Notations: Let $\mathbb{N}$ be the set of natural numbers, $\mathbb{Z}$ be the set of integers, $\mathbb{R}$ be the set of real numbers, and let $\mathbb{Q}$ be the set of rational numbers.

1. The number of one-one functions $f : \{1, 2, 3, 4, 5\} \to \{0, 1, 2, 3, 4, 5\}$ such that $f(1) \neq 0,\ 1$; is

   (A) 120 (B) 240 (C) 480 (D) 600

2. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function which assumes only rational values, and let $f(\frac{1}{2}) = \frac{1}{2}$. Then

   (A) $f(x) = \frac{1}{2}$, $x \in \mathbb{R}$. (B) $f(x) = x$, $x \in \mathbb{R}$.
   (C) range of f is uncountable (D) none of the above

3. The probability that at least one of the two sets A and B occur is 0.6, and that both A and B occur is 0.4. Then the probability that

   (A) none of the two sets occur is 0.6
   (B) A occurs is 1
   (C) B occurs is 0.2
   (D) only one of the sets A and B occurs is 0.2

4. The values of the constants A and B so that the function

$$f(x) = \begin{cases} Ax - B, & \text{if} \quad x \leq -1 \\ 2x^2 + 3Ax + B, & \text{if} \quad -1 < x \leq 1 \\ 4, & \text{if} \quad x > 1 \end{cases}$$

   is continuous for all values of x, are:

   (A) $A = \frac{3}{4}$, $B = -\frac{1}{4}$. (B) $A = \frac{3}{4}$, $B = \frac{1}{4}$.
   (C) $A = \frac{1}{4}$, $B = -\frac{3}{4}$ (D) $A = -\frac{1}{4}$, $B = \frac{3}{4}$

5. Suppose we have a variable X with mean=10, and variance=9. Consider a linear transformation $Y = 2X + 3$. Which of the following is true?

   (A) mean of Y is 20, variance of Y is 21
   (B) mean of Y is 20, variance of Y is 18
   (C) mean of Y is 23, variance of Y is 36
   (D) mean of Y is 23, variance of Y is 39

6. The equation $xe^x - 1 = 0$, for $x \in (0, 1)$, has

(A) infinitely many solutions (B) exactly two solutions
(C) exactly one solution (D) no solution

7. Let f be a real-valued function such that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x. The largest possible value for $f(2)$ is

(A) 7 (B) 8 (C) 10 (D) 13

8. The co-ordinates of the vertex of the parabola $x^2 + 4x + 2y + 8 = 0$

(A) (-2,1) (B) (-2,2) (C) (-2,-2) (D) (-2,-1)

9. Consider the function $f(x) = |x|$ on $\mathbb{R}$. Which one of the following is not correct?

(A) f is continuous on $\mathbb{R}$.
(B) f is differentiable on $\mathbb{R}$.
(C) for any real sequence $\{x_n\}$, $\{f(x_n)\} \to 0 \Rightarrow \{x_n\} \to 0$
(D) f is not one-one

10. Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ such that $f(x) = 5x$, if x is rational, and $f(x) = x^2 + 6$, if x is irrational. Then f is continuous at

(A) 2 only (B) 3 only (C) 2 and 3 (D) all x in $\mathbb{R}$ such that $x \neq 2, 3$

11. Suppose that 5% of men and 0.25% of women are colour-blind. A person is chosen at random and that person happens to be a colour-blind. What is the probability that the person is male? (Assume males and females to be in equal numbers).

(A) 0.75 (B) 0.90 (C) 0.95 (D) none of the above

12. The three straight lines $2x - 5y + 1 = 0$, $5x + 2y = 0$, and $x - y + 2 = 0$ form

(A) a right-angle triangle (B) an equilateral triangle
(C) no triangle (D) an acute-angle triangle

13. Let P and Q be constants such that the function $f(x) = x^2 + Px + Q$ has a maximum or minimum at $x = 1$, and $f(1) = 3$. Then the value of Q is

(A) -2 (B) 0 (C) 4 (D) 5

14. If $A = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix}$, then A^4 is equal to

(A) $\begin{bmatrix} 16 & 9 \\ 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 16 & 45 \\ 0 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 16 & 21 \\ 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 16 & 81 \\ 0 & 1 \end{bmatrix}$

15. The radius of the circle $x^2 + y^2 - 8x + 6y = 0$ is

(A) $\sqrt{5}$ (B) $\sqrt{10}$ (C) 5 (D) 10

16. Let A be a $m \times n$ matrix of rank r, where $r < \min\{m, n\}$. Let B be the matrix obtained from A by changing exactly one element. Then the rank of B is

(A) either r or $r - 1$
(B) either r or $r + 1$
(C) either $r - 1$ or $r + 1$
(D) either r, $r - 1$ or $r + 1$

17. The number of integers in $\{1, 2, 3, \ldots, 250\}$ that are divisible by at least one of the integers 2, 3 and 5, is

(A) 181 (B) 182 (C) 183 (D) 184

18. The number of ways to place 15 indistinguishable balls in three distinct boxes so that any two boxes together will contain more balls than the remaining one is

(A) 28 (B) 36 (C) 45 (D) 136

19. The values of λ and μ for which the system of equations $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda z = \mu$ has no solution, are

(A) $\lambda = 3,\ \mu = 10$
(B) $\lambda \neq 3,\ \mu = 10$
(C) $\lambda = 3,\ \mu \neq 10$
(D) $\lambda \neq 3$ and μ is arbitrary

20. Let x, y, z be positive integers such that $xyz = 8$, then the maximum value of $\dfrac{5xyz}{x + 2y + 4z}$ is

(A) $\dfrac{40}{35}$ (B) $\dfrac{40}{21}$ (C) $\dfrac{40}{16}$ (D) $\dfrac{40}{14}$

21. If $f(x) = \frac{x^2+(1-x)^2}{max\{x,1-x\}}$, then the value of $\int_0^1 f(x)dx$ is

(A) $2\log 2 + \frac{7}{8}$ (B) $2\log 2 - \frac{1}{2}$ (C) $2\log 2 - \frac{3}{8}$ (D) $2\log 2 + \frac{3}{8}$

22. If n distinct balls, numbered as $1, 2, 3, \dots, n$, are placed into n cells at random, what is the probability that exactly one cell remains empty?

(A) $\frac{n(n-1)}{2n^n}$ (B) $\frac{n!}{n^n}$ (C) $\frac{n(n!)}{n^n}$ (D) $\frac{(n!)n(n-1)}{2n^n}$

23. Let $f'(x) = g(x)$, $g'(x) = -f(x)$, $f(0) = 0$, $g(0) = 1$, then the function $f^2(x) + g^2(x)$ is

(A) constant and equal to 1 (B) constant and equal to 2
(C) strictly increasing (D) strictly decreasing

24. The set of all x that satisfy $x^5 - x^4 - 2x^3 \geq 0$ is

(A) $(-1, 0]$ (B) $(2, \infty)$ (C) $[-1, 0] \cup [2, \infty)$ (D) $[-1, -\frac{1}{2}] \cup [2, 3)$

25. Let $f_1(x) = \begin{cases} 8x - 1, & 0 \leq x \leq 1 \\ 4 + 3x, & 1 \leq x \leq 2 \end{cases}$

$f_2(x) = \begin{cases} 1 + 4x, & 0 \leq x \leq 1 \\ 3x + 2, & 1 \leq x \leq 2 \end{cases}$

The minimum value of $f(x) = f_1(x)f_2(x)$

(A) $-\frac{8}{9}$ (B) -1 (C) $-\frac{9}{8}$ (D) none of the above

26. In a game of dominoes, each piece is marked with two numbers. The pieces are symmetrical so that the number pair is not ordered, (for example, (2,6)=(6,2)). How many different pieces can be formed using the numbers $1, 2, 3, \dots, n$?

(A) n^2 (B) $\frac{n(n-1)}{2}$ (C) $\frac{n(n+1)}{2}$ (D) $\frac{(n+1)(n+2)}{2}$

27. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the roots of the equation $ax^2 + 2bx + 4c = 0$ will be

(A) $\alpha, 4\beta$ (B) $4\alpha, \beta$ (C) $2\alpha, 2\beta$ (D) $-2\alpha, -2\beta$

28. For qualifying a test, a student has to get a minimum (prefixed) score in each of the 6 subjects. In how many different ways a student can disqualify the test?

(A)6! (B) 5! (C) 2^6 (D) $2^6 - 1$

29. Consider the sets A and B, and $0 < P(A), P(B) < 1$. If $P(A|B) = P(A|B^c)$, then

(A) $P(B|A) = P(B|A^c)$.
(B) $P(B|A) = P(B^c|A)$
(C) $P(A \cap B) = P(A)P(B)$.
(D) $P(B) = P(B^c)$

30. If the points (x_1, y_1), (x_2, y_2), (x_3, y_3) are collinear, then which of the following is true?

(A) $(y_3 - y_1)(y_2 - y_1) = (x_3 - x_1)(x_2 - x_1)$
(B) $(y_3 - y_1)(x_2 - x_1) + (x_3 - x_1)(y_2 - y_1) = 0$
(C) $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$
(D) $(x_3 - x_1)(y_3 - y_1) = (x_2 - x_3)(y_2 - y_3)$