# JAM 2006

# GEOPHYSICS TEST PAPER



#### **IMPORTANT NOTE FOR CANDIDATES**

Select any <u>Two</u> Sections. Attempt ALL objective and subjective questions of the same Two Sections. Questions 1- 45 (objective questions) carry <u>three</u> marks each and questions 46 - 66 (subjective questions) carry <u>fifteen</u> marks each.

Write the answers to the objective questions in the Answer Table for Objective Questions provided on page 17 only)

#### **GEOLOGY SECTION**

- 1. Plutonic equivalent of Trachyte is
  - (A) diorite
  - (B) gabbro
  - (C) granite
  - (D) syenite
- 2. The river meanders at the mature stage with gentle gradient. The formation of pointbar will be on:
  - (A) outer zone of the bend
  - (B) inner zone of the bend
  - (C) straight channel segment
  - (D) steep bank of the channel.
- 3. Match the features in **Group 1** with the agents responsible for these features from **Group 2**

	Group 1	Grou	up 2
P.	Arete	1.	River
Q.	Backswamp	2.	Ground Water
R.	Yardangs	3.	Glacier
S.	Stalactites and stalagmites	4.	Wind
Cho	ose the correct answer from the following	lowing:	

- (A) P-4, Q-3, R-1, S-2
- (B) P-2, Q-1, R-3, S-4
- (C) P-3, Q-1, R-4, S-2
- (D) P-1, Q-2, R-4, S-3
- 4. The area bounded by two fault planes dipping away from each other with hanging walls going downward is called as:
  - (A) dome
  - (B) grabben
  - (C) horst
  - (D) klippe



5. Match the characteristics in Group 1 with the structures in Group 2

### Group 1 Group 2

- P. axial plane is horizontal 1. Isoclinal fold
- Q. hinges are sharp and angular 2. Parallel fold
- R. limbs are parallel 3. Recumbant fold
- S. thickness of bed remains constant 4. Cheveron fold

Choose the correct answer from the following:

- (A) P-2, Q-4, R-2, S-1
- (B) P-1, Q-3, R-2, S-4
- (C) P-4, Q-2, R-1, S-3
- (D) P-3, Q-4, R-1, S-2
- 6. Which of the following rock indicates initiation of metamorphism
  - (A) phyllite
  - (B) schist
  - (C) shale
  - (D) slate
- 7. Barrovian metamorphism of pelitic rocks is characterized by the first appearance of index minerals in a particular sequence. Which one of the following is the correct sequence.
  - (A) chlorite-garnet-biotite-kyanite-staurolite-sillimanite
  - (B) garnet-biotite-chlorite-staurolite-sillimanite-kyanite
  - (C) chlorite-biotite-garnet-staurolite-kyanite-sillimanite
  - (D) biotite-chlorite-garnet-kyanite-staurolite-sillimanite
- 8. A crystal has three crystallographic axes of 2 fold symmetry and mirror plane perpendicular to each of these crystallographic axes. The Herman-Manguin notation for crystal would be
  - (A) 2/m 2/m 2/m
  - (B) 2m
  - (C) 2mm
  - (D) 23
- 9. Indicate the correct order in terma of increasing Si : O ratio.
  - (A) phlogopite-beryl-plagioclase-epidote
  - (B) epidote-beryl-phlogopite-plagioclase
  - (C) beryl-phlogopite-plagioclase-epidote
  - (D) plagioclase-phlogopite-epidote-beryl
- 10. Sandstones and purple shales of Muree Series of Potwar region, equivalent to Dagshai and Kasauli beds of northwest Himalaya belongs to
  - (A) Upper Eocene
  - (B) Lower Eocene
  - (C) Middle Miocene
  - (D) Lower Miocene



- 11. A radiogenic isotope has half-life of 1 hour and we have 10000 atoms of that particular isotope in a particular system at a particular time. How much atoms of that isotope will be there after 6 hours?
  - (A) 78
  - (B) 156
  - (C) 313
  - (D) 625
- 12. Find the odd man out from the following
  - (A) stockwork
  - (B) ladder vein
  - (C) saddle reef
  - (D) banding
- 13. Sulfide chimneys are observed at
  - (A) vents of seafloor hotsprings around ridges
  - (B) inland hotspring vents in volcanic terrains
  - (C) sulfide mineral coatings on the chimneys of smelters
  - (D) mouths of explosive volcanoes
- 14. Within the mantle sudden density change produce seismic-wave discontinuities due to polymorphic transition or compositional change or a combination of both occur at a depth of
  - (A) 470 kms
  - (B) 570 kms
  - (C) 670 kms
  - (D) 760 kms
- 15. The estimated thickness of the moon's lithosphere is about
  - (A) 35 km
  - (B) 65 km
  - (C) 100 km
  - (D) 1000 km

#### PHYSICS SECTION

- 16. In case of an inelastic collision which one of the following is true
  - (A) Total energy is not conserved
  - (B) Momentum is not conserved
  - (C) Kinetic energy is conserved
  - (D) Kinetic energy is not conserved
- 17. The root mean square speed of an ideal gas, made up of molecules of molecular weight 0.0831 kg/mol, at temperature  $300^{\circ}$  K is (Take universal gas constant R = 8.31 J/mol K)
  - (A) 100 m/s
  - (B) 200 m/s
  - (C) 300 m/s
  - (D) 400 m/s



- 18. The temperature differences between hot  $(T_H)$  and cold  $(T_C)$  reservoirs of two Carnot engines A and B are the same. If the ratio of the respective efficiencies,  $\frac{\eta^A}{\eta^B}$ , is equal
  - to  $\frac{1}{2}$  then the ratio of the hot reservoir temperatures  $\frac{T_H^A}{T_H^B}$  is
    - (A) 0.25
  - (B) 0.5
  - (C)  $1 \cdot 0$
  - $(D) 2 \cdot 0$
- 19. Which one of the following phenomenon cannot be described by the particle nature of electromagnetic radiations.
  - (A) Blackbody radiations
  - (B) Compton scattering
  - (C) Photoelectric effect
  - (D) X-ray diffraction
- 20. If a semiconductor is doped with donor atoms then the impurity levels created in the semiconductor are close to the
  - (A) bottom of the conduction band
  - (B) top of the valence band
  - (C) bottom of the valence band
  - (D) top of the conduction band
- Binding energy per nucleon for the nuclei <sup>4</sup>He, <sup>56</sup>Fe, <sup>197</sup>Au and <sup>235</sup>U are given by B<sub>1</sub>, B<sub>2</sub>, B<sub>3</sub> and B<sub>4</sub>, respectively. These binding energies satisfy the order
  - (A)  $B_1 \le B_2 \le B_3 \le B_4$
  - (B)  $B_1 > B_2 > B_3 > B_4$
  - (C)  $B_2 \le B_3 \le B_4 \le B_1$
  - (D)  $B_2 > B_3 > B_4 > B_1$
- When a thin transparent sheet is introduced along the path of one of the slits in Young's double slit experiment, then the fringe width
  - (A) decreases
  - (B) increases
  - (C) does not change
  - (D) does not change but intensity becomes half



- 23. An infinite wire, lying along the z-axis, carries a current I in the positive z direction denoted by  $\hat{k}$ . The magnetic field at a point  $d \hat{i}$  is
  - (A)  $\frac{\mu_0 I}{2\pi d} \hat{j}$
  - (B)  $\frac{\mu_0 I}{2\pi d}\hat{i}$
  - (C)  $-\frac{\mu_0 I}{2\pi d}\hat{j}$
  - (D)  $-\frac{\mu_0 I}{2\pi d}\hat{i}$
- 24. The radius of curvature of curved surface of a plano-convex thin lens of glass (refractive index n = 1.5) of focal length 0.4 m is
  - (A) 0.1 m
  - (B) 0.2 m
  - (C) 0.4 m
  - (D) 0.8 m
- 25. The engine of a train, emitting the sound of frequency  $v_0$  approaches an observer with constant speed. If the observer measures the frequencies as  $v_1$  when it is approaching and  $v_2$  while it is going away, the relation between the frequencies is given by
  - (A)  $v_1 = v_2 = v_0$
  - (B)  $v_1 > v_0 > v_2$
  - $(\mathbf{C}) \qquad \nu_1 < \nu_0 < \nu_2$
  - (D)  $v_1 = v_2 \neq v_0$
- 26. In a dielectric sphere the polarization  $\stackrel{\rightarrow}{P}$  is given by  $\stackrel{\rightarrow}{P} = kr^3\hat{r}$ . The corresponding bound volume charge density is equal to
  - (A) -20k
  - (B) -10k
  - (C) 10k
  - (D) 20k
- An ideal fluid is flowing through a tube of cylindrical cross section with smoothly varying radius. The velocity of fluid particles at the point where tube's cross sectional area is  $1 \times 10^{-4}$  m<sup>2</sup> is given by 0.01 m/s. The velocity at a point where cross sectional area is  $2 \times 10^{-4}$  m<sup>2</sup> is given by
  - (A) 0.0025 m/s
  - (B) 0.005 m/s
  - (C) 0.02 m/s
  - (D) 0.04 m/s



- 28. The solution of Maxwell's equation for electric field in free space is given by  $E = E_0 \sin \omega (t x/c)$ , where  $E_0$  is a constant,  $\omega$  is the angular frequency and c is the speed of light. The corresponding solution for the magnetic field B is
  - (A)  $B = c E_0 \sin \omega (t x/c)$
  - (B)  $B = \frac{E_0}{c} \sin \omega (t x/c)$
  - (C)  $B = \frac{E_0}{c^2} \sin \omega (t x/c)$
  - (D)  $B = \frac{E_0}{c^3} \sin \omega (t x/c)$
- 29. The frequency of electron in n=1 Bohr orbit is given by  $f_1$  revolutions/s. The frequency of electron in the n-th orbit for n>1 is
  - (A)  $f_1/n$
  - (B)  $f_1/n^2$
  - (C)  $f_1/n^3$
  - (D)  $n f_1$
- 30. A signal of 1 mV is input to an amplifier circuit consisting of a transistor in common-emitter mode. What is the voltage gain if the collector current changes by 1 mA and the load resistance is equal to  $1 \text{ k}\Omega$ ?
  - (A) 10
  - (B)  $10^2$
  - (C)  $10^3$
  - (D)  $10^4$

## **MATHEMATICS SECTION**

- 11. Let  $\sum_{n\geq 1} a_n$ ,  $a_n > 0$  be a convergent series. Now, consider the following statements:
  - **P**: The series  $\sum_{n\geq 1} \sqrt{a_n}$  is always convergent.
  - Q: The series  $\sum_{n\geq 1} \left( \frac{a_1 + a_2 + \dots + a_n}{n} \right)$  is always divergent.

Then,

- (A) both **P** and **Q** are true
- (B) **P** is true and **Q** is false
- (C) both **P** and **Q** are false
- (D) **P** is false and **Q** is true



32. Let 
$$f : [0,1] \to [0,1]$$
 be defined by

$$f(x) = \begin{cases} \frac{1}{2} + \left(x - \frac{1}{2}\right)^2, & \text{if } x \text{ is rational} \\ \frac{1}{2}, & \text{if } x \text{ is irrational.} \end{cases}$$

Then,

- (A) f is continuous and differentiable only at  $x = \frac{1}{2}$
- (B) f is continuous only at  $x = \frac{1}{2}$  but not differentiable at  $x = \frac{1}{2}$
- (C) f is neither continuous nor differentiable at  $x = \frac{1}{2}$
- (D) f is continuous and differentiable for every x=[0,1]
- 33. The value of the integral  $\oint_C \frac{dz}{(z-i)^2(z+i)}$ , where  $C = \{z : |z-i|=1\}$ , is
  - (A) 1
  - (B)  $\pi$
  - (C)  $\frac{\pi}{2}i$
  - (D)  $\pi i$
- 34. The integral  $\int_{0}^{1} \int_{0}^{x} \int_{0}^{y} f(x, y, z) dz dy dx$  is equal to
  - (A)  $\int_{0}^{1} \int_{0}^{x} \int_{z}^{x} f(x, y, z) dy dz dx$
  - (B)  $\int_{0}^{1} \int_{0}^{x} \int_{0}^{z} f(x, y, z) dy dz dx$
  - (C)  $\int_{0}^{1} \int_{x}^{1} \int_{z}^{x} f(x, y, z) dy dz dx$
  - (D)  $\int_{0}^{1} \int_{x=0}^{1} \int_{0}^{z} f(x, y, z) \, dy \, dz \, dx$



- Consider the initial value problem (IVP): xy' y = 0, y(0) = 0. Now, consider the 35. following statements:
  - Picard's theorem is applicable to the above IVP.
  - **Q**: The above IVP has exactly one solution.
  - Then,
  - both P and Q are true (A)
  - (B) **P** is false but **Q** is true
  - both **P** and **Q** are false (C)
  - (D) **P** is true but **Q** is false
- 36. Let Q be the set of rational numbers in  $\Re$ . Then
  - Q is closed in  $\Re$ (A)
  - Q is open in  $\Re$ (B)
  - (C) Q is both open and closed in  $\Re$
  - Q is neither open nor closed in  $\Re$
- The radius of convergence of the power series  $\sum_{n\geq 0} \frac{(n!)^2}{(2n)!} x^{2n}$  is 37.

  - (A)  $\frac{1}{2}$ (B)  $\frac{1}{\sqrt{2}}$ (C)  $\sqrt{2}$ (D) 2
- Consider the differential equation y'' + 6y' + 25y = 0 with initial condition y(0) = 0. 38. Then, the general solution of the IVP is
  - $e^{-3x}(A\cos 4x + B\sin 4x)$
  - (B)  $B e^{-3x} \sin 4x$
  - (C)  $A e^{-4x} \sin 3x$
  - $e^{-4x}(A\cos 3x + B\sin 3x)$ (D)
- Let  $\vec{F}(x, y, z) = x^2 y \hat{i} + y \hat{j} + z^2 \hat{k}$ . If  $\vec{p} = curl \vec{F}$  and  $q = div \vec{F}$ , then  $(\vec{p}, q)$  is
  - (A)  $(-x^2 \hat{k}, 1 + 2xy + 2z)$
  - (B)  $(2xy\hat{i}+\hat{j}+2z\hat{k}, 1+2xy+2z)$
  - (C)  $(-x^2 \hat{k}, x^2 y + y + z^2)$
  - (D)  $(2xy\hat{i}+\hat{j}+2z\hat{k}, x^2y+y+z^2)$



- Let  $V = \{ (x, y, z, w) : x+y+z-3w = 0, x-y+z-w = 0, x-7y+z+5w = 0 \}$ 40. be a vector subspace of  $\Re^4$ . Then  $\dim(V)$  is
  - (A)

  - (D)
- Let  $T: \mathbb{R}^3 \to \mathbb{R}^3$  be a linear transformation defined by 41. T(x, y, z) = (x + y + z, y + z, z). Then  $T^{n}(x, y, z)$ , for  $n \ge 1$ , is

(A) 
$$\left(x+ny+\frac{n^2+n}{2}z, y+nz, z\right)$$

(B) 
$$(x+ny+(n^2-n+1)z, y+nz, z)$$

(A) 
$$\left( x + n y + \frac{n^2 + n}{2} z, y + n z, z \right)$$
(B) 
$$\left( x + n y + (n^2 - n + 1) z, y + n z, z \right)$$
(C) 
$$\left( x + n y + \frac{n^2 + 5n - 2}{4} z, y + n z, z \right)$$

(D) 
$$\left(x+ny+\frac{3n^2-n+2}{4}z, y+nz, z\right)$$

- Suppose that the moment generating function of a random variable X is 42.  $\frac{1}{2}e^{-3t} + \frac{1}{4}e^{-2t} + \frac{1}{4}e^{2t}$ . Then Var(X) is

  - (A)  $\frac{3}{2}$ (B)  $\frac{17}{4}$
  - (D)  $\frac{13}{2}$   $\frac{35}{4}$
- Perform Newton's method to the equation  $x^3 x 2 = 0$  starting with  $x_0 = 1$ . In this 43. operation, the value of  $x_2$  (the second iterate) is

  - (B)



44. The distribution function F of a random variable X is

$$F(x) = \begin{cases} 0, & \text{if } x < -1 \\ 1/8, & \text{if } -1 \le x < 0 \\ 1/4, & \text{if } 0 \le x < 1 \\ 1/2, & \text{if } 1 \le x < 2 \\ 1, & \text{if } x \ge 2. \end{cases}$$

If  $\alpha = P\left(-\frac{1}{2} < X \le 1\right)$  and  $\beta = P\left(0 \le X < 2\right)$ , then  $(\alpha, \beta)$  is

- (A)  $\left(\frac{3}{8}, \frac{3}{8}\right)$
- (B)  $\left(\frac{1}{8}, \frac{3}{8}\right)$
- (D)  $\left(\frac{1}{8}, \frac{1}{4}\right)$

Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample from a normal population  $N(\mu, \sigma^2)$ , 45. where  $\mu$  and  $\sigma^2$  are unknowns. Suppose that  $S^2 = \frac{1}{n-1} \sum_{i=1}^n (X_i - \overline{X})^2$ , where  $\overline{X}$  is the sample mean. It is known that  $cS^2$  follows a  $\chi^2$ -distribution with (n-1)degrees of freedom. Then c is equal to

- (A)  $\frac{n}{\sigma}$

- (B)  $\frac{n}{\sigma^2}$ (C)  $\frac{n-1}{\sigma}$ (D)  $\frac{n-1}{\sigma^2}$

#### **GEOLOGY SECTION**

- What is the relationship between an earthquake focus and the corresponding 46. (a) epicenter? **(6)** 
  - (9)What are the three kinds of Plate margins and associated magmatism? (b)
- **47.** What is dip slip fault? In an area a bed is dipping towards west at 42°. The area (a) had been affected by fault dipping toward east at 45°. With the help of neat diagrams show the relative movements of the blocks resulting in repetition of bed and and omission of bed (9)



- How you define monocline? A N-S trending bed is exposed on an easterly sloping ground with the bed-dipping west. Find the thickness of the bed, if the slope of the ground is 15° E; width of the bed measured perpendicular to strike is 100 m; dip of the bed is 30°W.

  (6)
- 48. (a) Why startovolcano like Mount Fuji in Japan has steep sides and shield volcano like Mauna Loa in Hawaii have gentle surface slopes? (6)
  - (b) How you define conformable and unconformable sequence? What geological events are indicated by angular unconformity? (9)
- 49. (a) Where do back-arc basins form and what is the necessary conditions for the formation of back-arc basins? How is the nature of magmatism different from that of a forearc? (9)
  - (b) Compare Airy's and Pratt's hypothesis on isostasy with the help of a neat diagram. (6)
- 50. (a) Distinguish between "perthitic" and "rapakivi" texture with the help of neat sketch. (6)
  - (b) Give the idealized Bouma sequence. Where do you find such a sequence of deposition of sediments? (9)
- Mention the broad tectonic regime and mode of occurrence of porphyry-copper deposits.

  (a) Mention the broad tectonic regime and mode of occurrence of porphyry-copper (6)
  - (b) Mention 3 locality of each of occurrences of Iron, Manganese and Copper deposits in India. (9)
- 52. (a) A grain of undeformed quartz is in contact with an untwined plagioclase, both showing first order gray interference color. How do you distinguish the two?

  (6)
  - (b) What is an optical indicatrix? Draw a positive biaxial indicatrix indicating the optic axes, the optic axial angle and circular sections. (9)

#### **PHYSICS SECTION**

- An ideal diatomic gas at pressure  $p_i$  and volume  $V_i$  doubles its volume adiabatically. Find
  - (a) the final pressure and (6)
  - (b) the work done by the gas. (9)
- A charge of magnitude  $9.8 \times 10^{-10}$  C and mass  $2.0 \times 10^{-6}$  kg is suspended through a silk thread along the line passing through the center and parallel to the length of two parallel plates with a spacing of 0.1 m. The plates are connected to a voltage source of 2000 V. (Take g =  $9.8 \,\mathrm{ms}^{-2}$ ). Find
  - (a) the electric field experienced by the charge and (6)
  - (b) the angle that the thread makes with the vertical when charge is in equilibrium. (9)



- 55. A cylinder of 1 kg mass and 0.02 m diameter left at the top of an inclined plane of height 1 m rolls down without slipping. (Take  $g = 9.8 \,\mathrm{ms}^{-2}$ )
  - (a) What is the kinetic energy of the cylinder when it reaches at the bottom of inclined plane? (6)
  - (b) Find the velocity of center of mass of cylinder on reaching the bottom of inclined plane. (9)
- Two waves described by  $y_1 = A \sin(\omega t + kx)$  and  $y_2 = A \sin(\omega t kx)$  are traveling along a string. Let A = 0.001 m, k = 3.142 m<sup>-1</sup> and  $\omega = 157.1$  s<sup>-1</sup> (Take  $\pi = 3.142$ )
  - (a) Find the magnitude and direction of velocity of these waves. (6)
  - (b) What shall be the amplitude of resultant wave on the string at x=0.5 m.

**(9)** 

- 57. Consider a monatomic FCC solid with lattice constant  $\sqrt{3}$  Å.
  - (a) Find the interplanar spacing of a set of parallel (111) planes. (6)
  - (b) For what incident angle  $\theta$  the first order Bragg peak would be observed if a monochromatic x-ray of wavelength  $\overset{0}{1}$  A is incident on these planes? (9)
- Consider an LR circuit with an inductor L, a resistor R, a battery of emf E and a switch S, all connected in series.
  - (a) Find an expression for current I in the circuit as a function of time after the switch S is closed. (9)
  - (b) What is value of I after a time that equals the time constant of this circuit?

**(6)** 

- 59. Take radius of hydrogen atom H to be  $5.3 \times 10^{-11}$  m. (Take  $\hbar = 1.054 \times 10^{-34}$  J s and  $m_e = 9.1 \times 10^{-31}$  kg) Assuming momentum of electron to be same as order of uncertainty in momentum,
  - (a) Find the order of kinetic energy that an electron in the hydrogen atom is expected to have based on the uncertainty principle. (9)
  - (b) If de Broglie wavelength of electron matches with the circumference of orbit, what is the velocity of electron? (6)

#### **Mathematics Section**

- Suppose that  $f:[a,b] \to \Re$ , a > 0 is continuous on the closed interval [a,b], that f is differentiable on the open interval (a,b), and that b f(a) = a f(b). Then prove that there exists  $c \in (a,b)$  such that f(c) = c f'(c).
  - (b) Let  $f:[0,2] \to \Re$  be defined by  $f(x) = \frac{x}{2} + (x-1)^{\frac{2}{3}}$ . Compute the absolute maximum and minimum value of f on [0,2]. (9)



- 61. (a) Let  $f:[0,1] \to \Re$  be continuous with  $\int_0^x f(t) dt = \int_x^1 f(t) dt$  for all  $x \in [0,1]$ . Does the above condition imply that  $f(x) \equiv 0$  on [0,1]? Explain.
  - (b) Let  $f:[0,1] \to \Re$  be defined by  $f(x)=x^3$ . Find the area of the surface generated by revolving the curve y=f(x) about the x-axis. (9)
- 62. (a) Let  $f(x)=1+3x^2+5x^4+7x^6+\cdots$ , for |x|<1, be a power series. Determine  $f\left(\frac{1}{2}\right)$ .
  - (b) Let V be a vector subspace of  $\Re^4$  spanned by the vectors (1,1,1,-1) and (1,-1,0,1). Let W be another vector subspace of  $\Re^4$  spanned by the vectors (1,1,-1,1) and (1,3,4,-5). Determine a basis for  $V \cap W$ .
- 63. (a) Consider the system of linear equations

$$x + y + z = 3$$

$$x + 2y + 3z = a$$

$$x + 3y + bz = 5.$$

Determine the values for a and b for which the above system has a unique solution, infinite number of solutions, and no solution. (9)

- (b) Solve:  $(4x^2y+5x^3y^2)dx + (2x^3+3x^4y)dy = 0.$  (6)
- 64. (a) Let C be the boundary of the triangle with vertices (0,1,0), (1,0,0) and (2,1,0). If  $\overrightarrow{F}(x,y,z) = -y\hat{i} + y^2z\hat{j} + zx\hat{k}$ , then use Stoke's theorem to evaluate  $\int_{C} \overrightarrow{F} \cdot d\overrightarrow{r}$  when C is traversed counter-clockwise when viewed from above. (9)
  - (b) Let  $u(x, y) = x^3 3xy^2 + x + 3$  be the real part of an analytic function f(x, y) on the entire complex plane. Determine the harmonic conjugate of u(x, y).

    (6)
- 65. (a) Let  $X_1, X_2, X_3, \dots, X_{20}$  be a random sample of size 20 from a normal population  $N(0, \sigma^2)$ . Find the best critical region of size  $\alpha = 0.05$  for testing  $H_0: \sigma^2 = 1$  against  $H_1: \sigma^2 = 2$ . (9)

[Given:  $\chi_{20}^2(0.95) = 31.4$ ,  $\chi_{19}^2(0.95) = 30.1$ ,  $\chi_{20}^2(0.05) = 10.9$ , and  $\chi_{19}^2(0.05) = 10.1$ ]

(b) Let  $X_1, X_2, X_3, \dots, X_n$  be a random sample of size n from a normal population  $N(\mu, 16)$ . Compute the minimum integral value of n such that  $P(\overline{X} - 2 < \mu < \overline{X} + 2) \ge 0.95$ , where  $\overline{X}$  is the sample mean. (6)

[For  $Z \sim N(0,1)$  and  $\Phi(z) = P(-\infty < Z < z)$ ,  $\Phi(1.645) = 0.95$  and  $\Phi(1.96) = 0.975$ ]



66. (a) Determine the value of c so that

$$f(x,y) = \begin{cases} c(x^2 - y^2), & for (x,y) \in D \\ 0, & otherwise, \end{cases}$$

where D is the triangle with vertices (0,0), (2,0) and (2,2), is the joint probability density function of the random variables X and Y. (6)

(b) The table below gives the values of f(x) for  $1 \le x \le 9$ .

0.0	1				00.	3
	x	1	3	5	7	9
2	f(x)	1	0	1	0	1

Compute the forward difference table and determine f(2) up to four decimal places. (9)

