Mechanics of Solids.

- Interatomic force – It is the force acting between the atoms of a molecule due to electrostatic interaction between the charges of the atom.
When two atoms are brought closer to distance comparable to the atomic size, they begin to interact as follows.

a. Attractive force between the nucleous of one atom and the electron of the other atom. The attraction tends to decrease the Potential energy of the two atoms.

b. Repulsive force between the two nuclei as well as between the electron of two atoms. The repulsive tends to increase the Potential energy of the two atoms.

- Intermoleculer forces :- It is the force acting between the two molecules of a substance due to the electrostatic interaction between their oppositely charged ends. These forces operate over distance of $10^{-9}m$.
- A molecule in which the center of mass of Positive and negative charges coincide is called a non-Polar molecule.
- A molecule in which center of mass of Positive and negative charges does not coincide with the center of mass is called a Polar molecule. It behave like an electric dipole.

Points of similarities —

1. Both the forces have electrical origin.
2. Both are short range forces.
3. The Potential energy versus distance and force versus distance curve have similar general shapes for both types of forces.
4. Both type of forces are attractive upto some seperation and become repulsive for smaller seperation.

— Points of dissimilarities —

1. Interatomic forces are 50 to 100 times stronger than inter molecular forces
2. Interatomic forces are due to sharing or transfer of electrons between two atoms. Intermolecular forces are the interaction between the oppositely charged ends of induced molecular dipoles.
3. As the molecules are not generally spherical, their potential energy or force depend on their seperation as well as relative orientation.
4. The equilibrium seperation of two molecules is large compared to the equilibrium seperation of two atoms.
5. Because of the larger value of equilibrium seperation. A molecule is not restricted to attract only one molecule but can attract many molecules towards itself till the size of the molecules prevents them getting closer.

— Solid — If the forces of thermal agitation are weaker than the intermolecular forces, than a substance remain in solid state. The molecules are not to free to move, they simply vibrate about their fixed mean position. The amplitude of vibration depend on temperature but is small. Due to the strong intermolecular forces. A solid has definite shape and volume.

— Liquid — If the forces due to thermal agitation are comparable to the intermolecular forces then a substance remain in liquid state. The molecules of a liquid can slide one another even at room temperature but they can not leave the company of other molecules. so liquid have a definite volume but no definite shape.

- Gases:- If the forces due to thermal agitation are stronger than intermoleculer forces, than a substance remains in gases state. The molecules of a gas are quite free to move. They keep on — colliding against each other and moving in random manner, So gas occupies all the space available to it. Gas has neither a definite shape nor a definite volume.

- Liquid crystal — A liquid crystal is a substance that flow like a liquid but has orderly arrangement of molecules like a solid.

- Crystalline Solids:- Crystalline Solids are those in which the atoms or molecules are arranged in a regular and repeated geometrical Pattern. There is a long range order in their Structure.

characteristics of crystalline solids.

1. The atoms or molecules are arranged in a definite geometrical order in crystalline solid. giving them definite geometrical form.
2. These Solids are bounded by flat surfaces.
3. these solids have sharp melting Points.
4. These Solids are anisotropic, their Physical Properties are different along different directions.
5. The crystalline state of these material is a stable state of minimum energy.
6. They are considered as a true solids.

- Amorphous Solids:- Amorphous Solids are those in which the atoms or molecules are not arranged according to certain definite geometrical order, the atoms or molecules are arranged in random manner

Characteristics of Amorphous solids.

1. They are not bounded by flat surface.
2. They donot have sharp melting point.
3. Amorphous solids are isotropic, their physical properties are same in different directions.
4. They are considered as super cooled liquids of high viscosity.

- Semi crystalline solids :- These are the solids in which the crystalline phase is interdispered in the amorphous phase.

- Deforming force — If a force is applied on a body which is neither free to move nor free to rotate, the molecules of the body are forced to undergo a change in their relative positions. As a result the body may undergo a change in length, volume or shape. A force which changes size or shape of a body is called a deforming force.

- Elasticity — If a body regain its origional size and shape after the removel of deforming force, it is said to be elastic body and this property is called elasticity.

- Perfectly elastic body — If a body regains its origional size and shape completely and immediately after the removel of deforming force, it is said to be a perfectly elastic body.

- Plasticity — If a body does not regain its origional size and shape even after the removel of deforming force, it is said to be a plastic body and this property is called plasticity

- Perfectly plastic body — If a body does not show any tendency to regain its origional size and shape even after removal of deforming force, it is said to be a Perfectly Plastic body.

- Stress – If a body gets deformed under the action of an external force, then at each section of the body as internal force of reaction is set up, which tends to restore the body into its original state. The internal restoring force set up per unit area of cross-section of the deformed body is called stress.

$$\text{Stress} = \frac{\text{Applied force}}{\text{Area}} = \frac{F}{A}.$$

The S.I. unit is Nm^{-2} and dimensional formula is $ML^{-1}T^{-2}$.

Types of stress.

1. Tensile stress – It is the restoring force set up per unit cross-section Area of a body when the length of the body increases in the direction of the deforming force. It is also known as longitudinal stress.
2. Compressional stress – It is the force set up per unit cross-section area of a body when its length decreases under a deforming force.
3. Hydrostatic stress – If a body is subjected to a uniform force from all sides then the corresponding stress is called hydrostatic stress.
4. Tangential or shearing stress – When a deforming force acts tangentially to the surface of a body, it produces a change in the shape of a body. The tangential force applied per unit area is equal to the tangential stress.

- Strain – When a deforming force acts on a body, the body undergoes a change in its shape and size. The ratio of the change in any dimension produced in the body to the original dimension is called strain.

$$\text{Strain} = \frac{\text{Change in dimension}}{\text{original dimension}}.$$

Types.

1. Longitudinal strain – It is defined as the increase in length per unit original length, when a body is deformed by external forces.

$$\text{Longitudinal Strain} = \frac{\text{Change in length}}{\text{original length}} = \frac{\Delta L}{L}$$

2. Volumetric strain – It is defined as the change in volume per unit original volume, when the body is deformed by external force.

$$\text{Volumetric strain} = \frac{\text{Change in volume}}{\text{original volume}} = \frac{\Delta V}{V}$$

3. Shear strain – It is defined as the angle θ, through which a face originally perpendicular to the fixed face get turned on applying tangential deforming force.

$$\text{Shear Strain} = \theta = \tan\theta \rightarrow = \frac{\text{Relative displacement between two parallel plane}}{\text{Distance between parallel plane}}.$$

– Elastic limit – The maximum stress within which the body regains its original size and shape after the removal of deforming force is called elastic limit.

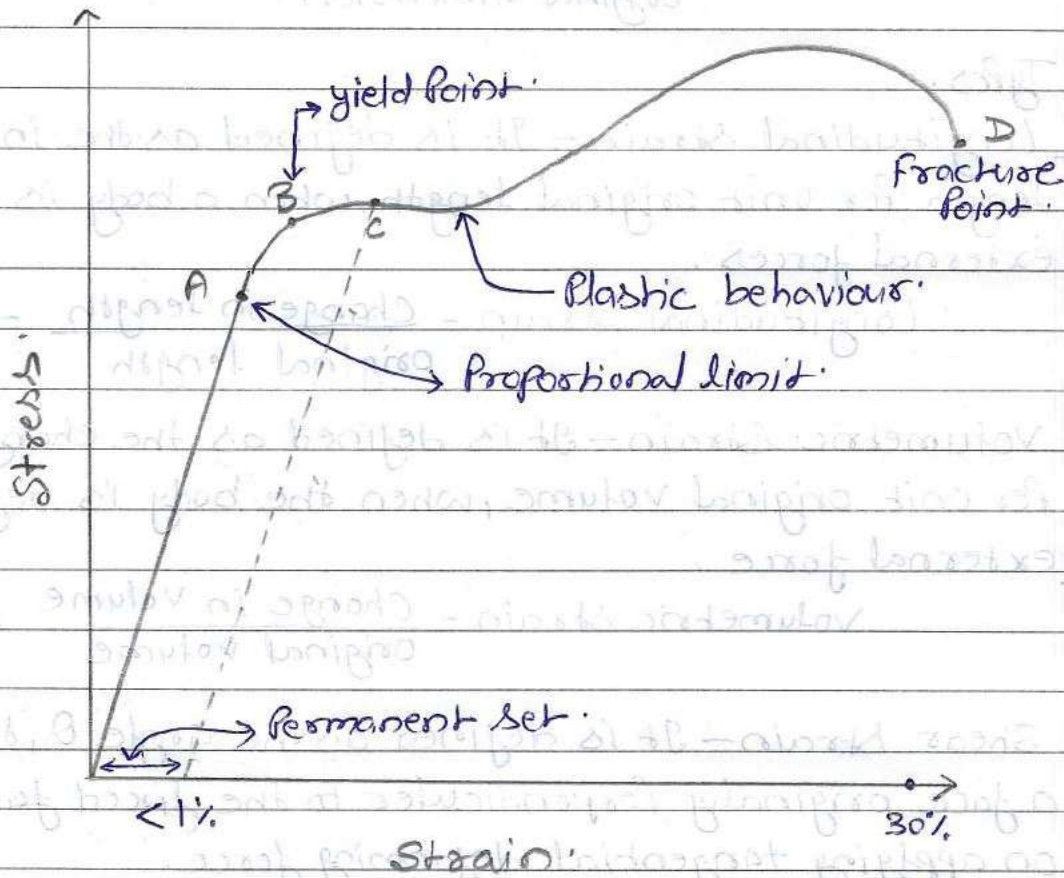
– Hook's law and modulus of elasticity – within the elastic limit, the stress is directly proportional to strain.

$$\text{Stress} \propto \text{Strain}.$$

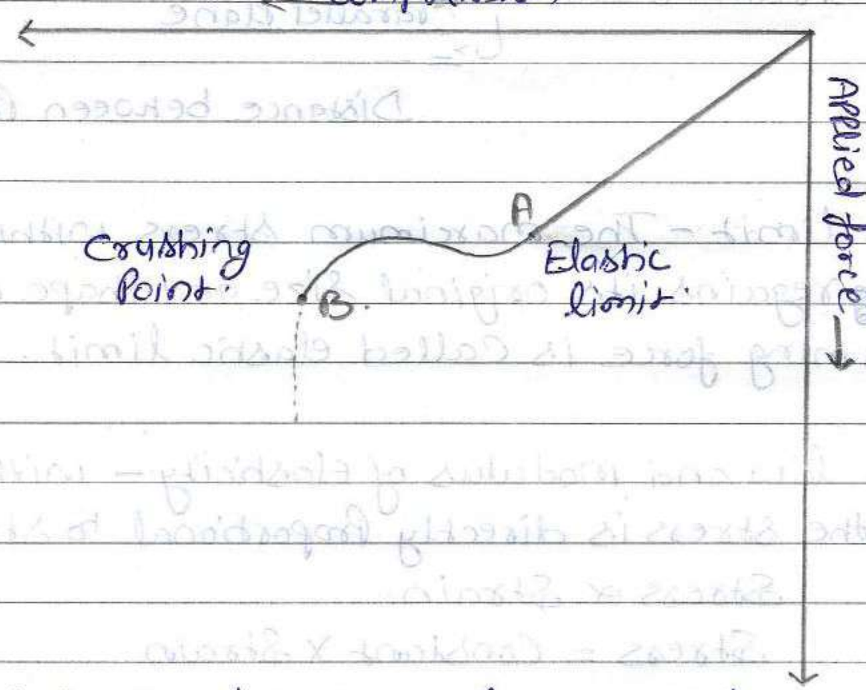
$$\text{Stress} = \text{Constant} \times \text{Strain}.$$

$$\frac{\text{Stress}}{\text{Strain}} = \text{Constant}.$$

the constant of proportionality is called modulus of elasticity.



Stress strain curve for a ductile material.



Load Compression curve for a metal.

thus the modulus of elasticity of a body is defined as the ratio of stress to the Corresponding strain, within the elastic limit.

The S.I. unit of modulus of elasticity is Nm^{-2} and its dimension formula is $ML^{-1}T^{-2}$

Types.

1 Young Modulus of elasticity — within the elastic limit, the ratio of longitudinal stress to the longitudinal strain is called Young's modulus of the material of the wire.

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{F}{A} \times \frac{L}{\Delta L} = \frac{FL}{A \Delta L} \text{ or } \frac{FL}{\pi r^2 \times \Delta L}$$

a Yield Point — The stress beyond which a solid flows is Called yield Point, example - water flows under its weight.

b. Breaking Point — The stress corresponding to which a wire break is called breaking stress.

Breaking force = Breaking stress x area of cross section of wire.

c. Plastic region — The region of stress - strain curve between the elastic limit and the breaking Point is Called Plastic region.

d. Ductile materials — The materials which have large Plastic range of extension are Called ductile material. Such materials Can be drawn into thin wires.

e. Brittle materials — The materials which have very small range of Plastic extension are Called brittle materials. Such materials break as soon as the stress is increase beyond the elastic limit.

f. malleability — when a solid is Compressed, a stage is reached beyond which it Cannot recover its original shape after the deforming force is removed. then solid behave like a Plastic body. The yield Point obtained under

Compression is called crushing point. After this stage, metals are said to be malleable. They can be hammered or rolled into thin sheets.

- Elastomers – The materials which can be elastically stretched to large values of strain are called elastomers.

- Bulk modulus of elasticity – within the elastic limit, the ratio of normal stress to the volumetric strain is called Bulk modulus of elasticity.

$$K = \frac{\text{Normal Stress}}{\text{Volumetric strain}} = \frac{F/A}{\Delta V/V} = \frac{F \times V}{A \times \Delta V}$$

- Compressibility – The reciprocal of the bulk modulus of a material is called its compressibility.

$$\text{Compressibility} = \frac{1}{K}.$$

- Modulus of rigidity or Shear modulus – within the elastic limit, the ratio of tangential stress to shear strain is called modulus of rigidity.

$$\eta = \frac{\text{Tangential Stress}}{\text{Shear strain}} = \frac{F/A}{\theta} = \frac{F}{A\theta} = \frac{F}{A} \times \frac{L}{\Delta L}$$

η of a material is smaller than its Y, This shows that it is eaiser to slide layers of atoms of solids over one another than to pull them apart or to squeeze them close together.

- Elastic after effect – the delay in regaining the original state by a body on the removel of the deforming force is called elastic after effect.
- Elastic fatigue – It is defined as loss in the strength of a material caused due to repeated alternating strains to which the material is subjected.

- Elastic hysteresis – The fact that the stress-strain curve is not retraced on reversing the strain is known as elastic hysteresis.

Q. Why any metallic part of a machinery is never subjected to a stress beyond the elastic limit.

An. Any metallic part of a machinery is never subjected to a stress beyond the elastic limit. because a stress beyond elastic limit will permanently deforms that matellic part.

Q. How is the knowledge of elasticity useful in selecting metal ropes used in crans for lifting heavy loads.

An. The thickness of metallic ropes used in cranes to lift heavy load is decided from the knowledge of the elastic limit of the material and the factor of safty.

Suppose a crane having steel ropes is required to lift load of 10 tons. The ultimate stress should not exceed the elastic limit (30×10^7 N/m²)

then. Ultimate stress $= \frac{F}{A} = \frac{10^4 \times 9.8}{\pi r^2}$.

$$\pi r^2 = \frac{10^4 \times 9.8}{30 \times 10^7}$$

$$r = .032 \text{ meter or } 3.2 \text{ cm}.$$

Q. Explain why should the beams used in the construction of bridges have large depth and small breath

or.

Explain why are garders given I shape.

Ans. The knowledge of elasticity is applied in designing a bridge such that it does not bend too much or break under the load of traffic, the force of wind and under its own weight. Consider a ranctangular bar of length l, breath b and thickness d, supported at both ends. when a load W is suspended at its middle, the bar gets depressed by an amount δ.

than

$$\delta = \frac{Wl^3}{4Ybd^3}$$

Bending can be reduced by using a material with a large Young modulus Y. As δ is proportional to d^{-3} and only b^{-1}, so depression can be reduced more effectively by increasing the depth rather than breath. But a deep bar has a tendency to bend under the weight of a moving traffic. This bending is called buckling. Hence a batter choice is to have a bar of I shape cross section. This section provides a large load bearing surface and enough depth to prevent bending. Also this shape reduces the weight of the beam without scrificing its strength and reduces the cost.

Q. How can the knowledge of elasticity be used to estimate the maximum height of a mountain on the earth.

An. The maximum height of mountain on earth depends upon shear modulus of rock. At the base of the mountain, the stress due to all the rocks on the top should be less than the critical shear stress at which the rock begins to flow.

Suppose the height of mountain is 'h' and density of its rock is 'ρ', then the force per unit area at the base is $= h\rho g$. Hence there is a tangential shear of order of $h\rho g$. The elastic limit for a typical rock is about 3×10^8 N/m^2 and its density is 3×10^3 kg/m^3.

than $h\rho g = 3\times10^8$

$$h = \frac{3\times10^8}{\rho g} = \frac{3\times10^8}{9.8\times3\times10^3} = 10{,}000\,m = 10\,km.$$

Q. Explain why hollow shafts are preferred to solid shafts for transmitting torque.

An. A hollow shaft is stronger than a solid shaft made of equal quantity of same material.

The torque required to produce unit twist in a solid shaft of radius r, length l and made of material of modulus of rigidity η is

$$\tau = \frac{\pi \eta r^4}{2l}.$$

The torque required to produce a unit twist in a hollow shaft of internal and external radius r_1 and r_2

$$\tau' = \frac{\pi \eta (r_2^4 - r_1^4)}{2l}$$

$$\frac{\tau'}{\tau} = \frac{\pi \eta (r_2^4 - r_1^4)}{2l} \times \frac{2l}{\pi \eta (r^4)} = \frac{r_2^4 - r_1^4}{r^4} \quad -(1)$$

If the two shafts are made from equal amount of material than.

$$\pi r^2 l = \pi (r_2^2 - r_1^2) l$$

$$\text{or } r_2^2 - r_1^2 = r^2 \quad -(2).$$

By (1) and (2).

$$\frac{\tau'}{\tau} = \frac{(r_2^2 + r_1^2)(r_2^2 - r_1^2)}{r^4} = \frac{(r_2^2 + r_1^2) r^2}{r^4}$$

$$= \frac{(r_2^2 + r_1^2)}{r^2}$$

As $r_2^2 - r_1^2 = r^2$ than $r_2^2 + r_1^2 > r^2$ then $\tau' > \tau$

Hence a hollow shaft is stronger than a solid shaft.

— Elastic Potential energy: when a wire is stretched, interatomic forces comes into play which oppose the change. Work has to be done against these restoring forces. The work done in stretching the wire is stored in it as its elastic potential energy.

– Expression for elastic Potential energy – Suppose a force 'F' applied on a wire of length l, increases its length by Δl. Initially, the internal restoring force in the wire is zero. When the length is increased by Δl, the internal force increases from zero to F

then Average internal force $= \frac{0+F}{2} = \frac{F}{2}$.

Work done on the wire is.

$$W = \text{Average force} \times \text{increase length}$$
$$= \frac{F}{2} \times \Delta l.$$

This work done is stored as elastic Potential energy U in the wire.

$$U = \frac{F}{2} \times \Delta l$$

Let A be the cross section Area of wire.

$$U = \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{l} \times Al.$$

$$= \frac{1}{2} \times \text{Stress} \times \text{strain} \times \text{volume of wire}.$$

Elastic Potential energy Per unit volume of the wire or elastic energy density is

$$u = \frac{U}{\text{Volume}} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

But Stress = Young modulus × Strain.

$$\therefore \; u = \frac{1}{2} \times \text{Young modulus} \times (\text{Strain})^2$$

– Poisson's ratio :- When a wire is loaded, its length increases but its dimeter decreases. The strain Produced in the direction of applied force is called.

longitudinal strain and that produced in the perpendicular direction is called lateral strain.

Within the elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's ratio.

Suppose the length of the loaded wire increases from l to $l+\Delta l$ and its diameter decreases from D to $D-\Delta D$

Longitudinal strain $= \frac{\Delta l}{l}$

Lateral strain $= \frac{\Delta D}{D}$

$$\text{Piossion's ratio } '\sigma' = \frac{\text{Lateral strain}}{\text{Longitudinal strain}} = \frac{-\Delta D}{D} \cdot \frac{l}{\Delta l}$$

$$= \frac{-l}{D} \cdot \frac{\Delta D}{\Delta l}$$

Negative sign indicates the longitudinal and lateral strain are in opposite sign.

In theoretical σ lie between -1 to 0.5

In actual practice σ lie between 0 and 0.5.

– Relation between Y, K, η, σ

I $Y = 3K(1-2\sigma)$

II $Y = 2\eta(1+\sigma)$

(III) $\sigma = \frac{3K-2\eta}{6K+2\eta}$

(IV) $\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}$

Q. Determine the Poisson's ratio of the material of a wire whose volume remains constant under an external normal stress.

Ans Volume of wire $= \pi \cdot \frac{D^2}{4} \cdot l$

As volume remains constant so

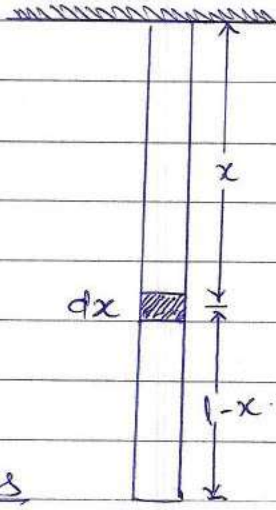
the differentiation of the equation.

$$0 = \frac{\pi l}{4} \cdot 2D\, dD + \frac{\pi D^2}{4} \cdot dl$$

$$-2l \cdot dD = D \cdot dl \quad \text{or} \quad \frac{dD}{D} = -\frac{1}{2} \cdot \frac{dl}{l}$$

then Poisson's ratio $\sigma = \frac{-dD/D}{dl/l} = \frac{\frac{1}{2} \cdot dl/l}{dl/l} = \frac{1}{2} = 0.5$.

Q. A uniform heavy rod of weight W, cross section Area A and length l, is hanging from a fixed support. Young modulus of the material of the rod is Y. Neglecting the lateral contraction find the alongation produced in the rod.



Ans. Consider a small element of thickness dx at distance x from the fixed support. Force acting on the element dx is.

F = weight of length $(l-x)$ of the rod.

$$= \frac{W}{l} \times (l-x).$$

Elongation of the element = original length $\times \frac{\text{Stress}}{Y}$

$$= dx \times \frac{F/A}{Y} = \frac{W(l-x) \cdot dx}{l \times A \times Y}$$

Total elongation produced in the rod.

$$= \frac{W}{lAY} \int_0^l (l-x)\,dx = \frac{W}{lAY} \left[lx - \frac{x^2}{2} \right]_0^l$$

$$= \frac{W}{lAY} \left[l^2 - \frac{l^2}{2} \right] = \frac{Wl^2}{2lAY} = \frac{Wl}{2AY}$$