DST
2019

1. The value of the integral

$$\int_m^n g''(y)dy,$$

where $g(y)$ is linear and $0 < m < n$, is

(A) 1. (B) $n - m$. (C) $\frac{n^2-m^2}{2}$. (D) 0.

2. The maximum value of $(4x + 4xy + 3)$, where $x + y = 2$, is

(A) 12. (B) 14. (C) 8. (D) 11.

3. Let $g(x) = x^2 + x + 1$ and $\frac{1}{3}g(f(x)) = 3x^2 - 3x + 1$. Then the form of $f(x)$ may be taken as

(A) $3x - 1$. (B) $3x - 2$. (C) $3x + 2$. (D) $3x + 1$.

4. Consider the function

$$f(x) = \begin{cases} k(x - [x]), & 0 \le x < 2 \\ 0, & \text{otherwise}, \end{cases}$$

where $[x]$ is the integer part of x. Then the value of k for which

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

is

(A) $\frac{1}{4}$. (B) $\frac{1}{2}$. (C) 1. (D) 2.

5. The four points (-2, -3), (0, 0), (2, 3) and (4, 6) are

(A) vertices of a parallelogram. (B) vertices of a rectangle.
(C) collinear. (D) lying on a circle.

6. Let $\mathbb{R}$ be the set of real numbers and let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying

$$f(0) = 1,\ f(1) = 2,\ f(2) = 7,\ f(3) = 12 \ \text{ and } \ f(4) = 18.$$

Which of the following must be in the range of f?

(A) 20 (B) 10 (C) Both 10 and 20 (D) Neither 10 nor 20.

7. Let x, y, z be consecutive positive integers such that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} > \frac{1}{10}$. Then the maximum value of $x + y + z$ is

(A) 84. (B) 93. (C) 90. (D) 87.

8. The number of functions $f\colon \{1,2,...,10\} \to \{1,2,...,10\}$ such that $f(x) \neq x$ for all x, is

(A) 10!. (B) 9^{10}. (C) 10^9. (D) $10^{10} - 1$.

9. Let $\{a_n\}$ be a sequence defined by

$$a_n = \frac{7^n}{n!}.$$

The maximum value of the sequence $\{a_n\}$ is attained at

(A) $n = 5, 6$. (B) $n = 6, 7$. (C) $n = 7, 8$. (D) $n = 8, 9$.

10. The values of x, y, z and w that satisfy

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

are, respectively,

(A) 2, 4, 1, 3. (B) 2, 4, 2, 3. (C) 1, 3, 2, 4. (D) 1, 2, 3, 4.

11. The value of

$$\lim_{x\to 2} \frac{e^{x^2} - e^{2x}}{(x-2)e^{2x}}$$

is

(A) 2. (B) 1. (C) 0. (D) ∞.

12. The set of real numbers x for which the function

$$f(x) = \Big(\frac{1}{1-\sqrt{1+2x}}\Big)^x$$

can be defined is

(A) $\{x : 1 < x < 2\}$. (B) $\{x : \frac{1}{2} < x < 1\}$.

(C) $\{x : 0 < x < \frac{1}{2}\}$. (D) $\{x : -\frac{1}{2} < x < 0\}$.

13. Let $f(x) = |x|^3 + a|x|^2 + b|x| + c$, where a, b and c are real numbers. Then the minimum number of zeros of the function $f(x)$ is

(A) 4. (B) 6. (C) 2. (D) 0.

14. How many 3-digit even numbers with distinct digits can be formed from the digits 1, 2, 3, 4, 5, 6 ?

(A) 60 (B) 72 (C) 75 (D) 108.

15. The range of values of x for which the inequality $\log_{|2-x|}(x-3) \geq -1$ holds is

(A) $x < \frac{5-\sqrt{5}}{2}$. (B) $\frac{5-\sqrt{5}}{2} < x < \frac{5+\sqrt{5}}{2}$.
(C) $x > \frac{5-\sqrt{5}}{2}$. (D) $x > \frac{5+\sqrt{5}}{2}$.

16. Let α, β and γ be the ith, jth and kth terms, respectively, of an arithmetic progression. Then the value of $\begin{vmatrix} i & \alpha & 1 \\ j & \beta & 1 \\ k & \gamma & 1 \end{vmatrix}$ is

(A) $\alpha+\beta+\gamma$. (B) 0. (C) 1. (D) $(\alpha-i)(\beta-j)(\gamma-k)$.

17. Let $\frac{1}{p}+\frac{1}{q}=1$, where p and q are positive real numbers. Then the value of $\frac{a^p}{p}+\frac{b^q}{q}$, where a and b are positive real numbers, is

(A) $\geq ab$. (B) $\leq ab$. (C) $\geq \frac{a}{b}$. (D) $\leq \frac{a}{b}$.

18. Let

$$I_1 = \int_1^2 \frac{e^x}{x} dx \quad \text{and} \quad I_2 = \int_e^{e^2} \frac{1}{\log_e x} dx.$$

Then which of the following is true?

(A) $I_1 - 2I_2 = 0$. (B) $2I_1 - I_2 = 0$.

(C) $I_1 - I_2 = 0$. (D) None of these.

19. The area (in sq. unit) of the region bounded by the curve $y=\sqrt{x-1}$, the y axis, and the lines $y=1$ and $y=5$ is

(A) $45\frac{1}{3}$. (B) $2\sqrt{3}-\frac{2}{3}$. (C) $2\sqrt{3}$. (D) $20-2\sqrt{3}$.

20. The time taken by a bacteria population to double its number is 12 hours. If the rate of increase of the bacteria population is proportional to the number of bacteria, then the time taken by the population to triple its number is

(A) 18 hours. (B) $60 \log_e 3$ hours.

(C) $\frac{12 \log_e 3}{\log_e 2}$ hours. (D) $\frac{600 \log_e 3}{\log_e 2}$ hours.

21. In a class of 80 students, there are 40 girls and 40 boys. Exactly 50 students among them wear glasses. Then the set of all possible numbers of boys without glasses is

(A) $\{10, 11, 12, ..., 40\}$. (B) $\{10, 11, 12, ..., 30\}$.
(C) $\{0, 1, 2, ..., 40\}$. (D) $\{0, 1, 2, ..., 30\}$.

22. The coefficient of $\frac{1}{x^3}$ in the expansion of $\left(\frac{1}{2\sqrt{x}} - \frac{1}{2}\right)^{10}$ is

(A) $\frac{105}{2^9}$. (B) $\frac{63}{2^8}$. (C) $\frac{63}{2^9}$. (D) $\frac{105}{2^{10}}$.

23. Let $g(x) = e^{x^3}$ and $g^{(n)}(x)$ be the n-th order derivative of $g(x)$ with respect to x. Then the value of $\frac{g^{(51)}(0)}{51!}$ is

(A) $\frac{51!}{17!}$. (B) $\frac{1}{17!}$. (C) $\frac{17!}{51!}$. (D) 0.

24. The five vowels A, E, I, O, U along with 15 X's are to be arranged in a row such that no X is at an extreme position. Also, between any two vowels there must be at least 3 X's. The number of ways in which this can be done is

(A) 1200. (B) 1800. (C) 2400. (D) 3000.

25. In a high school, the ratio of male to female students is 1 : 2. Further, the probability that a male student completes a course successfully is $\frac{7}{10}$ and that a female student does so is $\frac{4}{5}$. A student selected at random is found to have completed the course. Then the probability of the selected student being male is

(A) $\frac{16}{23}$. (B) $\frac{7}{30}$. (C) $\frac{23}{30}$. (D) $\frac{7}{23}$.

26. How many triangles can be formed with vertices of a 10-sided polygon so that no side of the triangle is a side of the polygon?

(A) $\binom{10}{3} - 60$ (B) $\binom{8}{3}$ (C) $\binom{10}{3} - 80$ (D) $\binom{10}{3} - 70$.

27. Let $f : \mathbb{R} \to \mathbb{R}$ be a strictly increasing function. Then which one of the following is always true?

(A) The limits $\lim_{x \to a+} f(x)$ and $\lim_{x \to a-} f(x)$ exist for all real numbers a.
(B) If f is differentiable at a, then $f'(a) > 0$.
(C) $\lim_{x \to \infty} f(x) = \infty$.
(D) $\lim_{x \to -\infty} f(x) = -\infty$.

28. Which of the following statements is **not** true?

(A) If $A = [1, \infty)$, then A is a closed set in $\mathbb{R}$.

(B) If A is the set of all natural numbers, then A is a closed set in $\mathbb{R}$.

(C) If A is a closed and bounded set in $\mathbb{R}$, then $s(= \sup A) \in A$.

(D) $A = \{x \in (0,1) : x \text{ is rational}\}$ is an open set in $\mathbb{R}$.

29. A regular polygon with 40 sides is inscribed in a circle. Suppose three vertices of the polygon are chosen at random with all such combinations being equally likely. What is the probability that three points form vertices of a right-angled triangle?

(A) $\frac{1}{40}$ (B) $\frac{2}{13}$ (C) $\frac{1}{13}$ (D) $\frac{3}{40}$.

30. Let A, B, C be three events such that $P(A) = 0.4,\ P(B) = 0.5,$

$$P(C) = 0.6,\ P(A \cup B) = 0.6 \text{ and } P(A \cap B \cap C^c) = 0.1,$$

where C^c denotes the complement of C. Then $P(A \cap B | C)$ is

(A) $\frac{1}{2}$. (B) $\frac{1}{3}$. (C) $\frac{1}{4}$. (D) $\frac{1}{5}$.