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JKPSC Exam

Mains Previous Paper

Simplifying
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MATHEMATICS-I

[15]

Time Allowed -3 Hours

Maximum Marks-300

INSTRUCTIONS

- i) Answers must be written in English.
- ii) The number of marks carried by each question is indicated at the end of the question.
- iii) The answer to each question or part there of should begin on a fresh page.
- iv) Your answer should be precise and coherent.
- v) The part/parts of the same question must be answered together and should not be interposed between answers to other questions.
- vi) Candidates should attempt any **Five** questions.
- vii) If you encounter any typographical error, please read it as it appears in the text book.
- viii) Candidates are in their own interest are advised to go through the General Instructions on the back side of the title page of the Answer Script for strict adherence.
- ix) No Continuation sheets shall be provided to any candidate under any circumstances.
- x) Candidates shall put a cross (X) on blank pages of Answer Script.
- xi) No blank page be left in between answer to various questions.
- xii) No programmable Calculator is allowed.
- xiii) No stencil (with different markings) is allowed.
- xiv) In no circumstances help of scribe will be allowed.

MATHEMATICS-I (12 Questions)

All questions carry same marks (60 each) and each part of every question carry the same weightage

1. Let U and W be subspaces of a vector space V . We define the sum $U+W$ as follows : $U+W=\{u+w : u \in U, w \in W\}$.
 - a) Show that $U+W$ is a subspace of V containing both U and W .
 - b) Show that $\text{span} \{u_1, u_2, \dots, u_r, w_1, w_2, \dots, w_s\} = \text{span} \{u_1, u_2, \dots, u_r\} + \text{span} \{w_1, w_2, \dots, w_s\}$ for any vectors u_i 's and w_j 's t from U and W respectively.

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(1)

[Turn Over

- c) If U and W are finite dimensional and if $U \cap W = \{0\}$, then show that $\dim(U+W) = \dim(U) + \dim(W)$.
2. Let $T:V \rightarrow V$ be a linear transformation of a finite dimensional vector space V .
- a) Prove that if $U \subset V$ is a vector subspace, then $T(U) = \{T(u):u \in U\}$ is a vector subspace of V .
- b) Prove that if T is onto, it sends linearly independent set of vectors to linearly independent set of vectors.
- c) Prove that if T is invertible and $U \subseteq V$ then $\dim T(U) = \dim U$.
3. Let A and B be $m \times n$ matrices.
- a) Show that $\text{rank}(A+B) \leq \text{rank}(A) + \text{rank}(B)$
- b) Show that $\text{null}(A) = \text{null}(UA)$ for any invertible $m \times m$ matrix U .
- c) Show that $\dim(\text{null}(A)) = \dim(\text{null}(AV))$ for any invertible $n \times n$ matrix V .
4. A complex matrix S is called skew-Hermitian, if $S^* = -S$
- a) Show that $Z - Z^*$ is skew-Hermitian for any square complex matrix Z .
- b) If S is skew-Hermitian, then show that S^2 and iS are Hermitian.
- c) If S is skew-Hermitian, then show that the eigenvalues of S are purely imaginary ($i\lambda$ for real λ).
5. Find the equation (s) of the tangent (s).
- a) to the parabola $y^2 = 2ax$ at the vertex and at the ends of Latus rectum.
- b) to the hyperbola $x^2 - 2y^2 = 1$ from the point $(7,5)$.
- c) to the ellipse $\frac{x^2}{32} + \frac{y^2}{18} = 1$ at the point whose x coordinate is 2.
6. Prove the following :
- a) Let $A \subset \mathbb{R}$. We say that a function $f:A \rightarrow \mathbb{R}$ is Lipschitz on A if there exists $L > 0$ such that $|f(x) - f(y)| \leq L|x - y|$ for all $x, y \in A$. Show that any Lipschitz function is continuous.

such that $|f(x)| > \frac{|f(c)|}{2}$ for all $x \in (c - \delta, c + \delta) \cap A$.

- c) Let $f: [0, 2\pi] \rightarrow [0, 2\pi]$ be continuous such that $f(0) = f(2\pi)$. Show that there exists $x \in [0, 2\pi]$ such that $f(x) = f(x + \pi)$.
7. a) Use Lagrange multipliers to find the maxima and minima of the function $f(x, y) = x^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 1$
- b) Find all the stationary points and local maxima and minima of the function $f(x, y) = e^{x+y} (x^2 + y^2 - xy)$.
8. Let $F(y) = \int_0^\infty \frac{\sin xy}{x(x^2 + 1)} dx$ if $y > 0$
- a) show that F satisfies the differential equation $F''(y) - F(y) + \pi/2 = 0$
- b) Deduce that $F(y) = \frac{1}{2} \pi (1 - e^{-y})$
- c) Deduce that for $y > 0$ and $a > 0$ $\int_0^\infty \frac{\cos xy}{x^2 + a^2} dx = \frac{\pi e^{-ay}}{2a}$
9. a) Solve the differential equation $y \cos x + 2xe^y + (\sin x + x^2 e^y - 1)y' = 0$
- b) A radioactive substance obeys the equation $y' = ky$ where $k < 0$ and y is the mass of the substance at time t . Suppose that initially, the mass of the substance is $y(0) = M > 0$. At what time does half of the mass remain?
- c) The half life of carbon-14 is 5730 years. If one starts with 100 milligrams of carbon-14; how much is left after 6000 years? how long do we have to wait before there is less than 2 milligrams?
10. Consider the initial value problem $y' + \frac{y}{4} = 3 + 2 \cos 2t, y(0) = y_0$
- a) Find the solution to the homogenous equation of this problem.
- b) Find the solution of this initial value problem and describe its behaviour for large t .

- c) Determine the value of t for which the solution first intersects the line $y=12$.
11. Let u and v be vector fields, ϕ is a scalar function and T is a tensor field. Show that
- $\text{curl}(\text{grad } v)^T = \text{grad}(\text{curl } v)$
 - $\text{curl } \text{curl } v = \text{grad}(\text{div } v) - \text{grad}^2 v$
 - $\text{curl}(\phi v) = \phi \text{curl } v + (\nabla \phi) \times v$
 - $\text{div}(u \otimes v) = (\text{div } v)u + (\text{grad } u)v$.
12. a) The tallest spot on Earth is Mt. Everest, which is 8857 m above sea level. If the radius of the Earth to sea level is 6369 km, how much does the magnitude of g change between sea level and the top of Mt. Everest?
- b) The value of g at the surface of the earth is 9.78 N/kg, and on the surface of Venus the magnitude of g is 8.6 N/kg. An astronaut has a mass of 70 kg on the surface of the earth. What will her weight be on the surface of Venus?
- c) A person in a kayak starts paddling, and it accelerates from 0 to 0.9 miles/hour in a distance of 0.9 km. If the combined mass of the person and the kayak is 80 kg, what is the magnitude of the net force acting on the kayak?



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