IIT - JEE 2017 (Advanced)

P2-17-6-7

PAPER-2

CODE 7

Time : 3 Hours **Maximum Marks : 183**

READ THE INSTRUCTIONS CAREFULLY

GENERAL

1. This sealed booklet is your Question Paper. Do not break the seal till you are told to do so.
2. The paper CODE is printed on the right hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The paper CODE is printed on the left part as well as the right part of the ORS. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator for change of ORS.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name, roll number and sign in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet at **2:00 pm**, verify that the booklet contains **36** pages and that all the **54** questions along with the options are legible. If not, contact the invigilator for replacement of the booklet.
8. You are allowed to take away the Question Paper at the end of the examination.

OPTICAL RESPONSE SHEET

9. The ORS (top sheet) will be provided with an attached Candidate's Sheet (bottom sheet). The Candidate's Sheet is a carbon-less copy of the ORS.
10. Darken the appropriate bubbles on the ORS by applying sufficient pressure. This will leave an impression at the corresponding place on the Candidate's Sheet.
11. The ORS will be collected by the invigilator at the end of the examination.
12. You will be allowed to take away the Candidate's Sheet at the end of the examination.
13. Do not tamper with or mutilate the ORS. **Do not use the ORS for rough work.**
14. Write your name, roll number and code of the examination center, and sign with pen in the space provided for this purpose on the ORS. **Do not write any of these details anywhere else** on the ORS. Darken the appropriate bubble under each digit of your roll number.

DARKENING THE BUBBLES ON THE ORS

15. Use a **BLACK BALL POINT PEN** to darken the bubbles on the ORS.
16. Darken the bubble ◯ **COMPLETELY.**
17. The correct way of darkening a bubble is as : ●
18. The ORS is machine-gradable. Ensure that the bubbles are darkened in the correct way.
19. Darken the bubbles **ONLY IF** you are sure of the answer. There is **NO WAY** to erase or "un-darken" a darkened bubble.

Please see the last page of this booklet for rest of the instructions.

DO NOT BREAK THE SEALS WITHOUT BEING INSTRUCTED TO DO SO BY THE INVIGILATOR

SEAL

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** 7

Solution to IIT JEE 2017 (Advanced) : Paper – II

PART I – PHYSICS

SECTION 1 (Maximum Marks:21)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +3 If only the bubble corresponding to the correct option is darkened.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –1 In all other cases.

1. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength $\lambda \left(\lambda < \frac{hc}{\phi_0}\right)$. The fastest photoelectron has a de Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then the ratio $\Delta\lambda_d/\Delta\lambda$ is proportional to

(A) λ_d^3 / λ^2 (B) λ_d^2 / λ^2 (C) λ_d/λ (D) λ_d^3 / λ

1. (A)

$$KE_{max} = \frac{hc}{\lambda} - \phi \qquad \ldots(i)$$

$$\lambda_d = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$

$$\Rightarrow K = \frac{h^2}{2m\lambda_d^2} \qquad \ldots(ii)$$

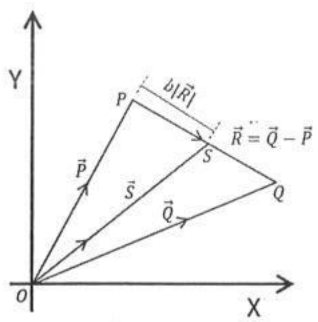
$$\frac{hc}{\lambda} - \phi = \frac{h^2}{2m\cdot\lambda_d^2}$$

Differentiating both sides

$$\frac{hc}{\lambda^2} d\lambda = \frac{h^2}{2m\lambda_d^3} \times d\lambda_d$$

$$\frac{\Delta\lambda_d}{\Delta\lambda} \propto \frac{\lambda_d^3}{\lambda^2}$$

2. Three vectors $\vec{P}$, $\vec{Q}$ and $\vec{R}$ are shown in the figure. Let S be any point on the vector $\vec{R}$. The distance between the points P and S is $b|\vec{R}|$. The general relation among vectors $\vec{P}, \vec{Q}$ and $\vec{S}$ is



(A) $\dot{S} = (1-b)\,\dot{P} + b^2\dot{Q}$ (B) $\dot{S} = (1-b^2)\,\dot{P} + b\dot{Q}$

(C) $\vec{S} = (1-b)\,\vec{P} + b\vec{Q}$ (D) $\vec{S} = (b-1)\,\vec{P} + b\vec{Q}$

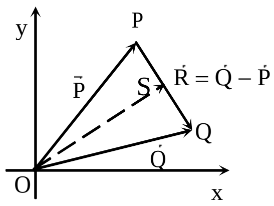
2. (C)

$|\vec{S}| = b|\vec{R}|$

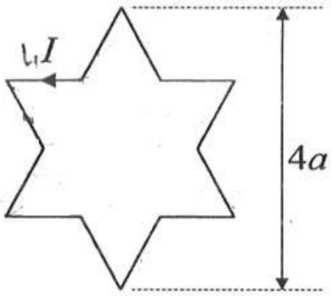
$\overrightarrow{OS} = \overrightarrow{OP} + \overrightarrow{PS}$

$\vec{S} = \vec{P} + b\vec{R} = \vec{P} + b(\vec{Q} - \vec{P})$

$\vec{S} = (1-b)\vec{P} + b\vec{Q}$



3. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is



(A) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}-1]$ (B) $\frac{\mu_0 I}{4\pi a}3[\sqrt{3}-1]$ (C) $\frac{\mu_0 I}{4\pi a}6[\sqrt{3}+1]$ (D) $\frac{\mu_0 I}{4\pi a}3[2-\sqrt{3}]$

3. (A)

Magnetic field at center of loop due to anyone section

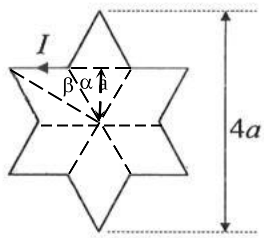
$B = \frac{\mu_0 I}{4\pi a}\left[\sin\beta - \sin\alpha\right]$

$\alpha = 30^\circ \quad \Rightarrow \sin\alpha = \frac{1}{2}$

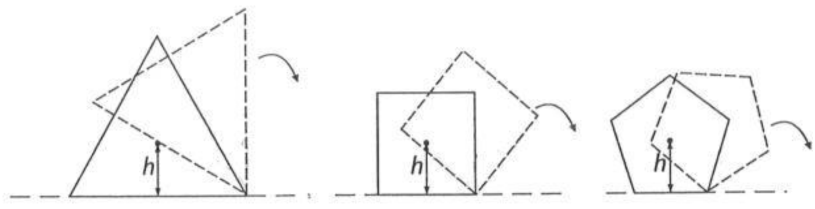
$\beta = 60^\circ \quad \Rightarrow \sin\beta = \frac{\sqrt{3}}{2}$

$B_{net} = 12B = \frac{12\mu_0 I}{4\pi a}\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$

$\boxed{B_{net} = 6\,\frac{\mu_0 I}{4\pi a}\left(\sqrt{3}-1\right)}$



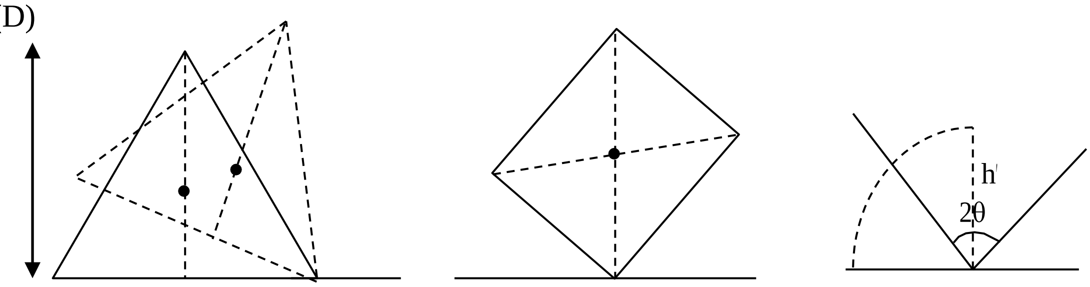
4. Consider regular polygons with number of sides n = 3, 4, 5 …… as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as



(A) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$ (B) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$

(C) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$ (D) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)}-1\right)$

4. (D)



$h' = 2h$ $h' = \sqrt{2}h$

$\Delta = h$ $\Delta = \left(\sqrt{2}-1\right)h$

for any regular polygon

$2\theta = \frac{2\pi}{n}$

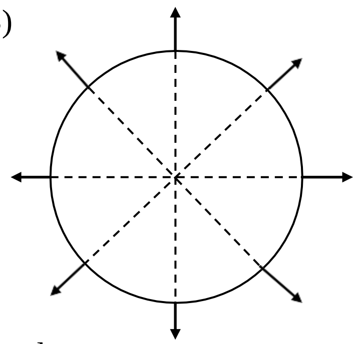
$h' = \frac{h}{\cos\theta} = \frac{h}{\cos\frac{\pi}{n}}$

$$\boxed{\Delta = h'-h = h\left(\frac{1}{\cos\frac{\pi}{n}}-1\right)}$$

5. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the instantaneous density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to

(A) $R^{2/3}$ (B) R (C) R^3 (D) $\frac{1}{R}$

5. (B)



Total mass = $\rho \times v$ = constant

$$\rho\frac{dV}{dt} = V\frac{d\rho}{dt} = 0$$

$$\rho\frac{dV}{dt} = -V\frac{d\rho}{dt}$$

$$\frac{dV}{dt} = -\frac{V}{\rho}\frac{d\rho}{dt}$$

$$\frac{dV}{dt} \propto V$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi t^2 \frac{dr}{dt} \propto \frac{4}{3}\pi r^3$$

Velocity of any point on surface

$$v = \frac{dr}{dt} \propto r$$

6. A person measures the depth of a well by measuring the time interval between dropping a stone and receiving the sound of impact with the bottom of the well. The error in his measurement of time is $\delta T = 0.01$ seconds and he measures the depth of the well to be L = 20 meters. Take the acceleration due to gravity $g = 10 ms^{-2}$ and the velocity of sound is $300\ ms^{-1}$. Then the fractional error in the measurement, $\delta L/L$, is closest to

(A) 0.2% (B) 5% (C) 1% (D) 3%

6. (C)

Time taken by stone to reach the surface of well.

$$t_1 = \sqrt{\frac{2gh}{g}} \Rightarrow t_1 = \sqrt{\frac{2\times 20}{10}} = 2\,\text{sec} \qquad \ldots\ldots\ldots\ldots (1)$$

time taken by sound to reach back

$$t_2 = \frac{h}{v} \Rightarrow t_2 = \frac{20}{300} = \frac{1}{15}\text{sec} \qquad \ldots\ldots\ldots\ldots (2)$$

Neglecting time taken by sound we have total time T = 2s

$$L = \frac{1}{2}gT^2$$

$$\frac{\Delta L}{L} = 2\cdot\frac{\Delta T}{T} = 2\times\frac{0.01}{2} = 0.01$$

$\therefore$ percentage error

= 0.01 × 100

= 1%

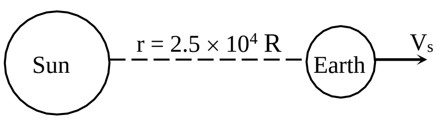
7. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun in 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $\nu_e = 11.2\ km\ s^{-1}$. The minimum initial velocity (ν_s) required for the rocket to be able to leave the Sun–Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet).

(A) $\nu_s = 62\ km\ s^{-1}$ (B) $\nu_s = 42\ km\ s^{-1}$ (C) $\nu_s = 72\ km\ s^{-1}$ (D) $\nu_s = 22\ km\ s^{-1}$

7. (B)

Energy conservation

$$\frac{1}{2}mV_s^2 - \frac{GM_e m}{R} - \frac{GM_s m}{r+2R} = 0$$



$$\frac{1}{2}mV_s^2 = \frac{GM_e m}{R} + \frac{G_s m}{r}$$

$$V_s^2 = 2G\left[\frac{M_e}{R} + \frac{3\times10^5\, M_e}{2.5\times10^4 R}\right]$$

$$V_s^2 = \frac{2GM_e}{R}\left[1+12\right] = 13\times\frac{2GM_e}{R}$$

$$V_e = \sqrt{\frac{2GM_e}{R}}$$

$$V_s = \sqrt{13}V_e = 3.6\times11.2 \approx 40.32 \text{ km/s}$$

SECTION – II (Maximum Marks:28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

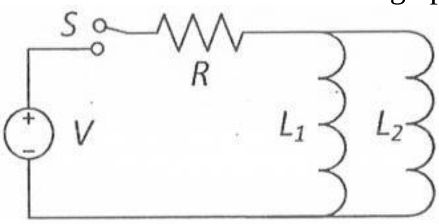
  Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

  Partial Marks : +1 For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

  Negative Marks : –2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

8. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t = 0, the switch is closed and current beings to flow. Which of the following options is/are correct?



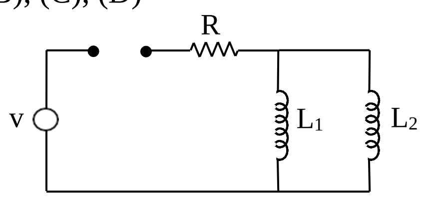
(A) At t = 0, the current through the resistance R is $\frac{V}{R}$

(B) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(C) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

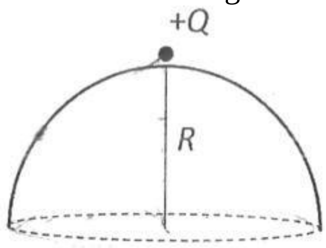
(D) The ratio of the currents through L_1 and L_2 is fixed at all time (t > 0)

8. (B), (C), (D)



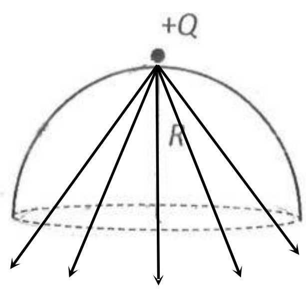
(A) at $t = 0$
Inductors behave as open switch
$i = 0$
(B), (C) After a long time
Inductors behave as closed switch
Net current $i = \frac{V}{R}$
If two inductors are parallel
flux $\phi_1 = \phi_2$
$L_1 i_1 = L_2 i_2$ ……………… (1)
$i_1 + i_2 = i$ ……………… (2)
on solving $i_1 = \frac{L_2\, i}{L_1 + L_2} = \frac{L_2}{L_1 + L_2} \times \frac{V}{R}$
$i_2 = \frac{L_1\, i}{L_1 + L_2} = \frac{L_1}{L_1 + L_2} \times \frac{V}{R}$
(D) $\frac{i_1}{i_2} = \frac{L_2}{L_1} =$ constant

9. A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct?

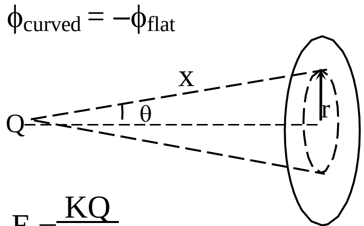


(A) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$
(B) The component of the electric field normal to the flat surface is constant over the surface
(C) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$
(D) The circumference of the flat surface is an equipotential

9. (A), (D)
Let hemisphere is closed.
Net Electric flux passing through hemisphere



$\phi = 0$
$\phi_{curved} + \phi_{flat} = 0$
$\phi_{curved} = -\phi_{flat}$



$E = \frac{KQ}{R^2 + r^2}$
$d\phi = EdA \cos\theta$

$$d\phi = \frac{KQ}{R^2+r^2} \times 2\pi r\, dr \times \frac{R}{\sqrt{R^2+r^2}}$$

$$\phi = \frac{QR}{2\in_0}\int_0^R \frac{r\,dr}{(R^2+r^2)^{3/2}}$$

$$\phi = \frac{QR}{2\in_0}\left[\frac{-1}{\sqrt{R^2+r^2}}\right]_0^R$$

$$\phi = \frac{QR}{2\in_0}\left[\frac{1}{R}-\frac{1}{R\sqrt{2}}\right]_0^R$$

$$\phi = \frac{Q}{2\in_0}\left[1-\frac{1}{\sqrt{2}}\right]$$

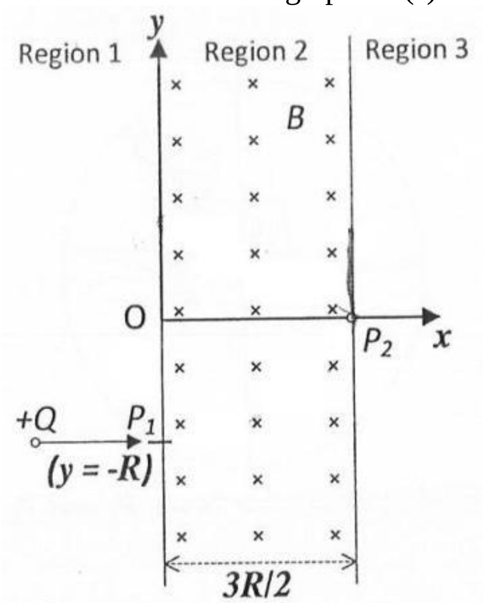
$$\boxed{\phi_{curved} = \frac{Q}{2\in_0}\left(1-\frac{1}{\sqrt{2}}\right)}$$

(B) Component of E.F. normal to flat surface

$$E\cos\theta = \frac{KQ}{R^2+r^2} \times \frac{R}{\sqrt{R^2+r^2}} \neq \text{Constant}$$

(D) All points on circumference are at same distance hence Equipotential.

10. A uniform magnetic field B exists in the region between x = 0 and x = $\frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x-axis enters region 2 from region 1 at point P_1 (y = –R). Which of the following options(s) is/are correct?



(A) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis.

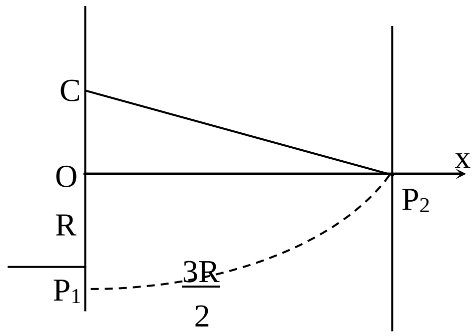
(B) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1

(C) For a fixed B, particles of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle.

(D) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the change in its linear momentum between point P_1 and the farthest point from y-axis is $p/\sqrt{2}$

10. (B)

Let C be the centre of the circle.



$CP_1 = CP_2 = r$ = radius of the circle = $\frac{p}{qB}$

We have $OC^2 + OP_2^2 = CP_2^2$

$\therefore \quad (CP_1 - OP_1)^2 + OP_2^2 = CP_2^2$

$\therefore \quad \left(\frac{P}{qB} - R\right)^2 + \left(\frac{3R}{2}\right)^2 = \left(\frac{p}{qB}\right)^2$

$\left(\frac{p}{qB}\right)^2 - \frac{2p}{qB}R + R^2 + \frac{9R^2}{4} = \left(\frac{p}{qB}\right)^2$

$\frac{-2pR}{qB} + R^2 + \frac{9}{4}R^2 = 0$

$\frac{13}{4}R^2 = \frac{2pR}{qB}$

$\therefore \quad B = \frac{8p}{13qR}$

$\therefore$ (A) is correct

Charge returns to region 1.

(B) $r < \frac{3R}{2}$

$\frac{p}{QB} < \frac{3R}{2}$

$\therefore B > \frac{2p}{Q}$

(B) is correct.

(C) radius of the circular path $r = \frac{m\upsilon}{qB}$

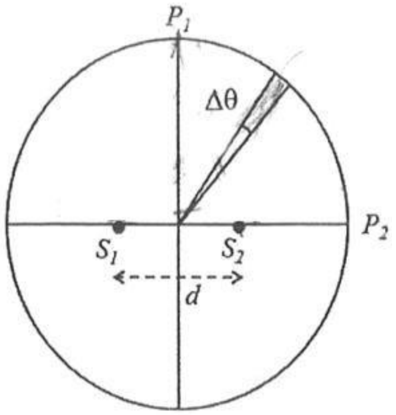
$r \propto m$

(C) is not correct.

(D) The momentum at P_1 and the farthest point on the x–axis are perpendicular to each other. The change in momentum is $\sqrt{2}p$.

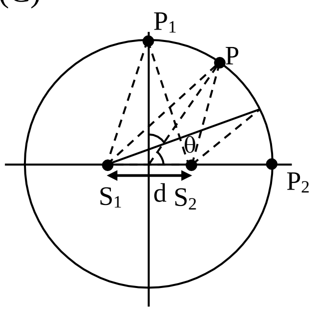
(D) is not correct.

11. Two coherent monochromatic point sources S_1 and S_2 of wavelength $\lambda = 600$ nm are placed symmetrically on either side of the center of the circle as shown. The sources are separated by a distance d = 1.8 mm. This arrangement produces interference fringes visible as alternate bright and dark spots on the circumference of the circle. The angular separation between two consecutive bright spots is $\Delta\theta$. Which of the following options is/are correct?



(A) The total number of fringes produced between P_1 and P_2 in the first quadrant is close to 3000
(B) A dark spot will be formed at the point P_2
(C) At P_2 the order of the fringe will be maximum
(D) The angular separation between two consecutive bright spots decreases as we move from P_1 to P_2 along the first quadrant.

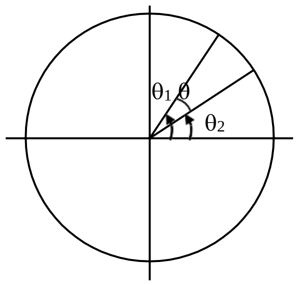
11. (A), (C)



(A) at point P_1 path difference $\Delta x = 0$ (central maxima)
at point P_2 path difference $\Delta x = d$

$$n = \frac{\Delta x}{\lambda} = \frac{1.8\times10^{-3}}{600\times10^{-9}}$$

n = 3000 (maxima)



$\theta = \theta_1 - \theta_2$

(C) As the path difference is maximum at P_2 the order of the fringe will be maximum.

(D) Angular fringe width increases as we more from P_1 to P_2 as shown below
path difference = x = d sin θ

$\therefore\ \Delta x = d\cos\theta \cdot \Delta\theta$

$\therefore\ \lambda = d\cos\theta \cdot \Delta\theta$

$$\Delta\theta = \frac{\lambda}{d\cos\theta}$$

as θ increases cos θ decreases and Δθ increases.

12. The instantaneous voltages at three terminals marked X, Y and Z are given by

$V_X = V_0 \sin \omega t$,

$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$ and

$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be

(A) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(B) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

(C) $V_{XY}^{rms} = V_0$

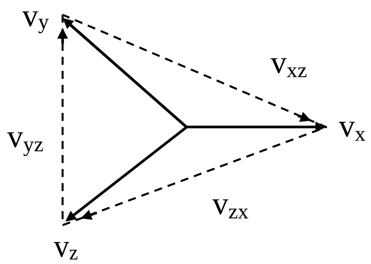
(D) independent of the choice of the two terminals

12. (B), (D)

$V_x = V_o \sin \omega t$

$V_y = V_o \sin\left(\omega t + \frac{2\pi}{3}\right)$

$V_z = V_o \sin\left(\omega t + \frac{4\pi}{3}\right)$



If voltmeter is connected between x and y

$V_{xy} = V_x - V_y$

$$\left(V_{xy}\right)_o = \sqrt{V_o^2 + V_o^2 - 2V_o^2 \cos\frac{2\pi}{3}}$$

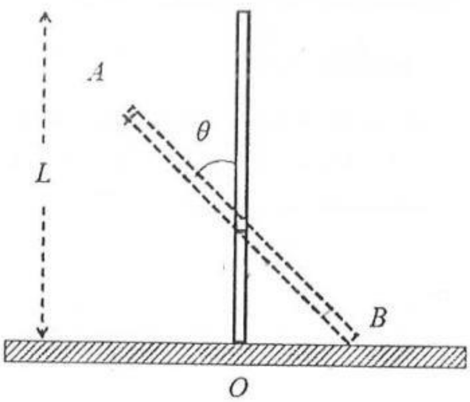
$$\left(V_{xy}\right)_o = 2V_o \sin\frac{\pi}{3} = V_o\sqrt{3}$$

$$V_{xy}^{rms} = \frac{\left(V_{xy}\right)_o}{\sqrt{2}} = V_o\sqrt{\frac{3}{2}}$$

Similarly $V_{yz} = 2V_o \sin\frac{2\pi}{3} = V_o\sqrt{3}$

$$V_{yz}^{rms} = V_o\sqrt{\frac{3}{2}}$$

13. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct?



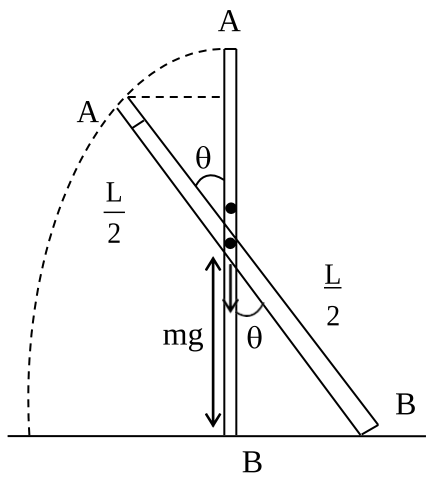
(A) The trajectory of the point A is a parabola

(B) instantaneous torque about the point in contact with the floor is proportional to sin θ

(C) The midpoint of the bar will fall vertically downward

(D) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to (1 – cos θ)

13. (A), (B), (C), (D)



(A) x Co-ordinate of point A

$$X = \frac{L}{2}\sin\theta$$

y Co-ordinate of point A

$$y = 2\left(\frac{L}{2}\cos\theta\right)$$

$$x^2 + \left(\frac{y}{2}\right)^2 = \frac{L^2}{4}$$

Point A moves on an elliptical path.

(B) Instantaneous Torque about point of contact

$$\tau = mg\sin\cdot\frac{L}{2}$$

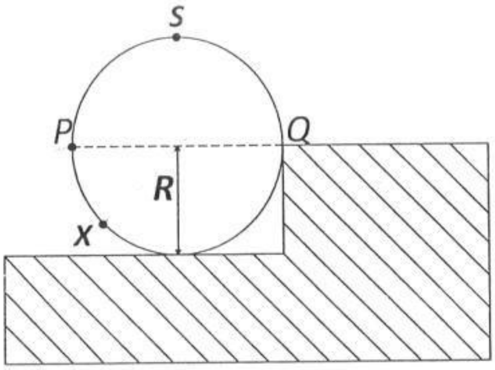
$\tau \alpha \sin\theta$

(C) Rod moves such that COM moves on a vertical line

(D) When rod is rotated by an angle θ decrease in height of com

$$\boxed{\Delta h = \frac{L}{2}\sin\theta}$$

14. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct?



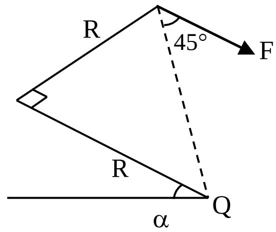
(A) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

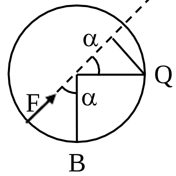
(B) If the force is applied normal to the circumference at point P then τ is zero

(C) If the force is applied normal to the circumference at point X then τ is constant

(D) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

14. (B), (C)





For part A : The torque due to F is always equal to FR. (about Q).

Torque due to weight = mg R cos α

$\left(FR\alpha - mgR\int_0^{\alpha}\cos\alpha \cdot dx\right)$ = work done by all the forces = change in K.E. of the ball

$\Rightarrow$ $\Delta KE = FR\alpha - mgR\sin\alpha$ [When the ball has turned by an angle 'α']

$\Rightarrow \Delta KE = FR\alpha\left[1-\left(\frac{mg}{F}\right)\left(\frac{\sin x}{\alpha}\right)\right]$

For F > mg ΔKE is always positive for all α. Thus it is possible for the ball to climb up the step [Assuming sufficient friction]. Hence (A) is False.

(B) is obvious True.

For part (C) : $\tau = FR\cos\alpha$, since α is fixed, τ is fixed. $\therefore$ (C) is true.

(D) is false because 'τ' is constant.

SECTION – III (Maximum Marks:12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 If all other cases.

Paragraph for Q. No. 15 & 16

Consider a simple RC circuit as shown in Figure 1.

Process 1 : In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2 : In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time T >> RC. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time T >> RC. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process I.

These to processes are depicted in Figure 2.

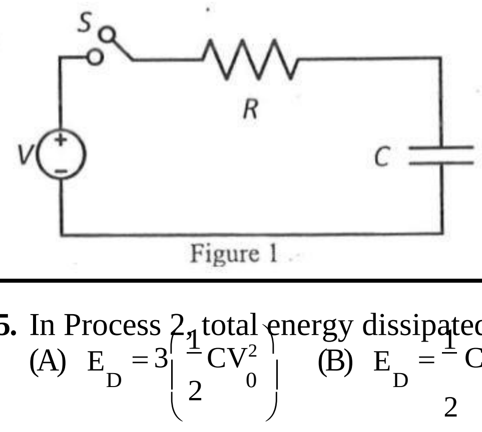


Figure 1

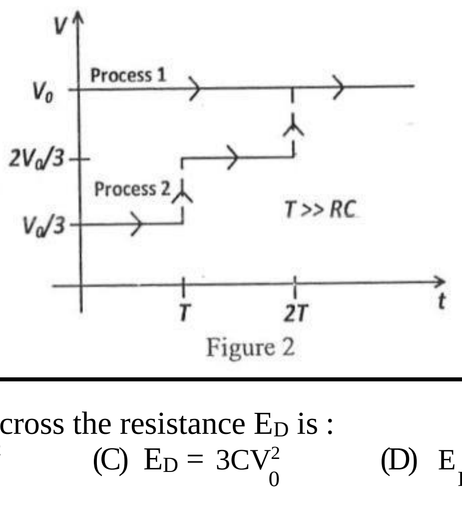


Figure 2

15. In Process 2, total energy dissipated across the resistance E_D is :

(A) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (B) $E_D = \frac{1}{2}CV_0^2$ (C) $E_D = 3CV_0^2$ (D) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

15. (D)

Charge flowing through the seat of emf in each of the three subparts (each of duration T) of process 2 is $\frac{CV_0}{3}$; the voltage across the seat of emf being $\frac{V_0}{3}, \frac{2V_0}{3}$ and V_0 respectively in the three intervals. The energy spent by emf source, therefore equals $\frac{CV_0}{3}\left[\frac{V_0}{3} + \frac{2V_0}{3} + V_0\right] = \frac{2CV_0}{3}$.

Out of this $\frac{1}{2}CV_0^2$ is visible in the finally charged capacitor.

The balance, i.e., $\frac{2}{3}CV_0^2 - \frac{1}{2}CV_0^2 = \frac{CV_0^2}{6}$ must then dissipate in the resistor.

16. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by :

(A) $E_C = \frac{1}{2}E_D$ (B) $E_C = 2E_D$ (C) $E_C = E_D$ (D) $E_C = E_D \ell n\, 2$

16. (C)

In process 1, the charge passing through the seat of emf = CV_0.

And, whole of it passes across potential difference V_0, making the cell spend energy $= CV_0^2$

Out of this, $\frac{1}{2}CV_0^2$ is stored in the capacitor,

i.e., $I_C = \frac{1}{2}CV_0^2$.

So, the energy dissipated in the resistor $E_d = CV_0^2 - \frac{1}{2}CV_0^2 = \frac{1}{2}CV_0^2$.

$\therefore E_c = E_d$

Paragraph for Q. No. 17 & 18

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g

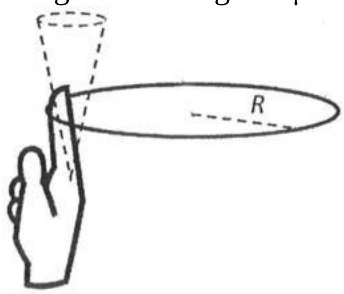


Figure 1

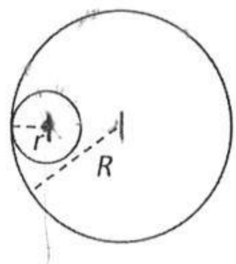


Figure.2

17. The minimum value of ω_0 below which the ring will drop down is

(A) $\sqrt{\frac{2g}{\mu(R-r)}}$ (B) $\sqrt{\frac{g}{\mu(R-r)}}$ (C) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (D) $\sqrt{\frac{g}{2\mu(R-r)}}$

17. (B)

Centre of mass of ring moves with angular velocity ω_0 in a circle of radius R−r centred about the centre of the circle of radius r.

$\therefore N = M\omega_o^2(R-r)$

Also $\mu N = Mg$

$\therefore \mu M\omega_0^2(R-r) = Mg$

$\Rightarrow \omega_0 = \sqrt{\frac{g}{\mu(R-r)}}$

18. The total kinetic energy of the ring is

(A) $\frac{3}{2}M\omega_0^2(R-r)^2$ (B) $\frac{1}{2}M\omega_0^2(R-r)^2$ (C) $M\omega_0^2(R-r)^2$ (D) $M\omega_0^2R^2$

18. (C)

Kinetic energy of rolling ring = $Mv^2 = M\omega_0^2(R-r)^2$.

PART II : CHEMISTRY

SECTION 1 (Maximum Marks:21)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

  Negative Marks : −1 In all other cases.

19. For the following cell,

$Zn(s) \mid ZnSO_4(aq) \| CuSO_4(aq) \mid Cu(s)$

when the concentration of Zn^{2+} is 10 times the concentration of Cu^{2+}, the expression for ΔG (in J mol^{-1}) is

[F is Faraday constant ; R is gas constant ; T is temperature ; E°(cell) = 1.1 V]

(A) 2.303 RT −2.2 F (B) 2.303 RT + 1.1 F

(C) 1.1 F (D) −2.2 F

19. (A)

$Zn(s) \mid ZnSO_4(aq) \| CuSO_4(aq) \mid Cu(s)$

Given : $[Zn^{+2}] = 10\,[Cu^{+2}]$

$$Zn(s) + Cu^{+2}_{(aq)} \longrightarrow Zn^{+2}_{(aq)} + Cu(s)$$

$\Delta G = \Delta G^0 + 2.303\,RT\log_{10}Q = -nFE^0 + 2.303\,RT\log_{10}Q$

$Q = \frac{[Zn^{+2}]}{[Cu^{+2}]} = 10$

$= -2F(1.1) + 2.303\,RT\log_{10}(10)$

$= -2.2\,F + 2.303\,RT$

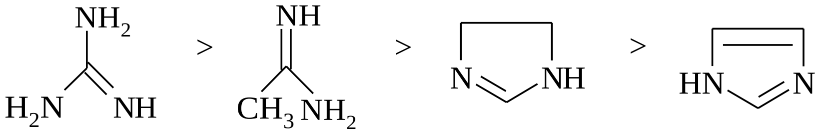
20. The order of basicity among the following compounds is

I II III IV

(A) I > IV > III > II (B) II > I > IV > III (C) IV > I > II > III (D) IV > II > III > I

20. (C)

Basicity order : IV > I > II > III



21. The standard state Gibbs free energies of formation of C (graphite) and C(diamond) at T = 298 K are

$\Delta_f G°[C(graphite)] = 0$ kJ mol^{-1}

$\Delta_f G°$ [C(diamond)] = 2.9 kJ mol^{-1}.

The standard state means that the pressure should be 1 bar, and substance should be pure at a given temperature. The conversion of graphite [C(graphite)] to diamond [C(diamond)] reduces its volume by 2×10^{-6} m^3 mol^{-1}. If C(graphite) is converted to C(diamond) isothermally at T = 298 K, the pressure at which C(graphite) is in equilibrium with C(diamond), is

[Useful information : 1J = 1 kg m^2 s^{-2} ; 1Pa = 1 kg m^{-1} s^{-2}; 1 bar = 10^5 Pa]

(A) 14501 bar (B) 29001 bar (C) 1450 bar (D) 58001 bar

21. (A)

$C_{(graphite)} \longrightarrow C_{(diamond)}$

$\Delta G° = \Delta_f G°_{(diamond)} - \Delta_f G°_{(graphite)}$

$\Delta G° = 2.9 - 0 = 2.9$ kJ/mol

$\Delta V = 2 \times 10^{-6}$ m^3 mol^{-1}

Isothermally ie $\Delta T = 0$ $\Rightarrow \Delta E = 0$

At equilibrium, $\Delta G = 0$

$\Delta G° = P\Delta V$

$2.9 \times 10^3 = P(2 \times 10^{-6})$

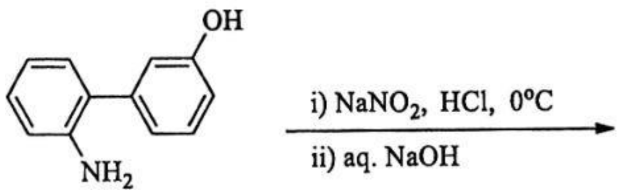
$$P - 1 = \frac{2.9 \times 10^3}{2 \times 10^{-6}} = 1.45 \times 10^9 \text{ pascal}$$

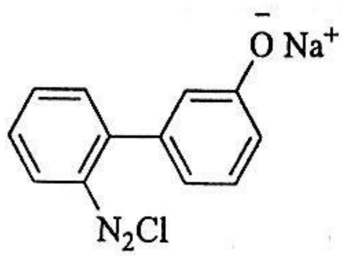
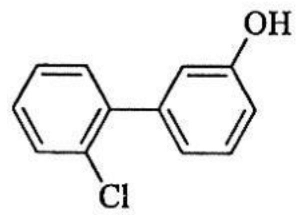
1 bar = 10^5 pa

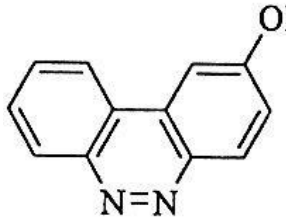
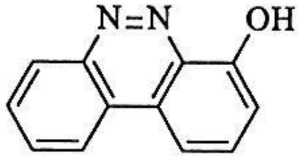
$\therefore$ P $= (1.45 \times 10^4$ bar + 1)

P = 14501 bar

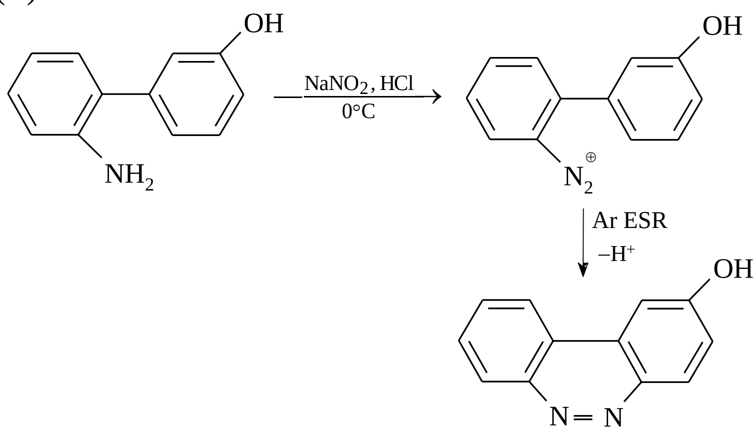
22. The major product of the following reaction is



(A) 
(B) 

(C) 
(D) 

22. (C)



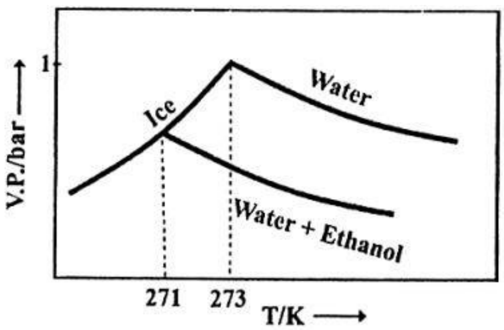
23. The order of the oxidation state of the phosphorus atom in H_3PO_2 , H_3PO_4, H_3PO_3, and $H_4P_2O_6$ is

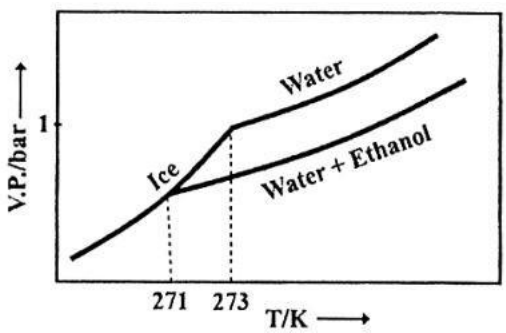
(A) $H_3PO_4 > H_3PO_2 > H_3PO_3 > H_4P_2O_6$
(B) $H_3PO_3 > H_3PO_2 > H_3PO_4 > H_4P_2O_6$
(C) $H_3PO_2 > H_3PO_3 > H_4P_2O_6 > H_3PO_4$
(D) $H_3PO_4 > H_4P_2O_6 > H_3PO_3 > H_3PO_2$

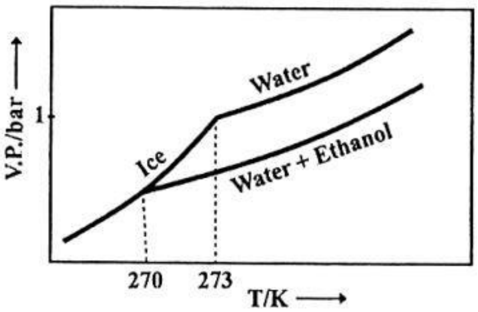
23. (D)

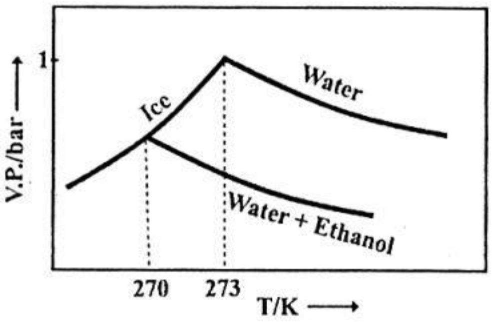
Compound State	**Oxidation State**
H_3PO_4	+5
$H_4P_2O_6$	+4
H_3PO_3	+3
H_3PO_2	+1

24. Pure water freezes at 273 K and 1 bar. The addition of 34.5 g of ethanol to 500 g of water changes the freezing point of the solution. Use the freezing point depression constant of water as 2 K kg mol^{-1}. The figures shown below represent plots of vapour pressure (V.P.) versus temperature (T). [molecular weight of ethanol is 46 g mol^{-1}]
Among the following, the option representing change in the freezing point is

(A) 

(B) 

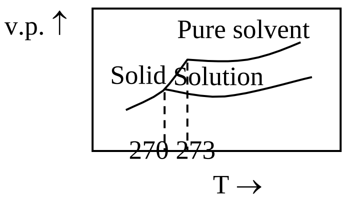
(C) 

(D) 

24. (C)

$\Delta T_f \quad = \quad K_f\, m$

$= \quad 2 \times \dfrac{\left(\dfrac{34.5}{46}\right)}{\left(\dfrac{500}{1000}\right)}$



$\Delta T_f \quad = \quad 3$

$T_f \quad = \quad 273 - 3 = 270$ K

25. Which of the following combination will produce H_2 gas

(A) Au metal and NaCN(aq) in the presence of air.

(B) Cu metal and conc.HNO_3

(C) Fe metal and conc. HNO_3

(D) Zn metal and NaOH(aq)

25. (D)

$Zn(s) + 2NaOH \rightarrow Na_2ZnO_2 + H_2 \uparrow$

SECTION – II (Maximum Marks:28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks	: +4	If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
Partial Marks	: +1	For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
Zero Marks	: 0	If none of the bubbles is darkened.
Negative Marks	: –2	In all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

26. The option(s) with only amphoteric oxides is (are)

(A) Cr_2O_3 , BeO, SnO, SnO_2 (B) Cr_2O_3 , CrO, SnO, PbO

(C) ZnO, Al_2O_3, PbO, PbO_2 (D) NO, B_2O_3, PbO, SnO_2

26. (A), (C)

(A) Cr_2O_3, BeO, SnO and SnO_2 are amphoteric oxides.

(C) ZnO, Al_2O_3, PbO and PbO_2 are amphoteric oxides.

27. The correct statement(s) about surface properties is(are)

(A) Cloud is an emulsion type of colloid in which liquid is dispersed phase and gas is dispersion medium.

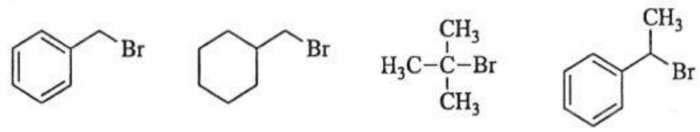
(B) Adsorption is accompanied by decrease in enthalpy and decrease in entropy of the system.

(C) The critical temperatures of ethane and nitrogen are 563 K and 126 K, respectively. The adsorption of ethane will be more than that of nitrogen on same amount of activated charcoal at a given temperature.

(D) Brownian motion of colloidal particles does not depend on the size of the particles but depends on viscosity of the solution.

27. (B), (C)

28. For the following compounds, the correct statement(s) with respect to nucleophilic substitution reactions is(are)



I II III IV

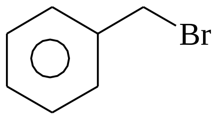
(A) **I** and **III** follow S_N1 mechanism

(B) Compound **IV** undergoes inversion of configuration

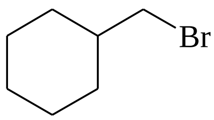
(C) **I** and **II** follow S_N2 mechanism

(D) The order of reactivity for **I**, **III** and **IV** is : **IV** > **I** > **III**

28. (A), (C), (D)



$S_N1 + S_N2$



S_N2

I II

29. Among the following, the correct statement(s) is(are)

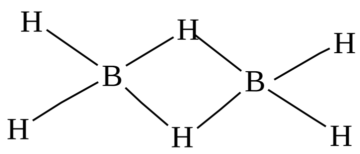
(A) BH_3 has the three-centre two-electron bonds in its dimeric structure

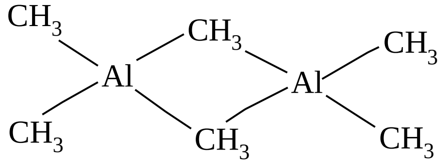
(B) $Al(CH_3)_3$ has the three-centre two-electron bonds in its dimeric structure

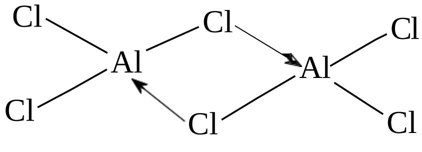
(C) $AlCl_3$ has the three-centre two-electron bonds in its dimeric structure

(D) The Lewis acidity of BCl_3 is greater than that of $AlCl_3$

29. (A), (B), (D)







30. In a bimolecular reaction, the steric factor P was experimentally determined to be 4.5. The correct option(s) among the following is(are)

(A) Experimentally determined value of frequency factor is higher than that predicted by Arrhenius equation

(B) The value of frequency factor predicted by Arrhenius equation is higher than that determined experimentally

(C) The activation energy of the reaction is unaffected by the value of the steric factor

(D) Since P = 4.5, the reaction will not proceed unless an effective catalyst is used

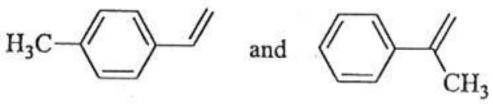
30. (A), (C)

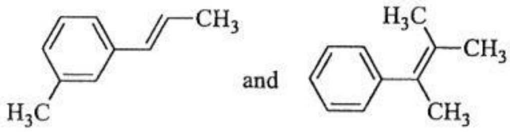
31. Compounds **P** and **R** upon ozonolysis produce **Q** and **S**, respectively. The molecular formula of **Q** and **S** is C_8H_8O. **Q** undergoes Cannizzaro reaction but not haloform reaction, whereas **S** undergoes haloform reaction but not Cannizzaro reaction.

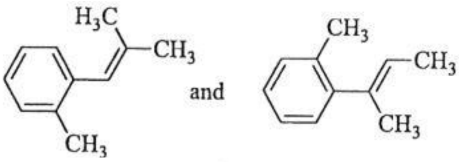
(i) $\mathbf{P} \xrightarrow[\text{ii) } Zn/H_2O]{\text{i) } O_3/CH_2Cl_2} \underset{(C_8H_8O)}{\mathbf{Q}}$

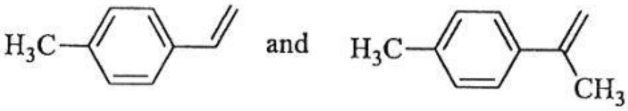
(ii) $\mathrm{R} \xrightarrow[\text{ii) } Zn/H_2O]{\text{i) } O_3/CH_2Cl_2} \underset{(C_8H_8O)}{\mathbf{S}}$

The option(s) with suitable combination of **P** and **R**, respectively, is(are)

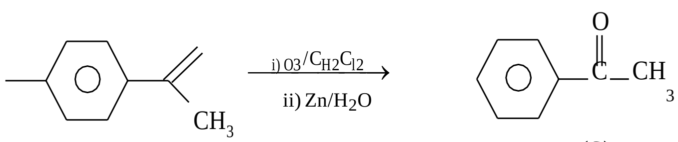
(A) 

(B) 

(C) 

(D) 

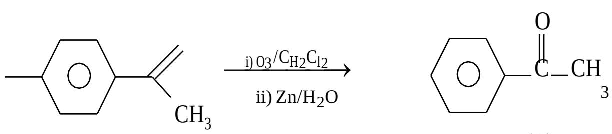
31. (A), (B)

(A) 

(R)

(S)

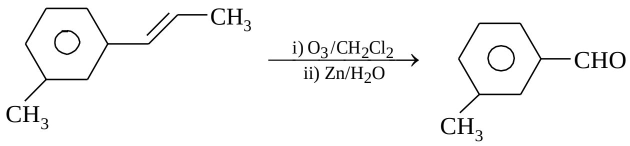
Shows Haloform Reaction
Doesnot gives Cannizzaro's Reaction



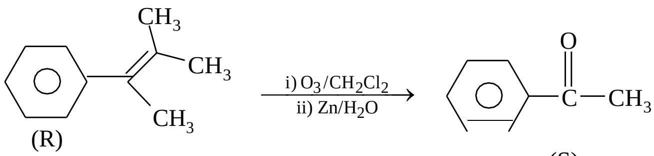
(R)

(S)

Shows Haloform Reaction
Doesnot gives Cannizzaro's Reaction

(B) 

(Q)



(R) (S)

Methyl ketone gives Halaform Reaction

32. For a reaction taking place in a container in equilibrium with its surroundings, the effect of temperature on its equilibrium constant K in terms of change in entropy is described by

(A) With increase in temperature, the value of K for exothermic reaction decreases because the entropy change of the system is positive

(B) With increase in temperature, the value of K for endothermic reaction increases because unfavourable change in entropy of the surroundings decreases

(C) With increase in temperature, the value of K for endothermic reaction increases because the entropy change of the system is negative

(D) With increase in temperature, the value of K for exothermic reaction decreases because favourable change in entropy of the surroundings decreases

32. (B)

(A) False (B) True (C) False (D) False

SECTION – III (Maximum Marks:12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.

  Zero Marks : 0 If all other cases.

PARAGRAPH 1

Upon heating $KClO_3$ in the presence of catalytic amount of MnO_2, a gas **W** is formed. Excess amount of **W** reacts with white phosphorus to give **X**. The reaction of **X** with pure HNO_3 gives **Y** and **Z**.

33. **W** and **X** are, respectively

(A) O_2 and P_4O_{10} (B) O_2 and P_4O_6 (C) O_3 and P_4O_6 (D) O_3 and P_4O_{10}

34. **Y** and **Z** are, respectively

(A) N_2O_5 and HPO_3 (B) N_2O_4 and HPO_3 (C) N_2O_4 and H_3PO_3 (D) N_2O_3 and H_3PO_4

33. (A) **34.** (A)

$$\underset{}{KClO_{3(s)}} \longrightarrow KCl_{(s)} + \underset{(W)}{1.5O_{2(g)}}$$

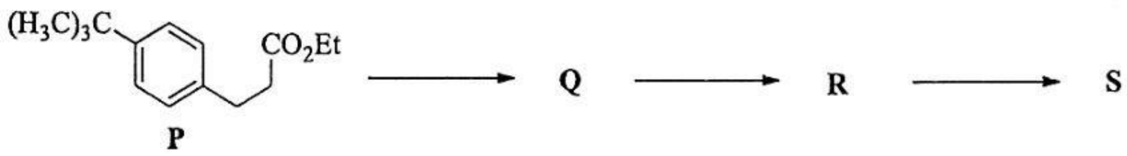
$$P_{4(s)} + 3O_2 \longrightarrow P_4O_6$$

$$P_4O_6 + 2O_2 \longrightarrow P_4O_{10}\ (X)$$

$$\underset{(X)}{P_4O_{10}} + 2HNO_3 \longrightarrow \underset{(Y)}{N_2O_5} + \underset{(Z)}{2\,HPO_3}$$

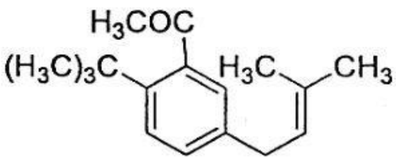
PARAGRAPH 2

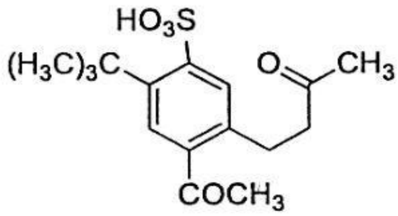
The reaction of compound **P** with CH_3MgBr (excess) in $(C_2H_5)_2O$ followed by addition of H_2O gives **Q**. The compound **Q** on treatment with H_2SO_4 at 0°C gives **R**. The reaction of **R** with CH_3COCl in the presence of anhydrous $AlCl_3$ in CH_2Cl_2 followed by treatment with H_2O produces compound S. [Et in compound **P** is ethyl group]

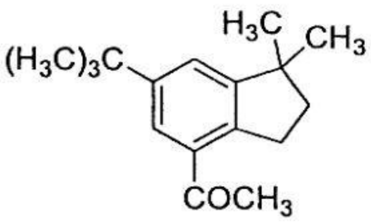


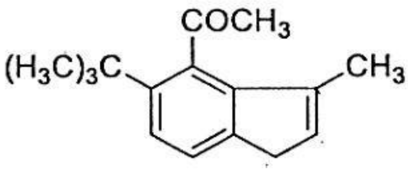
35. The reactions, **Q** to **R** and **R** to **S**, are
(A) Dehydration and Friedel–Crafts acylation.
(B) Friedel–Crafts alkylation and Friedel–Crafts acylation.
(C) Friedel–Crafts alkylation, dehydration and Friedel–Crafts acylation.
(D) Aromatic sulfonation and Friedel–Crafts acylation.

36. The product **S** is

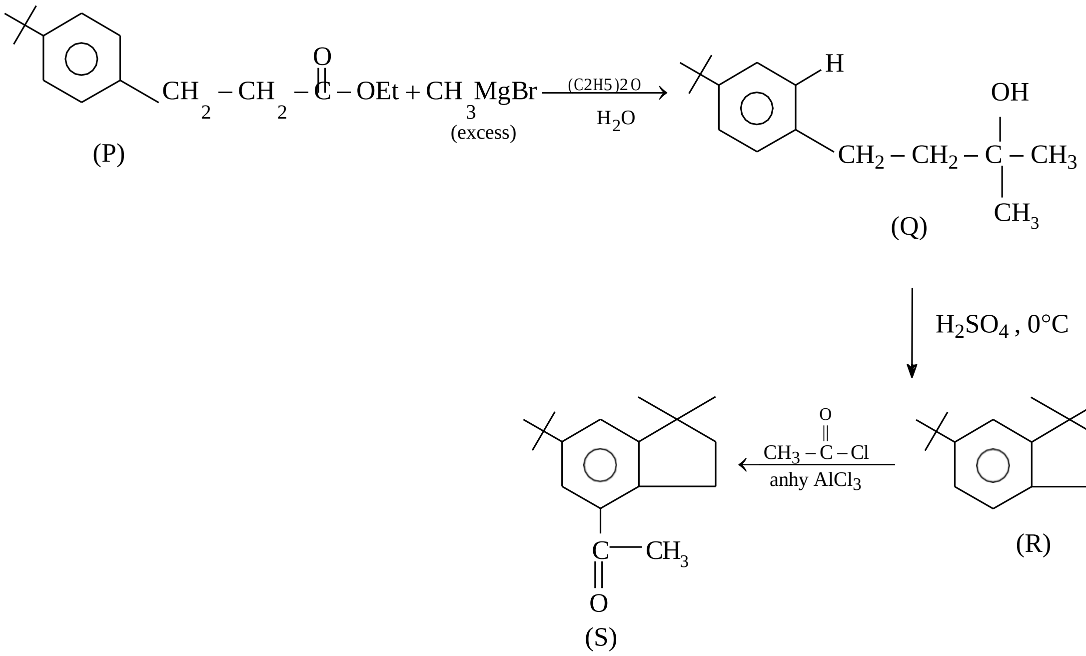
(A) 

(B) 

(C) 

(D) 

35. (B) **36.** (C)



PART III – MATHEMATICS

SECTION 1 (Maximum Marks:21)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 If only the bubble corresponding to the correct option is darkened.

  Zero Marks : 0 If none of the bubbles is darkened.

  Negative Marks : –1 In all other cases.

37. Three randomly chosen nonnegative integers x, y and z are found to satisfy the equation x + y + z = 10. Then the probability that z is even, is

(A) $\frac{5}{11}$ (B) $\frac{1}{2}$ (D) $\frac{36}{55}$

(C) $\frac{1}{1}$

37. (C)

$x + y + z = 10, \quad x, y, z \geq 0$

$n(s) = {}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

If z is even. $x + y + z = 10$

$x + y = 10 - z$

$$n(A) = \sum_{z\ \text{even}} {}^{10-z+2-1}C_{2-1} = \sum_{z\text{even}} {}^{11-z}C_1$$

$$= \sum_{z\text{even}} (11 - z)$$

$= 11 + 9 + 7 + 5 + 3 + 1 = 36$

$P(z \text{ is even}) = \frac{36}{66} = \frac{6}{11}$

38. If $f : \mathbb{R} \to \mathbb{R}$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in \mathbb{R}$, and $f\left(\frac{1}{2}\right) = \frac{1}{2}$, f (1) = 1, then

(A) $f'(1) > 1$ (B) $f'(1) \leq 0$ (C) $\frac{1}{2} < f'(1) \leq 1$ (D) $0 < f'(1) \leq \frac{1}{2}$

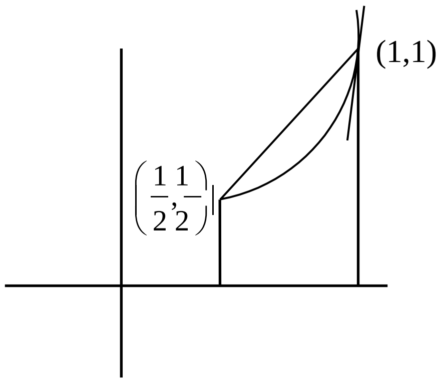
38. (A)

$f''(x) > 0 \quad \forall\, x \Rightarrow$ concave upward

$\Rightarrow$ f(x) is increasing.

$$f'(1) > \frac{1-1}{\frac{1}{2}-\frac{1}{2}}$$

$\Rightarrow f'(1) > 1$



39. The equation of the plane passing through the point (1, 1, 1) and perpendicular to the planes 2x + y – 2z = 5 and 3x – 6y – 2z = 7, is

(A) –14x + 2y + 15z = 3 (B) 14x + 2y – 15z = 1

(C) $14x + 2y + 15z = 31$ (D) $14x - 2y + 15z = 27$

39. (C)

Plane passing (1, 1, 1) is $a(x-1) + b(y-1) + c(z-1) = 0$ …(1)

As plane (1) is $\perp^r$ to both $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$

$\Rightarrow 2a + b - 2c = 0$

& $3a - 6b - 2c = 0$

$$\frac{a}{-14} = \frac{b}{-2} = \frac{c}{-15}$$

Put in (1)

$14(x-1) + 2(y-1) + 15(z-1) = 0$

$14x + 2y + 15z = 31$

40. Let 0 be the origin and let PQR be an arbitrary triangle. The point S is such that

$$\overrightarrow{OP}\cdot\overrightarrow{OQ} + \overrightarrow{OR}\cdot\overrightarrow{OS} = \overrightarrow{OR}\cdot\overrightarrow{OP} + \overrightarrow{OQ}\cdot\overrightarrow{OS} = \overrightarrow{OQ}\cdot\overrightarrow{OR} + \overrightarrow{OP}\cdot\overrightarrow{OS}$$

Then the triangle PQR has S as its

(A) circumcentre (B) orthocenter
(C) incentre (D) centroid

40. (B)

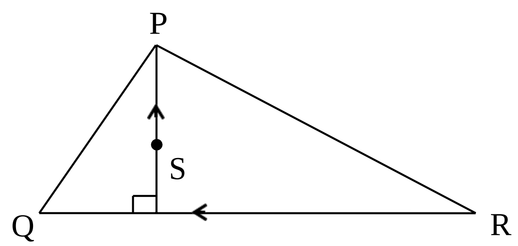
$$\overline{OP}\cdot\overline{OQ} + \overline{OR}\cdot\overline{OS} = \overline{OR}\cdot\overline{OP} + \overline{OQ}\cdot\overline{OS}$$

$$\overline{OP}\cdot(\overline{OQ} - \overline{OR}) + \overline{OS}\cdot(\overline{OR} - \overline{OQ}) = 0$$

$$(\overline{OQ} - \overline{OR})\cdot(\overline{OP} - \overline{OS}) = 0$$

$$\Rightarrow \left(\overline{OQ} - \overline{OR}\right) \perp^r (\overline{OP} - \overline{OS})$$

$\Rightarrow$ S is Orthocentre of the ΔPQR.



41. How many 3 × 3 matrices M with entries from {0, 1, 2} are there, for which the sum of the diagonal entries of $M^T M$ is 5?

(A) 126 (B) 198 (C) 162 (D) 135

41. (B)

$$M = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$$

$$M^T M = \begin{bmatrix} a_1 & a_4 & a_7 \\ a_2 & a_5 & a_8 \\ a_3 & a_6 & a_9 \end{bmatrix} \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}$$

Sum of diagonals of $M^T M$

$$= \left(a_1^2 + a_4^2 + a_7^2\right) + \left(a_2^2 + a_5^2 + a_8^2\right) + \left(a_3^2 + a_6^2 + a_9^2\right) = 5 \quad \text{(given)}$$

Possible if

- number of 2's is 1
  & number of 1's are 1
  & number of 0's are 7 = Total ways ${}^9C_2 \times 2!$
- number of 2's is 0
  & number of 1's are 5
  & number of 0's are 4 = ${}^9C_5$

Total ways = ${}^9C_2 \times 2! + {}^9C_5 = 198$

42. Let S = {1, 2, 3, …, 9}. For k = 1, 2, …, 5, let N_k be the number of subsets of S, each containing five elements out of which exactly k are odd. Then $N_1 + N_2 + N_3 + N_4 + N_5$ =

(A) 210 (B) 252 (C) 125 (D) 126

42. (D)

N_1 = {only 1 odd} = $^5C_1 \times {}^4C_4 = 5$

N_2 = {only 2 odd} = $^5C_2 \times {}^4C_3 = 40$

N_3 = {only 3 odd} = $^5C_3 \times {}^4C_2 = 60$

N_4 = {only 4 odd} = $^5C_4 \times {}^4C_1 = 20$

N_5 = {all odd} = $^5C_5 = 1$

Total = 126

43. If y = y(x) satisfies the differential equation

$$8\sqrt{x}\left(\sqrt{9+\sqrt{x}}\right) dy = \left(\sqrt{4+\sqrt{9+\sqrt{x}}}\right)^{-1} dx, \qquad x > 0$$

and y(0) = $\sqrt{7}$, then y(256) =

(A) 16 (B) 80 (C) 3 (D) 9

43. (C)

$$\frac{dy}{dx} = \frac{1}{\left(8\sqrt{x}\,\sqrt{9+\sqrt{x}}\right)\sqrt{4+\sqrt{9+\sqrt{x}}}}$$

Put y = $\sqrt{4+\sqrt{9+\sqrt{x}}}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4+\sqrt{9+\sqrt{x}}}} \bullet \frac{1}{2\sqrt{9+\sqrt{x}}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{8\sqrt{4+\sqrt{9+\sqrt{x}}}\,\sqrt{9+\sqrt{x}}\,\sqrt{x}}$$

$$\int dy = \int \frac{\square dx}{\sqrt{4+\sqrt{9+\sqrt{x}}}\left(8\sqrt{x}\sqrt{9+\sqrt{x}}\right)}$$

$$y = \sqrt{4+\sqrt{9+\sqrt{x}}} + C$$

put x = 0, y = $\sqrt{7}$

$\sqrt{7} = \sqrt{7} + C \qquad \Rightarrow \qquad C = 0$

y = $\sqrt{4+\sqrt{9+\sqrt{x}}}$

y(256) = $\sqrt{4+\sqrt{9+\sqrt{256}}} = 3$

SECTION – II (Maximum Marks:28)

- This section contains **SEVEN** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +4 If only the bubble(s) corresponding to all the correct option(s) is (are) darkened.
  Partial Marks : +1 For darkening a bubble corresponding **to each correct option,** provided NO incorrect option is darkened.
  Zero Marks : 0 If none of the bubbles is darkened.
  Negative Marks : –2 In all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will get +4 marks; darkening only (A) and (D) will get +2 marks; and darkening (A) and (B) will get –2 marks, as a wrong option is also darkened.

44. Let $f(x) = \frac{1 - x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$ for $x \neq 1$. Then

(A) $\lim_{x\to 1^-} f(x)$ does not exist (B) $\lim_{x\to 1^+} f(x)$ does not exist

(C) $\lim_{x\to 1^-} f(x) = 0$ (D) $\lim_{x\to 1^+} f(x) = 0$

44. (B), (C)

$$f(x) = \frac{1-x(1+|1-x|)}{|1-x|}\cos\left(\frac{1}{1-x}\right)$$

$$\text{L.H.L.} = \lim_{h\to 0}\frac{1-(1-h)(1+|1-(1-h)|)}{|1-(1-h)|}\cos\left(\frac{1}{1-(1-h)}\right)$$

$$= \lim_{h\to 0}\frac{1-(1-h^2)}{h}\cos\left(\frac{1}{h}\right) = \lim_{h\to 0} h\cos\left(\frac{1}{h}\right) = 0$$

$$\text{R.H.L.} = \lim_{h\to 0}\frac{1-(1+h)(1+|1-(1+h)|)}{|1-(1+h)|}\text{cps}\left(\frac{1}{1-(1+h)}\right)$$

$$= \lim_{h\to 0}\frac{1-(1+h)(1+h)}{h}\cos\left(-\frac{1}{h}\right)$$

$$= \frac{-2h-h^2}{h}\cos\left(\frac{1}{h}\right)$$

$$= (-2-h)\cos\left(\frac{1}{h}\right) = \text{doesn't exist.}$$

45. Let α and β be nonzero real numbers such that $2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$. Then which of the following is/are true?

(A) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) + \tan\left(\frac{\beta}{2}\right) = 0$ (B) $\sqrt{3}\tan\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\beta}{2}\right) = 0$

(C) $\tan\left(\frac{\alpha}{2}\right) + \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$ (D) $\tan\left(\frac{\alpha}{2}\right) - \sqrt{3}\tan\left(\frac{\beta}{2}\right) = 0$

45. (C), (D)

$2(\cos\beta - \cos\alpha) + \cos\alpha\cos\beta = 1$

$\cos\beta = \dfrac{1 + 2\cos\alpha}{2 + \cos\alpha}$

$\Rightarrow \dfrac{1 - \tan^2\left(\frac{\beta}{2}\right)}{1 + \tan^2\left(\frac{\beta}{2}\right)} = \dfrac{3 - \tan^2\left(\frac{\alpha}{2}\right)}{3 + \tan^2\left(\frac{\alpha}{2}\right)}$

$\Rightarrow \tan^2\left(\frac{\beta}{2}\right) = \dfrac{\tan^2\left(\frac{\alpha}{2}\right)}{3}$ (Componendo-Dividendo)

$\Rightarrow \tan\left(\frac{\alpha}{2}\right) = \pm\sqrt{3}\tan\left(\frac{\beta}{2}\right)$

46. If $f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos x & \cos x & -\sin x \\ \sin x & \sin x & \cos x \end{vmatrix}$, then

(A) f(x) = 0 at exactly three points in $(-\pi, \pi)$
(B) f(x) = 0 at more than three points in $(-\pi, \pi)$
(C) f(x) attains its maximum at x = 0
(D) f(x) attains its minimum at x = 0

46. (B), (C)

$f(x) = \begin{vmatrix} \cos(2x) & \cos(2x) & \sin(2x) \\ -\cos(x) & \cos(x) & -\sin(x) \\ \sin(x) & \sin(x) & \cos(x) \end{vmatrix}$

$= \begin{vmatrix} 0 & \cos(2x) & \sin(2x) \\ -2\cos(x) & \cos(x) & -\sin(x) \\ 0 & \sin(x) & \cos(x) \end{vmatrix}$ $(C_1 \to C_1 - C_2)$

$= 2\cos(x)\,[\cos(2x)\cos(x) - \sin(2x)\sin(x)]$

$f(x) = 2\cos x\cos(3x) = \cos(4x) + \cos(2x)$

$f(x) = -4\sin(4x) - 2\sin(2x) = 0$

$= -2\sin(2x)\,[4\cos(2x) + 1] = 0$

$\cos(2x) = \dfrac{-1}{4}$ or $\sin(2x) = 0$

$\therefore$ f(x) = 0 at more than three points in $(-\pi, \pi)$

$f(0^-) > 0$ and $f(0^+) < 0$

$\therefore$ f(x) attains maxima at x = 0

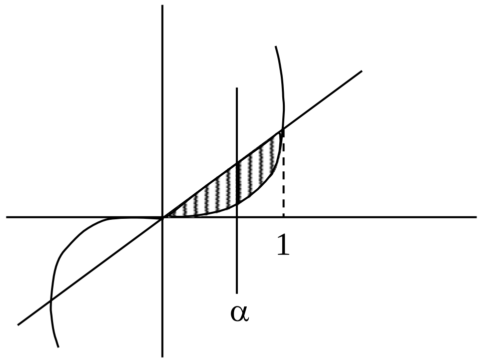
47. If the line $x = \alpha$ divides the area of region $R = \{(x, y) \in \mathbb{R}^2 : x^3 \le y \le x, 0 \le x \le 1\}$ into two equal parts, then

(A) $2\alpha^4 - 4\alpha^2 + 1 = 0$
(B) $\alpha^4 + 4\alpha^2 - 1 = 0$
(C) $0 < \alpha \le \dfrac{1}{2}$
(D) $\dfrac{1}{2} < \alpha < 1$

47. (A), (D)

$$2\int_0^{\alpha} (x-x^3)\,dx = \int_0^1 (x-x^3)\,dx$$

$$\Rightarrow 2\left(\frac{\alpha^2}{2}-\frac{\alpha^4}{4}\right)=\frac{1}{2}-\frac{1}{4}$$



$$\Rightarrow 2\alpha^4-4\alpha^2+1=0$$

$$\Rightarrow \alpha^2 = 1+\frac{1}{\sqrt{2}},\ 1-\frac{1}{\sqrt{2}}$$

But $\alpha < 1$

$$\therefore \alpha^2 = 1-\frac{1}{\sqrt{2}}$$

$$\alpha = \frac{\sqrt{4-2\sqrt{2}}}{2} > \frac{1}{2}$$

$$\therefore \frac{1}{2} < \alpha < 1$$

48. If $I = \sum_{k=1}^{98}\int_k^{k+1}\frac{k+1}{x(x+1)}dx$, then

(A) $I < \frac{49}{50}$ (B) $I > \log_e 99$ (C) $I > \frac{49}{50}$ (D) $I < \log_e 99$

48. (B), (C)

$$I = \sum_{k=1}^{98}\int_k^{k+1}\frac{(k+1)}{x(x+1)}dx$$

$$= \sum_{k=1}^{98}(k+1)\left(\int_k^{k+1}\frac{1}{x}-\frac{1}{x+1}dx\right)$$

$$= \sum_{k=1}^{98}(k+1)\left(\log\left(\frac{k+1}{k+2}\right)-\log\left(\frac{k}{k+1}\right)\right)$$

$$= \sum_{k=1}^{98}(k+1)\log_e\left(\frac{k+1}{k+2}\right)-k\log_e\left(\frac{k}{k+1}\right)-\log_e\left(\frac{k}{k+1}\right)$$

$$= \left(\sum_{k=1}^{98}(k+1)\log\left(\frac{k+1}{k+2}\right)-k\log\left(\frac{k}{k+1}\right)\right)-\left(\sum_{k=1}^{98}\log_e(k)-\log_e(k+1)\right)$$

$$= \left(99\log_e\left(\frac{99}{100}\right)-\log\left(\frac{1}{2}\right)\right)-\left(\log_e(1)-\log_e(99)\right)$$

$$= +\log_e(2)+\log_e(99) \qquad \left(\because \log_e\left(\frac{99}{100}\right)\approx\log_e(1)=0\right)$$

$$> \log_e(99) > \frac{49}{50}$$

$\therefore$ (B, C) options are true.

49. If $f:\mathbb{R}\to\mathbb{R}$ is a differentiable function such that $f'(x) > 2f(x)$ for all $x \in \mathbb{R}$, and $f(0) = 1$, then

(A) f(x) is decreasing in $(0, \infty)$ (B) $f'(x) < e^{2x}$ in $(0, \infty)$

(C) f(x) is increasing in $(0, \infty)$ (D) $f(x) > e^{2x}$ in $(0, \infty)$

49. (C), (D)

$f:\mathbb{R}\to\mathbb{R}$, $f(0) = 1$

$f'(x) > 2f(x)$

$e^{-2x} f'(x) - e^{-2x} . 2 f(x) > 0$

$\frac{d}{dx}\left(e^{-2x} . f(x)\right) > 0$

$\Rightarrow e^{-2x}$ f(x) is increasing.

Let $F(x) = e^{-2x} f(x)$

$F(0) = 1$

So, $F(x) > 1$, for $x > 0$

$e^{-2x} . f(x) > 1$

$f(x) > e^{2x}$ Hence option (D)

As $F(x) = \frac{f(x)}{e^{2x}}$

As e^{2x} is increasing and F(x) is also increasing.

$\Rightarrow$ f(x) is also increasing for $x > 0$.

50. If $g(x) = \int_{\sin x}^{\sin(2x)} \sin^{-1}(t)\, dt$, then

(A) $g'\left(\frac{\pi}{2}\right) = -2\pi$ (B) $g'\left(-\frac{\pi}{2}\right) = -2\pi$ (C) $g'\left(-\frac{\pi}{2}\right) = 2\pi$ (D) $g'\left(\frac{\pi}{2}\right) = 2\pi$

50. (A), (C)

$g'(x) = \sin^{-1}(\sin 2x)\{2(\cos 2x)\} - \sin^{-1}(\sin x).\cos x$

$= 2x(2 \cos 2x) - \sin^{-1}(\sin x) \cos x$

$g'\left(\frac{\pi}{2}\right) = 4\left(\frac{\pi}{2}\right)(-1) - 0 = -2\pi$

$g'\left(\frac{-\pi}{2}\right) = 2\pi$

Note : We feel that $\sin^{-1}(\sin 2x)$ is $\neq 2x$ for given value of x.

SECTION – III (Maximum Marks:12)

- This section contains **TWO** paragraphs.
- Based on each paragraph, there are **TWO** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in the ORS.
- For each question, marks will be awarded in one of the following categories:
  Full Marks : +3 If only the bubble corresponding to the correct answer is darkened.
  Zero Marks : 0 If all other cases.

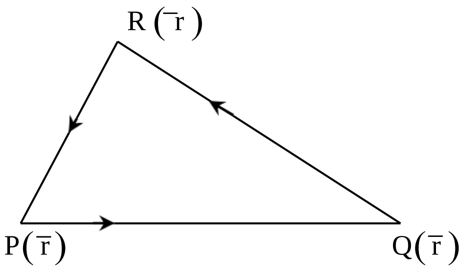
PARAGRAPH 1

Let O be the origin, and $\overrightarrow{OX}, \overrightarrow{OY}, \overrightarrow{OZ}$ be three unit vectors in the directions of the sides $\overrightarrow{QR}, \overrightarrow{RP}, \overrightarrow{PQ}$, respectively, of a triangle PQR

51. $|\overrightarrow{OX} \times \overrightarrow{OY}| =$

(A) $\sin(P + R)$ (B) $\sin 2R$ (C) $\sin(Q + R)$ (D) $\sin(P + Q)$

51. (D)



$$\left|\overrightarrow{OX} \times \overrightarrow{OY}\right| = \left|\overrightarrow{QR} \times \overrightarrow{RP}\right| = \sin(\pi - R)$$
$$= \sin\{P + Q + R - R\} \qquad [P + Q + R = \pi]$$
$$= \sin\{P + Q\}$$

52. If the triangle PQR varies, then the minimum value of

$$\cos(P + Q) + \cos(Q + R) + \cos(R + P)$$

is

(A) $-\frac{3}{2}$ (B) $\frac{5}{3}$ (C) $\frac{3}{2}$ (D) $-\frac{5}{3}$

52. (A)

$\cos(P + Q) + \cos(Q + R) + \cos(R + P)$

$= -\cos R - \cos P - \cos Q$

$= -[\cos P + \cos Q + \cos R]$

$= -\left[1 + \frac{r}{R}\right]$

$\geq -1 - \frac{1}{2}$

$= -\frac{3}{2}$

As we know

$R \geq 2r$

$\frac{1}{2} \geq \frac{r}{R}$

$\Rightarrow -\frac{1}{2} \leq -\frac{r}{R}$

$\Rightarrow -\frac{r}{R} \geq -\frac{1}{2}$

PARAGRAPH 2

Let p, q be integers and let α, β be the roots of the equation, $x^2 - x - 1 = 0$, where $\alpha \neq \beta$. For $n = 0, 1, 2, \ldots$, let $a_n = p\alpha^n + q\beta^n$.

FACT: If a and b are rational numbers and $a + b\sqrt{5} = 0$, then $a = 0 = b$.

53. If $a_4 = 28$, then $p + 2q =$

(A) 7 (B) 21 (C) 14 (D) 12

53. (D)

$x^2 - x - 1 = 0$

$\alpha + \beta = 1, \quad \alpha\beta = -1$

$x = \dfrac{1 \pm \sqrt{1+4}}{2} = \dfrac{1 \pm \sqrt{5}}{2}$

$a_0 = p + q$

$a_1 = p\alpha + q\beta$

$a_2 = p\alpha^2 + q\beta^2$

$a_3 = p\alpha^3 + q\beta^3$

$a_4 = p\alpha^4 + q\beta^4$

$28 = p\left(\dfrac{1+\sqrt{5}}{2}\right)^4 + q\left(\dfrac{1-\sqrt{5}}{2}\right)^4 \quad = \dfrac{p}{16}\cdot\left(6+2\sqrt{5}\right)^2 + \dfrac{9}{16}\left(6-2\sqrt{5}\right)^2$

$= \dfrac{p}{4}\left[\left(3+\sqrt{5}\right)^2\right] + \dfrac{q}{4}\left[\left(3-\sqrt{5}\right)^2\right] \quad = \dfrac{P\left[14+6\sqrt{5}\right]}{4} + \dfrac{9\left(14-6\sqrt{5}\right)}{4}$

$56 = p\left[7 + 3\sqrt{5}\right] + q\left[7 - 3\sqrt{5}\right]$

$56 = 7(p + q) + 3(p - q)\sqrt{5}$

Equate $7(p + q) = 56$ & $3(p - q) = 0$

$p + q = 8$ $p = q$

$\Rightarrow p = 4$

So, $p + 2q = 4 + 2 \times 4 = 12$

54. $a_{12} =$

(A) $a_{11} + a_{10}$ (B) $a_{11} - a_{10}$ (C) $a_{11} + 2a_{10}$ (D) $2a_{11} + a_{10}$

54. (A)

$a_n = 4\alpha^n + 4\beta^n$

$= 4(\alpha^n + \beta^n)$

$a_{12} = 4[\alpha^{12} + \beta^{12}] \qquad a_{11} = 4[\alpha^{11} + \beta^{11}], \qquad a_{10} = 4[\alpha^{10} + \beta^{10}]$

$a_{11} + a_{10} = 4[\alpha^{11} + \beta^{11} + \alpha^{10} + \beta^{10}] \quad = 4[\alpha^{10}(\alpha + 1) + \beta^{10}(\beta + 1)]$

$= 4[\alpha^{10}(\alpha^2) + \beta^{10}(\beta^2)]$

[as α is root of $x^2 - x - 1 = 0 \Rightarrow \alpha^2 = \alpha + 1$ & $\beta^2 = \beta + 1$]

$= 4[\alpha^{12} + \beta^{12}]$

$= a_{12}$

CODE **7**

QUESTION PAPER FORMAT AND MARKING SCHEME

20. The question paper has three parts: Physics, Chemistry and Mathematics.
21. Each part has three sections as detailed in the following table:

Section	Question Type	Number of Questions	Category-wise Marks for Each Question				Maximum Marks of the Section
			Full Marks	Partial Marks	Zero Marks	Negative Marks	
1	Single Correct Option	7	+3 If only the bubble corresponding to the correct option is darkened	—	0 If none of the bubbles is darkened	–1 In all other cases	21
2	One or more correct option(s)	7	+4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened	+1 For darkening a bubble corresponding to each correct option, provided NO incorrect option is darkened	0 If none of the bubbles is darkened	–2 In all other cases	28
3	Compre-hension	4	+3 If only the bubble corresponding to the correct option is darkened	—	0 In all other cases	—	12

NAME OF THE CANDIDATE ....

ROLL NO ....

I have read all the instructions and shall abide by them. ------------------------- Signature of the Candidate	I have verified the identity, name and roll number of the candidate, and that question paper and ORS codes are the same. ------------------------- Signature of the Invigilator

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