JEE MAINS-9-APRIL-2014
PHYSICS

1. An experiment is performed to obtain the value of acceleration due to gravity g by using a simple pendulum of length L .In this experiment time for 100 oscillations is measured by using a watch of 1 second least count and the value is 90.0 seconds .The length L is measured by using a meter scale of least count 1 mm and the value is 20.0 cm. The error in the determination of g would be :

(1) 1.7%

(2) 2.7%

(3) 4.4%

(4) 2.27%

2. The position of a projectile launched from the origin at t=0 is given by $\vec{r}=\left(40\hat{i}+50\hat{j}\right)$ m at 2=2 s. If the projectile was launched at an angle θ from the horizontal, then the θ is (take d=10 ms^{-1}).

(1) $\tan^{-1}\frac{2}{3}$

(2) $\tan^{-1} 3/2$

(3) $\tan^{-1} 7/4$

(4) $\tan^{-1} 4/5$

3. Water is flowing at a speed of 1.5 ms^{-1} through a horizontal tube of cross-sectional area 10^{-2} m^2 and you are trying to stop the flow by your palm. Assuming that the water stops immediately after hitting the palm, the minimum force that you must exert should be (density of water=10^3 kgm^{-3}).

(1) 15 N

(2) 22.5 N

(3) 33.7 N

(4) 45 N

4. A block A of mass 4 kg is placed on another block B of mass 5 kg, and the block B rests on a smooth horizontal table. If the minimum force that can be applied on A so that both the blocks move together is 12 N, the maximum force that can be applied on B for the blocks to move together will be :

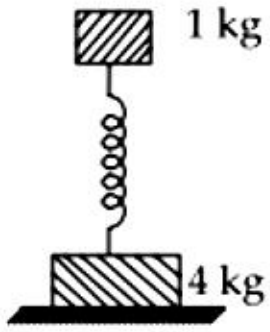
(1) 30 N

(2) 25 N

(3) 27 N

(4) 48 N

5. Two bodies of masses 1 kg and 4 kg are connected to a vertical spring, as shown in the figure. The smaller mass executes simple harmonic motion of angular frequency 25 rad/s, and amplitude 1.6 cm while the bigger mass remains stationary on the ground. The maximum force exerted by the system on the floor is (take g=10 ms^{-1})



(1) 20 N

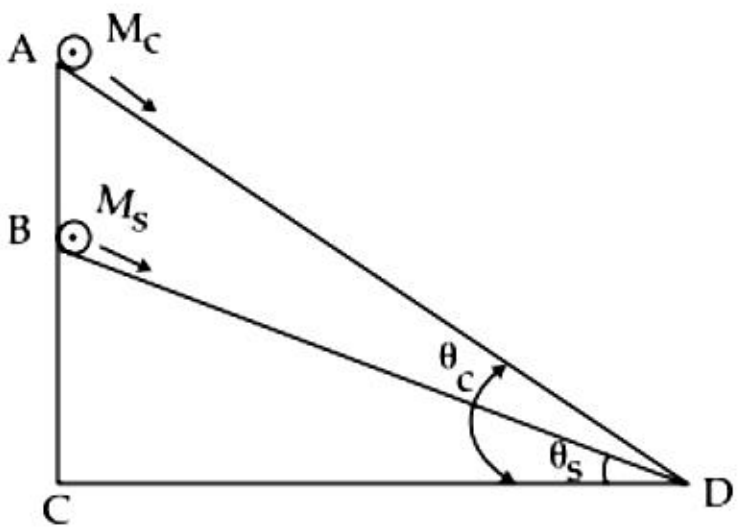
(2) 10 N

(3) 60 N

(4) 40 N

6. A cylinder of mass Mc and sphere of mass Ms are placed at points A and B of two inclines, respectively. (See Figure). If

they roll on the incline without slipping such that their accelerations are the same, then ratio $\frac{\sin\theta_C}{\sin\theta_S}$ is:



(1) $\sqrt{\frac{8}{7}}$

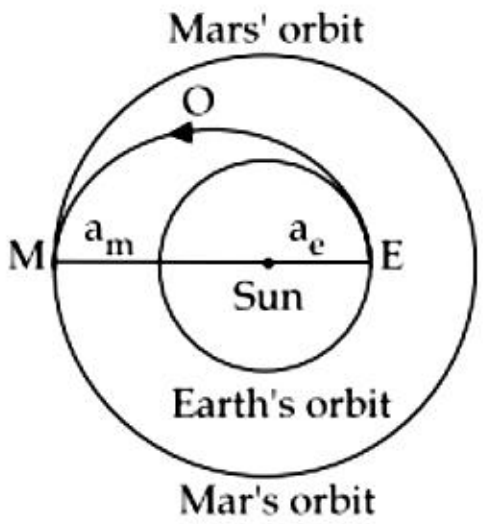
(2) $\sqrt{\frac{15}{14}}$

(3) $\frac{8}{7}$

(4) $\frac{15}{14}$

7. India's Mangalyan was sent to the Mars by launching it into a transfer orbit EOM around the sun. It leaves the earth at E and meets Mars at M. If the semi-major axis of Earth's orbit is $a_e = 1.5\times10^{11}$ m, that of Mar's orbit $a_m = 2.28\times10^{11}$ m, taken Kepler's

laws give the estimate of time for Mangalyan to reach Mars from Earth to be close to :



(1) 500 days

(2) 320 days

(3) 260 days

(4) 220 days

8. In materials like aluminium and copper, the correct order of magnitude of various elastic modulii is :

(1) Young's modulii < shear modulii < bulk modulii.

(2) Bulk modulii < shear modulii < Young's modulii.

(3) Shear modulii < Young's modulii < bulk modulii.

(4) Bulk modulii < Young's modulii < shear modulii.

9. The amplitude of a simple pendulum, oscillating in air with a small spherical bob, decreases from 10 cm to 8 cm in 40

seconds. Assuming that Stokes law is valid, and ratio of the coefficient of viscosity of air to that of carbon dioxide is 1.3, the time in which amplitude of this pendulum will reduce from 10 cm to 5 cm in, carbondioxide will be close to (ln 5 = 1.601, ln 2 = 0.693).

(1) 231 s

(2) 208 s

(3) 161 s

(4) 142 s

10. A capillary tube is immersed vertically in water and the height of the water column is x. When this arrangement is taken into a mine of depth d, the height of the water column is y. If R is the radius of earth, the ratio $\frac{x}{y}$ is :

(1) $\left(1-\frac{d}{R}\right)$

(2) $\left(1-\frac{2d}{R}\right)$

(3) $\left(\frac{R-d}{R+d}\right)$

(4) $\left(\frac{R+d}{R-d}\right)$

11. Water of volume 2 L in a closed container is heated with a coil f 1 kW. While water is heated, the container loses energy at a rate of 160 J/s. In how much time will the temperature of water rise from 27°C to 77°C ? (Specific heat of water is 4.2 kJ/kg and that of the container is negligible).

(1) 8 min 20 s

(2) 6 min 2 s

(3) 7 min

(4) 14 min

12. The equation of state for a gas is given by $PV = nRT + \alpha V$, where n is the number of moles and α is a positive constant. The initial temperature and pressure of one mole of the gas contained in a cylinder area T_o and P_o respectively. The work done by the gas when its temperature doubles isobarically will be:

(1) $\frac{P_o T_o R}{P_o - \alpha}$

(2) $\frac{P_o T_o R}{P_o + \alpha}$

(3) $P_oT_oR\ln 2$

(4) P_oT_oR

13. Modern vacuum pumps can evacuate a vessel down to a pressure of 4.0×10^{-15} atm. at room temperature (300 K). Taking $R = 8.3\ \text{JK}^{-1}\text{mol}^{-1}$, $1\ \text{atm} = 10^5\ \text{Pa}$ and $N_{\text{Avogadro}} = 6\times10^{23}\ \text{mol}^{-1}$, the mean distance between molecules of gas in an evacuated vessel will be of the order of :

(1) 0.2 μm

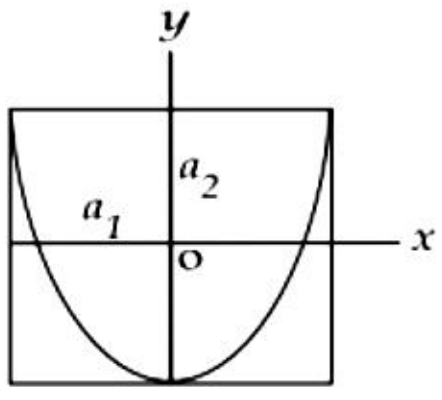
(2) 0.2 mm

(3) 0.2 cm

(4) 0.2 nm

14. A particle which is simultaneously subjected to two perpendicular simple harmonic motions t andωrepresented by ; $x = a_1\cos\omega t$ and $y = a_2\cos 2\omega t$ traces a curve given by:

(1)



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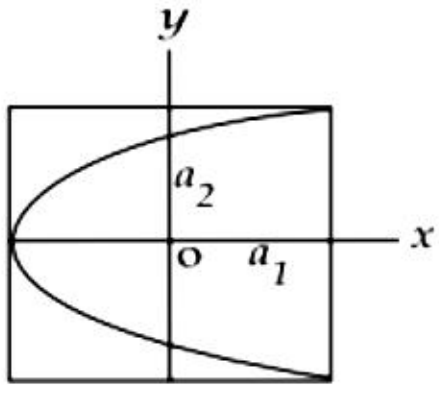

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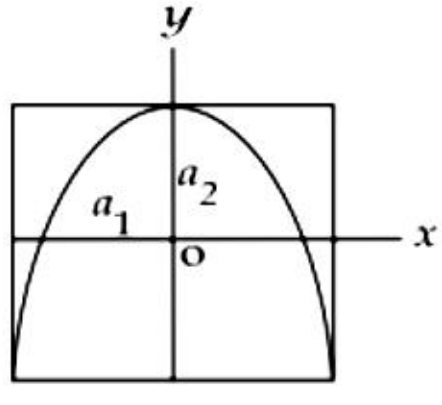

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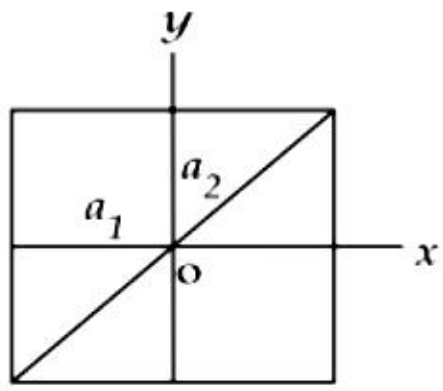


15. A transverse wave is represented by :

$$y = \frac{10}{\pi}\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

For what value of the wavelength the wave velocity is twice the maximum particle velocity?

(1) 40 cm

(2) 20 cm

(3) 10 cm

(4) 60 cm

16. The magnitude of the average electric field normally present in the atmosphere just above the surface of the Earth is about 150 N/C, directed inward towards the center of the Earth. This gives the total net surface charge carried by the Earth to be :

[Given $\in_o = 8.85 \times 10^{-12}$ C/N $\cdot$ m^2 $R_E = 6.37 \times 10^6$ m]]

(1) +670 kC

(2) −670 kC

(3) --−680 kC

(4) +680 kC

17. Three capacitances, each of 3 μF, are provided. These cannot be combined to provide the resultant capacitance of :

(1) 1 μF

(2) 2 μF

(3) 4.5 μF

(4) 6 μF

18. A d.c. main supply of e.m.f. 220 V is connected across a storage battery of e.m.f. 200 V through a resistance of 1 Ω. The battery terminals are connected to an external resistance 'R'. The

minimum value of 'R', so that a current passes through the battery to charge it is :

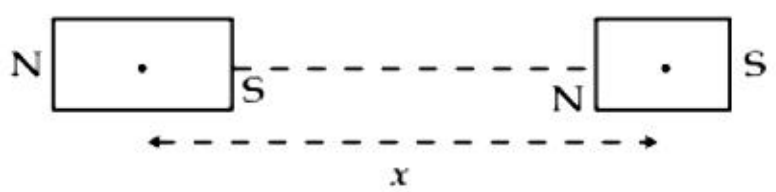
(1) 7 Ω

(2) 9 Ω

(3) 11 Ω

(4) Zero

19. The mid points of two small magnetic dipoles of length d end-on positions, are separated by a distance x ($x >> d$). The force between them is proportional to x^{-n} where n is:



(1) 1

(2) 2

(3) 3

(4) 4

20. The magnetic field of earth at the equator is approximately 4×10^{-5} T. The radius of earth is 6.4×10^{6} m. Then the dipole moment of the earth will be nearly of the order of:

(1) 10^{23} Am^2

(2) 10^{20} Am^2

(3) 10^{16} Am^2

(4) 10^{10} Am^2

21. When the rms voltages V_L, V_C and V_R are measured respectively across the inductor L, the capacitor C and the resistor R in a series LCR circuit connected to an AC source, it is found that the ratio $V_L : V_C : V_R = 1 : 2 : 3$. If the rms voltage of the AC source is 100 V, then V_R is close to:

(1) 50 V

(2) 70 V

(3) 90 V

(4) 100 V

22. Match **List I** (Wavelength range of electromagnetic spectrum) with **List II**. (Method of production of these waves) and select the **correct** option from the options given below the lists.

List I		**List II**	
(a)	700 nm to 1 mm	(i)	Vibration of atoms and molecules.
(b)	1 nm to 400 nm	(ii)	Inner shell electrons in atoms

			moving from one energy level to a lower level.
(c)	$< 10^{-3}$ nm	(iii)	Radioactive decay of the nucleus.
(d)	1 mm to 0.1 m	(iv)	Magnetron valve.

(1) (a)-(iv) (b)-(iii) (c)-(ii) (d)-(i)

(2) (a)-(iii) (b)-(iv) (c)-(i) (d)-(ii)

(3) (a)-(ii) (b)-(iii) (c)-(iv) (d)-(i)

(4) (a)-(i) (b)-(ii) (c)-(iii) (d)-(iv)

23. A diver looking up through the water sees the outside world contained in a circular horizon. The refractive index of water is $\frac{4}{3}$, and the diver's eyes are 15 cm below the surface of water. Then the radius of the circle is :

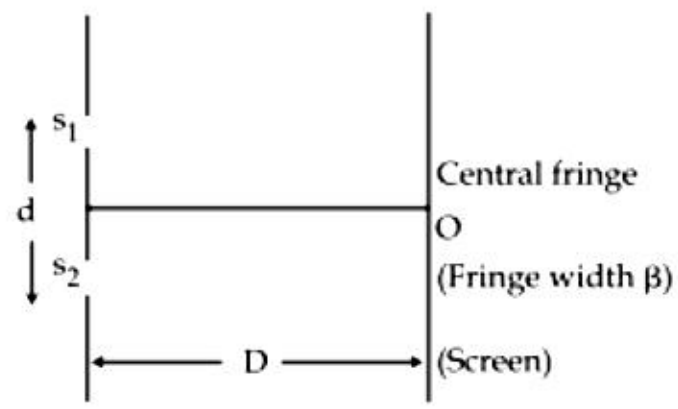
(1) $15 \times 3 \times \sqrt{5}$ cm

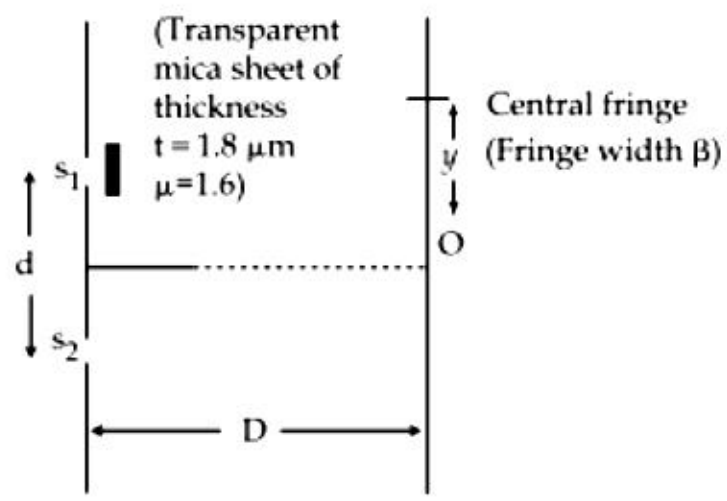
(2) $15 \times 3\sqrt{7}$ cm

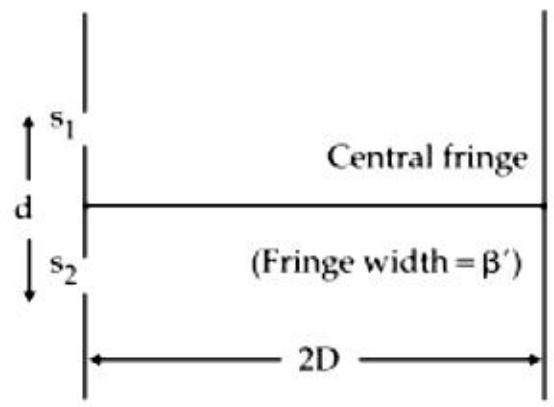
(3) $\frac{15 \times \sqrt{7}}{3}$ cm

(4) $\frac{15\times3}{\sqrt{7}}$ cm

24. Using monochromatic light of wavelength λ, an experimentalist sets up the Young's double slit experiment in three ways as shown. If she observes that $y=\beta'$, the wavelength of the light is:







(1) 520 nm

(2) 540 nm

(3) 560 nm

(4) 580 nm

25. The focal lengths of objective lens and eye lens of a Gallelian Telescope are respectively 30 cm and 3.0 cm. Telescope produces virtual, erect image of an object situated far away from it at least distance of distinct vision from the eye lens. In this condition, the magnifying Power of the Gallelian telescope should be :

(1) +11.2

(2) −11.2

(3) −8.8

(4) +8.8

26. For which of the following particles will it be most difficult to experimentally verify the de-Broglie relationship?

(1) an electron

(2) a proton

(3) an α-particle

(4) a dust particle

27. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{++} is :

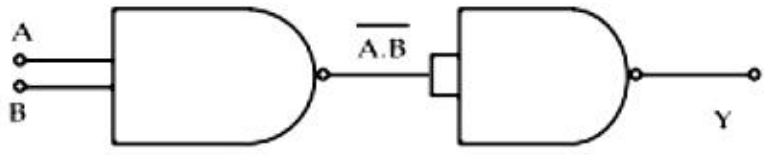
(1) 122.4 eV

(2) 30.6 eV

(3) 13.6 eV

(4) 3.4 eV

28. Identify the gate and match A, B, Y in bracket to check.



(1) AND (A=l, B = l, Y= l)

(2) OR (A = l, B=l, Y = 0)

(3) NOT (A=l, B = l, Y=l)

(4) XOR (A=0, B = 0. Y = 0)

29. A transmitting antenna at the top of a tower has a height 32 m and the height of the receiving antenna is 50 m. What is the maximum distance between them for satisfactory communication in line of sight (LOS) mode ?

(1) 55.4 km

(2) 45.5 km

(3) 54.5 km

(4) 455 km

30. An n-p-n transistor has three leads A, B and C. Connecting B and C by moist fingers, A to the positive lead of an ammeter, and C to the negative lead of the ammeter, one finds large deflection. Then, A, B and C refer respectively to :

(1) Emitter, base and collector

(2) Base, emitter and collector

(3) Base, collector and emitter

(4) Collector, emitter and base

PART-2

1. Sol:

The time period for simple pendulum is given by the equation,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

From this we get,

$$g = 4\pi^2 \frac{l}{T^2}$$

So, the error associated with the acceleration is given by,

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$$

Substituting the values,

$$\frac{\Delta g}{g}\% = \left[\left(\frac{0.1}{20}\right) + \left(2\frac{1}{90}\right)\right] \times 100\%$$

$$= 0.027 \times 100\%$$

$$= 2.7\%$$

2. Sol:

The horizontal distance travelled by the projectile is given by,

$$ut = (u\cos\theta)t + \frac{1}{2}(0)t^2$$

$$(u\cos\theta) \times 2 = 40$$

$$u\cos\theta = 20 \qquad (1)$$

Vertical distance travelled by the projectile is given by.

$$ut = (u\sin\theta)t - \frac{1}{2}gt^2$$

$$50 = (u\sin\theta) \times 2 - \frac{1}{2}(10)(4)$$

$$u\sin\theta = 35 \qquad (2)$$

The angle of projection of projectile.

$$\tan\theta = \frac{u\sin\theta}{u\cos\theta}$$

$$\tan\theta = \frac{35}{20}$$

$$\theta = \tan^{-1}\frac{7}{4}$$

3. Sol:

As the water stops following hitting the palm, the base power that ought to be applied will be the pace of progress of force.

The minimum force exerted is given by

$$F = \rho A V^2$$

$$= \left(10^3\right)\left(10^{-2}\right)\left(\left(1.5\right)^2\right)$$

$$= 22.5\ \text{N}$$

4. Sol:

The friction force between to surfaces is equal to the minimum forces exerted on block A.

The minimum acceleration of block A is given by,

$$a_{max} = \frac{12}{4}$$

$$= 3\ \text{m/s}^2$$

The maximum force on block B can be calculated by the equation,

$$F = ma_{max}$$

$$= 9 \times 3$$

$$= 27\ \text{N}$$

5. Sol:

The net compressive force of the spring is ,

$$x = A + \frac{mg}{K}$$

$$= A + \frac{mg}{m\omega^2}$$

$$= A + \frac{g}{\omega^2}$$

The magnitude of maximum force exerted on the floor,

$$F_{max} = Mg + K\left(A + \frac{g}{\omega^2}\right)$$

$$= Mg + m\omega^2\left(A + \frac{g}{\omega^2}\right)$$

$$= Mg + m\omega^2 A + mg$$

Substituting the values of parameters

$$F_{max} = 4 \times 10 + \left(1 \times \left(25^2\right) \times 0.016\right) + 1(10)$$

$$= 40 + 10 + 10$$

$$= 60\ N$$

6. Sol:

The equation for acceleration of cylinder is given as,

$$a_{cylinder} = \frac{M_c g \sin\theta_c}{M_c + \frac{1}{2}\frac{M_c R^2}{R^2}}$$

$$= \frac{2g\sin\theta_c}{3}$$

The equation for acceleration of sphere is given as,

$$a_{sphere} = \frac{M_s g \sin\theta_s}{M_s + \frac{I_s}{r^2}}$$

$$= \frac{M_s g \sin\theta_s}{M_s + \frac{2}{5}\frac{M_s R^2}{R^2}}$$

$$= \frac{5g \sin\theta_s}{7}$$

The acceleration of the sphere and the cylinder is equal.

$$a_{cylinder} = a_{sphere}$$

$$\frac{2g \sin\theta_c}{3} = \frac{5g \sin\theta_s}{7}$$

$$\frac{\sin\theta_c}{\sin\theta_s} = \frac{15}{14}$$

7. Sol:

The semi-major axis of the orbit of Mangalyan is,

$$a_y = \frac{1.5\times10^{11} + 2.28\times10^{11}}{2}$$

$$= 1.89\times10^{11}\ m$$

By the Kapler's rule,

$$T^2 \propto r^3$$

$$\left(\frac{T_y}{T_E}\right)^2 = \left(\frac{a_y}{a_E}\right)^3$$

Substituting the value in the above equation,

$$\left(\frac{T_y}{365}\right)^2 = \left(\frac{1.89\times10^{11}}{1.5\times10^{11}}\right)^3$$

$T_y \approx 518$ days

Therefore, the time required for Mangalyan to reach Mars is given as

$$T_{req} = \frac{T_y}{2}$$

$$\approx \frac{518}{2}$$

≈ 260 days

8. Sol

The misshapening of pliable materials like copper and aluminum can without much of a stretch be conceivable under elastic pressure. In this manner, the bendable material like have low shear modulii however has high mass modulii than shear modulii and Young's moduli.

Thus, the correct order of elastic modulii for a particular material is given as,

Shear modulii<Young's modulii<Bulk modulii

9. Sol:

The damped oscillation the equation of displacement is given by,

$$x = x_0 e^{-\lambda t}$$

Substitute the values for air

$$8 = 10e^{-40\lambda}$$

$$\ln\frac{4}{5} = -40\lambda$$

$$\lambda = 0.00537$$

Substitute the values for carbon dioxide

$$5 = 10e^{-\frac{\lambda t}{1.3}}$$

$$\ln\frac{1}{2} = -\frac{\lambda t}{1.3}$$

$$-0.693 = -\frac{0.00537}{1.3}t$$

$$t = 167.7 \text{ sec}$$

Thus, the closest time is 167 s.

10. Sol:

The pressures are equal

$$P_1 = P_2$$

$$\rho g_1 x = \rho g_2 y$$

$$g_1 x = g_2 y$$

The equation of change in g with depth is given by,

$$g_2 = \left(1 - \frac{d}{R}\right) g_1$$

By equating the two equations we'll get,

$$g_1 x = g_1 \left(1 - \frac{d}{R}\right) y$$

$$\frac{x}{y} = \left(1 - \frac{d}{R}\right)$$

11. Sol:

The heat absorbed by water in 1 second is,

$$Q = 1000 - 160$$

$$= 840 \text{ J/s}$$

The net amount of heat required to raise the temperature of water,

$$Q_{req} = m_w \times \text{specific heat} \times \Delta T$$

$$840 \times t = 2 \times 10^3 \times 4.2 \times (77 - 27)$$

$$t = \frac{2 \times 10^3 \times 4.2 \times 50}{840}$$

$$t = 8 \text{ min } 20 \text{ sec}$$

12. Sol:

The gas equation at a particular state of gas is given by,

$$PV = nRT + \alpha V$$

$$P = \frac{nRT}{V} + \alpha \qquad (1)$$

The ideal gas equation is given by,

$$PV = nRT$$

$$P_0 V_i = nRT_0$$

$$V_i = \frac{nRT_0}{P_0}$$

The final temperature is twice of the initial temperature,

$$T_f = 2T_0$$

The work done by the gas can be calculated as,

$$W = \int PdV$$

$$= \int \left(\frac{nRT}{V} + \alpha \right) dV$$

$$= \int_{T_0}^{2T_0} nRdT + \int_{V_i}^{V_f} \alpha dV$$

$$= nRT_0 + \alpha V_i$$

By solving the above expression we'll get,

$$W = nRT_0 + \alpha \left(\frac{nRT_0}{P_0} \right)$$

$$= nRT_0 \left(1 + \frac{\alpha}{P_0} \right)$$

Integrate the equation (1)

$$\int PdV = \int nRdT + \int \alpha dV$$

$$W = nRT_0 + \alpha\left(\frac{nRT_0}{P_0 - \alpha}\right)$$

$$W = nRT_0 + \left(1 + \frac{\alpha}{P_0 - \alpha}\right)$$

$$= \frac{nP_0T_0R}{P_0 - \alpha}$$

For one mole of gas substitute $n = 1$.

$$W = \frac{P_0T_0R}{P_0 - \alpha}$$

13. Sol:

Assume the average distance between the gas molecules is r . Then by ideal gas equation.

$$PV = \frac{nRT}{N_A}$$

$$P\left(\frac{4}{3}\pi r^3\right) = \frac{nRT}{N_A}$$

$$r = \left(\frac{3nRT}{4\pi PN_A}\right)^{1/3}$$

Substitute the values in the above expression,

$$r = \left(\frac{3(1)(8.3)(300)}{4\pi\left(4.0\times10^{-15}\times10^{5}\right)\left(6\times10^{23}\right)} \right)^{1/3}$$

$$= 0.2 \text{ mm}$$

14. Sol:

The displacement of particle which is under simple harmonic motion is given by,

$$x = a_1 \cos\omega t$$

$$\cos\omega t = \frac{x}{a_1}$$

The given equation is

$$y = a_2 \cos 2\omega t$$

$$= a_2 \left\{2\cos^2 \omega t - 1\right\}$$

$$= a_2 \left\{2\left(\frac{x}{a_1}\right)^2 - 1\right\}$$

$$y \propto x^2$$

Therefore, the curve in option (1) with square relation in correct.

15. Sol

The displacement of transverse wave is given as,

$$y = \frac{10}{\pi}\sin\left(\frac{2\pi}{T}t - \frac{2\pi}{\lambda}x\right)$$

The given condition is

$$v_w = 2\left(v_p\right)_{max}$$

Equation for calculation of velocity is,

$$v_w = f\lambda$$

$$2\omega A = f\lambda$$

$$2\left(\frac{2\pi}{T}\right)\left(\frac{10}{\pi}\right) = \left(\frac{1}{T}\right)\lambda$$

$$\lambda = 40\ \text{cm}$$

16. Sol:

Equation for electric field is,

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R^2}$$

$$= \frac{\sigma}{\varepsilon_0}$$

$$\sigma = \varepsilon_0 E$$

The net surface charge is given by,

$$Q = \varepsilon_0 E \times 4\pi R^2$$

By substituting the values we'll get,

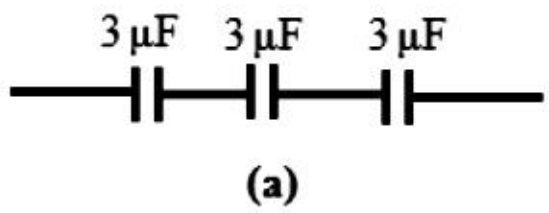
$$Q = \varepsilon_0 E \times 4\pi R^2$$

$$= 8.85\times10^{-12}\times150\times4\pi\left(6.37\times10^{6}\right)$$

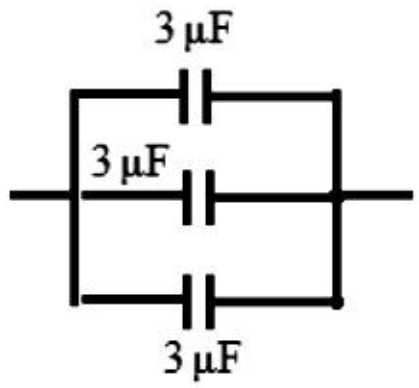
$$= 680\ \text{kC}$$

17. Sol:

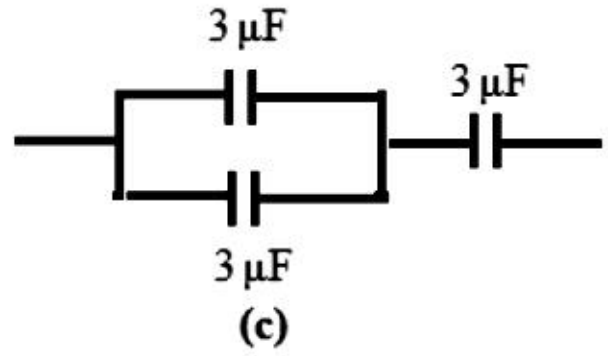
The capacitance of the given capacitors cannot give the given resultant capacitance. Below figure represents the possible arrangements of capacitors,



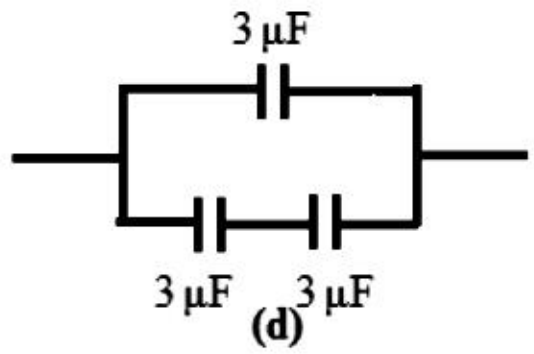
(a)



(b)



(c)



(d)

For case (a)

$$\frac{1}{C_{eq}} = \frac{3}{3}$$

$$C_{eq} = 1\,\mu F$$

For case (b)

$$C_{eq} = 3 + 3 + 3$$

$$= 9\,\mu F$$

For case (c)

$$C_{eq} = \frac{6 \times 3}{9}$$

$$= 2\,\mu F$$

For case (d)

$$C_{eq} = \frac{3\times 3}{6} + 3$$
$$= 4.5\,\mu F$$

Thus, option (c) is correct.

18. Sol:

The minimum value of R can be calculated by,

$$\frac{R}{R_1} = \frac{V}{E - V}$$
$$R = \frac{220}{220 - 200}(1)$$
$$= \frac{220}{20}$$
$$= 11\,\Omega$$

19. Sol:

The force between the magnetic poles is inversely proportional to the fourth power of the distance between the magnetic poles.

$$F \propto \frac{1}{r^4} \qquad (1)$$

The given equation is

$$F \propto x^{-n}$$
$$F \propto \frac{1}{x^n} \qquad (2)$$

Compare equation (1) with equation (2).

$n = 4$

20. Sol:

The equation of magnetic field of earth is,

$$B = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$$

$$M = \frac{4\pi r^3 B}{\mu_0}$$

Substitute the values, for dipole moment of earth,

$$M = \frac{4\pi\left(6.4\times10^{6}\right)^{3}\left(4\times10^{-5}\right)}{\left(4\pi\times10^{-7}\right)}$$

$$= 1.048\times10^{23}$$

$$\approx 10^{23}\ \text{Am}^2$$

21. Sol:

The given ratio of voltage is,

$$V_L : V_C : V_R = 1:2:3$$

The given ratio of reactance is,

$$X_L : X_C : X_R = 1:2:3$$

$$= x:2x:3x$$

The equation for current is,

$$I = \frac{V_{rms}}{Z}$$

$$= \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$= \frac{100}{\sqrt{90x^2 + x^2}}$$

$$= \frac{100}{\sqrt{10}x}$$

Equation for calculation of voltage across the resistor is,

$$V_R = (3x)I$$

$$= (3x)\frac{100}{\sqrt{10}x}$$

$$\approx 90 \text{ V}$$

22. Sol:

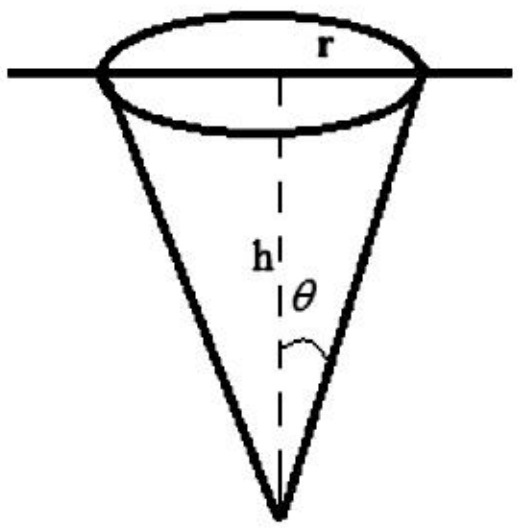
Vibrations of atoms and molecules - (700 nm to 1 mm)

Inner shell electrons in atoms moving from - (1 nm to 4nm)

Radioactive decay of the nucleus -

Magnetron valve generates wavelength of 1 mm to 0.1 m.

23. Sol:



The relation for θ is

$$\sin\theta = \frac{1}{\mu}$$

$$= \frac{1}{\left(\frac{4}{3}\right)}$$

$$= \frac{3}{4}$$

Equation for calculation of radius of circle is,

$$\tan\theta = \frac{r}{h}$$

$$\frac{3}{\sqrt{4^2 - 3^2}} = \frac{r}{15}$$

$$r = \frac{15 \times 3}{\sqrt{7}} \text{ cm}$$

24. Sol:

The fringe width is equal to the distance from the central fringe.

$$y = \beta'$$

$$d\sin\theta = (\mu - 1)t$$

$$d\theta = (\mu - 1)t$$

$$y = \frac{D(\mu - 1)t}{d}$$

The distance between the slits is doubled then for calculation of wavelength of light is,

$$\frac{(2D)}{d}\lambda = \frac{D(\mu - 1)t}{d}$$

$$\lambda = \left(\frac{1.6 - 1}{2}\right)\left(1.8 \times 10^{-6}\right) m$$

$$= 540 \times 10^{-9}\ m$$

$$= 540\ nm$$

25. Sol:

The equation for magnifying power is,

$$M = \frac{f_0}{f_c}\left(1 - \frac{f_c}{D}\right)$$

$$= \frac{30}{3.0}\left(1 - \frac{3.0}{25}\right)$$

$$= +8.8$$

26. Sol:

Among the given options the dust particle is difficult to verify with the de-Broglie relation.

27. Sol:

The energy required to remove electron from first orbit of Li^{++} is given by the equation,

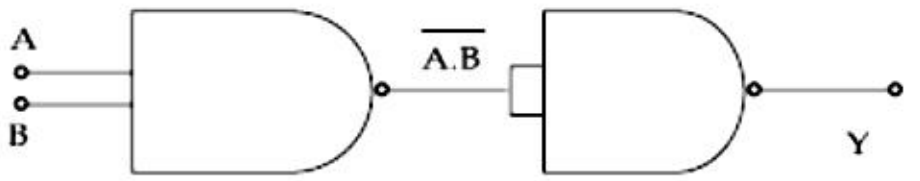
$$E_n = 13.6\frac{Z^2}{n^2}$$

$$= 13.6\left(\frac{3^2}{2^2}\right)$$

$$= 30.6\ eV$$

28. Sol:

The figure below represents the logic gate.



The value of output Y for the above logic gate is,

$$Y = \overline{\overline{AB}\cdot\overline{AB}}$$

$$= \overline{\overline{AB}} + \overline{\overline{AB}}$$

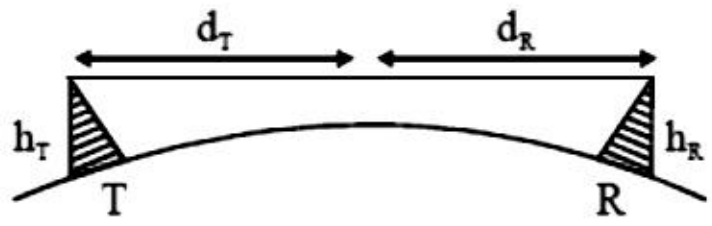
$$= AB + AB$$

$$= AB$$

Thus the given logic gate is an AND gate.

29. Sol:

The two towers are represented in the figure below,



The maximum distance between the towers is given by,

$$d = d_T + d_R$$

$$= \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

$$= \sqrt{2\left(6400\times10^3\right)(32)} + \sqrt{2\left(6400\times10^3\right)(50)}$$

$$= 45.5\ \text{km}$$

30. Sol:

In n-p-n or p-n-p transistor, the emitter, collector and base are,

- Collector is moderately doped.
- Emitter is heavily doped.
- Base is lightly doped.

The transistors are current controlled device in which high current flows between emitter and collector during conduction state.

Hence, in the given system, A refers to Emitter, B refers to base and C refers to collector.

31. In a face centered cubic lattice atoms A are at the corner points and atoms B at the face centered points. If atom B is missing from one of the face centered points, the formula of the ionic compound is :

(1) AB_2

(2) A_5B_2

(3) A_2B_3

(4) A_2B_5

32. Vander Wall's equation for a gas is stated as,

$$p = \frac{nRT}{V\text{-}nb} - a\left(\frac{n}{V}\right)^2.$$

This equation reduces to the perfect gas equation, $p = \frac{nRT}{V}$

when,

(1) temperature is sufficiently high and pressure is low

(2) temperature is sufficiently low and pressure is high.

(3) both temperature and pressure are very high

(4) both temperature and pressure are very low

33. the standard electrode potentials $\left(E^{o}_{M^{+}/M}\right)$ of four metals A, B, C and D are –1.2 V, 0.6 V, 0.85 V and –0.76 V, respectively. The sequence of deposition of metals on applying potential is :

(1) A,C,B,D

(2) B,D,C,A

(3) C,B,D,A

(4) D,A,B,C

34. At a certain temperature, only 50% HI is dissociated into H_2 and I_2 at equilibrium. The equilibrium constant is:

(1) 1.0

(2) 3.0

(3) 0.5

(4) 0.25

35. Dissolving 120 g of a compound of (mol. wt. 60) in 1000 g of water gave a solution of density 1.12 g/mL. The molarity of the solution is:

(1) 1.00 M

(2) 2.00 M

(3) 2.50 M

(4) 4.00 M

36. The half-life period of a first order reaction is 15 minutes. The amount of substance left after one hour will be:

(1) $\frac{1}{4}$ of the original amount

(2) $\frac{1}{8}$ of the original amount

(3) $\frac{1}{16}$ of the original amount

(4) $\frac{1}{32}$ of the original amount

37. A current of 10.0 A flows for 2.00 h through an electrolytic cell containing a molten salt of metal X. This results in the decomposition of 0.250 mol of metal X at the cathode. The oxidation state of X in the molten salt is : (F= 96,500 C)

(1) 1+

(2) 2+

(3) 3+

(4) 4+

38. The energy of an electron in first Bohr orbit of H-atom is – 13.6 eV. The energy value of electron in the excited state of Li^{2+} is :

(1) –27.2 eV

(2) 30.6 eV

(3) –30.6 eV

(4) 27.2 eV

39. The temperature at which oxygen molecules have the same root mean square speed as helium atoms have at 300 K is:

(Atomic masses: He =4 u, O =16 u)

(1) 300 K

(2) 600 K

(3) 1200 K

(4) 2400 K

40. The standard enthalpy of formation of NH_3 is –46.0 kJ/mol. If the enthalpy of formation of H_2 from its atoms is –436 kJ/mol and that of N_2 is –712 kJ/mol, the average bond enthalpy of N—H bond in NH_3 is :

(1) –1102 kJ/mol

(2) –964 kJ/mol

(3) +352 kJ/mol

(4) +1056 kJ/mol

41. The amount of oxygen in 3.6 moles of water is:

(1) 115.2 g

(2) 57.6 g

(3) 28.8 g

(4) 18.4 g

42. The gas evolved on heating CaF_2 and SiO_2 with concentrated H_2SO_4, on hydrolysis gives a white gelatinous precipitate. The precipitate is:

(1) hydrofluosilicic acid

(2) silica gel

(3) silicic acid

(4) calciumfluorosilicate

43. Chloro compound of Vanadium has only spin magnetic moment of 1.73 BM. This Vanadium chloride has the formula :

(at. no. of V = 23)

(1) VCl^2

(2) VCl^4

(3) VCl^3

(4) VCl^5

44. An octahedral complex of Co^{3+} is diamagnetic. The hybridisation involved in the formation of the complex is :

(1) Sp^3d^2

(2) dsp^2

(3) d^2sp^3

(4) dsp^3d

45. Which of the following is not formed when H_2S reacts with acidic $K_2Cr_2O_7$ solution ?

(1) $CrSO_4$

(2) $Cr_2(SO_4)_3$

(3) K_2SO_4

(4) S

46. Which of the following has unpaired electron(s) ?

(1) N_2

(2) O_2^-

(3) N_2^{2+}

(4) O_2^{2-}

47. In the following sets of reactants which two sets best exhibit the amphoteric character of $Al_2O_3.\ xH_2O$?

Set- 1 : $Al_2O_3.xH_2O(s)$ and $OH^-(aq)$

Set- 2 : $Al_2O_3.xH_2O(s)$ and $H_2O(l)$

Set- 3 : $Al_2O_3.xH_2O(s)$ and $H^+(aq)$

Set- 4 : $Al_2O_3.xH_2O(s)$ and $NH_3(aq)$

(1) 1 and 2

(2) 1 and 3

(3) 2 and 4

(4) 3 and 4

48. The number and type of bonds in C_2^{2-} ion in CaC_2 are:

(1) One σ bond and one π –bond

(2) One σ bond and two π –bonds

(3) Two σ bonds and two π –bonds

(4) Two σ bonds and one π –bond

49. The form of iron obtained from blast furnace is :

(1) Steel

(2) Cast Iron

(3) Pig iron

(4) Wrought Iron

50. The correct statement about the magnetic properties of $[Fe(CN)_6]^{3-}$ and $[FeF_6]^{3-}$ is :

($Z = 26$).

(1) $[Fe(CN)_6]^{3-}$ is paramagnetic

$[FeF_6]^{3-}$ is diamagnetic

(2) both are diamagnetic

(3) $[Fe(CN)_6]^{3-}$ is diamagnetic

$[FeF_6]^{3-}$ is paramagnetic

(4) both are paramagnetic

51. Which one of the following reactions will not result in the formation of carbon-carbon bond?

(1) Reimer-Tieman reaction

(2) Friedel Craft's acylation

(3) Wurtz reaction

(4) Cannizzaro reaction

52. In the hydroboration - oxidation reaction of propene with diborane, H2O2 and NaOH, the organic compound formed is :

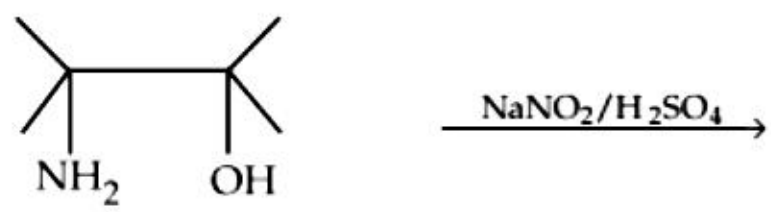
(1) CH_3CH_2OH

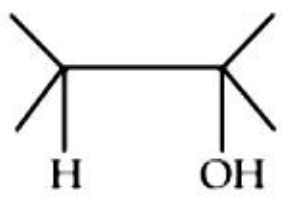
(2) $CH_3CHOHCH_3$

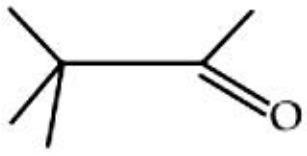
(3) CH_3CH2CH_2OH

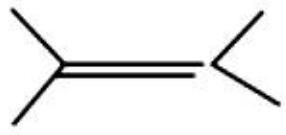
(4) $(CH_3)_3COH$

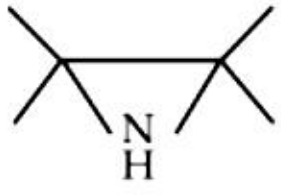
53. The major product of the reaction



(1) 

(2) 

(3) 

(4) 

54. For the compounds CH_3Cl, CH_3Br, CH_3I and CH_3F, the correct order of increasing C-halogen bond length is :

(1) $CH_3F < CH_3Br < CH_3Cl < CH_3I$

(2) $CH_3F < CH_3Cl < CH_3Br < CH_3I$

(3) $CH_3Cl < CH_3Br < CH_3F < CH_3I$

(4) $CH_3F < CH_3I < CH_3Br < CH_3Cl$

55. Allyl phenyl ether can be prepared by heating :

(1) $C_6H_5Br + CH_2 = CH - CH_2 - ONa$

(2) $CH_2 = CH - CH_2 - Br + C_6H_5ONa$

(3) $C_6H_5 - CH = CH - Br + CH_3 - ONa$

(4) $CH_2 = CH - Br + C6H_5 - CH_2 - ONa$

56. In a nucleophilic substitution reaction :

$$R - Br + Cl^- \xrightarrow{DMF} R\text{-}Cl + Br^-$$

which one of the following undergoes complete inversion of configuration?

(1) $C_6H_5CHC_6H_5Br$

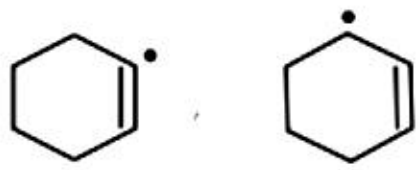
(2) $C_6H_5CHCH_3Br$

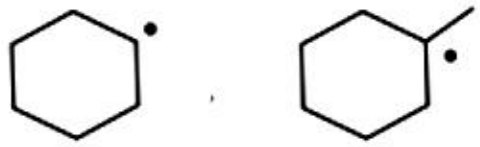
(3) $C_6H_5CHC_6H_5Br$

(4) $C_6H_5CCH_3C_6H_5Br$

57. In which of the following pairs A is more stable than B?

A **B**

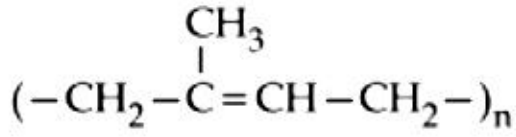
(1) 

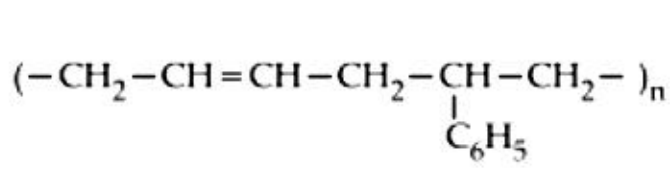
(2) 

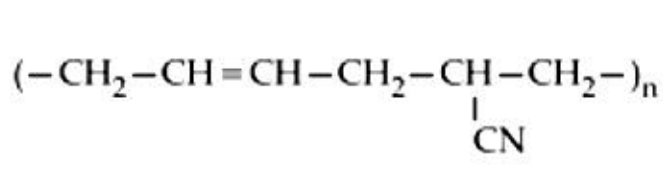
(3) 

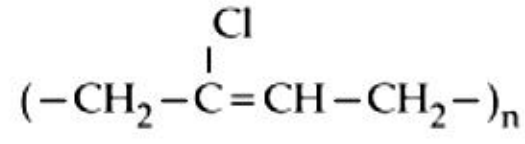
(4) $Ph_3C^{\bullet}$, $(CH_3)_3C^{\bullet}$

58. Structure of some important polymers are given. Which one represents Buna-S ?

(1) 

(2) 

(3) 

(4) 

59. Which is the major product formed when acetone is heated with iodine and potassium hydroxide ?

(1) Iodoacetone

(2) Acetic acid

(3) Iodoform

(4) Acetophenone

60. Which one of the following class of compounds is obtained by polymerization of acetylene ?

(1) Poly-eyne

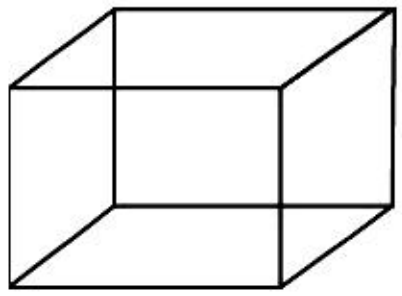
(2) Poly-ene

(3) Poly-ester

(4) Poly-amide

PART-2

31. Sol:



$$\text{Effective number of atoms at corner} = 8 \times \frac{1}{8}$$

$$= 1$$

$$\text{Effective number of atoms at tha face} = 6 \times \frac{1}{2} - 1 \times \frac{1}{2}$$

$$= \frac{5}{2}$$

The ratio of effective number of atoms.

$$A : B = 1 : \frac{5}{2}$$

$$= 2 : 5$$

Therefore, the formula of the compound is A_2B_5

32. Sol:

In the case when temperature is high enough and pressure is low the Vander wall's equation reduces to perfect gas equation i.e. $PV = nRT$.

33. Sol:

For higher value of reduction potential the rate of deposition is higher. Therefore the sequence is C,B,D,A.

34. Sol:

The HI is dissociated with the chemical equation,

$$2HI \rightleftharpoons I_2 + H_2$$

$$1-a \qquad \frac{a}{2} \quad \frac{a}{2}$$

Here, a is the concentration.

The equilibrium constant of the above chemical reaction can be calculated as follows.

$$K_{eq} = \frac{\left(\frac{a}{2}\right)^2}{(1-a)^2}$$

$$= \frac{a^2}{4(1-a)^2}$$

$$= \frac{\left(\frac{1}{2}\right)^2}{4(1/2)^2}$$

$$= \frac{1}{4}$$

Thus, the value of equilibrium is 0.25.

35. Sol:

The molarity of the given solution is

$$\text{Molarity} = \frac{\left(\frac{120}{60}\right)}{\frac{1120}{1.12} \times \frac{1}{1000}}$$

$$= 2\text{M}$$

Therefore, the molarity of the given solution is 2M.

36. Sol:

Reaction time for 50% completion is 15 min .

Then, the total number of half-life cycles is,

$$= \frac{60}{15}$$

$$= 4$$

Therefore, the amount of the substance left after one hour is,

$$= \frac{A_\circ}{(2)^n}$$

$$= \frac{A_\circ}{(2)^4}$$

$$= \frac{A_\circ}{16}$$

Thus, the amount left after 1 hour is $\frac{1}{16}$ of its initial amount.

37. Sol:

$$\text{Work done, } W = \frac{E}{96500} \times It$$

$$\text{Total number of moles} = \frac{It}{96500 \times (n - \text{factor})}$$

$$n - \text{factor} = \frac{720 \times 4}{965}$$

$$= +3$$

The value of X is $+3$.

38. Sol:

In the excited state of Lithium the energy of electron can be calculated as.

$$E = -13.6 \times \frac{Z^2}{n^2}$$

$$= -13.6 \times \frac{3^2}{2^2} eV$$

$$= -13.6 \times \frac{9}{4} eV$$

$$= -30.6 \text{ eV}$$

39. Sol:

The root mean square velocity of oxygen atom and helium atom is equal.

$$\left(U_{rms}\right)_{O_2} = \left(U_{rms}\right)_{He}$$

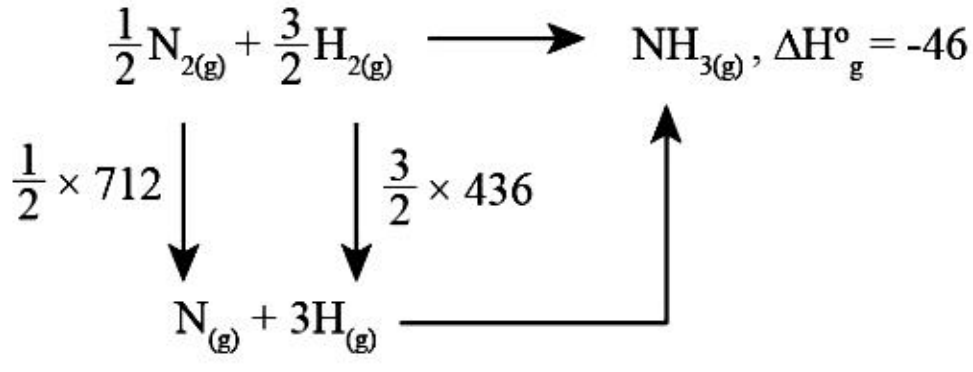
U_{rms} is root mean square velocity.

$$\left(U_{rms}\right)_{O_2} = \left(U_{rms}\right)_{He}$$

$$\frac{3RT_{O_2}}{32} = \frac{3RT_{He}}{4}$$

$$T_{O_2} = 8 \times 300 = 2400\,K$$

40. Sol:



The equation for calculation of bond enthalpy.

$$\text{Bond enthalpy} = \frac{1}{2}\times(-712)+\frac{3}{2}(-436)+(-46)$$
$$= -1056\,\text{kJ/mol}$$

The average enthalpy for three $N-H$ bonds is,

$$h = \frac{-1056\,\text{kJ/mol}}{3}$$
$$= -352\,\text{kJ/mol}$$

For bond disassociation the sign would be positive.

41. Sol:

There are 3.6 moles of oxygen in water. The mass in gram is,

$$= (3.6\,\text{moles})\times 16\,\text{g/mol}$$
$$= 57.6\,\text{g}$$

42. Sol:

The chemical reactions of CaF_2, SiO_2 and SiF_4 in concentration of H_2SO_4 are,

$$CaF_2 + H_2SO_4 \rightarrow H_2F_2 + Ca(HSO_4)_2$$
$$SiO_2 + 2H_2F_2 \rightarrow SiF_4 + 2H_2O$$
$$SiF_4 + H_2O \rightarrow H_2[SiF_6]$$

The precipitate is hydrofluosilicic acid.

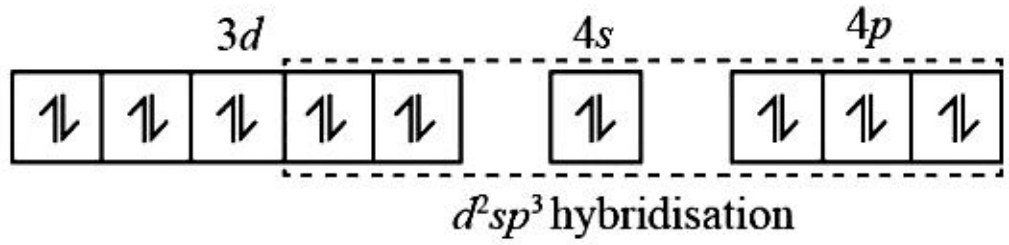
43. Sol:

The given estimation of magnetic moment 1.73 BM is that demonstrates that vanadium simply have just unpaired electron in this valence shell. In this way, VCl_4, vanadium has +4 oxidation state.

44. Sol:

The electronic configuration of Co^{3+} is $[Ar]3d^6$.Co^{3+} is diamagnetic in nature, it implies there is no unpaired electron present in the d-orbital of cobalt. Along these lines, the hybridization engaged with the development of the complex of must be (inward orbital complex) as demonstrated as follows.

Co^{3+} : $4s^0 3d^6$ diamagnetic



d^2sp^3 hybridisation

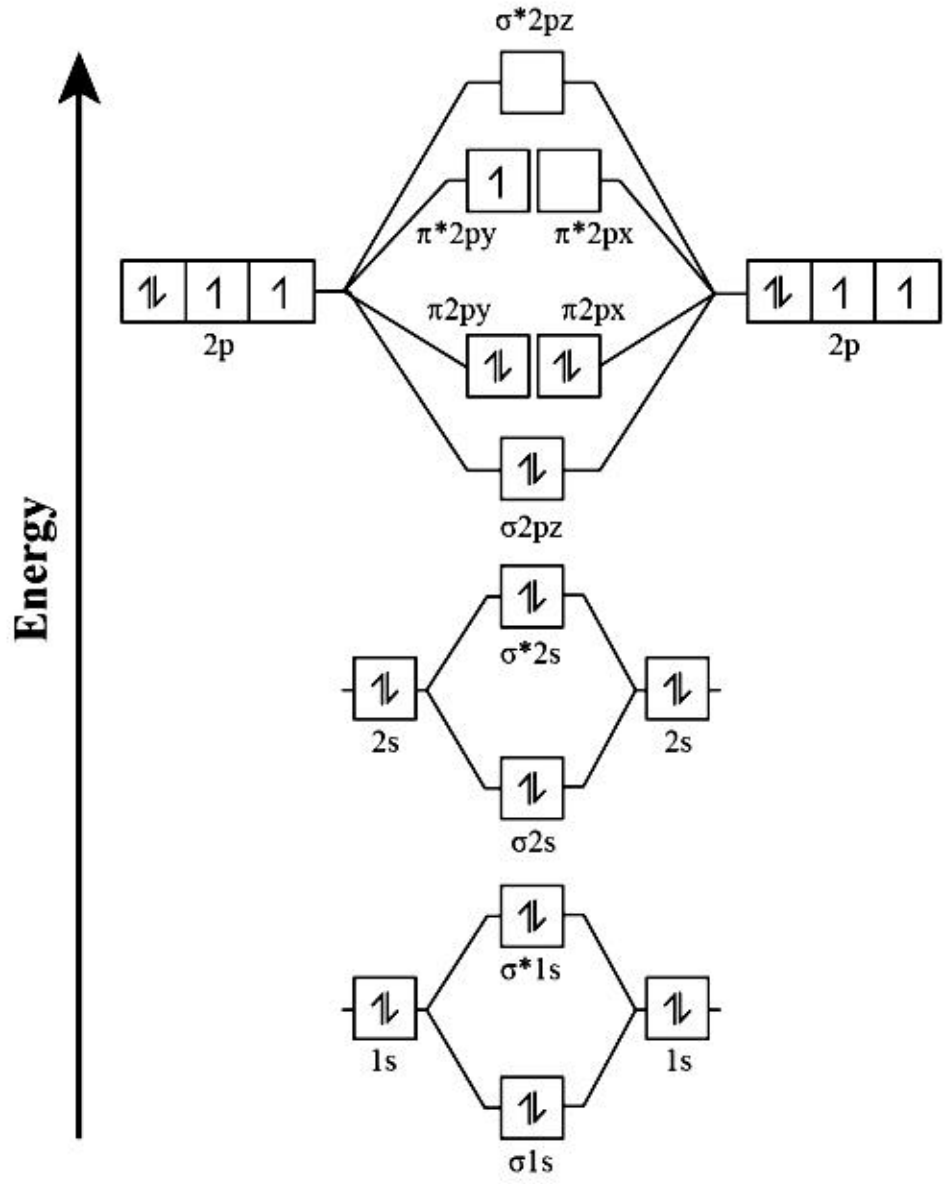
45. Sol:

The reaction of H_2S with $K_2Cr_2O_7$ is shows below.

$$K_2Cr_2O_7 + H_2S \rightarrow Cr_2(SO_4)_3 + S + K_2SO_4 + H_2O$$

46. Sol:

The molecule of O^{2-} has one unpaired electron in π^* orbital, as shown in figure below,



47. Sol:

Compound with amphoteric structure can act both as acid and base. Therefore, the set -1 is $Al_2O_3.xH_2O(s)$, that forms $Al(OH)_4^-$ in the solutions. The set-3 is of acidic nature $Al_2O_3.xH_2O(s)$ that shows Al^{3+} and H_2O.

48. Sol:

The CaC_2 has one σ bond and one π bonds.

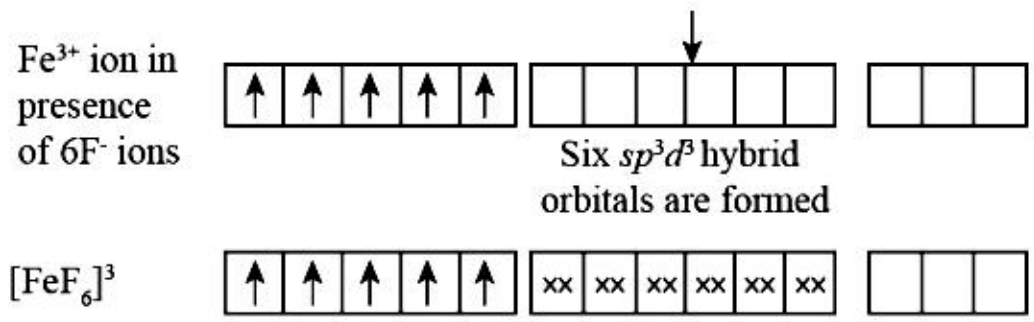
$$Ca^{2+}\left[C \equiv C\right]^{-2}$$

49. Sol:

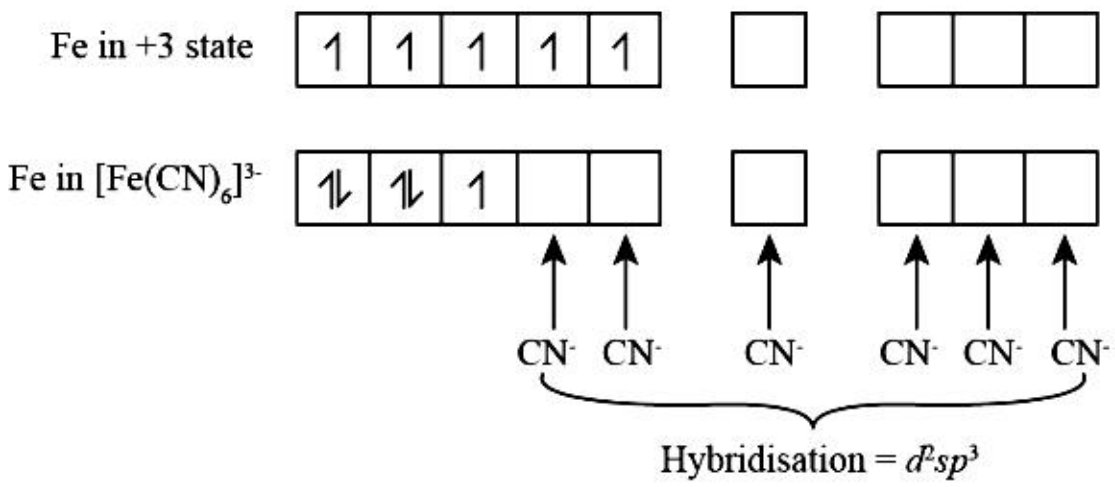
After processing the iron ore in blasé furnace, the form of iron is pig iron. Pig iron has 92-94% of iron and 3-5% of carbon.

50. Sol:

In Fluorine because of weak field ligand the pairing of electron is not possible. This distribution of electrons in case of Fluorine is shown below,



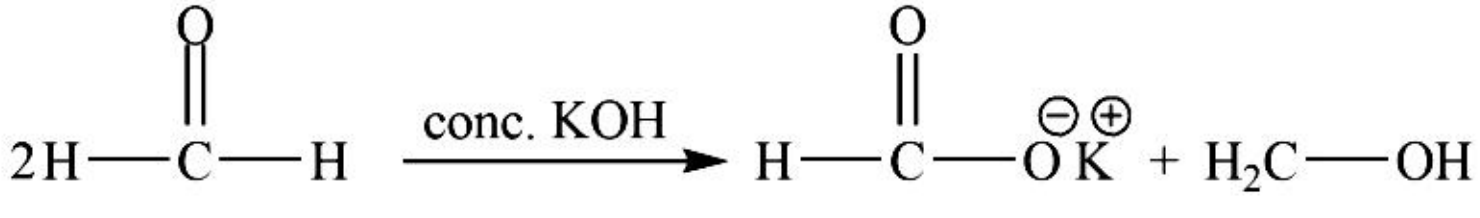
On the contrary Cyanide is a strong field ligand in which pairing of electron is east. The distribution of electron is shown below.



There are unpaired electrons in both elements. Thus, the nature of compound is paramagnetic.

51. Sol:

In the Cannizzaro reaction, the C-C bond does not form at all. The reaction is given below.



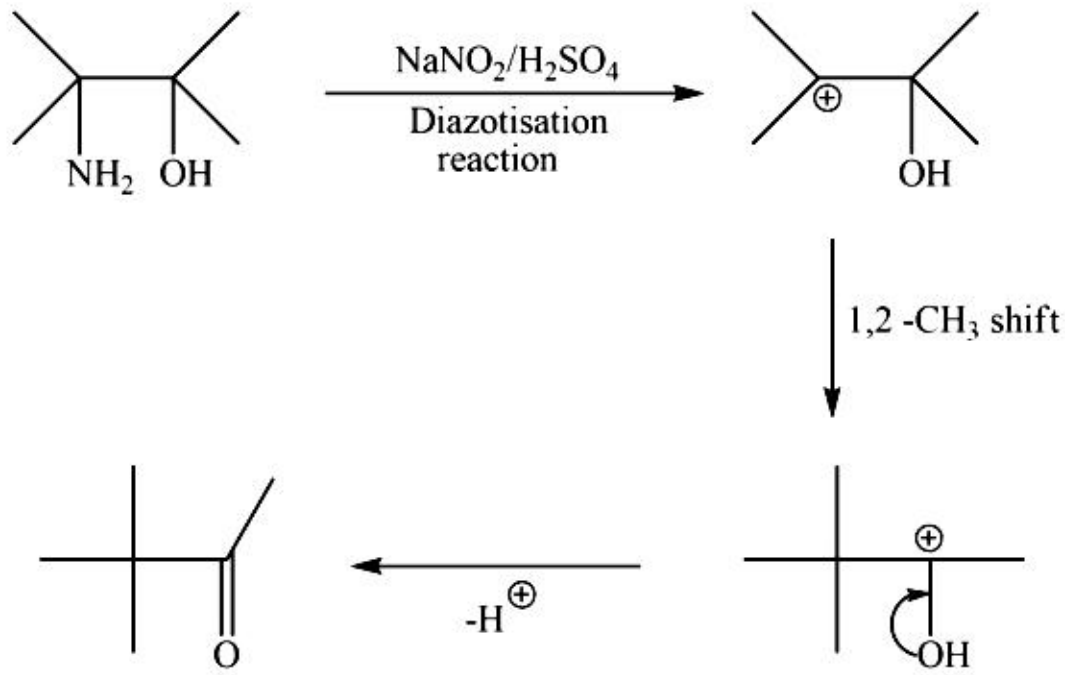
52. Sol

In the reaction hydroboration-oxidation of propane the product is 1-propanol.

$$CH_3CH=CH_2 \xrightarrow[H_2O_2/OH^-]{B_2H_6/THF} CH_3CH_2CH_2OH$$

53. Sol:

The chemical reaction is shown below:



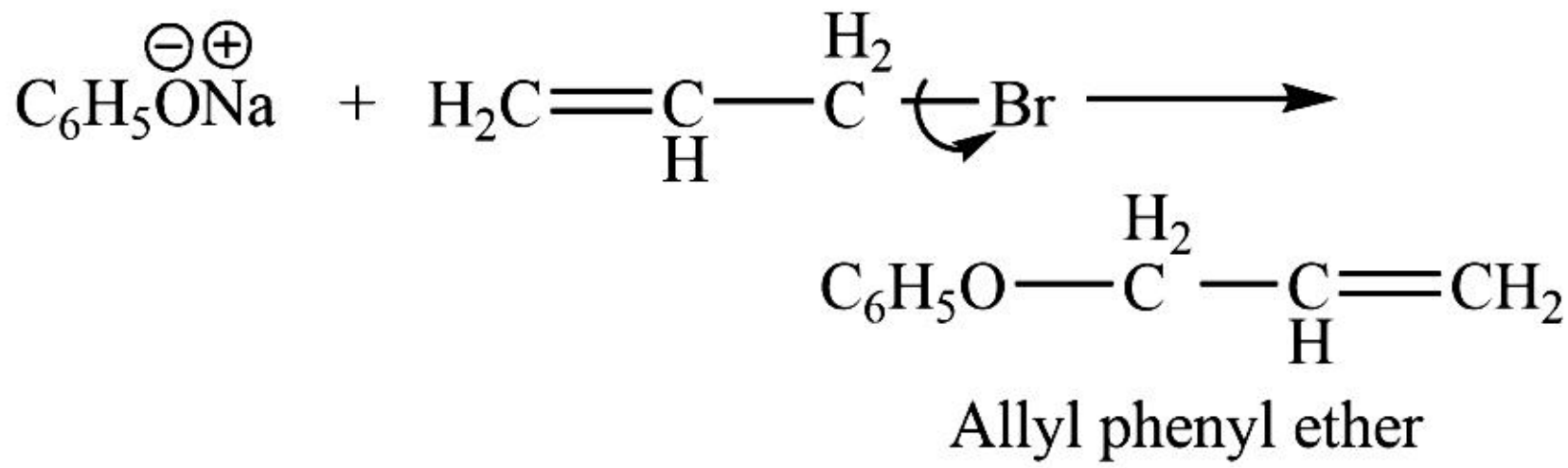
54. Sol:

The atomic radii increases from fluorine to iodine. Thus, the bond length also increases. Therefore the correct increasing order is

$CH_3F < CH_3Cl < CH_3Br < CH_3I$

55. Sol:

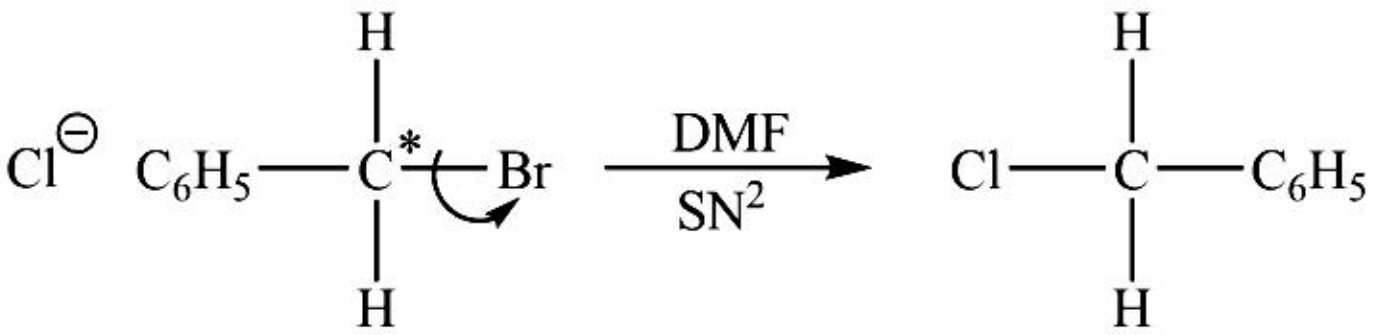
The chemical reaction of preparation of allyl phenyl ether is,



Allyl phenyl ether

56. Sol:

This is a nucleophilic substitution reaction. The compound $C_6H_5CH_2Br$ will completely undergoes to inversion.



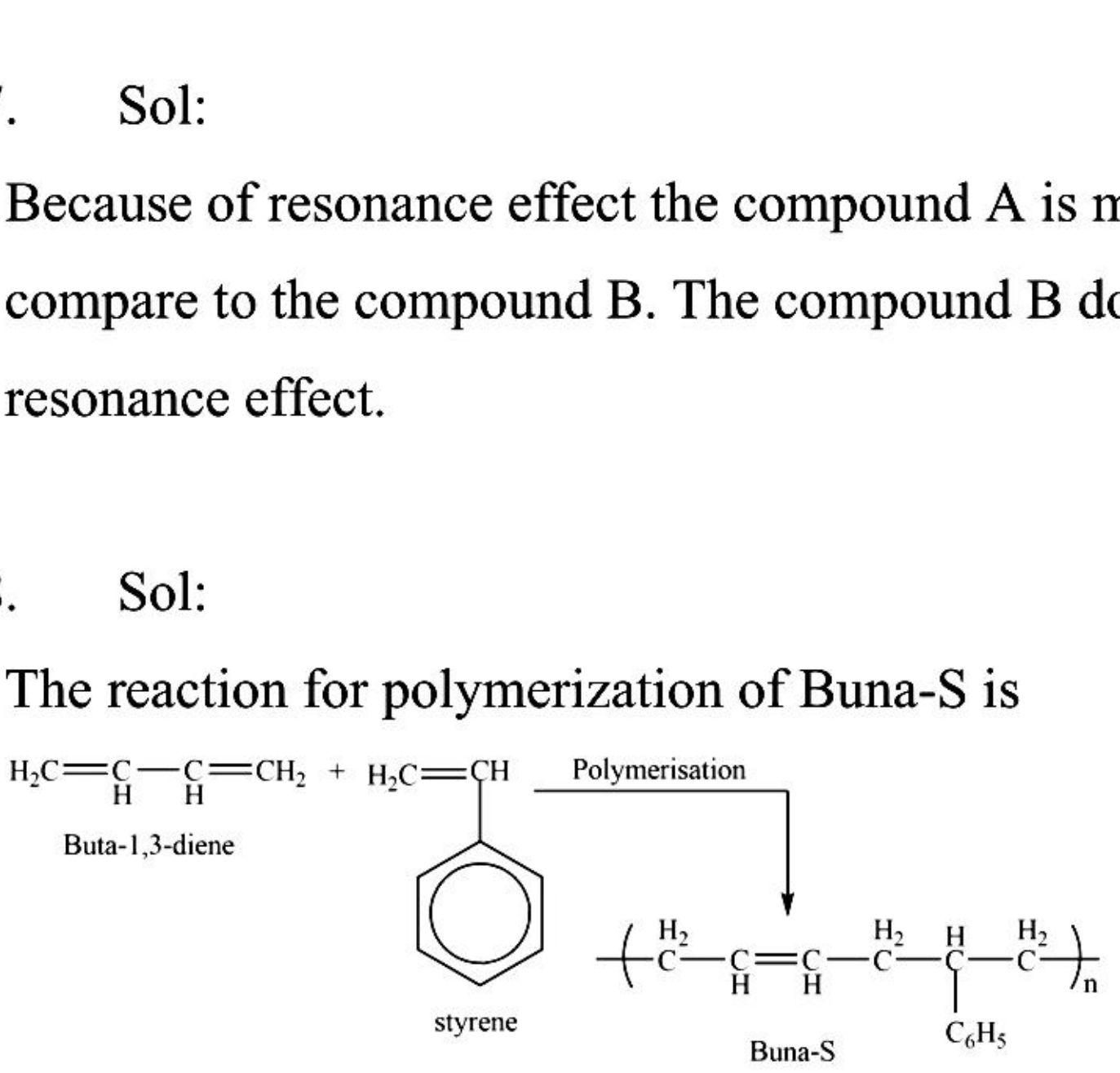
Inverted product

57. Sol:

Because of resonance effect the compound A is more stable compare to the compound B. The compound B does not have resonance effect.

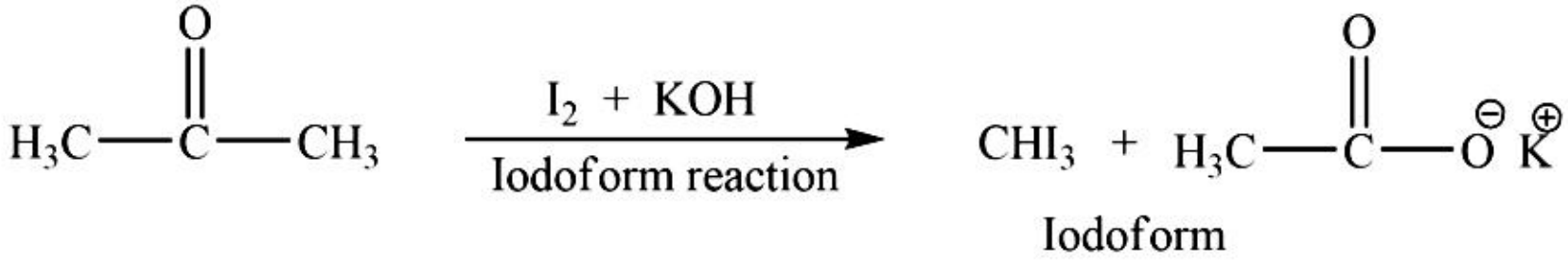
58. Sol:

The reaction for polymerization of Buna-S is



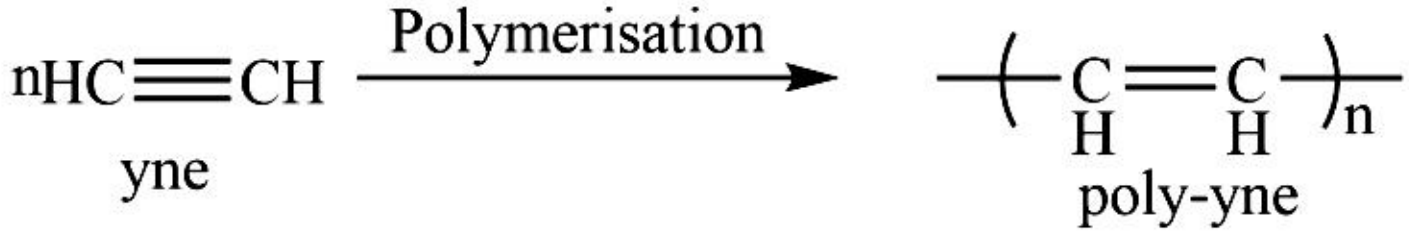
59. Sol:

When acetone heated with iodine and potassium hydroxide, it forms iodoform. The reaction is



60. Sol:

The chemical reaction of polymerization of acetylene gives poly-vne. The reaction is given below.



Answer keys

1	2	19	4	37	3	55	2	73	2
2	3	20	1	38	3	56	2	74	2
3	1	21	3	39	4	57	4	75	2
4	3	22	4	40	3	58	2	76	3
5	3	23	4	41	2	59	3	77	2
6	4	24	2	42	1	60	2	78	3
7	3	25	4	43	2	61	4	79	3
8	3	26	4	44	3	62	4	80	4
9	3	27	2	45	1	63	2	81	4
10	1	28	1	46	2	64	1	82	1
11	1	29	2	47	2	65	2	83	3
12	1	30	1	48	2	66	1	84	3
13	2	31	4	49	3	67	3	85	3
14	1	32	1	50	1	68	2	86	1
15	1	33	3	51	4	69	3	87	1
16	3	34	4	52	3	70	1	88	3
17	3	35	2	53	2	71	2	89	2
18	3	36	3	54	1	72	4	90	2

JEE MAINS-9-APRIL-2014
MATHEMATICS

61. Let P be the relation defined on the set of all real numbers such that

$P=\{(a,b):\sec^2 a-\tan^2 b=1\}$. Then P is:

(1) reflexive and symmetric but not transitive

(2) reflexive and transitive but not symmetric

(3) symmetric and transitive but not reflexive

(4) an equivalence relation

62. Let $w(\text{Im } w \neq 0)$ be a complex number. Then the set of all complex numbers z satisfying the equation $w-\bar{w}z=k(1-z)$, for some real number k, is :

(1) $\{z:|z|=1\}$

(2) $\{z:z=\bar{z}\}$

(3) $\{z:z\neq 1\}$

(4) $\{z:|z|=1, z\neq 1\}$

63. If equations $ax^2 + bx + c = 0$, $(a,b,c \in R, a \neq 0)$ and $2x^2 + 3x + 4 = 0$ have a common root, then a : b : c equals :

(1) 1 : 2 : 3

(2) 2 : 3 : 4

(3) 4 : 3 : 2

(4) 3 : 2 : 1

64. If $\frac{1}{\sqrt{\alpha}}$ and $\frac{1}{\sqrt{\beta}}$ are the roots of the equation, $ax^2 + bx + 1 = 0\,(a \neq 0, a, b \in R)$, then the equation, $x(x + b^3) + (a^3 - 3abx) = 0$ has roots :

(1) $\alpha^{3/2}$ and $\beta^{3/2}$

(2) $\alpha\beta^{1/2}$ and $\alpha^{3/2}\beta$

(3) $\sqrt{\alpha\beta}$ and $\alpha\beta$

(4) $\alpha^{-\frac{3}{2}}$ and $\beta^{-\frac{3}{2}}$

65. If a,b and c are non-zero real numbers and if the system of equations

$(a-1)x = y + z,$

$(b-1)y = z + x,$

$(c-1)z = x + y,$

has a non-trival solution, then ab+bc+ca equals :

(1) a+b+c

(2) abc

(3) 1

(4) -1

66. If B is a 3×3 matrix such that $B2 = 0$, then det. $\left[(I+B)^{50} - 50B\right]$ is equal to :

(1) 1

(2) 2

(3) 3

(4) 50

67. The number of terms in the expansion of $(1+x)^{101}(1+x^2-x)^{100}$ in powers of x is:

(1) 302

(2) 301

(3) 202

(4) 101

68. The sum of the digits in the unit's place of all the 4-digit numbers formed by using the numbers 3,4 and 6, without repetition, is

(1) 432

(2) 108

(3) 36

(4) 18

69. Given an A.P. whose terms are all positive integers. The sum of its first nine terms is greater than 200 and less than 220. If the second term in it is 12, then its 4^{th} term is:

(1) 20

(2) 16

(3) 8

(4) 24

70. If the sum $\frac{3}{1^2}+\frac{5}{1^2+3^2}+\frac{7}{1^2+2^2+3^2}+....+$ upto 20 terms mis equal to $\frac{k}{21}$, then k is eual to :

(1) 120

(2) 180

(3) 240

(4) 60

71. If $f(x)$ is continuous and $f\left(\frac{9}{2}\right)=\frac{2}{9}$, then $\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right)$ is equal to:

(1) $\frac{9}{2}$

(2) $\frac{2}{9}$

(3) 0

(4) $\frac{8}{9}$

72. If $y=e^{nx}$, then $\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^2x}{dy^2}\right)$ is equal to:

(1) $n\,e^{nx}$

(2) $n\,e^{-nx}$

(3) 1

(4) $-n\,e^{-nx}$

73. If the Rolle's theorem holds for the function $f(x)=2x^3+ax^2+bx$ in the interval [-1,1] for the point $c=\frac{1}{2}$, then the value of 2a+b is :

(1) 1

(2) -1

(3) 2

(4) -2

74. If $f(x)=\left(\frac{3}{5}\right)^x+\left(\frac{4}{5}\right)^x-1, x\in R,$ then the equation $f(x)=0$ has :

(1) no solution

(2) one solution

(3) two solutions

(4) more than two solutions

75. $\int \frac{\sin^8 x - \cos^8 x}{\left(1 - 2\sin^2 \cos^2 x\right)} dx$ is equal to :

(1) $\frac{1}{2}\sin 2x + c$

(2) $-\frac{1}{2}\sin 2x + c$

(3) $-\frac{1}{2}\sin x + c$

(4) $-\sin^2 x + c$

76. The integral $\int_0^{\frac{1}{2}} \frac{ln(1+2x)}{1+4x^2} dx$, equals :

(1) $\frac{\pi}{4} ln2$

(2) $\frac{\pi}{8} ln2$

(3) $\frac{\pi}{16} ln2$

(4) $\frac{\pi}{32} ln2$

77. Let $A=\{(x,y): y^2 \le 4x,\ y-2x \ge -4\}$. The area (in square units) of the region A is:

(1) 8

(2) 9

(3) 10

(4) 11

78. If the differential equation representing the family of all circles touching x-axis at the origin is $(x^2-y^2)\frac{dy}{dx}=g(x)y$, then $g(x)$ equals :

(1) $\frac{1}{2}x$

(2) $2x^2$

(3) $2x$

(4) $\frac{1}{2}x^2$

79. Let a and b be any two numbers satisfying $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4}$. Then, the foot of perpendicular from the origin on the variable line, $\frac{x}{a}+\frac{y}{b}=\frac{1}{4}$, lies on:

(1) a hyperbola with each semi-axis $=\sqrt{2}$

(2) a hyperbola with each semi-axis = 2

(3) a circle of radius = 2

(4) a circle of radius $=\sqrt{2}$

80. Given there points P, Q, R with P(5,3) and R lies on the x-axis. If equation of RQ is $x-2y=2$ and PQ is parallel to the x-axis, then the centroid of ΔPQR lies on the line :

(1) $2x+y-9=0$

(2) $x-2y+1=0$

(3) $5x-2y=0$

(4) $2x-5y=0$

81. If the point (1, 4) lies inside the circle $x^2+y^2-6x-10y+p=0$ and the circle does not touch or intersect the coordinate axes, then the set of all possible values of p is the interval:

(1) (0, 25)

(2) (25, 39)

(3) (9, 25)

(4) (25, 29)

82. If OB is the semi-minor axis of an ellipse, F_1 and F_2 are its foci and the angle between F_1B and F_2B is a right angle, then the square of the eccentricity of the ellipse is:

(1) $\frac{1}{2}$

(2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{2\sqrt{2}}$

(4) $\frac{1}{4}$

83. Equation of the plane which passes through the point of intersection of lines $\frac{x-1}{3}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x-3}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ and has the largest distance from the origin is:

(1) $7x+2y+4z=54$

(2) $3x+4y+5z=49$

(3) $4x+3y+5z=50$

(4) $5x+4y+3z=57$

84. A line in the 3-dimensional space makes an angle $\theta\left(0<\theta\leq\frac{\pi}{2}\right)$ with both the x and y axis. Then the set of all values of θ is the internal :

(1) $\left(0,\frac{\pi}{4}\right]$

(2) $\left[\frac{\pi}{6},\frac{\pi}{3}\right]$

(3) $\left[\frac{\pi}{4},\frac{\pi}{2}\right]$

(4) $\left(\frac{\pi}{3},\frac{\pi}{2}\right]$

85. If $|\vec{a}|=2,|\vec{b}|=3$ and $|2\vec{a}-\vec{b}|=5$, then $|2\vec{a}+\vec{b}|$ equals :

(1) 17

(2) 7

(3) 5

(4) 1

86. In a set of 2n distinct observations, each of the observation below the median of all the observations is increased by 5 and each of the remaining observations is decreased by 3. Then the mean of the new set of observations :

(1)) increases by 1.

(2)) decreases by 1.

(3)) decreases by 2.

(4) increases by 2.

87. If A and B are two events such that $P(A \cup B) = P(A \cap B)$, then the incorrect statement amongst the following statements is:

(1) A and b are equally likely

(2) $P(A \cap B') = 0$

(3) $P(A' \cap B) = 0$

(4) $P(A) + P(B) = 1$

88. The number of values of α in $[0, 2\pi]$ for which $2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$, is:

(1) 6

(2) 4

(3) 3

(4) 1

89. If $\cos\theta = \frac{p+q}{p-q}\left(p \neq q \neq 0\right)$, then $\left|\cot\left(\frac{\pi}{4} + \frac{\theta}{2}\right)\right|$ is equal to :

(1) $\sqrt{\frac{p}{q}}$

(2) $\sqrt{\frac{q}{p}}$

(3) $\sqrt{pq}$

(4) pq

90. The contrapositive of the statement "I go to school if it does not rain" is:

(1) If it rains, I do not go to school.

(2) If I do not go to school, it rains.

(3) If it rains, I go to school.

(4) If I go to school, it rains.

Part-2

61. Sol:

Since the given function is reflexive.

$\sec^2 a - \tan^2 b = 1 \quad \forall x \in R$

For being symmetric,

$\sec^2 a - \tan^2 b = 1$

$\sec^2 b - \tan^2 a = 1 \qquad (1)$

To prove this,

$\sec^2 b - \tan^2 a = 1 + \tan^2 b - \left(\sec^2 a - 1\right)$

$= 1 + \tan^2 b + 1 - \sec^2 a$

$= 2 - \left(\sec^2 a - \tan^2 b\right)$

$= 1$

Therefore, the function is symmetric.

For transitive,

$\sec^2 a - \tan^2 b = 1 \qquad (2)$

$\sec^2 b - \tan^2 c = 1 \qquad (3)$

Therefore, point P is reflexive, symmetric and transitive.'

Hence this is a equivalence relation.

62. Sol:

The given equation is

$$w - \bar{w}z = k - kz$$

$$kz - \bar{w}z = k - w$$

$$z = \frac{k - w}{k - \bar{w}} \qquad (1)$$

Value of $\bar{z}$ is,

$$\bar{z} = \frac{k - \bar{w}}{k - w} \qquad (2)$$

Solve equation (1) and (2).

$$z\bar{z} = 1$$

$$|z| = 1$$

Therefore, the correct option is (4).

63. Sol:

The equation is

$$ax^2 + bx + c = 0 \left(a, b, c \in R, a \neq 0\right)$$

And $2x^2 + 3x + 4 = 0$

For the above equation.

$$D \leq 0$$

It suggests roots of the given equation imaginary and common.

$$\frac{a}{2} = \frac{b}{3} = \frac{c}{4}$$

64. Sol:

The given equation is,

$$ax^2 + bx + 1 = 0 \left(a, b \in R, a \neq 0\right)$$

The product of the roots is,

$$\frac{1}{\sqrt{\alpha\beta}} = \frac{1}{a}$$

And,

The sum of the roots is,

$$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = -\frac{b}{a}$$

$$\frac{\sqrt{\beta} + \sqrt{\alpha}}{\sqrt{\alpha\beta}} = -\frac{b}{a}$$

$$\sqrt{\alpha} + \sqrt{\beta} = -b$$

Now,

Substitute the value on the equation,

$$x\left(x + b^3\right) + a^3 - 3abx = x^2 + \left(b^3 - 3ab\right)x + a^3$$

$$= x^2 + b\left(b^2 - 3a\right)x + a^3$$

Further, simplify the above equation,

$$\left[\begin{array}{l} = x^2 - \left(\sqrt{\alpha} + \sqrt{\beta}\right)\left\{\sqrt{\alpha} + \sqrt{\beta} + 2\sqrt{\alpha}\sqrt{\beta} - 3\sqrt{\alpha}\sqrt{\beta}\right\}x \\ +\alpha\beta\sqrt{\alpha}\sqrt{\beta} \end{array}\right]$$

$$= x^2 - \left(\alpha\sqrt{\alpha} + \beta\sqrt{\beta}\right)x + \alpha\beta\sqrt{\alpha\beta}$$

Hence, the roots of the above equation is $\alpha^{\frac{3}{2}}$ and $\beta^{\frac{3}{2}}$.

65. Sol:

For having non-trivial solutions,

$\Delta = 0$

Take the values given in the equations.

$$\begin{vmatrix} 1-a & 1 & 1 \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix} = 0$$

$$R_1 \to R_1 - R_3$$

$$\begin{vmatrix} -a & 0 & c \\ 1 & 1-b & 1 \\ 1 & 1 & 1-c \end{vmatrix} = 0$$

$$a\left\{(1-b)(1-c)-1\right\} + c\left\{1-(1-b)\right\} = 0$$

$$ab + ac + bc = abc$$

66. Sol:

$$\det\left[(1+B)^{50}-50B\right]=\det\left[1+50B+\frac{50\times49}{2!}B^2+....-50B\right]$$

$$=\det\left[1+B^2\left\{\frac{50\times49}{2!}+\frac{50\times49\times48}{3!}B+...\right\}\right]$$

$$=\det\left[1+0\left\{\frac{50\times49}{2!}+\frac{50\times49\times48}{3!}B+....\right\}\right]$$

$$=1$$

67. Sol:

$$(1+x)(1+x)^{100}\left(1+x^2-x\right)^{100}=(1+x)\left(1+x^3\right)^{100}$$

$$=\underbrace{1\left(1+x^3\right)^{100}}_{101\text{ terms}}+\underbrace{x\left(1+x^3\right)^{100}}_{101\text{ terms}}$$

Therefore, the number of terms in the expression is,

101+101 = 202 terms

68. Sol:

If the unit place is 3 then remaining three palaces can be filled in 3! ways. Thus 3 appears in unit place in 3! times.

Similarly, each digit appear in unit place times.

Hence, the sum of the digits in units place is.

$$(6+5+4+3)(4-1)! = 18\times 3!$$
$$= 18\times 6$$
$$= 108$$

69. Sol:

Use expression for the n^{th} term of an AP.

$$a+(2-1)d = 12$$
$$a+d = 12$$

According to the given condition its 9^{th} term

$$200 < \frac{9}{2}(2a+8d) < 220$$
$$\frac{92}{27} < d < \frac{112}{27}$$

But d has to be a integer.

Therefore,

$$d = 4$$

Then,

$$a+4 = 12$$
$$a = 8$$

The 4^{th} term of the AP is

$$a+3d = 8+(3\times 4)$$
$$= 20$$

70. Sol:

The n^{th} term of the given series is

$$t_n = \frac{2n+1}{\frac{n(n+1)(2n+1)}{6}}$$

$$= \frac{6}{n(n+1)}$$

$$= 6\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

Substitute $n = 1, , 2, 3, ..., 20$ in the above equation.

$$S_n = 6\left\{\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} + ... - \frac{1}{21}\right\}$$

$$= 6\left\{\frac{1}{1} - \frac{1}{21}\right\}$$

$$= \frac{120}{21}$$

Hence, k=120

71. Sol:

Simplifying the given expression

$$\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right) = \lim_{x\to 0}\left(\frac{1-2\sin^2\frac{3x}{2}+1}{x^2}\right)$$

$$= \lim_{x\to 0} f\left\{\left(\frac{2\sin^2\frac{3x}{2}}{\frac{3x}{2}\times\frac{3x}{2}}\right)\times\frac{9}{4}\right\}$$

$$= \lim_{x\to 0} f\left\{\left(\frac{\sin\frac{3x}{2}}{\frac{3x}{2}}\right)^2\left(2\times\frac{9}{4}\right)\right\}$$

$$= f\left(\frac{9}{2}\right)$$

Therefore, the required value is

$$\lim_{x\to 0} f\left(\frac{1-\cos 3x}{x^2}\right) = f\left(\frac{9}{2}\right)$$

$$= \frac{2}{9}$$

72. Sol:

$$y = e^{nx} \qquad (1)$$

Upon differentiation

$$\frac{dy}{dx} = ne^{nx}$$

$$\frac{d^2y}{dx^2} = n^2e^{nx} \qquad (2)$$

Take log both sides in equation (1).

$$\frac{1}{n}\log y = x$$

$$\frac{1}{n}\left(\frac{1}{y}\right) = \frac{dx}{dy}$$

$$-\frac{1}{ny^2} = \frac{d^2x}{dy^2} \qquad (3)$$

Multiply the equation (2) and (3).

$$\frac{d^2y}{dx^2}\frac{d^2x}{dy^2} = n^2e^{nx} \times \left(-\frac{1}{ny^2}\right)$$

$$= -\frac{n}{e^{nx}}$$

$$= -ne^{-nx}$$

73. Sol:

Substitute the values in the function.

$$f(-1) = -2 + a - b$$

$$f(1) = 2 + a + b$$

By the rolls theorem

$$f(-1) = f(1)$$

$$-2 + a - b = 2 + a + b$$

$$-2 = b$$

Differentiation of the given function is

$$f'(x) = 6x^2 + 2ax + b$$

At $x = \frac{1}{2}$,

$$f'\left(\frac{1}{2}\right) = 6 \times \frac{1}{4} + 2a \times \frac{1}{2} + b$$

$$= 0$$

$$\frac{3}{2} + a + b = 0 \quad (\because b = -2)$$

$$a = \frac{1}{2}$$

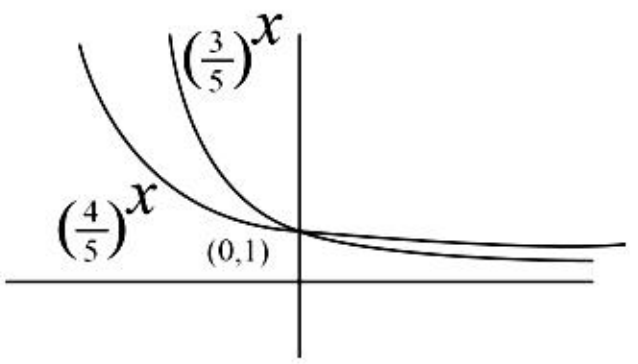
Therefore, the required value is

$$2a + b = 2\frac{1}{2} + (-2)$$

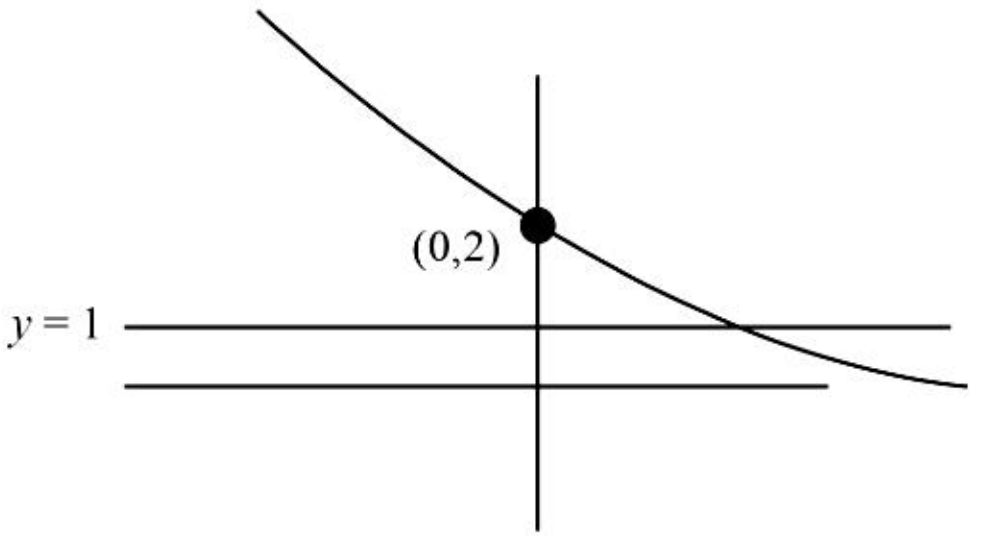
$$= -1$$

74. Sol:

By graph method



By adding both the curve $\left(\frac{3}{5}\right)^x$ and $\left(\frac{4}{5}\right)^x$



The figure shows that there is one intersection point. Therefore, the number of solution is one.

75. Sol:

By integrating the given function,

$$I = \int \frac{\left(\sin^8 x - \sin^8 x\right)}{\left(1 - 2\sin^2 x \cos^2 x\right)} dx$$

$$= \int \frac{\left(\sin^4 x + \sin^4 x\right)\left(\sin^2 x + \sin^2 x\right)\left(\sin^2 x - \sin^2 x\right)}{\left\{\left(\sin^2 x + \sin^2 x\right)^2 - 2\sin^2 x \cos^2 x\right\}} dx$$ By

$$= \int \frac{\left(\sin^4 x + \sin^4 x\right)\left(\sin^2 x - \sin^2 x\right)}{\left(\sin^4 x + \sin^4 x\right)} dx$$

$$= \int -\cos 2x dx$$

integrating it further.

$$I = -\frac{\sin 2x}{2} + c$$

76. Sol:

Put,

$2x = \tan\theta$

Differentiate the above expression.

$$dx = \frac{1}{2}\sec^2\theta$$

At $x = 0,\ \theta = 0$

And at $x = \frac{1}{2},\ \theta = \frac{\pi}{4}$

Substitute the values in the given function.

$$I = \int_0^{\frac{\pi}{4}} \frac{\ln(1+\tan\theta)}{1+\tan^2\theta}\frac{1}{2}\sec^2\theta d\theta$$

$$= \frac{1}{2}\int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)d\theta$$

$$= \frac{1}{2}I_1$$

Solve I_1

$$I_1 = \int_0^{\frac{\pi}{4}} \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right)d\theta$$

$$= \int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan\theta}\right)d\theta$$

$$= \int_0^{\frac{\pi}{4}} \ln 2 d\theta - \int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)d\theta$$

$$= \frac{\pi}{4}\ln 2 - I_1$$

Further simplify for I_1.

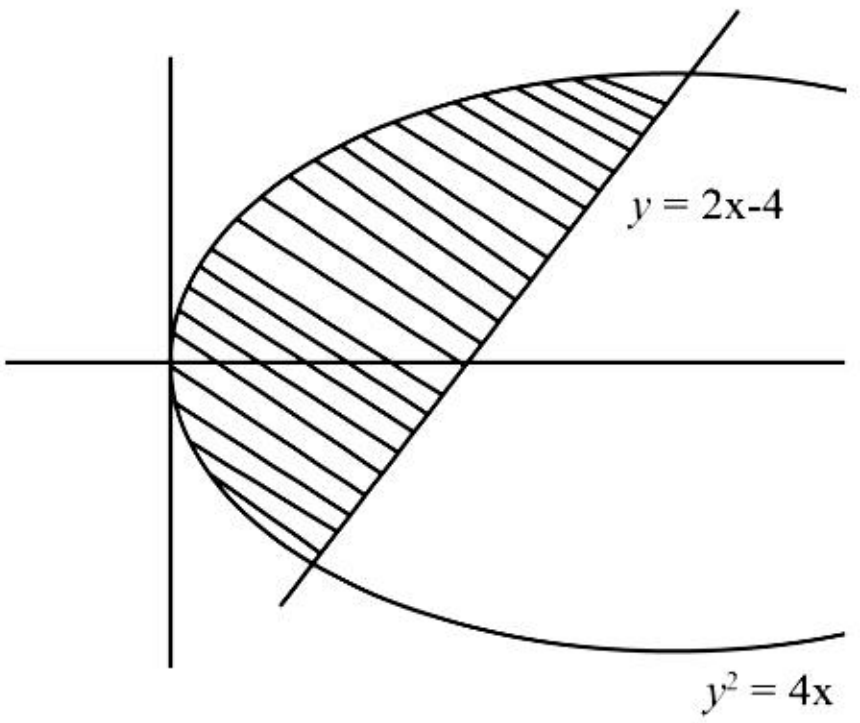
$$I_1 = \frac{\pi}{8}\ln 2$$

The integration of the required function is

$$I = \frac{\pi}{16}\ln 2$$

77. Sol:

The bounded curve is shown in figure below.



Solve the equations for y.

$$y = -2, 4$$

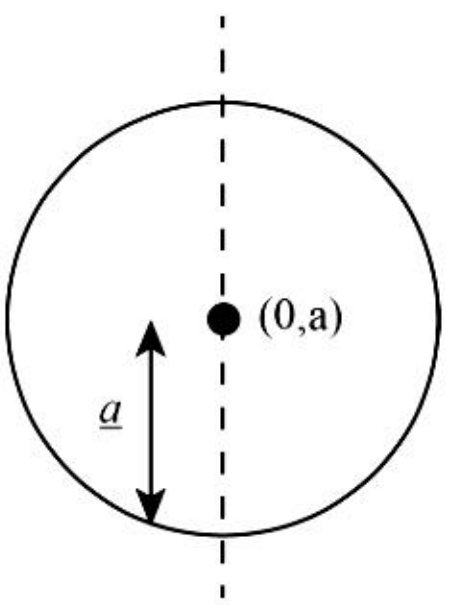
Calculating the area of the triangle,

$$\text{Area} = \int_{-2}^{4} \left(\frac{y+4}{2} - \frac{y^2}{4} \right) dy$$

$$= \left(\frac{y^2}{4} + 2y - \frac{y^3}{12} \right)_{-2}^{4}$$

$$= 9$$

78. Sol:



The equation of circle is

$$x^2 + (y-a)^2 = a^2$$

$$x^2 + y^2 - 2ay = 0 \qquad (1)$$

Differentiate the above equation

$$2x + 2y\frac{dy}{dx} - 2a\frac{dy}{dx} = 0$$

$$a = \frac{x + yy'}{y'}$$

Put value of a in equation (1).

$$x^2 + y^2 - 2y\left(\frac{x + yy'}{y'}\right) = 0$$

$$\left(x^2 - y^2\right)y' = 2xy$$

Compare the above equation with

$$\left(x^2 - y^2\right)\frac{dy}{dx} = g(x)y$$

Therefore, the value of $g(x)$ is

$$g(x) = 2x$$

79. Sol:

Equation of normal is

$$\frac{h-0}{\frac{1}{a}} = \frac{k-0}{\frac{1}{b}} = \frac{1}{\frac{1}{a^2} + \frac{1}{b^2}}$$

$$a = \frac{h}{4} \qquad (1)$$

$$b = \frac{k}{4} \qquad (2)$$

Solve equation (1) and (2).

$$\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{4}$$

$$\frac{h^2}{16}+\frac{k^2}{16}=\frac{1}{4}$$

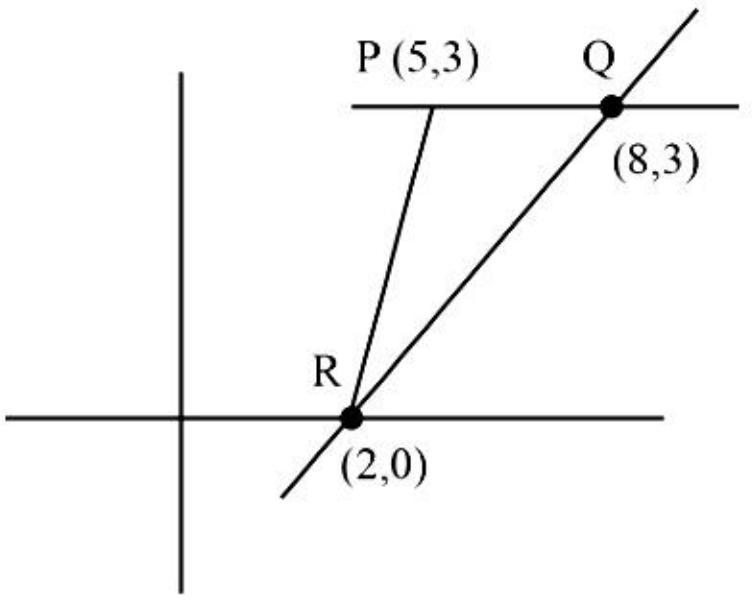
Therefore, the locus is.

$$\frac{x^2}{16}+\frac{y^2}{16}=\frac{1}{4}$$

$$x^2+y^2=4$$

80. Sol:

The figure below represents the given points.



The equation of line RQ is,

$$x-2y=2$$

The equation of line PQ is,

$$y=3$$

Substitute 3 for y in equation of RQ.

$$x - 2(3) = 2$$

$$x = 8$$

$$\Rightarrow R(8, 3)$$

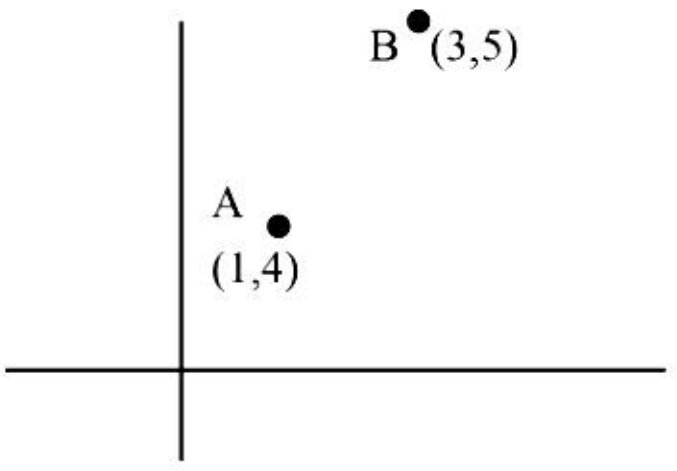
Therefore, the centroid of the triangle is.

$$\left(\left(\frac{2+8+5}{3}\right), \left(\frac{0+3+3}{3}\right)\right) \equiv (5, 2)$$

Only the line $2x - 5y$ satisfy this point.

81. Sol:

The points are represented in the figure below



The distance between AB is

$$AB = \sqrt{2^2 + 1}$$

$$= \sqrt{5}$$

Hence, from the given relation

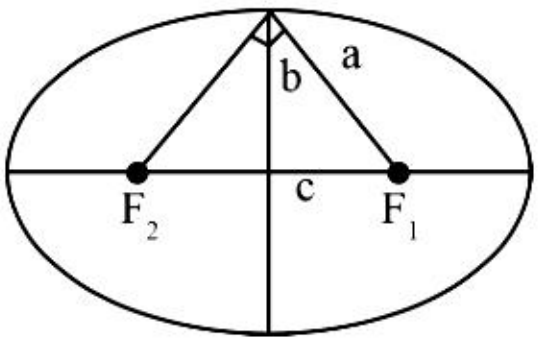
$$\sqrt{5} < \sqrt{\left(3^2 + 5^2 - p\right)} < q$$

$$5 < 34 - p < q$$

$$-29 < -p < -25$$

$$29 > p > 25$$

82. Sol:



From Pythagoras theorem,

$$a^2 = b^2 + c^2$$

Substituting the given values,

$$a^2 + a^2 = (2c)^2$$

$$2a^2 = 4c^2$$

$$a^2 = 2c^2$$

From the relation of eccentricity,

$$c = ae$$

$$\frac{c}{a} = e$$

$$\frac{c^2}{a^2} = e^2$$

$$\frac{c^2}{2c^2} = e^2$$

Hence, the value of square of eccentricity is

$$e^2 = \frac{1}{2}$$

83. Sol:

The equation of planes is.

$$\frac{x-1}{3} = \frac{y-2}{1} = \frac{z-3}{2} = \alpha$$

$$\frac{x-3}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \beta$$

Consider a plane with direction cosine l, m, n and distances from origin is d is.

$$lx + my + nz = d$$

Hence, the equation of the plane is,

$$\frac{4}{\sqrt{50}}x + \frac{3}{\sqrt{50}}y + \frac{5}{\sqrt{50}}z = \sqrt{50}$$

$$4x + 3y + 5z = 50$$

84. Sol:

The given angle θ which makes by the plane in the 3-dimensional space is,

$$0 < \theta \leq \frac{\pi}{2}$$

The condition for minimum value of angle θ is:

If the line lies on x, y plane, it makes angle of $45°$.

The condition for maximum value of angle θ is:

If line at $z-$axis, it makes an angle of $90°$.

Hence, the required interval is,

$$\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$$

85. Sol:

$$\left|2\vec{a}-\vec{b}\right| = 5$$

Simplifying the above equation,

$$\left|2\vec{a}-\vec{b}\right|^2 = 25$$

$$4\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 - 4\vec{a}.\vec{b} = 25$$

$$16 + 9 - 4\vec{a}.\vec{b} = 25$$

$$4 \times \vec{a}.\vec{b} = 0$$

Then,

$$\left|2\vec{a}+\vec{b}\right| = k$$

$$\left(2\vec{a}+\vec{b}\right)\left(2\vec{a}+\vec{b}\right) = k^2$$

$$4\left|\vec{a}\right|^2 + \left|\vec{b}\right|^2 + 4\vec{a}.\vec{b} = k^2$$

$$\sqrt{16+9+0} = k$$

Thus the value of k is 5.

86. Sol:

Mean value of the set of 2n observations.

$$\frac{t_1+t_2+t_3+.....t_n+t_{n+1}+.....+t_{2n}}{2n}=M$$

Substitute the values in the above equation.

$$N=\frac{t_1+5+t_2+5+.....t_n+5+t_{n+1}-3+.....+t_{2n}-3}{2n}$$

$$=\frac{t_1+t_2+.....t_n+5(n)+t_{n+1}+.....+t_{2n}-3(n)}{2n}$$

$$=\frac{t_1+t_2+.....t_n+t_{n+1}+.....+t_{2n}}{2n}+\frac{5n-3n}{2n}$$

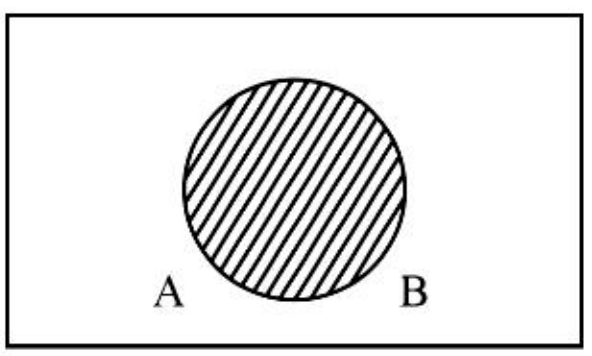
$$=\frac{t_1+t_2+.....t_n+t_{n+1}+.....+t_{2n}}{2n}+1$$

Then the mean value is,

$$\frac{t_1+t_2+.....t_n+t_{n+1}+.....+t_{2n}}{2n}+1=M+1$$

87. Sol:

The van diagram of events is given below.



$$P(A\cup B)=P(A\cap B)$$

From van diagram it is clear that option (1) is incorrect.

88. Sol:

The trigonometric expression is

$2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$

Simplify the above expression.

$$2\sin^3\alpha - 7\sin^2\alpha + 7\sin\alpha = 2$$

$$2\sin^3\alpha - 2 = 7\sin^2\alpha - 7\sin\alpha$$

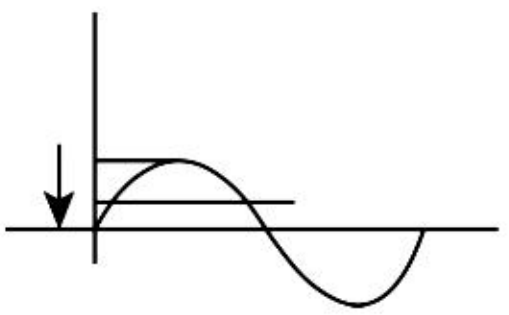
$$2(\sin\alpha - 1)(\sin^2\alpha + 1 + \sin\alpha) = 7\sin\alpha(\sin\alpha - 1)$$ By

$$(\sin\alpha - 1)\left(\sin^2\alpha + 1 + \sin\alpha - \frac{7}{2}\sin\alpha\right) = 0$$

equating it to zero.

$\sin\alpha = 1$ or $\sin\alpha = \frac{1}{2}, -2$ $(\sin\alpha \neq -2)$

Below figure represent the sine curve.



Therefore, the number of solution here is 3.

89. Sol:

Simplify the given expression

$$\left|\cot\left(\frac{\pi}{4}+\frac{\theta}{2}\right)\right| = \left|\frac{1-\tan\frac{\theta}{2}}{1+\tan\frac{\theta}{2}}\right|$$

$$= \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}} \times \frac{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}$$

$$= \frac{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\cos\frac{\theta}{2}\sin\frac{\theta}{2}}{\cos\theta}$$

$$= \frac{1-\sin\theta}{\cos\theta}$$

Upon further simplification

$$\frac{1-\sin\theta}{\cos\theta} = \frac{1-\frac{p-q}{p+q}}{\sqrt{1-\left(\frac{p-q}{p+q}\right)^2}}$$

$$= \frac{\sqrt{q}}{\sqrt{p}}$$

90. Sol:

Consider, p is equal to the statement “if it does not rain”

And, q is equal to the statement “I go to school”

According to the Contrapositive law,

$p \rightarrow q = \sim q \rightarrow \sim p$

Thus,

Negation of p is “its rains”

And is “if I do not go to school”

Hence, is “If I do not go to school, it rains.”