QUESTION PAPER CODE 65/3/RU

EXPECTED ANSWERS/VALUE POINTS

SECTION - A

Marks

1. $\frac{dx}{dy} + \frac{2y}{1+y^2} . x = \cot y$ ½ m

Integrating factor = $e^{\log(1+y^2)}$ or $(1+y^2)$ ½ m

2. $\Delta = \begin{vmatrix} x+y+z & x+y+z & x+y+z \\ z & x & y \\ -3 & -3 & -3 \end{vmatrix}$ ½ m

$= 0$ ½ m

3. order 2, degree 1 (any one correct) ½ m

sum = 3 ½ m

4. $\frac{x - 3/5}{1/5} = \frac{y + 7/15}{1/15} = \frac{z - 3/10}{-1/10}$ ½ m

Direction cosines are $\frac{6}{7}, \frac{2}{7}, \frac{-3}{7}$ or $\frac{-6}{7}, \frac{-2}{7}, \frac{3}{7}$ ½ m

5. $\left| 2\hat{a} + \hat{b} + \hat{c} \right|^2 = (2\hat{a})^2 + (\hat{b})^2 + (\hat{c})^2 + 2\left(2\hat{a}\cdot\hat{b} + \hat{b}\cdot\hat{c} + \hat{c}\cdot 2\hat{a}\right)$ ½ m

$\therefore \left| 2\hat{a} + \hat{b} + \hat{c} \right| = \sqrt{6}$ ½ m

6. $\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{vmatrix} = -\hat{i} + \hat{j}$ ½ m

unit vector is $-\frac{\hat{i}}{\sqrt{2}} + \frac{\hat{j}}{\sqrt{2}}$ ½ m

SECTION - B

7. $\vec{a}-\vec{b} = -\hat{i}+\hat{j}+\hat{k}\,;\ \vec{c}-\vec{b} = \hat{i}-5\hat{j}-5\hat{k}$ 1½ m

$$\left(\vec{a}-\vec{b}\right)\times\left(\vec{c}-\vec{b}\right) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & -5 & -5 \end{vmatrix} = -4\hat{j} + 4\hat{k}$$ 1½ m

$\therefore$ Unit vector perpendicular to both of the vectors $= -\dfrac{\hat{j}}{\sqrt{2}} + \dfrac{\hat{k}}{\sqrt{2}}$ 1 m

8. let the equation of line passing through $(1, 2, -4)$ be

$\vec{r} = \hat{i}+2\hat{j}-4\hat{k} + \lambda\left(a\hat{i}+b\hat{j}+c\hat{k}\right)$ 1 m

Since the line is perpendicular to the two given lines $\therefore$

$\therefore$ $3a - 16b + 7c = 0$

$3a + 8b - 5c = 0$ 1½ m

Solving we get, $\dfrac{a}{24} = \dfrac{b}{36} = \dfrac{c}{72}$ or $\dfrac{a}{2} = \dfrac{b}{3} = \dfrac{c}{6}$ 1 m

$\therefore$ Equation of line is : $\vec{r} = \hat{i}+2\hat{j}-4\hat{k} + \lambda\left(2\hat{i}+3\hat{j}+6\hat{k}\right)$ ½ m

OR

Equation of plane is : $\begin{vmatrix} x+1 & y-2 & z \\ 2+1 & 2-2 & -1 \\ 1 & 1 & -1 \end{vmatrix} = 0$ 3 m

Solving we get, $x+2y+3z-3=0$ 1 m

9. Let x = No. of spades in three cards drawn

x	:	0	1	2	3	1 m
P(x)	:	$^3C_0\left(\frac{3}{4}\right)^3 = \frac{27}{64}$	$^3C_1\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)^2 = \frac{27}{64}$	$^3C_2\left(\frac{1}{4}\right)^2\frac{3}{4} = \frac{9}{64}$	$^3C_3\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)^0 = \frac{1}{64}$	2 m
x . P(x)	:	0	$\frac{27}{64}$	$\frac{18}{64}$	$\frac{3}{64}$	½ m

$$\text{Mean} = \sum x \cdot P(x) = \frac{48}{64} = \frac{3}{4}$$ ½ m

OR

let p = probability of success ; q = Probability of failure

then, $9\,P(x=4) = P(x=2)$

$\Rightarrow \quad 9 \cdot {}^6C_4\, p^4 \cdot q^2 = {}^6C_2 \cdot p^2 \cdot q^4$ 2 m

$\Rightarrow \quad 9p^2 = q^2 \quad \therefore \quad q = 3p$ 1 m

Also, $p + q = 1 \Rightarrow p + 3p = 1 \quad \therefore \quad p = \frac{1}{4}$ 1 m

10. $y = e^{m \sin^{-1} x}$, differentiate w.r.t. "x", we get $\frac{dy}{dx} = \frac{m\, e^{m \sin^{-1} x}}{\sqrt{1-x^2}}$ 1½ m

$\Rightarrow \sqrt{1-x^2}\, \frac{dy}{dx} = my$, Differentiate again w.r.t. "x"

$\Rightarrow \sqrt{1-x^2}\, \frac{d^2y}{dx^2} - \frac{x}{\sqrt{1-x^2}} \frac{dy}{dx} = m \frac{dy}{dx}$ 1½ m

$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = m\left(\sqrt{1-x^2}\, \frac{dy}{dx}\right) = m\,(my)$ ½ m

$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - m^2 y = 0$ ½ m

11. $f(x) = \sqrt{x^2+1}$, $g(x) = \frac{x+1}{x^2+1}$, $h(x) = 2x - 3$

Differentiating w.r.t. "x", we get

$f'(x) = \frac{x}{\sqrt{x^2+1}}$, $g'(x) = \frac{1-2x-x^2}{(x^2+1)^2}$, $h'(x) = 2$ 1+1½+1 m

$\therefore \quad f'(h'(g'(x))) = \frac{2}{\sqrt{5}}$ ½ m

12. $\int (3-2x)\sqrt{2+x-x^2}\,dx = 2\int \sqrt{\left(\frac{3}{2}\right)^2 - \left(x-\frac{1}{2}\right)^2}\,dx + \int (1-2x)\sqrt{2+x-x^2}\,dx$ 2 m

$= 2\cdot\left\{ \frac{x-\frac{1}{2}}{2}\sqrt{2+x-x^2} + \frac{9}{8}\sin^{-1}\left(\frac{x-\frac{1}{2}}{\frac{3}{2}}\right) \right\} + \frac{2}{3}\left(2+x-x^2\right)^{3/2} + c$ 2 m

or $\left(\frac{2x-1}{2}\sqrt{2+x-x^2} + \frac{9}{4}\sin^{-1}\left(\frac{2x-1}{3}\right) + \frac{2}{3}\left(2+x-x^2\right)^{3/2} + c \right)$

OR

$\int \frac{x^2+x+1}{(x^2+1)(x+2)}\,dx = \frac{1}{5}\int \frac{2x+1}{x^2+1}\,dx + \frac{3}{5}\int \frac{1}{x+2}\,dx$ 2 m

$= \frac{1}{5}\int \frac{2x}{x^2+1}\,dx + \frac{1}{5}\int \frac{1}{x^2+1}\,dx + \frac{3}{5}\int \frac{1}{x+2}\,dx$ ½ m

$= \frac{1}{5}\log\left|x^2+1\right| + \frac{1}{5}\tan^{-1}x + \frac{3}{5}\log\left|x+2\right| + c$ 1½ m

13. $\int_0^{\pi/4} \frac{1}{\cos^3 x\sqrt{2\sin 2x}}\,dx = \int_0^{\pi/4} \frac{1}{\cos^4 x\, 2\sqrt{\tan x}}\,dx$ 1 m

$= \int_0^{\pi/4} \frac{(1+\tan^2 x)}{2\sqrt{\tan x}}\sec^2 x\,dx$ 1 m

$= \frac{1}{2}\int_0^1 \frac{1+t^2}{\sqrt{t}}\,dt$ Taking, $\tan x = t$; 1 m

$= \frac{1}{2}\left[2\sqrt{t} + \frac{2}{5}t^{5/2}\right]_0^1$ ½ m

$= \frac{1}{2}\left[2+\frac{2}{5}\right] = \frac{6}{5}$ ½ m

14. $\int \log x \cdot \frac{1}{(x+1)^2}\, dx = \log x \cdot \frac{-1}{x+1} + \int \frac{1}{x} \cdot \frac{1}{x+1}\, dx$ 2 m

$= \frac{-\log x}{x+1} + \int \frac{1}{x} dx - \int \frac{1}{x+1}\, dx$ 1 m

$= \frac{-\log x}{x+1} + \log x - \log(x+1) + c$ 1 m

or $\frac{-\log x}{x+1} + \log\left(\frac{x}{x+1}\right) + c$

15. $\begin{pmatrix} 400 & 300 & 100 \\ 300 & 250 & 75 \\ 500 & 400 & 150 \end{pmatrix} \begin{pmatrix} 50 \\ 20 \\ 40 \end{pmatrix} = \begin{pmatrix} 30000 \\ 23000 \\ 39000 \end{pmatrix}$ 2 m

cost incurred respectively for three villages is Rs. 30,000, Rs. 23,000, Rs. 39,000 1 m

One value : Women welfare or Any other relevant value 1 m

16. $\tan^{-1}\left(\frac{x+1+x-1}{1-(x+1)(x-1)}\right) = \tan^{-1}\left(\frac{8}{31}\right)$ 2 m

$\Rightarrow \frac{2x}{2-x^2} = \frac{8}{31} \quad \therefore \quad 4x^2 + 31x - 8 = 0$ 1 m

$\therefore \quad x = \frac{1}{4}$, -8 (Rejected) 1 m

OR

L.H.S. $= \tan^{-1}\left(\frac{x-y}{1+xy}\right) + \tan^{-1}\left(\frac{y-z}{1+yz}\right) + \tan^{-1}\left(\frac{z-x}{1+zx}\right)$ 2 m

$= \tan^{-1}x - \tan^{-1}y + \tan^{-1}y - \tan^{-1}z + \tan^{-1}z - \tan^{-1}x$

$= 0 = \text{RHS}$ 2 m

17. $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = abc \begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$

Taking a, b & c common from C_1, C_2 and C_3 1 m

$= 2\,abc \begin{vmatrix} a+c & c & a+c \\ a+b & b & a \\ b+c & b+c & c \end{vmatrix}$

$C_1 \to C_1 + C_2 + C_3$ and taking 2 common from C_1 1 m

$= 2\,abc \begin{vmatrix} a+c & c & 0 \\ a+b & b & -b \\ b+c & b+c & -b \end{vmatrix}$ $C_3 \to C_3 - C_1$ 1 m

$= 2\,abc \begin{vmatrix} a+c & c & 0 \\ a-c & -c & 0 \\ b+c & b+c & -b \end{vmatrix}$ $R_2 \to R_2 - R_3$ ½ m

Expand by C_3, $= 2\,abc\,(-b)\,(-ac - c^2 - ac + c^2) = 4a^2\,b^2\,c^2$ ½ m

18. $\text{Adj } A = \begin{pmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{pmatrix}$; $|A| = 27$ 2+1 m

$A.\ \text{Adj } A = \begin{pmatrix} -1 & -2 & -2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{pmatrix} \begin{pmatrix} -3 & 6 & 6 \\ -6 & 3 & -6 \\ -6 & -6 & 3 \end{pmatrix} = 27 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = |A|\ I_3$ 1 m

19. $f(x) = |x-1| + |x+1|$

$L\,f'(-1) = \lim_{x \to (-1)^-} \frac{\{-(x-1)-(x+1)\}-2}{x-(-1)} = \lim_{x \to (-1)^-} \frac{-2\,(x+1)}{x+1} = -2$ 1 m

$R\ f'(-1) = \lim_{x \to (-1)^+} \frac{\{-(x-1)+(x+1)\}-2}{x-(-1)} = \lim_{x \to (-1)^+} \frac{0}{x+1} = 0$ 1 m

$-2 \neq 0 \quad \therefore\ f(x)$ is not differentiable at $x = -1$

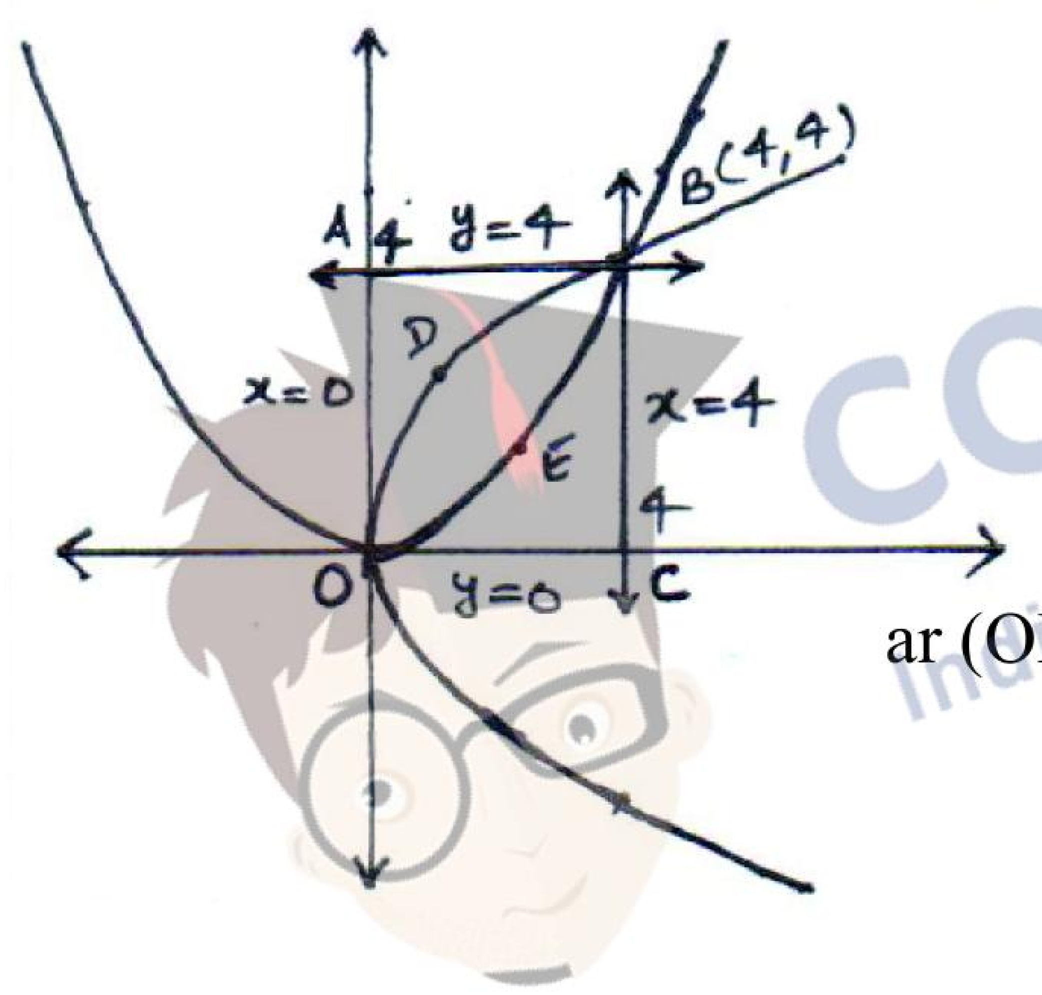
$L\ f'(1) = \lim_{x \to 1^-} \frac{\{-(x-1)+(x+1)\}-2}{x-1} = \lim_{x \to 1^-} \frac{0}{x-1} = 0$ 1 m

$R\ f'(1) = \lim_{x \to 1^+} \frac{\{x-1+x+1\}-2}{x-1} = \lim_{x \to 1^+} \frac{2(x-1)}{x-1} = 2$ 1 m

$0 \neq 2 \quad \therefore\ f(x)$ is not differentiable at $x = 1$

SECTION - C

20.



correct figure 1½ m

$ar(ABDOA) = \frac{1}{4}\int_0^4 y^2 dy = \left.\frac{y^3}{12}\right]_0^4 = \frac{16}{3}$ ......(i) 1½ m

$ar(OEBDO) = \int_0^4 2\sqrt{x}\,dx - \int_0^4 \frac{x^2}{4}\,dx = \left[\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right]_0^4$

$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$ ....(ii) 1½ m

$ar(OEBCO) = \frac{1}{4}\int_0^4 x^2\,dx = \left.\frac{x^3}{12}\right]_0^4 = \frac{16}{3}$ ....(iii) 1½ m

From (i), (ii) and (iii) we get ar (ABDOA) = ar (OEBDO) = ar (OEBCO)

21. $\frac{dy}{dx} = \frac{y^2}{xy - x^2} \Rightarrow \frac{dy}{dx} = \frac{\left(\frac{y}{x}\right)^2}{\frac{y}{x} - 1}$, Hence the differential equation is homogeneous 1 m

Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$, we get $v + x\frac{dv}{dx} = \frac{v^2}{v-1}$ 1+1 m

$$\therefore \; x\frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v}{v-1}$$ 1 m

$$\int \frac{v-1}{v}\,dv = \int \frac{1}{x}\,dx \;\Rightarrow\; v - \log v = \log x + c$$ 1 m

$$\therefore \; \frac{y}{x} - \log\frac{y}{x} = \log x + c \quad \left(\text{or, } \frac{y}{x} = \log y + c\right)$$ 1 m

OR

Given differential equation can be written as $\frac{dx}{dy} + \frac{1}{1+y^2}x = \frac{\tan^{-1}y}{1+y^2}$ 1 m

Integrating factor $= e^{\tan^{-1}y}$ and solution is : $x\,e^{\tan^{-1}y} = \int \frac{\tan^{-1}y \cdot e^{\tan^{-1}y}}{1+y^2}\,dy$ 1+1½ m

$x\,e^{\tan^{-1}y} = \int te^t\,dt = te^t - e^t + c = e^{\tan^{-1}y}(\tan^{-1}y - 1) + c$ (where $\tan^{-1}y = t$) 1½ m

$x = 1, y = 0 \Rightarrow c = 2 \;\therefore\; x \cdot e^{\tan^{-1}y} = e^{\tan^{-1}y}(\tan^{-1}y - 1) + 2$ 1 m

or $x = \tan^{-1}y - 1 + 2e^{-\tan^{-1}y}$

22. E_1 : Bolt is manufactured by machine A

E_2 : Bolt is manufactured by machine B

E_3 : Bolt is manufactured by machine C

A : Bolt is defective

$$P(E_1) = \frac{30}{100};\; P(E_2) = \frac{50}{100};\; P(E_3) = \frac{20}{100};$$

$$P(A/E_1) = \frac{3}{100};\; P(A/E_2) = \frac{4}{100};\; P(A/E_3) = \frac{1}{100}$$ 3 m

$$P(E_2/A) = \frac{\frac{50}{100}\times\frac{4}{100}}{\frac{30}{100}\times\frac{3}{100}+\frac{50}{100}\times\frac{4}{100}+\frac{20}{100}\times\frac{1}{100}} = \frac{200}{90+200+20} = \frac{20}{31}$$ 2 m

$$P(\overline{E}_2/A) = 1 - P(E_2/A) = \frac{11}{31}$$ 1 m

23. Equation of line through A and B is $\frac{x-3}{-1} = \frac{y+4}{1} = \frac{z+5}{6} = \lambda$ (say) 2 m

General point on the line is $(-\lambda+3, \lambda-4, 6\lambda-5)$ 1 m

If this is the point of intersection with plane $2x+y+z=7$

then, $2(-\lambda+3)+\lambda-4+6\lambda-5=7 \Rightarrow \lambda=2$ 1 m

$\therefore$ Point of intersection is $(1, -2, 7)$ 1 m

Required distance = $\sqrt{(3-1)^2+(4+2)^2+(4-7)^2} = 7$ 1 m

24. Let the two factories I and II be in operation for x and y

days respectively to produce the order with the minimum cost

then, the LPP is :

Minimise cost : $z = 12000x + 15000y$ 1 m

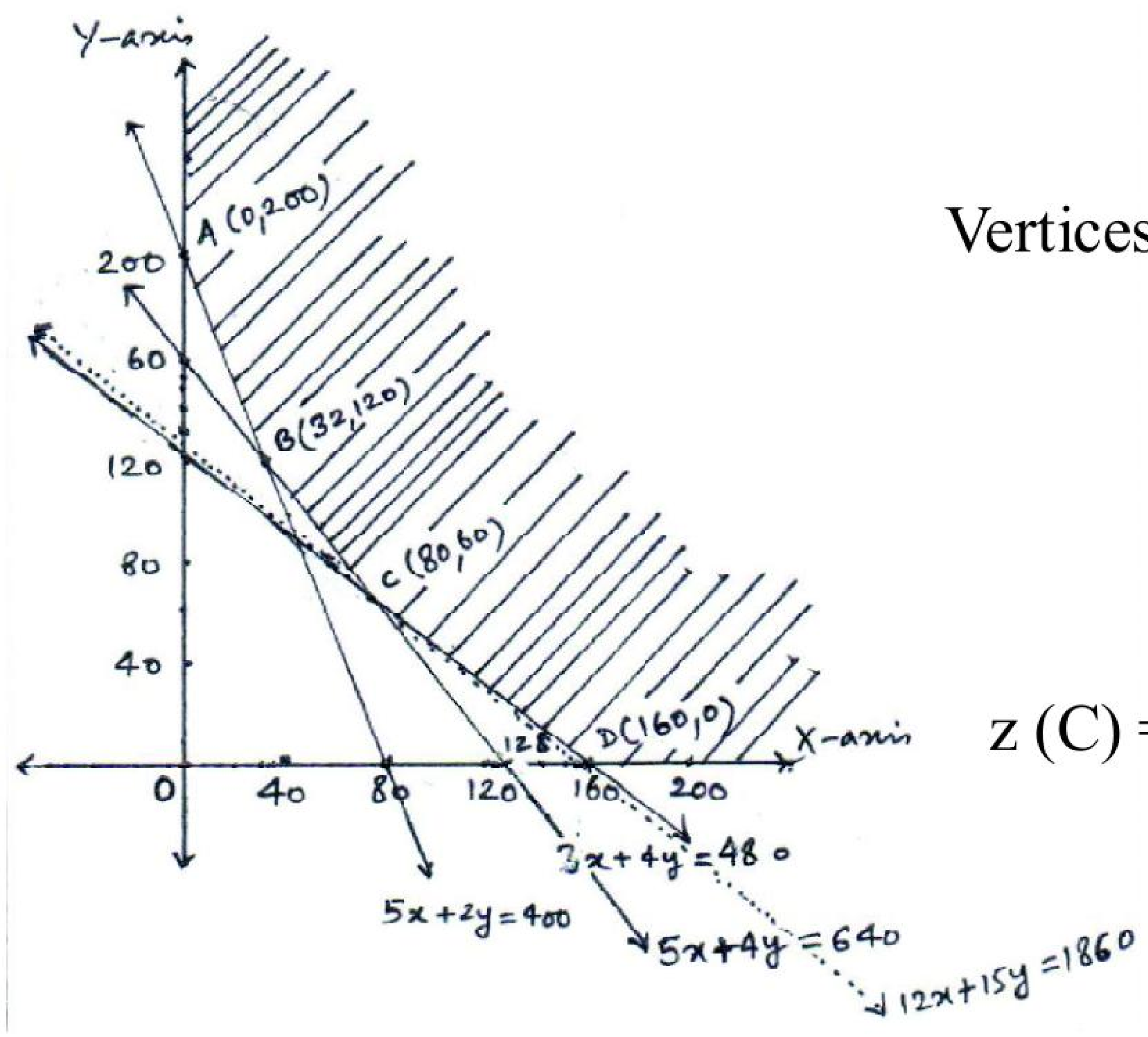
Subject to :

$50x + 40y \geq 6400$ or $5x + 4y \geq 640$

$50x + 20y \geq 4000$ or $5x + 2y \geq 400$ 2 m

$30x + 40y \geq 4800$ or $3x + 4y \geq 480$

$x, y \geq 0$



correct graph 2 m

Vertices are A (0, 200) ; B (32, 120)

C (80, 60) ; D (160, 0) ½ m

z (A) = Rs. 30,00,000; z (B) = Rs. 21,84,000;

z (C) = Rs. 18,60,000 (Min.); z (D) = Rs. 19,20,000;

On plotting z < 1860000

or 12x + 15y < 1860, we get no

point common to the feasible region

$\therefore$ Factory I operates for 80 days

Factory II operates for 60 days ½ m

25. $f: R_+ \to [-9, \infty)$; $f(x) = 5x^2 + 6x - 9$; $f^{-1}(y) = \dfrac{\sqrt{54+5y} - 3}{5}$

$$fof^{-1}(y) = 5\left\{\frac{\sqrt{54+5y}-3}{5}\right\}^2 + 6\left\{\frac{\sqrt{54+5y}-3}{5}\right\} - 9 = y$$ 3 m

$$f^{-1}of(x) = \frac{\sqrt{54+5\left(5x^2+6x-9\right)}-3}{5} = x$$ 2½ m

Hence 'f' is invertible with $f^{-1}(y) = \dfrac{\sqrt{54+5y}-3}{5}$ ½ m

OR

(i) commutative : let $x, y \in R - \{-1\}$ then

$x * y = x + y + xy = y + x + yx = y * x \quad \therefore\ *$ is commutative 1½ m

(ii) Associative : let $x, y, z \in R - \{-1\}$ then

$x * (y * z) = x * (y + z + yz) = x + (y + z + yz) + x(y + z + yz)$

$= x + y + z + xy + yz + zx + xyz$ 1½ m

$(x * y) * z = (x + y + xy) * z = (x + y + xy) + z + (x + y + xy) \cdot z$

$= x + y + z + xy + yz + zx + xyz$ 1 m

$x * (y * z) = (x * y) * z \therefore *$ is Associative

(iii) Identity Element : let $e \in R - \{-1\}$ such that $a * e = e * a = a \ \forall \ a \in R - \{-1\}$ ½ m

$\therefore \quad a + e + ae = a \Rightarrow e = 0$ ½ m

(iv) Inverse : let $a * b = b * a = e = 0$; $a, b \in R - \{-1\}$ ½ m

$\Rightarrow \quad a + b + ab = 0 \quad \therefore \quad b = \frac{-a}{1+a}$ or $a^{-1} = \frac{-a}{1+a}$ ½ m

26. Solving the two curves to get the points of intersection $\left(\pm 3\sqrt{p}, 8\right)$ 1½ m

m_1 = slope of tangent to first curve $= \frac{-2x}{9p}$ 1½ m

m_2 = slope of tangent to second curve $= \frac{2x}{p}$ 1½ m

curves cut at right angle iff $\frac{-2x}{9p} \times \frac{2x}{p} = -1$ ½ m

$\Leftrightarrow \quad 9p^2 = 4x^2$ (Put $x = \pm 3\sqrt{p}$)

$\Leftrightarrow \quad 9p^2 = 4(9p)$

$\therefore \quad p = 0 ; \ p = 4$ 1 m