Sample Paper 4

ANSWERKEY																			
1	(d)	2	(d)	3	(c)	4	(a)	5	(b)	6	(c)	7	(b)	8	(d)	9	(a)	10	(c)
11	(c)	12	(c)	13	(a)	14	(c)	15	(c)	16	(b)	17	(b)	18	(b)	19	(c)	20	(a)
21	(d)	22	(a)	23	(b)	24	(a)	25	(c)	26	(d)	27	(b)	28	(a)	29	(b)	30	(b)
31	(b)	32	(d)	33	(c)	34	(b)	35	(c)	36	(d)	37	(a)	38	(b)	39	(b)	40	(b)
41	(a)	42	(d)	43	(a)	44	(b)	45	(c)	46	(a)	47	(c)	48	(a)	49	(b)	50	(b)

SOLUTIONS

1. **(d)** We have, $\frac{\tan\theta - \cot\theta}{\sin\theta\cos\theta}$

$= \frac{\tan\theta}{\sin\theta\cos\theta} - \frac{\cot\theta}{\sin\theta\cos\theta}$

$= \frac{\sin\theta}{\cos\theta\sin\theta\cos\theta} - \frac{\cos\theta}{\sin\theta\cos\theta\cos\theta}$

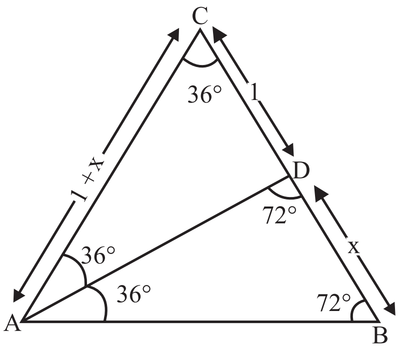
$= \frac{1}{\cos^2\theta} - \frac{1}{\sin^2\theta} = \sec^2\theta - \text{cosec}^2\theta$

$= 1 + \tan^2\theta - 1 - \cot^2\theta = \tan^2\theta - \cot^2\theta$

2. **(d)** L.C.M × H.C.F = First number × second number

Hence, required number = $\frac{36 \times 2}{18} = 4$.

3. **(c)** Let BD = x cm



Since AC = BC, therefore ΔABC is an isoscele triangle.

$\Rightarrow$ ∠B = ∠CAB = 72°

Since AD bisects ∠A

$\therefore$ ∠DAB = 36° so, In ΔADB, ∠ADB = 72°

$\Rightarrow$ ΔADB is an isoscele triangle

$\therefore$ AB = AD = 1cm

$\Rightarrow$ AB = 1 cm

Similarly, ΔADC is also an isoscele triangle.

$\therefore$ AD = CD $\Rightarrow$ AD = 1 cm

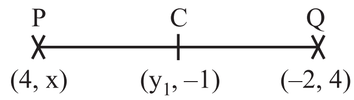
Now $\frac{AC}{AB} = \frac{CD}{BD}$

$\Rightarrow \frac{1+x}{1} = \frac{1}{x} \Rightarrow x + x^2 - 1 = 0$

$\Rightarrow x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

BD = $\frac{\sqrt{5} - 1}{2}$

4. **(a)** Since, $C(y, -1)$ is the mid-point of $P(4, x)$ and $Q(-2, 4)$.



We have, $\frac{4-2}{2} = y$ and $\frac{4+x}{2} = -1$

$\therefore$ $y = 1$ and $x = -6$

5. **(b)** Area of sector = 240°/360° × $\pi(100)^2$ = 20933 cm^2.

Let r be the radius of the new circle, then

$$20933 = \pi r^2 \Rightarrow r = \sqrt{\frac{20933}{\pi}} = 81.6 \text{ cm.}$$

6. **(c)** Required probability $= \frac{5}{25} = \frac{1}{5}$.

7. **(b)** Value of $n = 2$.

8. **(d)** Put $a = b$ in given polynomial. Remainder comes to be 0.

9. **(a)** Let the radii of the outer and inner circles be r_1 and r_2 respectively; we have

Area $= \pi r_1^2 - \pi r_2^2 = \pi(r_1^2 - r_2^2)$

$= \pi(r_1 - r_2)(r_1 + r_2)$

$= \pi(5.7 - 4.3)(5.7 + 4.3) = \pi \times 1.4 \times 10$ sq. cm

$= 3.1416 \times 14$sq. cm. = 43.98 sq. cms.

10. **(c)** Given, area of two similar triangles,

$A_1 = 81\text{cm}^2$, $A_2 = 49\text{ cm}^2$

Ratio of corresponding medians $= \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{81}{49}} = \frac{9}{7}$

11. **(c)** We have, $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

$$\Rightarrow \cos\theta\left(\frac{1+\sin\theta+1-\sin\theta}{1-\sin^2\theta}\right) = 4$$

$$\Rightarrow \frac{2\cos\theta}{\cos^2\theta} = 4 \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

12. **(c)** Let the required ratio be $k : 1$

Then, $2 = \frac{6k - 4(1)}{k+1}$ or $k = \frac{3}{2}$

$\therefore$ The required ratio is $\frac{3}{2}:1$ or $3:2$

Also, $y = \frac{3(3)+2(3)}{3+2} = 3$

13. **(a)** Since, p_1 and p_2 are odd primes and sum of two odd number is an even number.

So, $p_1 + p_2$ is an even number.

Since, multiple of even number is always even.

Therefore, $(p_1 + p_2)(p_1 - p_2)$ is even

Hence, $p_1^2 - p_2^2 = (p_1 + p_2)(p_1 - p_2)$ is an even number.

14. **(c)** Let unit's digit : x, tens digit : y

then $x = 2y$, number $= 10y + x$

Also $10y + x + 36 = 10x + y$

$\therefore 9x - 9y = 36$ or $x - y = 4$

Solve, $x = 2y$, $x - y = 4$

Substitute $x = 2y$ in $x - y = 4$

we get, $2y - y = 4 \Rightarrow y = 4$

and $x = 8$

So, the number $= 10y + x = 48$

15. **(c)** Total outcomes = HH, HT, TH, TT

Favourable outcomes = HT, TH, TT

P(at most one head) $= \frac{3}{4}$.

16. **(b)** We have,

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta - 1)}$$

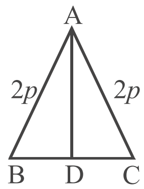
$$= \tan\theta\left[\frac{1-2(1-\cos^2\theta)}{2\cos^2\theta - 1}\right] = \tan\theta\left[\frac{(2\cos^2\theta - 1)}{2\cos^2\theta - 1}\right]$$

$= \tan\theta$

17. **(b)** Given an equilateral triangle ABC in which

$AB = BC = CA = 2p$

and $AD \perp BC$.



$\therefore$ In ΔADB,

$AB^2 = AD^2 + BD^2$

(By Pythagoras theorem)

$\Rightarrow (2p)^2 = AD^2 + p^2 \Rightarrow AD^2 = \sqrt{3}\,p$.

18. **(b)** $x^2 + 4x + 2 = (x^2 + 4x + 2) - 2 = (x + 2)^2 - 2$

Lowest value = – 2 when $x + 2 = 0$

19. **(c)** $-\frac{3(1)+4(2)-7}{3(-2)+4(1)-7} = -\frac{4}{-9} = \frac{4}{9}$

20. **(a)** Required area $= \left(7^2 - \frac{1}{4} \times \frac{22}{7} \times 7^2\right)$ cm^2

$= (49 - 38.5)\text{ cm}^2 = 10.5\text{ cm}^2$

21. **(d)** We have, $\frac{2\tan 30°}{1+\tan^2 30°}$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{2\times 3}{\sqrt{3}\times 4} = \frac{\sqrt{3}}{2}$$

Alternate method:

$\left(\text{Using identity}, \sin 2A = \dfrac{2\tan A}{1+\tan^2 A}\right)$

$\sin 60° = \dfrac{2\tan 30°}{1+\tan^2 30°} = \dfrac{\sqrt{3}}{2}$

22. **(a)** The largest number of four digits is 9999. Least number divisible by 12, 15, 18, 27 is 540.
On dividing 9999 by 540, we get 279 as remainder.
Required number = (9999 – 279) = 9720.

23. **(b)** Let $P(x, 0)$ be a point on X-axis such that $AP = BP$

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x+2)^2 + (0-3)^2 = (x-5)^2 + (0+4)^2$

$\Rightarrow x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16$

$\Rightarrow 14x = 28 \Rightarrow x = 2$

Hence, required point is (2, 0).

24. **(a)** Let side of a square = x cm

$\therefore$ By Pythagoras theorem, $x^2 + x^2 = (16)^2 = 256$

$\Rightarrow 2x^2 = 256 \Rightarrow x^2 = 128 \Rightarrow x = 8\sqrt{2}$ cm.

25. **(c)** $\dfrac{3x+4y}{x+2y} = \dfrac{9}{4}$

$\Rightarrow 4(3x + 4y) = 9(x + 2y)$

Hence, $12x + 16y = 9x + 18y$ or $3x = 2y$

$\therefore x = \dfrac{2}{3}y$.

Substitute $x = \dfrac{2}{3}y$ in the required expression.

i.e. $3x + 5y : 3x - y$

$= 3\left(\dfrac{2}{3}y\right) + 5y : 3\left(\dfrac{2}{3}y\right) - y$

$= 2y + 5y : 2y - y$

$= 7y : y = 7 : 1$

26. **(d)**

27. **(b)** $10x = 7.\overline{7}$ or $x = 0.\overline{7}$

Subtracting, $9x = 7 \quad \therefore \quad x = \dfrac{7}{9}$

$2x = \dfrac{14}{9} = 1.555........ = 1.\overline{5}$

28. **(a)** There are 2 favourable choice (3, 7) for unit place.

$P = 1 \times 1 \times \dfrac{2}{5} = \dfrac{2}{5}$

29. **(b)**

30. **(b)** The numbers that can be formed are xy and yx. Hence $(10x + y) + (10y + x) = 11(x + y)$. If this is a perfect square then $x + y = 11$.

31. **(b)** We know that

$13! = 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \times 11 \times 12 \times 13$

$= 2^{10} \times 3^5 \times 5^2 \times 7 \times 11 \times 13 \Rightarrow 24^k = (2^3 \times 3)^k$

where k is largest non-negative integer

When 13! is an divided by 24^k, we get

$\dfrac{2^{10} \times 3^5 \times 5^2 \times 7 \times 11 \times 13}{2^{3k}.3^k}$

$= 2^{10-3k} . 3^{5-k} . 5^2 \times 7 \times 11 \times 13$

$\therefore$ $10 - 3k$ is integer.

Then, maximum value of $k = 3$.

32. **(d)** Since, $\Delta ABC \sim \Delta PQR$

$\therefore \dfrac{ar(\Delta ABC)}{ar(\Delta PQR)} = \dfrac{BC^2}{QR^2} \Rightarrow \dfrac{9}{16} = \dfrac{(4.5)^2}{QR^2}$

$\Rightarrow QR^2 = \dfrac{16 \times (4.5)^2}{9} \Rightarrow QR = 6$ cm

33. **(c)**

34. **(b)** Centroid is $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$

i.e. $\left(\dfrac{3 + (-8) + 5}{3}, \dfrac{-7 + 6 + 10}{3}\right) = \left(\dfrac{0}{3}, \dfrac{9}{3}\right) = (0, 3)$

35. **(c)** Required probability $= \dfrac{1+2+1}{11} = \dfrac{4}{11}$.

36. **(d)** The expressions $(x - 1)(x + 1)$ and $(x - 1)(x - 1)$ which vanish if $x = 1$.

37. **(a)** **Given:** The natural number, when divided by 13 leaves remainder 3

The natural number, when divided by 21 leaves remainder 11

So, 13 – 3 = 21 – 11 = 10 = k

Now, LCM (13, 21) = 273

But the number lies between 500 and 600

$\therefore$ 2 LCM (13, 21) – k = 546 – 10 = 536

$536 = 19 \times 8 + 4 \quad \therefore$ remainder = 4

38. **(b)** $\sin\theta + \sin^3\theta = \cos^2\theta$

$\sin\theta(1 + 1 - \cos^2\theta) = \cos^2\theta$

$\Rightarrow \sin^2\theta(2 - \cos^2\theta)^2 = \cos^4\theta$

$\Rightarrow (1 - \cos^2\theta)(4 + \cos^4\theta - 4\cos^2\theta) = \cos^4\theta$

$\Rightarrow 4 + \cos^4\theta - 4\cos^2\theta - 4\cos^2\theta - \cos^6\theta + 4\cos^4\theta = \cos^4\theta$

$\Rightarrow \cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

39. **(b)** Since, $\Delta ABC \sim \Delta APQ$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta APQ)} = \frac{BC^2}{PQ^2}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{4 \cdot ar(\Delta ABC)} = \frac{BC^2}{PQ^2} \Rightarrow \left(\frac{BC}{PQ}\right)^2 = \frac{1}{4}$$

$$\Rightarrow \frac{BC}{PQ} = \frac{1}{2}$$

40. **(b)** If the two digits are x and y, then the number is $10x + y$.

Given that, $\frac{5}{6}(10x + y) = 10y + x$. Solving it,

we get $44x + 55y \Rightarrow \frac{x}{y} = \frac{5}{4}$.

Also $x - y = 1$. Solving them, we get $x = 5$ and $y = 4$.

Therefore, number is 54.

41. **(a)** Area of $\Delta ABC = \frac{\sqrt{3}}{4}a^2$

$$17320.5 = \frac{\sqrt{3}}{4}a^2$$

$$a^2 = \frac{17320.5 \times 4}{1.73205} = 40000$$

a = 200 cm

42. **(d)** Radius of circle $= \frac{200}{2}$

= 100 cm

43. **(a)** Area of each sector

$$= \frac{60}{360} \times \pi r^2$$

$$= \frac{1}{6} \times 3.14 \times 10000 = 5233.3 \text{ cm}^2$$

44. **(b)** Area of the shaded region

= Area of ΔABC – 3 × Area of each sector

$$= 17320.5 - 3 \times \frac{31400}{6} = 1620.5 \text{ cm}^2$$

45. **(c)** Perimeter of ΔABC = 3 × 200 = 600 cm

46. **(a)** **47.** **(c)** **48.** **(a)** **49.** **(b)** **50.** **(b)**