MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. $\int_{\pi/6}^{\pi/3} \frac{2+3\sin x}{\sin x(1+\cos x)}dx$ is equal to

(1) $\ln(\sqrt{3}+2)-\frac{\ln 3}{2}+6\sqrt{3}-\frac{28}{3}$

(2) $\ln(\sqrt{3}+2)-\frac{\ln 3}{2}$

(3) $\ln(\sqrt{3}+2)-\frac{\ln 3}{2}-\frac{28}{3}$

(4) $6\sqrt{3}-\frac{28}{3}$

Answer (1)

Sol. $I=\int \frac{2}{\sin x(1+\cos x)}dx+\int \frac{3}{1+\cos x}dx$

$$=\underbrace{\int \frac{2\sin x}{\sin^2 x(1+\cos x)}dx}_{I_1}+\underbrace{\int \frac{3}{2\cos^2 \frac{x}{2}}dx}_{I_2}$$

Let $\cos x = t$

$$I_1=\int \frac{-2dt}{(1-t^2)(1+t)}$$

$$=-2\left(\frac{\ln|t+1|}{4}+\frac{1}{2t+2}-\frac{\ln|t-1|}{4}\right)+C$$

$$=-2\left(\frac{\ln|\cos x+1|}{4}+\frac{1}{2\cos x+2}-\frac{\ln|\cos x-1|}{4}\right)+C$$

$$I_2=\frac{3}{2}2\tan\frac{x}{2}+C$$

So,

$$\int_{\pi/6}^{\pi/3} \frac{2+3\sin x}{\sin x(1+\cos x)}dx=\ln(2+\sqrt{3})-\ln\frac{3}{2}+6\sqrt{3}-\frac{28}{3}$$

2. The product and sum of first four terms of G.P. is 1296 and 126 respectively, then sum of the possible values of common difference is ___.

(1) 14 (2) $\frac{10}{3}$

(3) $\frac{7}{2}$ (4) 3

Answer (4)

Sol. $\frac{a}{r^3}\cdot\frac{a}{r}\cdot ar\cdot ar^3=1296$

$\Rightarrow$ $a = 6$

Now, $\frac{a}{r^3}+\frac{a}{r}+ar+ar^3=126$

$\Rightarrow$ $\frac{1}{r^3}+\frac{1}{r}+r+r^3=21$

$\Rightarrow$ $\left(r+\frac{1}{r}\right)\left(\left(r+\frac{1}{r}\right)^2-3\right)+\left(r+\frac{1}{r}\right)=21$

Let $r+\frac{1}{r}=t$

$t^3-3t+t=21$

$\Rightarrow$ $t^3-2t-21=0$

$\Rightarrow$ $t=3$

$\Rightarrow$ $r+\frac{1}{r}=3$

$\Rightarrow$ $r^2-3r+1=0$

$r_1+r_2=3$

Sum of possible values of r is 3

3. If $B = \ln(1-a)$ and $P(a)$

$$=\left(a+\frac{a^2}{2}+\frac{a^3}{3}+.....+\frac{a^{50}}{50}\right)$$

then $\int_0^a \frac{t^{50}}{1-t}dt$ equals

(1) $-(B+P(a))$ (2) $-B+P(a)$

(3) $B-P(a)$ (4) $B+P(a)$

Answer (3)

Sol. $\int_0^a \left(\frac{t^{50}-1}{1-t}+\frac{1}{1-t}\right)dt$

$\Rightarrow \int_0^a \left(-(1+t+t^2+....+t^{49})+\frac{1}{1-t}\right)dt$

$\Rightarrow \ln(1-t)-\left(t+\frac{t^2}{2}+\frac{t^3}{3}+.....\frac{t^{50}}{50}\right)\Big|_0^a$

$\Rightarrow \ln(1-a)-\left(a+\frac{a^2}{2}+\frac{a^3}{3}+....+\frac{a^{50}}{50}\right)$

$\Rightarrow B-P(a)$

4. $\sin^{-1}\left(\frac{a}{17}\right)+\cos^{-1}\left(\frac{4}{5}\right)-\tan^{-1}\left(\frac{77}{36}\right)=0$, then the value of $\sin^{-1}(\sin a)+\cos^{-1}(\cos a)$ is

(1) 0 (2) $16-2\pi$

(3) π (4) 5

Answer (3)

Sol. $\sin^{-1}\left(\frac{a}{17}\right)=-\cos^{-1}\left(\frac{4}{5}\right)+\tan^{-1}\left(\frac{77}{36}\right)$

Let $\cos^{-1}\frac{4}{5}=\beta$ and $\tan^{-1}\left(\frac{77}{36}\right)=\alpha$

$\Rightarrow \sin\left(\sin^{-1}\frac{a}{17}\right)=\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\Rightarrow \frac{a}{17}=\frac{77}{85}\cdot\frac{4}{5}-\frac{36}{85}\cdot\frac{3}{5}$

$\Rightarrow a=\frac{200}{25} \Rightarrow a=8$

$\therefore \sin^{-1}\sin 8+\cos^{-1}\cos 8=3\pi-8+8-2\pi$

$=\pi$

5. The range of $\frac{[x]}{x^2+1}$ if domain is [–2, 6] is ([.] represents G.I.F)

(1) $\left[-4,\frac{-5}{2}\right]\cup\left[\frac{5}{37},\frac{1}{2}\right]\cup\{0\}$

(2) $\left(-1,\frac{-2}{5}\right)\cup\left[0,\frac{5}{37}\right]$

(3) $\left(-1,\frac{-2}{5}\right]\cup\left[\frac{5}{37},\frac{1}{2}\right)\cup\{0\}$

(4) $\left[-1,\frac{-2}{5}\right)\cup\left[0,\frac{5}{37}\right)$

Answer (3)

Sol. $x\in[-2,-1), f(x)=\frac{-2}{x^2+1}, f(x)\in\left[\frac{-2}{5},-1\right]$

$x\in[-1,0), f(x)=\frac{-1}{x^2+1}, f(x)\in\left[\frac{-1}{2},-1\right]$

$x\in[0,1), f(x)=0$

$x\in[1,2), f(x)=\frac{1}{x^2+1}, f(x)\in\left(\frac{1}{5},\frac{1}{2}\right]$

$x\in[2,3), f(x)=\frac{2}{x^2+1}, f(x)\in\left(\frac{1}{5},\frac{2}{5}\right]$

$x\in[3,4), f(x)=\frac{3}{x^2+1}, f(x)\in\left(\frac{3}{17},\frac{3}{10}\right]$

$x\in[4,5), f(x)=\frac{4}{x^2+1}, f(x)\in\left(\frac{4}{26},\frac{4}{17}\right]$

$x\in[5,6), f(x)=\frac{5}{x^2+1}, f(x)\in\left(\frac{5}{37},\frac{5}{26}\right]$

$x=6, f(x)=\frac{6}{37}$

$\therefore$ range of $f(x)$ is $\left(-1,\frac{-2}{5}\right]\cup\{0\}\cup\left(\frac{5}{37},\frac{1}{2}\right]$

6. If maximum distance of a normal to the ellipse $\frac{x^2}{4}+\frac{y^2}{b^2}=1$ from (0, 0) is 1, then find the eccentricity of the ellipse.

(1) $\frac{\sqrt{3}}{4}$ (2) $\frac{1}{\sqrt{2}}$

(3) $\frac{1}{2}$ (4) $\frac{\sqrt{3}}{2}$

Answer (4)

Sol. Equation of normal is

$(2\sec\theta)x-(b\,\mathrm{cosec}\theta)y=4-b^2$

Perpendicular distance from (0, 0) is

$D=\left|\frac{4-b^2}{\sqrt{4\sec^2\theta+b^2\mathrm{cosec}^2\theta}}\right|$

$=\frac{4-b^2}{\sqrt{(4+b^2)+4\tan^2\theta+b^2\cot^2\theta}}$

$\le\frac{4-b^2}{\sqrt{b^2+4+4b}}$ [Using $AM\ge GM$]

$=\frac{4-b^2}{(2+b)}=2-b$

$D_{max} = 2 - b = 1 \Rightarrow b = 1$

$$e^2 = 1 - \frac{1}{4}$$

$$e = \frac{\sqrt{3}}{2}$$

7. Let the curve C_1 be represented by $|z| = 2$ and C_2 by $\left|z + \frac{1}{z}\right| = \frac{15}{4}$ then

(1) C_1 lies inside C_2

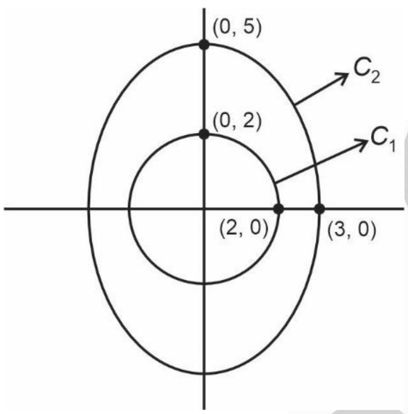
(2) C_2 lies inside C_1

(3) C_1 and C_2 has 2 points of intersection

(4) C_1 and C_2 has 4 points of intersection

Answer (1)

Sol. Let $z = x + iy$



$$C_1 \Rightarrow x^2 + y^2 = 4$$

$$C_2 \Rightarrow \left|x + iy + \frac{1}{x + iy}\right| = \frac{15}{4}$$

OR $\left|x + iy + \frac{x - iy}{4}\right| = \frac{15}{4}$

OR $\left(\frac{5x}{4}\right)^2 + \left(\frac{3y}{4}\right)^2 = \frac{225}{16}$

OR $\frac{x^2}{9} + \frac{y^2}{25} = 1$

$$e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$

8. Find the number of real solution(s) of $\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 6}$ is

(1) 1 (2) 2

(3) 3 (4) 4

Answer (1)

Sol. $x^2 - 4x + 3 \geq 0$

$(x - 1)(x - 3) \geq 0$

$x \in (-\infty, 1] \cup [3, \infty)$ …(i)

$x^2 - 9 \geq 0 \Rightarrow x \in (-\infty, -3] \cup [3, \infty)$ …(ii)

$4x^2 - 14x + 6 \geq 0 \Rightarrow (2x - 1)(x - 3) \geq 0$

$\Rightarrow x \in \left(-\infty, \frac{1}{2}\right] \cup [3, \infty)$ …(iii)

(i) $\cap$ (ii) $\cap$ (iii)

$x \in (-\infty, -3] \cup [3, \infty)$

Now squaring both sides of given equation:

$(x^2 - 4x + 3) + (x^2 - 9) + 2\sqrt{(x^2 - 4x + 3)(x^2 - 9)} = 4x^2 - 14x + 6$

$\Rightarrow 2\sqrt{(x^2 - 4x + 3)(x - 3)(x + 3)} = 2(x^2 - 5x + 6)$

$\Rightarrow (x^2 - 4x + 3)(x - 3)(x + 3) = (x - 3)^2 (x - 2)^2$

$x = 3$ is one solution

$\Rightarrow (x^2 - 4x + 3)(x + 3) = (x^2 - 4x + 4)(x - 3)$

$\Rightarrow x^3 - 4x^2 + 3x + 3x^2 - 12x + 9$

$= x^3 - 4x^2 + 4x - 3x^2 + 12x - 12$

$\Rightarrow 6x^2 - 25x + 21 = 0$

$\Rightarrow x = 3, \frac{7}{6}$

$\therefore$ Only one real solution $x = 3$ as $x = \frac{7}{6}$, is not is the domain.

9. If $f(x) = \sin^3\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)$ then $f'(1)$ is

(1) $\frac{3\pi^2}{8}$ (2) $\frac{3\pi^2}{4}$

(3) $\frac{3\pi^2}{16}$ (4) $\frac{\pi^2}{2}$

Answer (3)

Sol. $f'(x) = 3\sin^2\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)$

$$\cdot\cos\left(\frac{\pi}{3}\cos\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)$$

$$\cdot\frac{\pi}{3}\left(-\sin\left(\frac{\pi}{3\sqrt{2}}\left(-4x^3 + 5x^2 + 1\right)^{\frac{3}{2}}\right)\right)$$

$$\cdot\frac{\pi}{3\sqrt{2}}\cdot\frac{3}{2}\left(-4x^3+5x^2+1\right)^{\frac{1}{2}}\cdot\left(-12x^2+10x\right)$$

$$f'(1)=3\sin^2\left(\frac{\pi}{3}\cos\left(\frac{2\pi}{3}\right)\right)\cdot\cos\left(\frac{\pi}{3}\cos\left(\frac{2\pi}{3}\right)\right)$$

$$\cdot\frac{\pi}{3}\left(-\sin\frac{2\pi}{3}\right)\cdot\frac{\pi}{2\sqrt{2}}\left(\sqrt{2}\right)(-2)$$

$$=3\sin^2\left(\frac{-\pi}{6}\right)\cdot\cos\left(\frac{-\pi}{6}\right)\cdot\frac{\pi}{3}\left(-\frac{\sqrt{3}}{2}\right)(-\pi)$$

$$=\frac{3}{4}\cdot\frac{\sqrt{3}}{2}\cdot\frac{\pi}{3}\cdot\frac{\sqrt{3}}{2}\cdot\pi=\frac{3\pi^2}{16}$$

10. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\left|\vec{a}+\vec{b}+\vec{c}\right|=\left|\vec{a}+\vec{b}-\vec{c}\right|$ and $\vec{b}\cdot\vec{c}=0$ then

Statement-I : $|\vec{a}+\lambda\vec{c}|\ge 0$ for all $\lambda\in\boldsymbol{R}$.

Statement-II : $\vec{a}$ is always parallel to $\vec{c}$.

(1) Statement-I is true, statement-II is false
(2) Statement-I is true, statement-II is true
(3) Statement-I is false, statement-II is true
(4) Statement-I is false, statement-II is false

Answer (1)

Sol. $|\vec{a}+\vec{b}+\vec{c}|^2=|\vec{a}+\vec{b}-\vec{c}|^2$

$\Rightarrow\ \vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}=\vec{a}\cdot\vec{b}-\vec{b}\cdot\vec{c}-\vec{c}\cdot\vec{a}$

$\Rightarrow\ \vec{c}\cdot\vec{a}=0$

$\therefore\ \vec{a}$ is perpendicular to $\vec{c}$

$|\vec{a}+\lambda\vec{c}|\ge 0$ (However, it is always true)

and statement-II is false.

11. $A=\begin{bmatrix}2&0&0\\0&3&0\\0&1&1\end{bmatrix}$, then find sum of diagonal elements of $(A-I)^{11}$

(1) 4096 (2) 4097
(3) 2048 (4) 2049

Answer (2)

Sol. $A-I=\begin{bmatrix}1&0&0\\0&2&0\\0&0&0\end{bmatrix}$

$$(A-I)^{11}=\begin{bmatrix}\underline{1^{11}}&&\\&\underline{2^{11}}&\\&&\underline{0^{11}}\end{bmatrix}$$

(Calculating only for diagonal elements)

trace $(A-I)^{11}=2^{11}+1^{11}$

$=4097$

12. Circle $x^2+y^2-4x-6y+11=0$ is rolled up by 4 units along a tangent to it at the point (3, 2).

Let this be circle C_1, C_2 is the mirror image of circle C_1 about the tangent. *A* and *B* are centres of circles C_1 and C_2. *C* and *D* are the feet of perpendicular from *A* and *B* respectively upon *X*-axis. The area of the trapezium *ABCD* equals to

(1) $4\left(1+\sqrt{2}\right)$ (2) $2\left(1+\sqrt{2}\right)$
(3) $3\left(1+\sqrt{2}\right)$ (4) $1+\sqrt{2}$

Answer (1)

Sol. Given circle is

$x^2+y^2-4x-6y+11=0$, centre $E(2, 3)$

Tangent at (3, 2) is

$x-y-1=0$

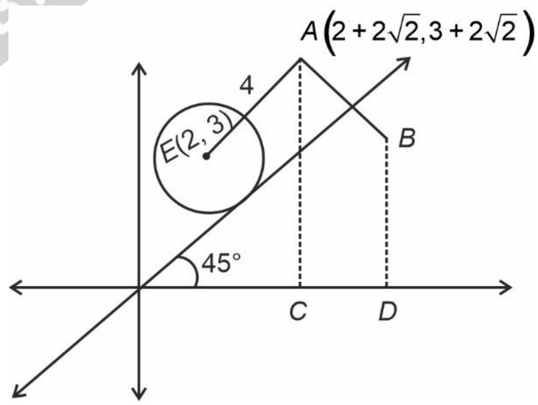
After rolling up by 4 units,

Centre of C_1 is *A*

Where $A\equiv\left(2+4\times\frac{1}{\sqrt{2}},3+4\times\frac{1}{\sqrt{2}}\right)$

$\equiv\left(2+2\sqrt{2},3+2\sqrt{2}\right)$

B is image of *A*



$$\frac{x-\left(2+\sqrt{2}\right)}{1}=\frac{y-\left(3+2\sqrt{2}\right)}{-1}=-2\left(\frac{-2}{2}\right)=2$$

$B\left(4+2\sqrt{2},1+2\sqrt{2}\right)$

Area of *ABCD*

$$=\frac{1}{2}\times\left(4+4\sqrt{2}\right)\times\left(\left(4+2\sqrt{2}\right)-(2+2\sqrt{2})\right)$$

$$=4\left(1+\sqrt{2}\right)$$

13. Let the relation R, $(a, b)\ R\ (c, d)$ be such that $ab(d - c) = cd(a - b)$, then R is

(1) Reflexive only

(2) Symmetric

(3) Transitive but not symmetric

(4) Reflexive and symmetric but not transitive

Answer (2)

Sol. Checking for reflexive

$\therefore\ (a, b)\ R\ (a, b)$

$\Rightarrow ab(b - a) = ab(a - b)$

$1 = -1 \qquad \therefore$ Not reflexive

Checking for $(a, b)\ R\ (c, d)$ then $(c, d)\ R\ (a, b)$

$\Rightarrow cd(b - a) = ab(c - d)$

$\Rightarrow ab(d - c) = cd(a - b) \quad \therefore$ Symmetric

$\therefore$ R is symmetric

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Find the number of 5-digit numbers formed using the digits 0, 3, 4, 7, 9 when repetition of digits is allowed is

Answer (2500)

Sol. | | | | | |

$4 \times 5 \times 5 \times 5 \times 5$

Total number of numbers = $4 \times 5 \times 5 \times 5 \times 5$

= 2500

22. Find remainder when 5^{99} is divided by 11.

Answer (09)

Sol. $5^{99} = (5^5)^{19} \cdot 5^4$

$= (3125)^{19} \cdot 5^4$

$= (11\lambda + 1)^{19} \cdot 5^4$

$= (11K + 1)\ 5^4$

$= 11K_1 + 5^4$

When 5^4 is divided by 11, we get remainder = 9

23. If $f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}$ then value of $12f(8)$ equals.

Answer (17)

Sol. Differentiating both sides we get

$$f'(x) + \frac{f(x)}{x} = \frac{1}{2\sqrt{x+1}}$$

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x} = \frac{1}{2\sqrt{x+1}}$$

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = x$$

$$\therefore\ xy = \frac{1}{2}\int \frac{x}{\sqrt{x+1}} dx$$

$$\Rightarrow xy = \frac{1}{2}\int \left(\sqrt{x+1} - \frac{1}{\sqrt{x+1}}\right) dx$$

$$\Rightarrow xy = \frac{1}{2}\left(\frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1}\right) + c$$

Put $x = 3$ in $f(x) + \int_3^x \frac{f(t)}{t} dt = \sqrt{x+1}$ we get $f(3) = 2$

$$\therefore\ c = \frac{16}{3}$$

$$\Rightarrow 8f(8) = \frac{1}{2}\left(\frac{27}{3} \cdot 2 - 2 \cdot 3\right) + \frac{16}{3}$$

$\Rightarrow 12f(8) = 17$

24. $y = f(x)$ is a parabola with focus $\left(-\frac{1}{2}, 0\right)$ and directrix $y = -\frac{1}{2}$

Given that $\tan^{-1}\sqrt{f(x)} + \sin^{-1}\sqrt{f(x)+1} = \frac{\pi}{2}$

Then number of solutions for x is

Answer (02)

Sol. $SP = SQ$

$$\left(x+\frac{1}{2}\right)^2+y^2=\left(y+\frac{1}{2}\right)^2$$

$$x^2+\frac{1}{4}+x+y^2=y^2+\frac{1}{4}+y$$

$\Rightarrow$ Eqn. of parabola $y = x^2 + x$

$f(x) = x^2 + x$

$$\tan^{-1}\sqrt{x^2+x}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{2}$$

$$\tan^{-1}\sqrt{x^2+x}=\cos^{-1}\sqrt{x^2+x+1}$$

$$\cos^{-1}\frac{1}{\sqrt{x^2+x+1}}=\cos^{-1}\sqrt{x^2+x+1}$$

$$\Rightarrow \quad \frac{1}{\sqrt{x^2+x+1}}=\sqrt{x^2+x+1}$$

$$\Rightarrow \quad x^2+x+1=1$$

Or $x = 0, -1$

25. The direction ratio's of two lines which are parallel are given by < 2, 1, –1> and < $\alpha + \beta$, $1 + \beta$, 2 >. Find $|2\alpha+3\beta|$

Answer (11)

Sol. $\frac{\alpha+\beta}{2}=\frac{1+\beta}{1}=\frac{2}{-1}$

$\Rightarrow \alpha + \beta = -4,\ 1 + \beta = -2$

So, $\beta = -3,\ \alpha = -1$

$|2\alpha + 3\beta| = 11$

26. Given $|\vec{a}|=\sqrt{14}, |\vec{b}|=\sqrt{6}, |\vec{a}\times\vec{b}|=\sqrt{48}$. Find $(\vec{a}\cdot\vec{b})^2$

Answer (36.00)

Sol. $(\vec{a}\cdot\vec{b})^2=|\vec{a}|^2|\vec{b}|^2-(\vec{a}\times\vec{b})^2$

$= 14\cdot6 - 48$

$= 36$

27.

28.

29.

30.

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