' school. He spent $15 \%$ and $10 \%$ of the remaining on groceries and vacation in Spain. If he saved Rs. 518400 in the entire year, then find his monthly salary?
A. Rs. 90000
B. Rs. 108000
C. Rs. 98000
D. Rs. 136000
E. None of these

Ans-(A)

Q14. Two villages Rampur and Jamnagar had the same population 2 years ago. Population of Rampur decreased at R\% p.a. while the population of Jamnagar increased at R\% p.a. Today, the difference between their population is 1000R, then what was the population of any village $\mathbf{2}$ years ago?
A. 15000
B. 20000
C. 25000
D. Data insufficient
E. None of these

Ans- (C)

Q15. The bank deposit of Rama is $100 \%$ more than that of Ajay and $75 \%$ more than that of Jatin. Rama's deposits are what percent of the total deposits of Ajay and Jatin together?
A. $93.67 \%$
B. $92.67 \%$
C. $93.33 \%$ s
D. $91.33 \% \mathrm{~s}$
E. None of these

Ans- (C)

Q16. In a school, $40 \%$ of students are in high school or above and rest are in junior high school or below. Of those who are in high school or above, the ratio of boys to girls is $\mathbf{7 : 3}$, and those in junior high school or below have boys to girls in ratio 7 : 5. Ratio of boys in high school or above to junior high school or below:
A. 2: 3
B. 4: 3
C. 3: 4
D. 4: 5
E. None of these

Ans- (D)

Q17. In an examination of SBI SO, Ramu scored 92\% marks, Naveen scored 56\% and Samarth scored 634 marks out of the total marks. Average marks scored by them was 643. What percentage of the total marks did Samarth get in the SBI SO exam?
A. $66.23 \%$
B. $68.34 \%$
C. $72.45 \%$
D. $76.67 \%$
E. None of these

Ans- (C)

Q18. The speed ratio of $A, B$ and $C$ is 5: 4: 3. All of them start running together on a track and match their respective wrist watches when they finish the race. $C$ completes the race in 20 min . When B finishes the race the wrist watch of A shows 7:27PM. When C finishes the race, his watch shows 7:30PM and wrist watch of $B$ shows 7:16PM. At the start of the race what is the difference between the time in the wrist watch of $A$ and $B$ ?
A. 15 min
B. 16 min
C. 12 min
D. 6 min
E. None of these

Ans- (B)

